Three Field Ontologies for QFT

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Abstract

Relativistic quantum field theory (QFT) is ostensibly a quantum mechanical theory of *fields*, but determining exactly what these are is a thorny metaphysical task in the face of no-go arguments given by Baker (2009). This paper explores three possible answers according to which quantum fields are (I) superpositions of classical fields, (II) fields of expectation values for local observables, or (III) fields of local quantum states. I argue that each of these ontologies has resources available to respond to Baker's challenge, though all three face residual puzzles.

1 Where are the Fields?

Relativistic quantum field theory (QFT) is ostensibly a quantum mechanical theory of *fields*. On the usual interpretation, a field is a continuous entity extending across spacetime. Its possible configurations are given by continuous assignments of certain properties or relations to spacetime points. Dynamical laws, usually cast as differential equations, constrain these assignments. This picture draws heavily on classical field theory, however, and the transition from classical to quantum mechanics greatly obscures the underlying metaphysics.

As a simple example, consider the classical Klein-Gordon field. Its configurations are represented by $\mathbb{R}$-valued functions on spacetime, $\varphi(x)$, that solve the Klein-Gordon equation, $(\Box + m^2)\varphi(x) = 0$. Specifying a real scalar representing the value of a single physical quantity at each spacetime point determines a configuration of the field. The values of all other physical quantities characterizing the field supervene on these scalars.

When we quantize a field theory, several things happen that subvert our classically-honed intuitions. Classical field values, $\varphi(x)$, are replaced by (unbounded)

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linear operators, $\hat{\varphi}(x)$, acting on a Hilbert space representing states of the quantum field. Unlike classical field values, these operators do not represent determinate properties/relations. Rather they represent determinable properties/relations. We must specify a state (or perform a measurement) to know what value they have. Further complicating matters, no-go results tell us that it is actually impossible to assign field operators to spacetime points (Halvorson and Müger, 2006, §6). Quantum fields must be reinterpreted as operator-valued *distributions*, $\hat{\varphi}(f)$, assigning operators to test functions, f, over spacetime. Finally, the Hilbert space that the field operators act on is typically assumed to be *Fock space*, the infinite direct sum of n-particle Hilbert spaces

$$\mathcal{F} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3 \oplus \ldots .$$

This enables direct contact with particle physics experiments, but it makes it difficult to understand what kinds of field configurations gives rise to multiparticle states.

By the end, instead of a continuous assignment of determinate properties to spacetime points, we are faced with a continuous assignment of operators to test functions acting on a multi-particle Hilbert space. Unsurprisingly, many physics students scratch their heads at this point and ask "where are the fields?" In the face of this quandary, we might be tempted to equip relativistic QFT with a particle ontology, but here several no-go theorems present significant obstacles (Halvorson and Müger, 2006, §4). The situation is not much better for fields. Baker (2009) gives four compelling arguments against common field ontologies. Upon deeper reflection, it is not at all clear what sort of ontologies relativistic QFT is compatible with.

This paper explores three possible answers. According to the first, so-called *wavefunctional interpretations*, quantum fields are superpositions of classical fields (e.g., Huggett 2003, Wallace 2006, Sebens 2020). According to the second, they are fields of expectation values for local observables. Such an idea has been adopted by much of the recent philosophical literature on *algebraic QFT* (e.g., Halvorson and Müger 2006, Baker 2009, Ruetsche 2011). According to the third, quantum fields are fields of local quantum states. This is a relativistic version of *spacetime state realism* (Wallace and Timpson 2010, Swanson 2020). I argue that each of these ontologies has resources available to respond to Baker's challenge, though all three face residual puzzles.

To keep the discussion manageable, we restrict attention to a single theoretical framework, algebraic QFT on Minkowski spacetime, and we will not address the measurement problem head on. The distinct interpretive challenges we're interested in arise within the context of unifying special relativity and quantum mechanics in a fundamental theory, hence we focus on universal QFTs defined at all energy scales.

