Philosophical lesson from the controversy on the consistency of classic electrodynamics

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Abstract

At the beginning of the 21st century, a peculiar discussion about the possible existence of unresolvable contradictions in the conceptual bases of classical electrodynamics was carried out. The arguments put forward to point out such alleged inconsistencies, as well as the replies they received, constitute an excellent example of scientific controversy from which electromagnetic theory emerged unscathed. However, the details of the debate show that classical physics, far from being devoid of interesting problems, can still accommodate various and profound lines of research of a fundamental nature.

Keywords: Classical electrodynamics, electron model, theoretical inconsistencies, matter and field

I. INTRODUCTION

Contrary to the widespread belief that the only physical problems worthy of interest arise in disciplines that emerged during the 20th century, electromagnetic theory, in its most traditional formulation, poses some of the most challenging challenges for the foundations of physics. These are no longer issues related to the frontiers of research, such as plasma physics, magnetohydrodynamic phenomena or nonlinear photonics, which exist in all scientific disciplines. In this case, we have to deal with questions that reach the conceptual roots of the theory under discussion, perhaps weakening its own internal coherence.

This was the case with the debate that developed during the first decade of the 21st century on the fundamental inconsistency of classical electrodynamics. The dialectical fire was opened by M. Frisch, of the University of Hannover, who shortly afterwards received a forceful response from F. A. Muller, of the University of Utrecht, followed in the same vein by G. Belot, of the University of Michigan.

At times the disagreements seemed to shift to the question of whether the contenders were really referring to the same body of theory, or whether each was referring to his own version of classical electrodynamics. This in turn led, if only indirectly, to the question of the different formulations of the same scientific theory, and whether or to what degree they are all interchangeable.

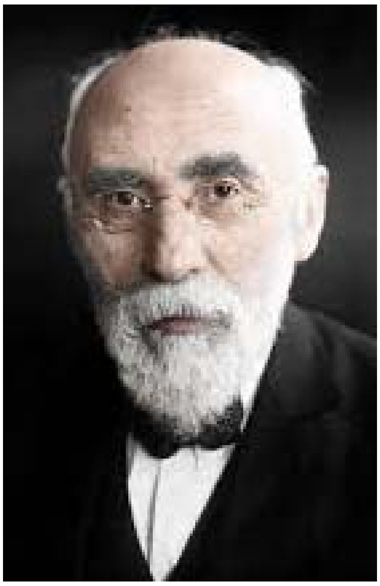
The turmoil eventually subsided, although the allegations made during the dispute were of such magnitude that one might wonder whether the matter was settled once and for all, or whether it is simply dormant until it is revived as soon as better arguments appear. Therefore, in order to obtain an overview of this deep and multifaceted discussion, this article will begin by going back, in the second and third sections, to the historical origins of this controversy, situated in the scientific understanding of electromagnetic phenomena developed throughout the 19th century. Then, in the fourth section, the main theses of Frisch's attack on the foundations of classical electrodynamics will be presented, counterbalanced by the corresponding responses that he himself received. The fifth and final section will be devoted to the attempt to draw up a balance of the arguments put forward in this dispute.

II. ELECTRODYNAMICS EMERGES

The great work of the Scotsman James Clerk Maxwell (1831-1879) achieved the unification of electrical, magnetic and optical phenomena at the high price of simultaneously abandoning certain features of the world views of Faraday and Newton (two conceptions in many respects opposed to each other) on which this author claimed to be based. Like the vast majority of his contemporaries, Maxwell assumed the existence of an invisible and imponderable medium that occupied all space –- the ether –- and acted as a support for the propagation of light waves, electromagnetic and gravitational forces [1]. The main mission of the ether was to satisfy a fundamentally metaphysical requirement, which was the desire to avoid actions at a distance. The idea of a field due to Faraday resolved the question of the lack of a material substratum, apparently indispensable to justify the physical reality of the lines of force of the fields. This was the role that was then assigned to the ether in the nineteenth century.

Maxwell, however, built his own model of ether, assimilating it to a mechanical system of rollers and bearings, which dispensed with properties that today seem so basic to us, such as electric charge. Nevertheless, he obtained field equations substantially identical to those we use today, considering them fully correct in the macroscopic domain [2]. This does not mean that at the end of the 19th century there was no lack of authors willing to retouch Maxwell's theory by specifying the properties of ether, whether as a perfect fluid or as an elastic solid. Despite the efforts of researchers of the calibre of Lord Kelvin, Lodge, Larmor, Heaviside or Fitzgerald, nothing definitive was achieved in this regard [3].

