IN SEARCH OF COSMIC TIME: COMPLETE OBSERVABLES AND THE CLOCK HYPOTHESIS

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Abstract. This paper consider a new and deeply challenging face of the problem of time in the context of cosmology drawing on the work of Thiemann (2006). Thiemann argues for a radical response to cosmic problem of time that requires us to modify the classical Friedmann equations. By contrast, we offer a conservative proposal for solution of the problem by bringing together ideas from the contemporary literature regarding reference frames (Bamonti 2023; Bamonti and Gomes 2024), complete observables (Gryb and Thébault 2016, 2023), and the model-based account of time measurement (Tal 2016). On our approach, we must reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement in order to preserve the Friedmann equations as the dynamical equations for the universe.

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1. Introduction

Spacetime symmetry and time evolution are not straightforward to reconcile. Most pointedly, the spacetime diffeomorphism invariance of general relativity would appear to imply, recalling Minkowski (1908)'s famous remark, that time by itself is

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doomed to fade away into mere a shadow.[1] In the context of cosmology, however, it is standard to make reference to a notion of cosmic time.[2] One application of this time is of course in discussions of the age of the universe. Cosmic times can be shown to be well-defined in any relativistic cosmology with stable causal structure (Hawking 1969). Moreover, when they can be defined, such times manifestly *are* consistent with the spacetime symmetries of general relativity since they are spacetime diffeomorphism invariant. There are, however, plausible formal and physical reasons to doubt the status of a cosmic time as an independent physical observable. First, on very general grounds, we might demand of an observable that it corresponds to a measurable quantity of the theory and it is not immediately clear how one would measure cosmic time. Furthermore, we might also demand that genuine observables are *relational* in the sense that they correspond to *correlations* between measurable quantities. On such an understanding of what is to be an observable we would not expect that cosmic time is observable since it is does not appear to have the right form to be a correlation between two measurable quantities.

This potential challenge to the observable status of cosmic time becomes all the more acute in the context of the canonical formalism for general relativity. A remarkable achievement of the canonical gravitational formalism, pioneered by Dirac and ADM (Dirac 1958b; Arnowitt et al. 1959, 1962), was to show that one can represent spacetime symmetries within a 3D+1 space and time formalism in *constrained Hamiltonian terms*, with the spacetime diffeomorphism symmetry of the 4D covariant theory represented via *hyper-surface deformation algebroid of constraints*. Although it is restricted to spacetimes admitting globally hyperbolic topology, there is no expectation that such a limitation will conflict with the application of the formalism to cosmology (the black hole case is more subtle due to the Kerr solutions not being globally hyperbolic).[3] A further attractive feature of the canonical formulation of general relativity is that it comes equipped with a formal criterion for a quantity to be an observable in terms of the *Dirac criterion*. Furthermore, in a groundbreaking and highly influential analysis it was shown that a particular approach to relational concept of an observable, as a correlation between two *partial observables* leading to a *complete observable* (Rovelli 2002b), provides a construction of objects that satisfy the Dirac criterion. An unambiguous criterion for functions to be observables is of course indispensable staging material needed

[1]The implications of diffeomorphism invariance and the related notions of general covariance and background independence is much debated. See Pooley (2017); James Read (2023).

[2]A cosmic time function on a spacetime manifold M assigns to any $p \in M$ the supremum of the durations of all future-directed continuous timelike curves ending at p (Andersson et al. 1998; Fletcher 2025). See Smeenk (2013); Callender and McCoy (2021) for philosophical overviews of time in cosmology. See Rugh and Zinkernagel (2009) for a discussion focused on cosmic time.

[3]"There are some good reasons for believing that all physically realistic spacetimes must be globally hyperbolic (see Penrose (1980))" (Wald 1984, p. 200).

towards the pursuit of canonical quantization. It is the algebra of classical observables that one seeks to faithfully represent as operators on a physical Hilbert space. Such a criterion is also useful for settling questions of observability in the classical formalism. The problem, however, is that it is not at all clear that canonical representations of cosmic time would in fact satisfy the Dirac criterion. In particular, prima facie, cosmic time does not appear to be the right kind of relational object to fit with the complete observables approach. It is not at all clear that cosmic time is either a partial or complete observable in Rovelli's terms. This is, ultimately, a cosmological manifestation of the infamous problem of time, which we name the cosmic problem of time and upon which we will have more to say later.