2 Baker's Challenge

2.1 Wavefunctionals

Baker's arguments target wavefunctional interpretations of QFT. Letting Q be an infinite dimensional space of classical field configurations, a wavefunctional, $\Psi : Q \to \mathbb{C}$ assigns a complex number to each point in $q \in Q$, with $|\Psi(q)|^2$ interpreted as the probability that the field is in classical configuration q.

This idea is simple and intuitive, but making rigorous sense of it in the context of universal relativistic QFT is tremendously hard. The only way we currently know appeals to the existence of a special representation of the field operators, the *real wave representation* (Baez et al., 1992). We generally construct relativistic QFTs by first specifying algebraic relations between abstract field operators (or other closely related quantities), and then looking for representations of this structure in terms of concrete operators acting on some Hilbert space. In algebraic QFT, this two-stage process is made explicit and mathematically rigorous.

For example, in order to construct the quantum Klein-Gordon field, we begin with a certain abstract algebra, the *Weyl C^*-algebra*, $\mathfrak{W}$, encoding a rigorous form of the canonical commutation relations. $\mathfrak{W}$ is generated by unitary operators $W(f)$, associated with test functions, f, in some symplectic vector space (V, σ). (These test functions are usually interpreted as solutions of the classical Klein-Gordon equation with spatially compact boundary conditions.) The physical interpretation of $W(f)$ only becomes clear once we specify a particular Hilbert space representation, i.e., a *-homomorphism, $\pi : \mathfrak{W} \to B(\mathcal{H})$, mapping $\mathfrak{W}$ into a subalgebra of bounded operators on $\mathcal{H}$. Within certain physically well-behaved representations, it is possible to identify self-adjoint field operators, $\hat{\varphi}(f)$, and conjugate field operators, $\hat{\pi}(f)$, which act as infinitesimal generators of 1-parameter groups of Weyl unitaries. This allows us to view them as exponentiated versions of more familiar field operators.

One important example is the standard Fock representation. In order to construct it, we turn (V, σ) into a "single particle" Hilbert space by choosing a σ-compatible complex structure, J. There is great flexibility here, but it turns out that there is a unique Lorentz-invariant choice — all inertial observers agree on a splitting of the space of solutions to the Klein-Gordon equation into positive and negative frequency subspaces, and this splitting defines a unique, Lorentz-invariant complex structure. Having defined the one-particle Hilbert space, Fock space is then built in the usual fashion. To complete the construction, we prove that we can generate $\mathfrak{W}$ as functions of creation and annihilation operators on $\mathcal{F}$.

The real wave representation is a different representation constructed using the same complex structure. Intuitively, we want to construct a Hilbert space of wavefunctionals,

$\Psi : Q_\Sigma \to \mathbb{C}$, which assign complex numbers to classical field configurations on some Cauchy surface, Σ. There are two important subtleties. First, since the configuration space is infinite dimensional, there is no translation-invariant measure, and standard Lebesgue integration theory cannot be deployed. To circumvent this problem, the complex structure J can be used to define a unitary-invariant *isonormal distribution*, d, and develop a suitable integration theory (Baez et al., 1992, Ch. 1.4). Second, in order for this procedure to work, we have to replace the classical configuration space of smooth $\mathbb{R}$-valued functions with compact support on Σ, with a smeared configuration space, $\tilde{Q}_\Sigma$ (usually the space of tempered distributions on Σ). We can then go on to construct a space of square integrable wavefunctionals, $L^2(\tilde{Q}_\Sigma, d)$, and find a representation of the Weyl algebra in terms of operators acting on this space.

Despite being constructed using very different mathematical procedures, the Fock and real wave representations are unitarily equivalent (Baez et al., 1992, Cor. 1.10.3). This is widely seen as a sufficient condition for their physical equivalence (Ruetsche, 2011, Ch. 2.2). Intuitively, we can view the two different representations as a change of basis: the Fock representation diagonalizes the particle number operator, while the real wave representation diagonalizes the field operators on some Cauchy surface.