The next step was taken by the Dutchman Hendrik Antoon Lorentz (1823 – 1958), who decided to reinterpret Maxwell's electrodynamics in 1892 by adding two postulates [4]: (1) Electromagnetic fields have their origin in a property, –the electric charge –possessed by the particles that constitute matter at a microscopic level; (2) The ether remains perfectly motionless in the face of the movement of matter and the electromagnetic fields associated with it. When the British Joseph John Thomson (1856 – 1940) experimentally discovered the particle that we now call "electron", Lorentz's hypotheses acquired an irresistible appeal and the equations of the electromagnetic field came to be called "Maxwell-Lorentz equations".



Hendrik Antoon Lorentz

To the four Maxwell equations describing the reciprocal variations of the electric and magnetic fields in space and time (eqs. 1 – 4), Lorentz added a fifth (the "Lorentz force"), referring to the electromagnetic force acting on a moving charged particle (eq. 5). As usual, **E** and **B** are, respectively, the electric and magnetic fields,ρ is the electric charge density, $\boldsymbol{J}$ is the current density, v the speed of the charged particle and q the value of its electric charge.

$\nabla\cdot\mathbf{E} = 4\pi\rho$ (ec. 1)

$\nabla\times\mathbf{B} -(\partial\mathbf{E} / \partial t) = \boldsymbol{J}$ (eq. 2)

$\nabla\times$**AND** $-(\partial\mathbf{B} / \partial t) = 0$ (eq. 3)

$\nabla\cdot\mathbf{B} = 0$ (eq. 4)

$\mathbf{F} = q (\mathbf{E} + v \times \mathbf{B})$ (ec. 5)

The fact that the charged particle on which electromagnetic forces act carries its own field, which interacts with external fields, heralded the advent of serious problems, as researchers of the stature of Poincaré soon realized. The great French scientist did not miss the consequences of electromagnetic self-induction, in relation to the phenomenon called "radiative reaction" [5].

An electric charge subjected to an electromagnetic field accelerates and emits radiation, altering the Lorentz force on it, which in turn modifies the acceleration of the particle and its irradiation. In this way, the electromagnetic field will depend on the derivatives of the position with respect to time in all orders up to infinity. Poincaré immediately realized that this peculiar effect resulted in an added component to the total value of the field proportional to the acceleration, which in turn could be interpreted as an electromagnetic contribution to the inertia of the particle.

Considerations such as these animated the so-called "electromagnetic program," which consisted of the attempt to reduce the purely mechanical characteristics of matter, such as inertial mass, to by-products of its electromagnetic properties. From such a point of view, the genuine substratum of physical reality would be none other than the electromagnetic field and, therefore, Newtonian classical mechanics should be subsumed in some way in the suitably extended Maxwell-Lorentz electrodynamics. The champions of this line of thought were the Germans Wilhelm Wien (1864 – 1928) and Max Abraham (1875 – 1922).

The electromagnetic program was exhausted in itself, swamped by the tide of theoretical and experimental advances that occurred at the beginning of the 20th century, leaving open the question of the complete reconciliation between mechanics and electrodynamics. Even in a classic text like that of Landau and Lifshitz, the difficulty of taking rigorously into account certain effects that are difficult to treat is recognized. This occurs when we try to explain the loss of energy by radiation in charges accelerated by an external field by introducing the so-called "Lorentz friction forces" and we arrive at the conclusion that a charge passing through an electromagnetic field, upon leaving it, would experience unlimited self-acceleration [6]:

> «The question may be raised as to how electrodynamics, which obeys the law of conservation of energy, can lead to an absurd result in which a free particle increases its energy indefinitely. The root of this contradiction actually lies in the alleged infinite electromagnetic "own" mass of the elementary particles we are talking about […]. By writing the infinite mass of the charge in the equations of motion, we actually formally assign to it an infinite negative "own mass" of non-electromagnetic origin, which together with the electromagnetic mass would result in the finite mass of the particle. But the subtraction of an infinite magnitude from another also infinite magnitude is not a correct mathematical operation, and this gives rise to a whole series of contradictions among which is the one we have just indicated.»