The acuteness of the problem of cosmic time can be illustrated most clearly in the context of the Friedmann equations that are the cornerstone of the standard model of cosmology. These equations describe the expansion of the universe in terms of the dynamics of the spatial scale factor a with respect to the cosmic time parameter t. These equations can be straightforwardly represented in canonical terms since they turn out to simply be equivalent to the Hamilton equations. In this case it is simple to observe that neither the cosmic time nor the scale factor are Dirac observables. By the Dirac criterion, observables must be phase space functions which have (weakly) vanishing Poisson bracket with the Hamiltonian constraint that generates the dynamics of the Friedmann equation. However, clearly neither a nor t can satisfy such a criterion: a has non-zero Poisson bracket and t is not even a phase space function. Though simple to articulate and difficult to ignore once it is articulated, this problem has received almost no discussion in physics or philosophy. The notable exception is the treatment of Thiemann (2006), who explicitly argues that 'it is incorrect to interpret the FLRW equations as evolution equations of observable quantities' (p. 9). Rather, he suggests, we should follow the complete observables procedure and re-write the equations relationally. The true evolution equations would then acquire *observable modifications* when compared to the Friedmann equations. We are thus lead from a formal-conceptual problem regarding time and observability to a physical proposal for a new approach to cosmology with empirical consequences.

This paper will consider this chain of argument and evaluate whether there is an alternative to Thiemann's radical cosmological revisionism. We formulate and critically appraise a conservative approach to finding cosmic time based upon the application of the clock hypothesis in the context of the concept of *Hubble flow*. The development of this alternative approach will lead us, in turn, to reconsider the methodological status of the clock hypothesis in cosmology and the meaning of 'measurability' in the context of the complete observables programme. Our analysis draws crucially on the different types of reference frames in physical theory and the insights that this can deliver for clarifying foundational questions regarding the construction of 'complete observables' as correlations between 'partial observables'. Our goal is to synthesise key ideas from the contemporary literature regarding reference frames (Bamonti 2023; Bamonti and Gomes 2024), complete observables (Gryb and Thébault 2016, 2023), and the model-based account of time measurement (Tal 2016), whilst drawing attention to a new and deeply challenging face of the problem of time in the context of cosmology. The cosmic problem of time leads to a dilemma: we can apply a conservative understanding of Dirac observables, downplay the significance of the clock hypothesis, and modify the Friedmann equations; or we can reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement, and preserve the Friedmann equations. Whatever option we take, something must change, for things to stay as they are.

Our arguments are structured as follows. In Section 2 we provide a brief overview of the complete and partial observables programme as a response to the problem of time that seeks to preserve the Dirac criterion for observables. This will include an explicit toy model construction of a complete observable to give the reader a clear formal intuition regarding the mathematical form of these objects. We then apply recent work on reference frames to disambiguate two important details in the definition of a partial observable and better understand what it means for a physical variable to play the role of a clock in the context of a complete observable. The following Section 3 considers the status of time in the Friedmann equations, poses Thiemann's challenge to the standard interpretation of these equations as dynamical equations and reviews his solution in terms of a de-parameterization via a *phantom field* through the well-known Brown and Kuchař (1995) mechanism. Section 4 then introduces the crucial idea of Hubble flow and seeks to reframe comic time as a proper time parameter τ along the Hubble flow. In this context, we consider the question of the observability and measurability of Hubble parameter (i.e. $H = \dot{a}/a$) and the question of whether we can consider $H(\tau)$ to be a complete observable whose dynamics is described by the unmodified Friedman equations. We articulate the specific problem of there being no physical system which can be understood to *even approximately* measure proper time along the Hubble flow. This leads, in Section 5, to the final step in the articulation of our proposed solution: the introduction of a more liberalised sense of 'measurement procedure' in the context of cosmic time. This more liberalised notion draws upon the model-based account of time measurement developed in the context of atomic clocks and the measurement of Coordinated Universal Time due to Tal (2016). An appendix provides a formal analysis of the status of cosmic proper time as a gauge invariant quantity and relevance of gauge fixings which supplements and supports the the analysis provided in the body of the paper.

2. Complete Observables and Real Reference Frames

The problem of time is best understood as a cluster of formal, physical and conceptual challenges to the isolation of the physical degrees of freedom in theories which display temporal diffeomorphism symmetry.[4] In the canonical representation, many of the challenges stem from lack of an unambiguous phase space representation of re-foliation symmetry and the implications that this has for quantization. However, the problem is not restricted to canonical representations, and reoccurs in covariant form, for example, in terms of the challenge of finding an appropriate measure in path-integral approaches. One particular pressing aspect of the problem is the tension between the standard definition of a gauge invariant observable and the seemingly obvious fact that observable quantities change. In particular, in the context of constrained Hamiltonian theories, following Dirac (1950, 1958a, 1964) the criterion to be an observable is to have (weakly) vanishing Poisson bracket with first class (primary) constraints.[5] Even for simple theories, temporal diffeomorphism immediately leads to a problem of time since, in such theories, we have that the Hamiltonian is a sum of first class constraints. Application of the Dirac criterion then immediately implies that observables are condemned to be frozen as constants of the motion. In general relativity this problem re-occurs in a more complex fashion but with essentially the same elements. There are an infinite family of Hamiltonian constraints and if we insist that Dirac observables commute with them, then the observables of general relativity are frozen.