Taken together, these ideas yield the ingredients for our first field ontology:

> **Ontology I:** QFT describes a superposition of classical field configurations. Field operators, $\hat{\varphi}(f)$, represent fundamental quantities. Field configurations are given by wavefunctionals in the real wave representation, $L^2(\tilde{Q}_\Sigma, d)$.

2.2 Against Fields

We are now in a position to examine Baker's four arguments against ontology I.

The first argument employs the Unruh effect (Unruh, 1976). An immortal, uniformly accelerating observer is causally confined to a certain wedge-shaped region of Minkowski spacetime, the so-called *Rindler wedge*. For them, the flow of time is naturally described by the subgroup of wedge-preserving Lorentz boosts (which generate proper time translations along their worldline). In contrast, an immortal, inertial observer naturally describes the flow of time using the subgroup of time-like translations. This disagreement leads to a different splitting of (V, σ) into positive and negative frequency subspaces, and thus to different complex structures $J \neq J'$, and ultimately to unitarily inequivalent Fock representations (Halvorson and Clifton, 2001). In the Minkowski vacuum state, the inertial observer detects no particles, while the accelerating observer detects infinitely many!

Baker observes that because of the equivalence between Fock and real wave representations, the two observers must also employ unitarily inequivalent real wave representations. They don't just attribute inequivalent particle content to the vacuum state, they describe it as physically inequivalent superpositions of classical field configurations. Ultimately, the disagreement centers on the conjugate field operators, $\hat{\pi}(f)$. One can prove that $\hat{\pi}(f) = \hat{\varphi}(Jf)$, and because the observers employ different complex structures, they employ different conjugate field operators.

Baker's second argument relies on the phenomenon of *spontaneous symmetry breaking* (SSB) and employs the massless Klein-Gordon field as an example. Classically, if $\varphi(x)$ is a solution to the $m = 0$ Klein-Gordon equation, then so is $\varphi(x)' = \varphi(x) + \eta$ where η is any real constant. Quantum mechanically, this symmetry can be represented by a 1-parameter group of automorphisms of the Weyl algebra, $\alpha_\eta : \mathfrak{W} \to \mathfrak{W}$, which commutes with the dynamics. As a result, if ω is a translation-invariant vacuum state, so is $\omega \circ \alpha_\eta := \omega(\alpha_\eta(\cdot))$. Each of these vacuum states have natural Hilbert space representations given by the Gelfand-Naimark-Segal (GNS) construction. But these representations are unitarily inequivalent!

The problem, according to Baker, is that in each of these representations, different operators represent the fields — $\hat{\varphi}(f)$ in the ω-representation, versus $\hat{\varphi}(f)' = \hat{\varphi}(f) + \eta \int d^4x f(x) I$ in the $\omega \circ \alpha_\eta$-representation. This is true despite the fact that the operators $\hat{\varphi}(f)$ and $\hat{\varphi}(f)'$ are well-defined in both representations. Since which operators play the role of the fields is a representation-dependent fact, and which representation we're in depends on the outcome of SSB, a physically contingent fact, Baker concludes that the field operators cannot be fundamental quantities.[1]

Baker's third argument has essentially the same structure, but with an added twist. It involves an example of SSB involving coherent states which break rotational invariance, an external spacetime symmetry. In this example, not only are the identities of the field operators representation-dependent, they are not Lorentz-invariant. Fundamental quantities must be Lorentz-invariant, and so the field operators cannot represent fundamental quantities.

Baker's final argument employs Haag's theorem (Haag, 1955). Widely interpreted as a no-go theorem for the standard interaction picture, Haag's theorem has an important corollary: any interacting representation of $\mathfrak{W}$ is unitarily inequivalent to a Fock representation. It immediately follows from the equivalence between Fock and real wave representations that any interacting representation is inequivalent to the latter too!

[1]Here Baker's argument relies on the following metaphysical principle: "Which quantities are fundamental (if instantiated) is a physically necessary fact" (p. 598).