III. THE CLASSICAL ELECTRON THEORY

Not only were there concerns about the interactions between fields and particles, but the very concept of an electrically charged corpuscle faced very serious obstacles. If we assume that the electron is a point particle, a simple look at Coulomb's law reveals that the electric field would tend to infinity, since as we approach a point, $r \to 0$ and obviously $F \propto 1/r^2 \to \infty$. This seems logical, since we are assigning a finite electric charge to a zero volume with the result of an infinite charge density. This did not seem very tolerable, which pushed theorists to deal with non-point-like models for electrons [7, 8, 9, 10].

The assumption that the electron could be an extended object, a sphere of finite radius, was not without its problems. In such a case, it would be necessary to explain how the electron does not break up due to the electrostatic repulsion between its parts. Poincaré pointed out this obstacle and proposed as a solution the existence of non-electromagnetic forces that compensate for this repulsion, the so-called "Poincaré tensions." Quantum electrodynamics also requires non-electromagnetic forces that balance the electrical repulsions between the various parts of an extended electron. That is, neither in classical nor in quantum electrodynamics does the electron appear to be a self-sufficient entity, since compensating fields for the internal forces are always needed, which cannot themselves be electromagnetic.

Jules Henri Poincaré

Essentially, we are faced with the same problem mentioned by Landau and Lifshitz at the end of the previous section. The electron mass is supposed to be composed of an electromagnetic part and a non-electromagnetic part, –thus breaking Abraham's purely electromagnetic paradigm, –the latter with a negative sign. The use of Poincaré tensions to compensate the internal repulsion of a spherically uniform charge distribution (as assumed in the pre-relativistic case), on the other hand, was irrelevant in the analysis of the non-point model of the electron [11]:

> «The arbitrariness of the Lorentz-Poincaré electron is much greater than the freedom we have in choosing the 00-component of the energy-momentum tensor for the mechanism stabilizing a spherical surface charge distribution. For starters, we can choose a (surface or volume) charge distribution of any shape we like –a box, a donut, a banana, etc. As long as this charge distribution is subject to the Lorentz-FitzGerald contraction, we can turn it into a system with the exact same energy-momentum-mass-velocit and relations as the Lorentz-Poincaré electron by adding the appropriate non-electromagnetic stabilizing mechanism . Of course, as the analysis in this section, […], shows, any closed static system will have the same energy-momentum-mass-velocity relations as the Lorentz-Poincaré electron, no matter whether it consists of charges, electromagnetic fields, and Poincaré pressure or of something else altogether. The only thing that matters is that whatever the electron is made of satisfies Lorentz-invariant laws. The restriction to static closed systems, moreover, is completely unnecessary. Any closed system will do. In short, there is nothing we can learn about the nature and structure of the electron from studying its energy-momentum-mass-velocity relations.»

The classical model of the electron was, in any case, energetically unstable in the absence of a bond to counterbalance the electrostatic repulsion, even though Weisskopf showed that its charge appeared to be dispersed over a region on the Compton wavelength scale [12]. As might be expected, the physical significance of the introduction of negative masses aroused reluctant skepticism [13]. However, it seemed justified to expect that the energy responsible for maintaining the integrity of the electron would be negative if we take into account that the energy of the electromagnetic field is defined as the work necessary to assemble an elementary charge from its infinitesimal portions infinitely separated from each other. In fact, the search for classical models for the electron that prove energetically stable remains an open question today, despite the various answers offered to date [14, 15].

The fact is that the ontology of classical electrodynamics makes a strict distinction between particles and fields, which in turn divides the laws of the theory into two halves: on the one hand, Maxwell's equations, which are intended to explain how charged particles create fields, and on the other, the Lorentz force, which concerns the effect of fields on charges. There is little to discuss about the Maxwellian equations because the problem lies in the Lorentz force, which is in principle composed of two terms, an external component and an internal one. The electromagnetic fields external to the charged particle are obviously responsible for the external component of the total Lorentz force, while the internal, or "own" component originates from the action of the field on the particle that generates it. That is, $\mathbf{F}_{tot} = \mathbf{F}_{ext} + \mathbf{F}_{int}$.

