Cantor's illusion-part 2

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abstract

This analysis shows a simple change in the rule of formation for Cantor's horizontal enumeration of the infinite set M, eliminates the diagonal sequence and the false exclusion of a single sequence used to prove the cardinality of M is greater than the cardinality of the set of integers N.

keywords: Cantor, diagonal, infinite

1. the argument

Translation from Cantor's 1891 paper [1]:

Namely, let *m* and *n* be two different characters, and consider a set [*Inbegriff*] *M* of elements

$E = (x_1, x_2, \dots , x_v, \dots)$

which depend on infinitely many coordinates $x_1, x_2, \dots , x_v, \dots$, and where each of the coordinates is either *m* or *w*. Let *M* be the totality [*Gesamtheit*] of all elements *E*.
To the elements of *M* belong e.g. the following three:

$E^I = (m, m, m, m, \dots),$
$E^{II} = (w, w, w, w, \dots),$
$E^{III} = (m, w, m, w, \dots).$

I maintain now that such a manifold [*Mannigfaltigkeit*] *M* does not have the power of the series 1, 2, 3, …, *v*, ….

This follows from the following proposition:
"If $E_1, E_2, \dots, E_v, \dots$ is any simply infinite [*einfach unendliche*] series of elements of the manifold *M*, then there always exists an element E_0 of *M*, which cannot be connected with any element E_v."
For proof, let there be

$E_1 = (a_{1.1}, a_{1.2}, \dots , a_{1,v}, \dots)$
$E_2 = (a_{2.1}, a_{2.2}, \dots , a_{2,v}, \dots)$
$E_u = (a_{u.1}, a_{u.2}, \dots , a_{u,v}, \dots)$
…………………………

where the characters $a_{u,v}$ are either *m* or *w*. Then there is a series $b_1, b_2, \ldots b_v, \ldots$, defined so that b_v is also equal to *m* or *w* but is *different* from $a_{v,v}$.
Thus, if $a_{v,v} = m$, then $b_v = w$.
Then consider the element

$$E_0 = (b_1, b_2, b_3, \ldots)$$

of *M*, then one sees straight away, that the equation

$$E_0 = E_u$$

cannot be satisfied by any positive integer u, otherwise for that u and for all values of v.

$$b_v = a_{u,v}$$

and so we would in particular have

$$b_u = a_{u,u}$$

which through the definition of b_v is impossible. From this proposition it follows immediately that the totality of all elements of *M* cannot be put into the sequence [*Reihenform*]: $E_1, E_2, \ldots, E_v, \ldots$ otherwise we would have the contradiction, that a thing [*Ding*] E0 would be both an element of M, but also not an element of M.
(end of translation)

2. Cantor's enumeration

The symbols {0, 1} will be substituted for {m, w} for visual clarity.
Cantor defines an infinite set M consisting of elements En. Each En is an infinite one dimensional horizontal sequence composed of two symbols 0 and 1. He does not specify a rule of formation for sequences, thus they are assumed to result from a random process such as a coin toss. There is one sequence per row, and all sequences are unique differing in one or more positions. He then assigns coordinates to the array of symbols using a two dimensional (u, v) grid.
Cantor then defines a diagonal sequence D composed of symbols with coordinates (u, u). The negation of a sequence differs in all positions. Using D as a template, he interchanges all 0's and 1's to produce E_0 as the negation of D or (not D). He declares, E_0 as a horizontal sequence, cannot be in the enumeration since it will conflict with each coordinate (u, u).

3. issue

For a random list of 10 sequences, there are 10! possible lists. If Cantor's argument was true and applied to all of those lists, there would be more missing sequences than listed sequences, which is a contradiction.

4. The solution

u \ v	1	2	3	4	5	6	
1	0	1	0	1	0	1	...
2	0	1	0	1	0	1	...
3	0	1	0	1	0	1	...
4	0	1	0	1	0	1	...
5	0	1	0	1	0	1	...
6	0	1	0	1	0	1	...
7							

D	0	1	0	1	0	1	...
E_0	1	0	1	0	1	0	...

fig.1

In fig.1 identical sequences of alternating (0 and 1) are deliberately entered in rows 1 to 6. The purpose is to show the red diagonal D is not complete until the list is complete, and D is redundant. It is not needed to form its negation E_0 since any of the 6 are complete as D.

u \ v	1	2	3	4	5	6	
1	0	1	0	1	0	1	...
2	1	0	1	0	1	0	...
3	1	1	1	1	1	1	...
4	0	0	0	0	0	0	...
5							
6							
7							

fig.2

All that is required is an additional rule of formation. Enter a sequence and its negation in pairs, as shown in fig.2.

conclusion: E_0 is not missing.

ref: [1] THE LOGIC MUSEUM Copyright © E.D.Buckner 2005