3 Evading Baker's Challenge

3.1 Local Observables

In the face of Baker's challenge, where else might we look for candidate field ontologies? The mathematical architecture of algebraic QFT supplies one natural idea (which Baker suggests at the end of his paper). Each region of spacetime is assigned an abstract C^*-algebra, $\mathfrak{A}(O)$, representing functional relations between quantities, called *local observables*. These algebras look like subalgebras of bounded Hilbert space operators, but at the outset, we do not assume that they act on a Hilbert space in any particular manner. States are represented by (normalized, positive) linear functionals, $\rho : \mathfrak{A}(O) \to \mathbb{C}$. On self-adjoint elements, states are $\mathbb{R}$-valued and the number $\rho(A)$ gives the expectation value of observable $A \in \mathfrak{A}(O)$ in state ρ.

Historically, the local observables were given an operationalist interpretation, but recently, philosophers of QFT have given them a realist interpretation according to which they are physical quantities characterizing the local, intrinsic determinable properties of the field in region O (e.g., Halvorson and Müger 2006, Baker 2009, Ruetsche 2011, Swanson 2017). This realist interpretation motivates our second ontology.

> **Ontology II:** QFT describes a field of expectation values on spacetime. Self-adjoint elements of local observable algebras, $\mathfrak{A}(O)$, represent intrinsic properties of the field in region O. State functionals $\rho : \mathfrak{A}(O) \to \mathbb{C}$ specify possible field configurations.

Ontology II has a number of strengths. It assigns properties directly to spacetime regions, rather than test function intermediaries. It also inherits the generality and mathematical rigor of algebraic QFT. An added bonus: the local algebras, $\mathfrak{A}(O)$, are typically gauge-invariant. (In contrast, the field operators, $\hat{\varphi}(f)$, are often gauge-dependent.)

It also has resources to respond directly to Baker's four arguments. In the Klein-Gordon model, the local observable algebra assigned to spacetime region O is given by $\mathfrak{W}(O)$, the subalgebra of $\mathfrak{W}$ generated by Weyl unitaries $W(f)$ where f has support in O. In the Unruh case, even though inertial and accelerating observers employ unitarily inequivalent representations of $\mathfrak{W}(W)$ (the local observable algebra assigned to the Rindler wedge), they agree on the structure of $\mathfrak{W}(W)$ itself. Similarly in SSB cases, unitarily inequivalent vacuum representations agree on the structure of $\mathfrak{W}$, the full Weyl algebra across all of spacetime. In neither case is there a disagreement over which quantities are fundamental. Moreover, the Haag-Kastler axioms ensure that the assignment of algebras to spacetime regions is covariant under Poincaré transformations. The identities of the local observables is thus a Lorentz-invariant fact. Finally, ontology

II is not dependent on the existence of the Fock or real wave representations. There are models of interacting QFTs satisfying the Haag-Kastler axioms (Summers, 2012), and any such model can be directly equipped with ontology II.

Despite these strengths, there are at least three lingering worries about ontology II. Perhaps most glaringly, it is overly plenitudinous. There are way too many self-adjoint operators in the local observable algebras to be fundamental quantities. (Note that for any subset of observables which might be deemed fundamental, the local algebras contain arbitrary bounded functions of them.) And the algebras do not capture which are fundamental and which are emergent in any obvious way. In order to do this, ontology II must be amended by identifying a subset of self-adjoint operators $\mathfrak{B} \subset \mathfrak{A}(O)$ such that (a) $\mathfrak{B}$ generates $\mathfrak{A}(O)$, (b) states on $\mathfrak{B}$ extend uniquely to states on $\mathfrak{A}(O)$, and (c) $\mathfrak{B}$ is sparse enough to plausibly be fundamental.

A second worry pulls in the opposite direction. Many physically significant quantities including energy-momentum, particle number, temperature, and charge will not typically be elements of $\mathfrak{A}(O)$. Instead they are only well-defined relative to particular representations where we can extend the local algebras by taking their closure, $\overline{\mathfrak{A}(O)}^w$, in the weak operator topology. These extended *local von Neumann algebras*, $\overline{\mathfrak{A}(O)}^w$, include *parochial observables* not contained in the original C^*-algebras (Ruetsche, 2011).