We can hardly characterize this internal Lorentz force without having a model of the structure of the charged particle that tells us how the fields affect the behavior of their own source. What we could call "self-action" is due to the fact that the Coulomb field of different parts of an electron, –let us suppose, –affects other parts of that same electron as external fields, creating forces that do not balance out when the electron accelerates. Therefore, only when we have an approximation to the structure of the particle can we calculate the internal Lorentz force because only then can we add up the effects on each of the different parts of the electron of the fields produced by the rest of the particle (and even then we should ask ourselves if there is any self-action of each part on itself).

The alternative to such an abstruse situation would be to take the energy radiated by the accelerated electron and calculate the force on the particle by means of energy conservation. This strategy leads to the Abraham-Lorentz equation, whose deduction, unfortunately, entails a significant range of problems due to its non-Newtonian character, since it is a third-order equation in the derivative of the position with respect to time [16]. The main obstacle arises from the existence of non-physical solutions that violate energy conservation, as is the case of the self-accelerations that the equation predicts even when the external force on the particle is completely cancelled.

A generation later the requirements of special relativity were explicitly incorporated into the so-called Lorentz-Dirac equation. Despite its name, the British Paul Dirac (1902-1984) did not derive this equation as the point-like limit of an extended electron, but assumed the point-like character of the electron from the start, assuming the validity of Maxwell's equations at all distance scales (except at the very point occupied by the electron). Nevertheless, the great English physicist always insisted that this equation had a purely phenomenological scope and even then with restrictions, since certain conditions must be imposed on the external forces present if we want to avoid the appearance of solutions without physical meaning [17].

IV. IS CLASSICAL ELECTRODYNAMICS INCONSISTENT?

The accusations of inconsistencies that Frisch was going to formulate against classical electrodynamics in the first decade of the 21st century, therefore, had venerable precedents and a long history of discussions on the subject. However, the most novel aspect of his objections came from the irrefutable character with which he clothed them, –denying the possibility of future theoretical extensions overcoming them –and the epistemological consequences that he intended to extract from them. From the alleged inconsistencies that he detected in the foundations of electromagnetic theory, Frisch concluded that a certain degree of inconsistency must be inherent to scientific knowledge, so rather than fighting it we would have to learn to adapt to it.

The first move in this direction took place in a 2004 article with the suggestive title *Inconsistency in Classical Electrodynamics* [18]. The topic of this work was expanded a year later, giving rise to a full book entitled *Inconsistency, Asymmetry, and Non-Locality: A Philosophical Investigation of Classical Electrodynamics* [19]. In 2009, five years after the start of the controversy, Frisch published another article [20] where he summarized the main ideas contained in his 2005 book without adding substantially new content.

The subtitle of Frisch's book ("A Philosophical Investigation of Classical Electrodynamics") clearly shows that the author's intention is to draw all the physical and metaphysical conclusions that can be obtained from the premises he defends in the text. Perhaps for this reason its content is divided into two large sections: the first part deals with particles and the second with fields.

The chapters on fields deal with the absence of electromagnetic waves that are advanced in time (chronoretrograde), despite their being allowed by Maxwell's equations, as well as the connections of this problem with entropy and thermodynamics. Towards the end of the work, nonlocal effects sponsored by the Aharanov-Bohm effect are also mentioned in relation to the quantum description of electromagnetic potentials. These issues could rarely be described as inconsistencies, but rather as open questions under investigation, which in turn connect with much broader physical questions, the outcome of which is unlikely to threaten the coherence of the theory.

In fact, it is in the chapters dealing with particles within electrodynamics that the central objections to the theoretical edifice of classical electrodynamics are found. Frisch begins by asserting that the contradictions he criticises derive directly from what is, in his opinion, the universally used theoretical scheme by electrodynamics specialists. This formulation is summarised in four basic assumptions: (1) There are electrically charged particles capable of experiencing accelerations; (2) Maxwell's equations correctly describe electromagnetic fields; (3) the expression of the Lorentz force on charged particles is valid; and (4) energy is conserved in the interactions between charges and fields.

Energy conservation, rather than being an initial postulate, is usually deduced from the symmetries of the theory itself, whose dynamic equations can be written in Lagrangian or Hamiltonian form to immediately obtain - using Noether's theorems - the corresponding conservation laws.

It does not seem justified, therefore, to place the conservation of energy among the theoretical premises of classical electrodynamics; to do so would put us in the same position as the nineteenth-century scientists who lacked the overall vision that we have today of the theory and its subsequent evolution. In the same sense, we now know that the microscopic behaviour of electrons does not strictly belong to the classical regime but to the quantum one, where it is not surprising that classical physics offers poor results.