The standard approach to the problem of reconciling the Dirac definition of observables with the necessity to describe dynamics is the *partial and complete observables* approach. This approach was pioneered by Rovelli (1991a,b, 2002b, 2004, 2007) and later formalized by Dittrich (2006, 2007).[6] The essence of such

[4]See Kuchař (1992); Isham (1993); Anderson (2017) for scientific overview. Casadio et al. (2024) provides an overview of a family of alternative approaches in which first-class phase-space constraints may be relaxed based on an interpretation of them as fixing the values of new degrees of freedom. Technically informed discussion in the philosophical literature include Belot and Earman (2001); Belot (2007); Gryb and Thébault (2016); Thébault (2021b). A hybrid formal and philosophical monograph length treatment of the global problem of time is Gryb and Thébault (2023). Further references will be given where relevant below.

[5]This idea also traces back to the discussions of Bergmann (1956); Bergmann and Komar (1960); Bergmann (1961b,a); Bergmann and Komar (1962) and so one might plausibly use the term Bergmann-Dirac observables. However, Bergmann changed his view at various points. See Pitts (2019) for discussion.

[6]For detailed overview see (Thiemann 2007; Tambornino 2012). Important developments of the approach include (Gambini and Porto 2001; Gambini et al. 2009) and (Bojowald et al. 2011a,b; Höhn 2019). Critical responses include (Kuchař 1991, 1992; Kuchar 1999; Dittrich et al. 2017). For a review of the various notions of observable, that includes discussion of the limitations of the partial and complete observables approach, see (Anderson et al. 2014; Anderson 2017). For philosophical analysis of the ontological implications of the partial and complete observables approach an excellent extended discussion can be found in (Rickles 2007, pp. 161–171). For a further overview see (Thébault 2021b), which contains further references and discussion.

approach is to designate a subset of measurable quantities or 'partial observables' as internal clocks, and then use these clocks to construct 'complete observables' that are both predicable and measurable, and which correspond to Dirac observables. More specifically, the formal application of the approach requires one to consider, for each Hamiltonian constraint, one physical variable to play the role of a physical clock. One first constructs expressions for the clock and non-clock variables in terms of the parameterized flow of the constraints; the simplest way to do this is via the relevant Hamilton-Jacobi equation (Rovelli 2004; Gryb and Thébault 2016). One next inverts the flow equation for the clock variable and substitutes it into flow equations for the other variables to construct an algebraic expression for their correlation that is parameter free. Finally, one considers the correlation between the clock variable and the other variables at a particular value of the clock variable. This is a complete observable and corresponds to the value of the non-clock partial observables when the clock partial observables takes a particular value.

Let us provide the simplest possible physical example so the reader can conceptualise clearly how the procedure works. Consider two free particles moving in one dimension and described by a theory with a single Hamiltonian constraint. We can write an expression for the integral of motion in terms of the time parameter of the flow of the constraint, by solving the Hamilton-Jacobi equation. This will give us an expression for the position of each particle, q_i, as a functions of the constants of motion (i.e. inital position and momenta), Q_i and P_i, and parameter time, t. This takes the form:

$$q_i(t) = Q_i + \frac{P_i}{m_i}t \tag{1}$$

for $i = 1, 2$. These variables are partial observables and do not commute with the Hamiltonian constraint $\mathcal{H}$, since we have that $\{\mathcal{H}, q_i\} = \dot{q}_i \neq 0$. However, we can combine the two expressions to describe the correlation between the values of the position of each particle. We do this by inverting the expression for one variable such that we obtain t as a function of (q_i, Q_i, P_i), and then inserting this expression into the expression for the other. The first variable is then playing the role of a physical clock and we evaluate the second variable for a given value of the second variable, say $s \in \mathbb{R}$. In this way we get a family of complete observables, one for each value of s.[7]

For even slightly complicated physical systems the inversion step may run into significant obstacles and is typically such that we can only define the relevant expressions for restricted values of the time parameter. Dittrich (2007) provides

[7]Note that this is also the construction of the so-called evolving constant of motions (Rovelli 1991a). In fact, a complete observable formally coincides with an evolving constant. The difference between the two concepts lies mainly in the fact that for evolving constants, the focus is on the evolution of the quantity with respect to the parameter s that serves as 'internal time'.

a detailed treatment of such a case. In our case, by contrast, since the physical dynamics is trivial and we are able to solve Hamilton equations for the considered system, the inversion is simply given by:

$$t = \frac{m_1}{P_1}(q_1 - Q_1) \tag{2}$$

Re-inserting this into (1), we get:

$$q_2(q_1) = Q_2 - \frac{P_2}{m_2}\frac{m_1}{P_1}(q_1 - Q_1) \tag{3}$$

Finally, we evaluate our expression $q_2(q_1)$ at $q_1 = s$ to get the parameterized family of complete observables:

$$q_2(q_1)|_{q_1=s} = Q_2 - \frac{P_2}{m_2}\frac{m_1}{P_1}(q_1 - Q_1)|_{q_1=s} \tag{4}$$

This is a complete observable constructed according to the Rovelli-Dittrich procedure. It is also a Dirac observable since for any specification of s we have $q_2(q_1)|_{q_1=s} : \Gamma \to \mathbb{R}$ and $\{\mathcal{H}, q_2(q_1)|_{q_1=s}\} = 0$, where Γ is four dimensional phase space $(q_i, p_i) \in \Gamma$ for $i = 1, 2$.