If even some parochial observables are among the list of fundamental quantities, then ontology II must again be amended. The trouble is, once we let parochial observables in, the response to Baker's first two arguments is significantly weakened. In the Unruh case, although observers agree about the structure of the C^*-algebra $\mathfrak{W}(W)$, they disagree about the extended von Neumann algebra $\overline{\mathfrak{W}(W)}^w$. Similarly, each vacuum representation in SSB cases supports a different von Neumann algebra, $\overline{\mathfrak{W}}^w$. Thus prima facie, the inclusion of parochial observables reignites disagreement over which quantities are fundamental.[2]

A third worry concerns whether ontology II has identified the right sort of fundamental properties. If the numbers assigned to local observables really are expectation values, then trouble looms. The 'propensity to take on a certain average value when measured' is strangely operationalistic for a fundamental property. We might skirt the issue, insisting that we need a solution to the measurement problem to understand what expectation values are, but if we take this route, we are resigned to providing an emergent rather than fundamental ontology for QFT at best. Alternatively,

[2] Baker suggests that the situation for parochial observables is not as bad as the situation for field operators since parochial observables are not instantiated in other representations (p. 598). Mathematically speaking, it's not clear how to maintain this line. Parochial observables are elements of the extended von Neumann algebras, while field operators are affiliated with them, i.e., their spectral projections are elements. They are in the same boat. Our discussion in section 3.3 suggests another possible way out: perhaps different parochial *operators* represent the same physical quantity in different representations.

we might try to reinterpret the numbers as sui generis scalar quantities. The task then becomes to give an explanation of measurement processes according to which expectation values emerge in the end and correspond neatly to these local scalars.

3.2 Local States

A different ontology allows us to make progress on this third worry while retaining much of the flexibility of ontology II. Motivated by wavefunction realism, but wary of the challenges involved with extending it to relativistic QFT, Wallace and Timpson (2010) develop a field ontology called *spacetime state realism*. Each spacetime region is assigned a Hilbert space, $\mathcal{H}(O)$, whose density operators represent possible local states of the field in that region. A field configuration is then an assignment of density operators to regions across spacetime. Expectation values recede into the background, only arising during the analysis of measurement processes.

Unfortunately, this version of spacetime state realism cannot be applied to universal relativistic QFTs. Swanson (2020, Lem. 1-2) proves that in such theories we cannot rigorously defined the concept of a local Hilbert space $\mathcal{H}(O)$ in the manner Wallace and Timpson require. Instead, Swanson proposes an alternative version modeled more closely on algebraic QFT.

The key idea is to replace the local Hilbert space, $\mathcal{H}(O)$, with the *local state space*, $\mathcal{S}(O)$, the space of (normalized, positive) linear functionals over $\mathfrak{A}(O)$. $\mathcal{S}(O)$ is a convex set with an order structure inherited from $\mathfrak{A}(O)$ and a lattice of exposed faces that mirrors the algebra's lattice of projection operators. By equipping $\mathcal{S}(O)$ with a certain orientation structure, it is possible to reconstruct $\mathfrak{A}(O)$ (Alfsen et al., 1980). Swanson argues that we can dualize the standard Haag-Kastler axioms, recasting algebraic QFT as a framework describing a presheaf of oriented convex sets across spacetime.

These ideas motivate our third field ontology.

> **Ontology III:** QFT describes a state-valued field on spacetime. Local state spaces, $\mathcal{S}(O)$, represent possible values of the field in region O. Sections of the presheaf of oriented local state spaces specify field configurations.

Because the mathematical formalism of ontology III is dual to the usual algebraic QFT framework, it has similar strengths to ontology II, including the same resources for responding to Baker's arguments. At the same time, it also suffers from versions of our first two worries. We don't expect all of the geometric structure in the presheaf of local statespaces to reflect fundamental features of the field. Some method for whittling this down to its fundamental core is required. Moreover, it is distinctly possible that some of these fundamental features will not be captured by the local state spaces dual to $\mathfrak{A}(O)$, but rather by the local normal state spaces dual to the extended von Neumann algebras

$\overline{\mathfrak{A}(O)}^w$. If so, disagreements about which features are fundamental are expected to arise in the Unruh and SSB cases.