The crux of the contradiction that Frisch points out is found in the fact that, in his opinion, Lorentz's law of electromagnetic forces stipulates that the variation in energy of a charge is due only to the effect of the external component of the force, $\mathbf{F}_{ext}$; that is, $\mathbf{F}_{int} = 0$. Under this condition, Frisch arrives at a contradiction when he deduces that an accelerated charge emits and does not emit, at the same time, energy in the form of radiation [19].

A reply that may immediately arise among those who do not support these objections would be to emphasize that it is precisely these inconsistencies, or the violation of the conservation of energy, that reveal that Frisch's position is untenable [21]. Selfconsistency as a logical requirement and the conservation of energy as a regulative principle serve to filter out those proposals that aspire to be accepted as full-fledged scientific theories. Approaches that do not meet such preconditions will be discarded from the outset. If such a point of view were adopted, the pertinent question would then be why Frisch arrives at the conclusions that he claims to obtain from his version of classical electrodynamics. Finding out the roots of these apparent inconsistencies also seems interesting as an exercise in clarifying the bases on which electromagnetic theory rests [22].

In the vast majority of real cases, classical electrodynamics has been shown to be capable of providing extremely precise responses, regardless of self-actions ($\mathbf{F}_{int} \approx 0$), a circumstance that does not prevent us from recognizing its character as a mere approximation [23]. In the most general case - as we know - the accelerated motion of the electron causes variations in the fields that affect the electron's own motion, which in turn modifies the fields, and so on. In other words, both the fields and their sources are unknown, even though we know that their variations are coupled, which allows us to understand the complexity of the problem. There are many ways of approaching it, although all of them are based on a wide range of idealizations and simplifications adapted to a wide variety of cases [24].

In [20] Frisch makes some observations that are surprising at first glance:

> «The problem is that any theory with self-interactions has to posit a model for charged particles and arguably there is no physically 'well-behaved' model of a discrete finitely charged particle that does not involve what *by the theory's own lights* are idealizations.»

The question that remains floating in the reader's mind would be where the problem suggested by the preceding paragraph lies. Naturally, all scientific theories, as constructions of the human intellect, involve idealizations to a greater or lesser degree and at any level of sophistication. For example, the ideal gas equation, –as its name indicates, –is a high-level idealization that can be replaced under certain conditions by the Van der Waals equation, which is supposedly closer to the behavior of real gases. But we would never criticize fluid physics for offering us theories that never fit perfectly with the real nature of the objects it studies.

Frisch goes into more detail later, pointing out that the idealizations he criticizes are dead ends that lead to insurmountable contradictions . For example, when treating the electron as a point particle, we come up against infinite values of the field at the particle's own location. And when we consider extended particle models, we avoid infinite quantities at the cost of violating the relativistic restrictions that prohibit the propagation of superluminal signals. However, we must take these claims with extreme caution, as there are reasons to distrust them.

In classical electrodynamics - and presumably also in quantum electrodynamics - we usually encounter two kinds of problems: those in which, knowing the fields and their variation with time, we have to calculate the effect produced by them on charges and currents, and those in which, given the distribution of charges and currents, we have to determine the generated electromagnetic fields. However, there is a third type of problem, much more complicated, in which we would have to solve a set of coupled partial differential equations in which both external and internal components of the electromagnetic fields are taken into account using the Lorentz force.

Frisch acknowledges that if this were to be the case, the inconsistencies he denounces would be greatly weakened, although he points out that this is an unviable path, undermined by a multitude of conceptual entanglements such as self-accelerations or the infinite value for the electric potential of a point charge. But there is no such thing, since the electrostatic energy of a point charge at rest can be obtained by means of an integral over all space except for a tiny sphere of radius $d > 0$ as small as desired [25]. That is, except for a sphere of arbitrarily tiny radius d , the integral that defines the electrostatic energy of a point charge does not diverge. What happens at $d = 0$? In this case, the divergence of the calculations tells us that the electromagnetic theory has no place for charged particles of zero size. We simply note that point charges are not entities proper to classical electrodynamics.