There is a specific tension within the physics literature regarding the interpretation of the partial observables. This tension will prove crucial in the context of application of the approach to cosmology. Consider, in particular, that according to the original approach of Rovelli (2002b), by definition, a partial observable is 'a physical quantity with which we can associate a (measuring) procedure leading to a number' (p. 2). By contrast, following Thiemann (2007) we have that 'a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to non-measurable quantities...which we will simply call non-observables' (p. 78). A third view is advocated by Gryb and Thébault (2016, 2023) in the context of theories with a single Hamiltonian constraint. On this approach, one can think of the complete and partial observables programme as allowing us de-parameterize evolution purely in terms of observable quantities. However, this evolution is fundamentally controlled by the evolution equations generated by the Hamiltonian constraint and is *always* well-defined, even when a particular deparameterization breaks down. On this approach, even if one wishes to use parameter-free complete observable expressions, one is still required to retain the full partial observables representation. This supports the Rovelli (2002b) perspective, in which partial observables are measurable quantities, rather than the Thiemann (2007) perspective, where the partial observables are understood as non-measurable.

A further important disambiguation can be made based upon the connection between partial and complete observables and reference frames. The role of reference frames in general relativity has an extensive philosophical literature.[8] Most relevant to our analysis is the distinction made by Bamonti (2023) between: 'Idealised Reference Frames' (**IRF**s), in which any dynamical interaction of the material system represented by the reference frame is ignored; 'Dynamical Reference Frames' (**DRF**s), in which the set of equations that determine the dynamics of the matter field is included but the the stress-energy tensor of the matter field used as reference frame is neglected; and 'Real Reference Frames' (**RRF**s) in which both the dynamics of the chosen material system and its stress-energy tensor are taken into account.

This distinction allows us to disambiguate two important details in the definition of a partial observable, which is often not stressed in the relevant literature. Following Bamonti and Gomes (2024), we should understand partial observables to be *relational but gauge-variant* quantities that are nevertheless associated measuring procedure. We can understand this seeming contradiction in terms of the fact that partial observables are defined relative to an Idealised Reference Frame. In particular, the parameter of flow equation acts as an **IRF** and, as such, partial observables are relational in the sense that they describe the correlation between a physical variable and the **IRF**. Consider our expression for the partial observables (1) above. The variables $q_i(t)$ are measurable quantities of the theory but they are not measurable independently of a specification of the value of the flow parameter t. Furthermore, since "all measurements are comparisons between different physical systems" (Anderson 1967, p.128), t itself represents a physical system, whose dynamics is neglected as a result of approximations. The second relevant remark is that not every pair of physical quantities to which measuring instruments can be associated can play the role of partial observables. *Bona fide* partial observables must be dynamically coupled to each other, in order for their relation to constitute a *bona-fide* complete observable. (Bamonti and Gomes 2024).

Let us then consider the status of the complete observables. In this context, the partial observable that is chosen as the clock observable is playing the role of a reference frame. The point above suggests that to construct a complete observable, we must use **DRFs** or **RRFs**. Since we are considering finite dimensional particle mechanics there is no stress-energy tensor to consider. However, the distinction between Dynamical Reference Frames and Real Reference Frames can still be made. That is, a clock variable is *always* a **DRF** since its dynamics are always relevant via the flow equation. However, it is only an **RRF** when the coupling between the clock

[8]See Earman and Friedman (1973); Earman (1974), Norton (1989, 1993), DiSalle (2020). Recently, a community of scholars has also emerged in the field of the so-called quantum reference frames, see Giacomini (2021); Kabel et al. (2024) and reference therein.

variable and the other non-clock partial observables is included. Our simple system with free particles is thus an implementation the complete observables programme in terms of a **DRF** rather than an **RRF**.[9] However, in cases where the coupling is included, complete observables admit an interpretation in terms of an **RRF**. This is precisely the application of the complete observables approach that we will consider in the context of cosmology in the following section.

3. Time and the Friedmann Equations

The universe is estimated to be 13.7 billion years old. This estimation is made based upon the standard model of cosmology in which the spacetime structure of the universe is described via general relativity. The base-model of modern cosmology, upon which more sophisticated models are built, is one in which the spatial structure is extremely simple. In particular, following the Cosmological Principle, we assume the universe is homogeneous and isotropic on large scales ($\geq 10^2 MPc$). This means that, from any location, the distribution of matter and energy appears the same and without any preferred direction. Formally, this corresponds to use of the Friedman (1922)-Lemaître (1931)-Robertson (1935)-Walker (1937) (FLRW) metric, which describes the geometry of spacetime under these symmetries and forms the basis for modelling the large-scale structure and dynamics of the universe.