In addition, it might seem that the version of spacetime state realism captured by ontology III represents a step backwards. After all, the field values are just functionals assigning expectation values to local observables! But expectation values are no more a part of the basic structure of ontology III than they are a part of Wallace and Timpson's original proposal. The possible values of the field in region O are represented by points in a geometrically rich convex set. The physics are encoded in how these values vary across spacetime. Expectation values only enter when we consider observables, represented by $\mathbb{R}$-valued (bounded, affine) functions on the local statespaces, $A : \mathcal{S}(O) \to \mathbb{R}$. Thus from the standpoint of ontology III, it is observables rather than states which assign expectation values to local quantities.

The mathematical details of ontology III require significant development. Still, for philosophers convinced that we should treat the quantum state as a real physical entity, ontology III represents a promising way to implement this idea in universal relativistic QFTs. Others may balk. Maudlin (2013) and Halvorson (2019) both argue that reifying the quantum state in this fashion represents a kind of category mistake. Whether represented by a density operator, a wavefunction, or a point in a convex set, the quantum state is just not the right sort of thing to represent a fundamental property. From this perspective, while ontology III may avoid positing a field of brute expectation values, it has the same Achilles heel as ontology II.

3.3 Wavefunctionals (Again)

These difficulties prompt us to re-evaluate the status of ontology I. After all, one of its unique strengths is that it identifies a sparse set of fundamental quantities represented by the field operators. Baker's first three arguments hinge on the fact that these field operators are parochial. (Though they are unbounded, their spectral projections are elements of the extended von Neumann algebras $\overline{\mathfrak{W}(O)}^w$.) As a result different representations featuring in the Unruh and SSB cases will apparently disagree about which quantities are fundamental.

Upon closer inspection, I think we should draw a different conclusion about these cases. In the Unruh case, observers agree about the field operators. They only disagree about which operators, $\hat{\pi}(f) = \hat{\varphi}(Jf)$ or $\hat{\pi}'(f) = \hat{\varphi}(J'f)$, are canonically conjugate to $\hat{\varphi}(f)$. Baker acknowledges that we might deny that these conjugate operators represent fundamental quantities. But defenders of wavefunctional interpretations have good reason to do so! Classically, conjugate fields are defined in terms of the fields, $\pi(x) = \frac{\partial}{\partial \dot{\varphi}} \mathcal{L}_{KG} = \dot{\varphi}(x)$, where $\mathcal{L}_{KG}$ is the Klein-Gordon Lagrangian and $\dot{\varphi}(x)$ is the partial derivative of the field in some chosen time-like direction. Arguably, conjugate fields are emergent, not fundamental. Moreover, they depend on a choice of reference frame. Since inertial and accelerating observers disagree about the flow of time, it is not at all surprising that they disagree about conjugate fields. What is surprising is that this apparently benign disagreement leads to unitarily inequivalent representations of $\mathfrak{W}(W)$! There is a lingering puzzle here for the wavefunctional view to sort out, but much of the force of Baker's first argument has been deflected.[3]

In SSB cases, an even stronger response is available. Classically, the symmetry $\varphi \mapsto \varphi + \eta$ of the massless Klein-Gordon field is perfectly comprehensible. It relates field configurations with different global boundary conditions and fewer symmetries than the dynamical laws. Fundamental quantities remain fixed. When we switch to QFT, φ, is transformed into an operator, $\hat{\varphi}$. Suddenly the symmetry $\hat{\varphi} \mapsto \hat{\varphi} + \eta I$ appears to relate distinct fundamental quantities. But the mystery is illusory. We use the same linear functional, ω, to represent two different vacuum states with different boundary conditions by applying it to two different algebras, $\mathfrak{W}$ versus $\alpha_\eta(\mathfrak{W})$. That is, we let the symmetry in question operate on quantities rather than states.