The alternative to this approach involves renormalization techniques, which obtain the measured experimental mass of the electron m_{exp} from two quantities: the mass equivalent to the energy of its own field (including, in the quantum case, all kinds of virtual particles), m_f, and the mass that would remain if such fields were to disappear, m_d. To reach the desired conclusion, m_d must be negative and carefully adjusted, since calculations indicate that both m_f and m_d tend to infinity when all the correction orders are taken into account.

Frisch judges this as an insurmountable problem when, in principle, it is not. It can be cumbersome - and often is - to calculate $m_{exp} = m_f - m_d$, but the truth is that the difference of two divergent series may well converge by itself [26]. Fortunately, this happens often enough for renormalization techniques to be a useful and versatile tool in theoretical physics, despite their questionable interpretative background.

After careful analysis, it is also possible to regard pre-acceleration and auto-acceleration as computational artifacts caused by the complexities inherent to problems where we consider mutual interactions between charges and fields. This kind of unphysical accelerations disappear when we reformulate the Abraham-Lorentz equation in integro-differential form and impose the appropriate boundary conditions [27, 28, 29]. And this is not the only example; the modified Abraham equation [15] and the Yaghjian equation [30] do not suffer from these problems either.

V. CONCLUSIONS

Classical electrodynamics introduced for the first time an irreducible duplicity into its ontological basis, formed by particles and fields. Particles idealized discrete portions of matter as volumeless points, perfectly localizable in physical space, while fields constituted a new kind of continuous and extended physical entity –present throughout space, in principle –whose source was matter itself, which was also affected by its influence. The properties of particles and fields were so disparate that we should almost be surprised by the beauty and power of the unification carried out by Maxwell with the later refinements of Lorentz.

However, the double role of matter as an active subject (it creates them) and as a passive object (it suffers them) of the fields could not fail to bring complications at some point. And in fact, the development of electromagnetic theory confirmed the existence of problems without a closed solution, especially in cases where it was desired to simultaneously take into account this double role mentioned above, inferring from it a complete dynamic description of fields, charges and currents.

The subtlety of the paths opened to overcome such obstacles became so great that it was mistaken for the existence of paradoxes, contradictions and inconsistencies by authors such as Frisch. A deeper analysis, however, reveals that electrodynamics has unsolved problems, like all sciences worthy of the name, but it lacks true inconsistencies at a fundamental level. The conceptual difficulties that it undoubtedly has are transferred in its specific modality to the quantum world, to such an extent that quantum electrodynamics, rather than a fundamental theory from which the classical version is derived, could be taken as an extension of classical electrodynamics to a domain in which its concepts and strategies lose validity [31].

Nevertheless, the debate that Frisch has maintained with the rest of the scientific community regarding the alleged inconsistencies of classical electrodynamics has also served to revive the old controversy over the fidelity with which our scientific theories represent the properties and evolution of the natural world. The meaning of a theory, the semantic hypotheses that allow us to interpret its formalism, are not excluded from the debates of the professional community involved. Granting our confidence in the veracity, more or less approximate, of certain premises is always a question of degree. And it is often debatable whether an inference deserves the same –provisional confidence –as its premises, which, in turn, can present a very variable level of confirmation depending on the historical moment.

Hence, a strictly Lakatosian image of scientific theories, with a hard core and a protective belt of auxiliary hypotheses, does not seem very representative of real research. It is precisely through these fissures between fundamental and accessory assumptions that misunderstandings such as Frisch's penetrate. It is in these blurred boundaries that implicit postulates can nest and are tacitly accepted (such as $\mathbf{F}_{int} = 0$, Sommerfeld's boundary condition for radiation, or the slow variation of fields in the so-called adiabatic limit), taking their validity for granted or applying them to a range of phenomena to which they do not correspond.

We now know that classical electrodynamics must be replaced in certain areas by its quantum version, of which the classical theory behaves, in a certain way, as an excellent approximation. It is true that different approximations to the same underlying theory may turn out to be mutually incompatible depending on the assumptions adopted in the simplification. But beyond the mutual inconsistency between partial approximations, what can never occur is the appearance of flagrant contradictions within the same theoretical framework, as Frisch claimed to have established in the framework of classical electrodynamics.

In short, Frisch's radical disqualification of classical electrodynamics cannot be upheld as an insurmountable objection to any form of electromagnetic theory. It has merit as a reminder of the formal problems that still surround theoretical constructions generally considered free of difficulties, but it does not constitute irrefutable proof of the inconsistency of one of the most important legacies of 19th century physics whose benefits we still enjoy today.

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