In the context of the FLRW metric the Einstein Field Equations take on the remarkably simple form given by the Friedmann equations. These equations describe the evolution of a single geometric variable, the scale factor $a(t)$, and take the form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{6}$$

where t is the so-called cosmological time, $\rho(t)$ and $p(t)$ are the density and pressure of the matter, $k = 0, -1, +1$ is the curvature spatial parameter, and the constants have their usual meaning.[10] For a given specification of matter we can solve these equations to get dynamical expressions for the scale factor. For the matter mix that we take to correspond to our universe (including dark energy and dark matter)

[9]This connection also points to the sense in which the idea of inertial reference frames as discussed in the late nineteenth century by Lange, Neumann, Tait and others are examples of a **DRF** and not an **RRF**. This is one way of thinking about Mach's criticisms. See Thébault (2021a).

[10]The value of k does not fix the overall topology. In fact, different topological choices are possible for the same k: for example, a hyperplane (closed topology) is characterised by curvature parameter $k = 0$, like a hyperplane (open topology).

the relevant expressions describe an expanding universe which matches our observational data to a remarkable degree.[11] This model provides a standard, textbook level story of the expansion of the universe that is assumed by almost all scientists to be unproblematic, at least back to the (presumed) inflationary epoch.

Remarkably, however, when the story regarding the Friedman equations and the expansion of the universe is combined with the Dirac criterion for observables we run into an immediate and deeply problematic conflict. As just noted, the Friedmann equations describe the evolution of the scale factor a and this appears to provide a clear description of the time evolution of spatial geometric structure of the universe. However, the Friedmann equations are equivalent to those generated by a Hamiltonian constraint. So, *if* the evolution equations generated by a constraint are interpreted as gauge transformations, *then* we should be understood the Friedmann equations not as dynamical equations by as gauge equations.[12] Quantities such as a might appear to evolve over time in cosmology. However, they are not gauge-invariant, and so this evolution is not to be understood as physical. Remarkably, this conflict between the Friedman equations and the Dirac criterion for observables has received almost not detailed discussion in the physics or philosophy literature.

The major exception is the discussion of Thiemann (2006), who explicitly argues that 'it is incorrect to interpret the FLRW equations as evolution equations of observable quantities' (p. 9) although he does 'not doubt the validity the Einstein equations' he wants to 'stress that their interpretation as physical evolution equations of observables is fundamentally wrong' (p. 9). Moreover on this view:

> All textbooks on classical GR incorrectly describe the Friedmann equations as physical evolution equations rather than what they really are, namely gauge transformation equations. The true evolution equations acquire possibly observable modifications to the gauge transformation equations whose magnitude depends on the physical

[11] Specifically, observations of the Cosmic Microwave Background (CMB), a relic radiation from the early Universe, provided strong support for the model's predictions of a hot, dense origin characterising the so-called ΛCDM Cosmological Model. Data from missions like COBE, WMAP, and Planck confirmed the homogeneity and isotropy of the CMB within the order of one part in 10^5, in line with the Cosmological Principle. Observations of large-scale structure, such as those from the Sloan Digital Sky Survey (SDSS), further corroborate the predictions made by FLRW cosmology, particularly regarding the distribution of galaxies. These experimental validations solidify the FLRW model as the cornerstone of our understanding of cosmology.

[12] We might, of course, simply reject Dirac's argument connecting gauge transformations to Hamiltonian constraints. In particular, his theorem that first class constraints generate gauge transformations does not apply to Hamiltonian constraints, see (Barbour and Foster 2008) and (Gryb and Thébault 2023, S7.3) for details. Moreover, rigorous formal analysis of these constraints indicate that there are distinct gauge generating and dynamics generating roles that can be explicitly disentangled in the case of theories with a single Hamiltonian constraint. See (Gryb and Thébault 2023, §13) for details.

clock that one uses to deparametrise the gauge transformation equations.' (p. 3)

A simple approach to formalising this idea is to note that $a(t)$ does not Poisson commute with the Hamiltonian constraint of the theory. Mathematically, the time derivative of the scale factor is given by: $\dot{a}(t) = \{\mathcal{H}, a\} \neq 0$, where $\mathcal{H}$ is the Hamiltonian constraint in canonical GR, and $\{,\}$ denotes the Poisson bracket. When the theory is understood in these terms it is indisputable that the scale factor is not a Dirac observable. This leads to an apparent contradiction with the physical observations of the universe's expansion.

The natural response to this problem is apply the complete and partial observable scheme to construct Dirac observables base upon the Friedmann system of equations. This is precisely what Thiemann suggests to do. His explicit proposal involves introducing a scalar field as a clock, capable of deparametrizing the theory through the Brown and Kuchař (1995)'s mechanism. This approach allows for the construction of a physical Hamiltonian, which generates the evolution of gauge-invariant Dirac observables. Since the dynamics of the chosen material system and its stress-energy tensor are taken into account, it is also to explicitly implement a deparametrization in terms of a Real Reference Frames (**RRF**) as per the discussion of the last section.