This "Heisenberg picture" of symmetries is used widely in QFT. It treats symmetries as algebraic automorphisms while leaving states fixed. A more physically transparent "Schrödinger picture" would treat the Weyl algebra as fixed and let symmetries act on states. The fact that we don't usually do this, I suspect, comes from an overreliance on the traditional mathematical picture of algebraic QFT that assigns local observables rather than states to regions. The presheaf of local state spaces appearing in ontology III offers a path forward here.[4]

If this idea is right, then different vacuum representations don't actually disagree about which quantities are fundamental. They merely disagree about which mathematical objects represent fundamental quantities in the Heisenberg picture.

This observation diffuses Baker's second argument, and deflects much of the force of his third. Coherent states have boundary conditions that break rotational invariance. (For example, coherent charged states in quantum electrodynamics include fluxes generated by soft photon clouds that single out a preferred spacelike direction.) We expect rotations to preserve fundamental quantities, but not asymmetric boundary conditions. This is transparent in the Schrödinger picture. In the Heisenberg picture it is masked by representing a change in boundary conditions as a shift in which operators

[3]This puzzle takes the following form: even though inertial and accelerating observers agree about the fundamental quantities, $\hat{\varphi}(f)$, they ascribe different states with incompatible boundary conditions to the Klein-Gordon field in the Rindler wedge. One possibility is that only the inertial observer gets the boundary conditions right (e.g., Arageorgis et al. 2003, Buchholz and Verch 2015). Another possibility is that this incompatibility merely concerns the values of emergent quantities and is therefore benign.

[4]Alfsen and Shultz (2001, Cor. 4.20) establish a 1-1 correspondence between Jordan automorphisms of C^*-algebras and affine homeomrphisms of their abstract statespaces, providing raw ingredients to formulate this Schrödinger picture of symmetries.

represent the fundamental quantities. Again, there is a lingering puzzle here. Although the real wave representation is unitarily equivalent to the Fock representation, which is usually constructed in a Lorentz-invariant 4-dimensional manner, the real wave representation assigns field operators to 3-dimensional Cauchy surfaces, requiring a foliation of spacetime. Although these field operators are Lorentz-covariant, one might reasonably worry that the Lorentz-invariant ontology remains hidden from view.

Baker's fourth argument represents a bigger obstacle, but there are reasons for optimism. Even though the real wave representation is equivalent to the standard Fock representation, the class of wavefunctional representations is likely much broader than the class of Fock representations. Both Jackiw (1987) and Hatfield (2018) employ wavefunctional representations that contain non-Fock states, but these have not yet been rigorously constructed.

Even though real wave and Fock representations employ the same complex structure, it's not clear that the physical arguments justifying the choice of J are parallel. In the Fock case, we need J to build a Lorentz-invariant single-particle Hilbert space, and this is plausibly tied to the "right" Lorentz-invariant choice of frequency splitting. But in the real wave case, we're already breaking Lorentz-invariance by choosing a particular Cauchy surface. It's less clear why the choice of an isonormal distribution defined by J is the "right" one, beyond allowing the Schrödinger dynamics to take a particularly simple form. In interacting theories, there is no Lorentz-invariant frequency splitting. Fraser (2008) argues that this is ultimately why Haag's theorem rules out Fock representations. But complex structures abound, and one might serve as a suitable choice to define an appropriate isonormal distribution. Ashtekar and Magnon (1975) develop one possible alternative method for choosing a complex structure in interacting theories.

Interestingly, the real wave representation was originally introduced by Segal (1967) as part of a program to construct nonlinear QFTs using real-time functional integration methods. Although this program has had limited success to date, Baez et al. (1992, Ch. 8) use it to construct a simple 2-dimensional nonlinear QFT on $S_1 \times \mathbb{R}$. Of course, since the spatial dimension is compactified, the complications arising from Haag's theorem are moot. At best this example provides weak evidence for the viability of the program. Taken together, though, these three observations suggest that defenders of ontology I have more room to maneuver than Baker's fourth argument implies. Constructing interacting QFTs is notoriously difficult, and from this angle, the main challenge for wavefunctional interpretations is everyone's challenge.

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