Let us introduce a spatially homogeneous, scalar field ϕ, which acts as a 'phantom' field, not directly observable but capable of generating a physical notion of time.[13] The key innovation of this approach is to deparametrize the Hamiltonian constraint of GR, transforming it from a constraint equation into a physical Hamiltonian. The Hamiltonian constraint $\mathcal{H}$ is rewritten as: $\mathcal{H} = \pi + \mathfrak{h}$, where where π is the conjugate momentum to the scalar field and $\mathfrak{h}$ is called the *physical Hamiltonian* generating the temporal physical evolution of observables. Using this scalar field Thiemann constructs a framework in which the universe's time evolution is generated by the physical Hamiltonian rather than the Hamiltonian constraint. This reformulation allows for the deparametrization of the theory, with $\mathfrak{h}$ now acting as a physical Hamiltonian that generates time evolution for gauge-invariant observables. The crucial difference here is that $\mathfrak{h}$ is not constrained to vanish, as is the case with the traditional Hamiltonian constraint in GR.

Once the theory is deparametrized, the time evolution of observables such as the scale factor $a(\phi)$ can be computed using the physical Hamiltonian:

[13]It is worth nothing here that there are two importantly different senses of phantom that coincide in Thiemann's usage. First, 'phantom' in the sense of 'missing physics' that is not directly observable. Second, 'phantom' in the more formal sense used by cosmologists as indicating a field with a first order kinetic term in the Lagrangian with a coefficient which has a sign opposite to the sign in the Klein–Gordon Lagrangian. See (Thiemann 2006, p. 4).

$$\frac{da(\phi)}{d\phi}|_{\phi=s} = \{\mathfrak{h}, a(\phi)\}|_{\phi=s} \neq 0$$

for $s \in \mathbb{R}$. Of course,

$$\{\mathcal{H}, a(\phi)\}|_{\phi=s} = 0.$$

This evolution is now consistent with the Dirac criterion, as $O_a(\phi)$ is a Dirac observable that Poisson commutes with all constraints, unlike the original scale factor $a(t)$, which did not Poisson commute with the Hamiltonian constraint.

Crucially, however, in this framework, the evolution of $O_a(\phi)$ is governed by a *modified version* of the Friedmann equations, which includes additional terms due to the presence of the scalar field. In particular, the first Friedmann equations reads as:

$$\left(\frac{da/d\phi}{a(\phi)}\right)^2 = \left[\frac{8\pi G}{3}[\rho_m(\phi) + \rho_{\text{phantom}}(\phi)] + \frac{\Lambda}{3}\right]\left(1 + \frac{1}{x}\right),$$

where

$$x = \frac{E^2}{\alpha^2 a(\phi)^6}$$

is a deviation parameter, used to quantify how much the dynamics of the universe, governed by the modified Friedmann equation, differs from the standard cosmological model; E is a constant of motion, representing the energy of the universe; α is a model parameter characterising the influence of the phantom field. We chose $k = 0$ to adhere to Thiemann's formalism.

We thus arrive at an observationally distinct formulation of the theory which implements the Dirac observable prescription. The implication is then that we can either have the standard Friedmann equation and give up on our formalism for gauge invariant observables or we can keep our formalism for observables gauge invariant observables and modify the Friedmann equations. We cannot have both. Thiemann emphasises the gravity of this problem, stating that either the mathematical formalism of GR is inappropriate for cosmology, or we are missing some new physics. In the following section we seek to extricate ourselves from Thiemann's dilemma based upon the use of Einstein's famous clock hypothesis: that physical clocks measure proper time along their world-lines.

4. Hubble Flow and the Clock Hypothesis

Let us return to the derivation of the Friedman equations and seek an alternative physical interpretation of the t in the equations. One crucial aspect of the model we have not yet explicitly considered is the idea of *Hubble flow*. This flow describes the large-scale motion of matter, driven by the expansion of spacetime itself.

More formally, Hubble flow is the component of recessional velocity of matter due to the expansion, separating it from peculiar velocities caused by local gravitational interactions.

One way of understanding the derivation of the FLRW metric is via the adoption of the so-called synchronous frame (Landau and Lifshitz 1987). In this frame there is a common, global cosmological time for all observers comoving with the Hubble flow. Crucially, this means that the synchronous frame is a *geodetic reference frame*. This means that time-like trajectories, orthogonal to space-like 3D hypersurfaces are geodesics of space-time and the four-velocity of each observer $U^\mu = (1, \vec{0})$ automatically satisfy the geodesic equation.

From this perspective the Friedmann equations are *not* gauge transformation equations. Rather, they are evolution equations *in* a specific gauge. We can see this as follows. Recall that in the Arnowitt-Deser-Misner (ADM) formalism (Arnowitt et al. 1960) for canonical general relativity the metric is expressed in terms of the lapse function $N(t)$, the shift vector $N^i(t)$, and a spatial metric h_{ij}.[14] The lapse function N formalises the temporal separation between two infinitesimally close hypersurfaces, measured in the normal direction to the first hypersurface. The shift vector N^i measures the displacement between the coordinates x^i of a point $P \in \Sigma_t$ and its orthogonal projection $Q \in \Sigma_{t+dt}$. The connection between N and temporal diffeomorphisms and N^i and 3-diffeomorphisms emerges. It is specified that, in order to have a future-directed foliation, the lapse function N must be positive. In this formalism the general line element becomes:

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right).$$

In the specific case of FLRW cosmology, the lapse function $N(t) = 1$ and the shift vector $N^i(t) = 0$. These choices defines the synchronous gauge, reflecting the absence of preferred locations and directions and simplifying the FLRW metric. Since in the synchronous gauge we can assume the coordinate time t to coincide with the proper time τ measured by observers comoving with the Hubble flow, we will have that the t in the equations will coincide with the proper time of the relevant bundle of geodesics following the Hubble flow. This means we can re-write the equations in terms of proper time τ simply by equating $t = \tau$. It is worth noting that it is not necessary to show that $\{\mathcal{H}, a(\tau)\} \neq 0$, since we already gauge-fixed (deparametrised). So $a(\tau)$ can be seen as a gauge-fixed observable. A gauge-fixed observable is defined in that particular gauge, and need not commute with the constraints because there are no more gauge transformations to refer to. In other words, the gauge freedom has already been eliminated and we no longer have any gauge constraints left, because we have chosen a specific reference frame. To

[14]From Geroch (1970)'s Theorem follows that a globally hyperbolic spacetime can be foliated, that is decomposed into spatial slices parameterized by a global parameter t. The ADM formalism can thus be applied to any globally hyperbolic spacetime.

clarifiy what (Dittrich 2007, p.1914) claims, complete observables and gauge-fixed observables *are* the same, since a choice of reference frame can be seen as a gauge choice (a choice of section on a principle fibre bundle). Or, at least, it is always true that a gauge-fixed observable is a complete observable and only the reverse is less immediately obvious. A more detailed discussion of the status of gauge fixings, reference frames and gauge invariant observables in the context of cosmic proper time, within the fibre bundle formalism, is provided in Appendix A. The important point for our present purposes is to consider the physical consequences of this gauge fixed perspective on the Friedmann equations.

Consider, first that the Hubble flow should be understood to be the flow of a perfect fluid whose stress-energy tensor in the synchronous gauge is $T_{\mu\nu} = \text{diag}(\rho(\tau), -p(\tau))$, where $\rho(\tau)$ is the fluid's energy density, $p(\tau)$ its pressure. The Friedmann equations are then understood to describe how the expansion rate changes with the proper time of the observers comoving with the fluid. They can be written in terms of the Hubble parameter $H(\tau) = \dot{a}(\tau)/a(\tau)$. The first Friedmann equation is then:

$$H(\tau)^2 = \frac{8\pi G}{3}\rho(\tau) - \frac{kc^2}{a(\tau)^2} + \frac{\Lambda c^2}{3}.$$

and the second is:

$$\dot{H}(\tau) + H(\tau)^2 = -\frac{4\pi G}{3}\left(\rho(\tau) + \frac{3p(\tau)}{c^2}\right) + \frac{\Lambda c^2}{3}$$

where we have used the fact that $\frac{\ddot{a}}{a} = \dot{H} + H^2$ and assumed differentiation with respect to proper time.

For a given matter model we can then write the density and pressure in terms of the scale factor allowing us to for example re-write the first equation as:

$$H^2(\tau) = H_0^2\left[\Omega_{m,0}\frac{1}{a(\tau)^3} + \Omega_{r,0}\frac{1}{a(\tau)^4} + \Omega_\Lambda + \Omega_k\frac{1}{a(\tau)^2}\right]$$

where H_0 is the Hubble constant and we have introduced *experimentally measurable* density parameters *at the current time*; $\Omega_{0,R}$ for the radiation density, $\Omega_{0,M}$ for the matter (dark plus baryonic) density, $\Omega_{0,k}$ for the spatial curvature density, and $\Omega_{0,\Lambda}$ for the cosmological constant density. The measurement of H_0 suffers from the so-called *Hubble-tension*: a 5σ discrepancy between the experimental values obtained from the method using Cepheids and Type Ia Supernovae as standard candles and the CMB measurements via PLANCK experiment. A 5σ tension means that, statistically, the probability of the two measurements being compatible with each other is extremely low (less than one chance in a million). For a review see Smeenk (2022).

The question is then whether Hubble parameter $H(\tau)$ and the proper time along the Hubble flow can be understood to be *measurable quantities*. Let us consider each in turn.

First, the Hubble parameter. Experimentally, we cannot of course *directly* measure the Hubble parameter with an 'H-meter'. However, we can surely measure it *indirectly*, using astronomical observations that allow us to trace the expansion of the universe at different cosmic epochs. Let's just consider one of the various experimental possibilities. Type Ia supernovae are considered *standard candles* in cosmology because they have a well-known absolute luminosity. By measuring the apparent luminosity of a supernova, we can estimate its distance (luminosity distance d_L), while the redshift z is a direct indicator of how much the universe has expanded from the time τ of light emission of an object to the present day. The luminosity distance d_L is related to the Hubble parameter $H(z)$ through the following relation:

$$d_L(z) = (1+z)\int_0^z \frac{c\,dz'}{H(z')}$$

By measuring the redshift z through spectrometers and the distance d_L through large-field telescopes, such as the Hubble Space Telescope, we can obtain $H(z)$, from which the value of the Hubble parameter at different times τ can be derived. In particular, since τ is related to the redshift through the scale factor $a(\tau) = 1/(1+z)$, we can thus obtain information about $H(\tau)$.

Second, and more subtle, is the question of whether we can measure the proper time along the Hubble flow τ. Again we evidently cannot measure *directly* the proper time of our galaxy following the Hubble flow. If we say that we use *whatever* periodic physical system as a clock 'attached to the galaxy', it will not follow the Hubble flow. Actually, the concept of Hubble flow can be valid only at cosmological scales, so even for our Galaxy we should account for the effects of peculiar velocities. Completely eliminating peculiar motions from measurements of galaxy recession velocities is not possible, but it is possible to correct them in an approximate way. Therefore, strictly speaking, experimentally we cannot measure *directly* with a clock the proper time of *any* object following the Hubble flow. What we can do is appeal to the clock hypothesis. This amounts to the *hypothetical* assumption there is a clock that measures proper time along any given world-line. Proper time can be rigorously defined in the context of a relativistic spacetime (M, g_{ab}) as follows (Malament 2012, §2.3): let $\gamma : [s_1, s_2] \to M$ be a smooth future-directed timelike curve in the manifold M with tangent ξ^a. Then the proper time associated with the curve relative to the metric g_{ab} is given by:

$$||\gamma|| = \int_{s_1}^{s_2} \left(g_{ab}\xi^a\xi^b\right)^{\frac{1}{2}} ds \tag{7}$$

s where ds is the line element. Since the clock hypothesis applies also to the world-lines of observers following the Hubble flow, we seem to have solved the problem by stipulation.

Since we can associate measurements with partial observables and partial observables with reference frames, we can wonder: what kind of Reference Frame the cosmic proper time is? And it is a *bona-fide* partial observable? Given the clock hypothesis, we can stipulate a clock that measures cosmic time. Furthermore, cosmic time will always be dynamically coupled with the expansion rate of the Universe, parametrised by $H(\tau)$, since both quantities depend on the same FLRW metric. The cosmic time value, being a proper time, is a structural property of the gravitational field given by equation (7). This is very similar to the the sense in which the Hubble parameter measures the rate of expansion of the volume of the universe as a geometric quantity, which is derived from the gravitational field: $dV = \sqrt{-g}dx^{\mu}$. τ is a geodesic reference clock, since the four-velocity of the cosmological fluid is associated to the geodesic dynamics of a dust fluid, whose energy-momentum tensor $T_{ab} = \rho U_a U_b$ is source of the EFEs and give rise to the FLRW solution. Thus, τ is a **RRF** in (Bamonti 2023)'s classification. The nature of the **RRF** clock comes from the fact that τ is the proper time of the cosmological fluid, whose back-reaction on gravity is taken into account and gives rise to the FLRW metric which in turn determines the proper time τ (this is the essence of the non-linear feedback of EFEs). This **RRF** clock provides the privileged representation in which the cosmic microwave background radiation is represented as perfectly homogeneous and isotropic, in absence of small inhomogeneities of the primordial universe.

We thus have that cosmic proper time: i) is an **RRF** since it involved back-reaction; ii) is a *bona-fide* partial observable according to Bamonti and Gomes (2024); and iii) corresponds to a measurable quantity by the clock hypothesis. Is this enough for us to conclude that $H(\tau)$ is a complete observable and thus have solved Thiemann's dilemma? Almost.

As stated above, the problem is that *on a practical level*, it is not possible to have an experimentally accessible clock, (i.e. with which we can exchange signals), that follows the Hubble flow. In general, distant galaxies are considered to follow the Hubble Flow, as for very distant galaxies, the contribution of their peculiar motions is negligible compared to the recession velocity due to the expansion of the universe. Thus, let's consider a galaxy in our past light cone as a satellite sending radiation towards us. In this way, we would use it as a kind of Rovelli (2002a)'s 'GPS clock that would allow us to define local quantities, such as the Hubble parameter, in its proper time.[15] Again, however, the cosmic time would be the time measured by some stipulated and not further defined clock 'attached' to that galaxy-satellite and

[15]Note also that clusters of matter represent inhomogeneities that are assumed to evolve following the underlying FLRW background structure. So, their evolution does not influence the global FLRW evolution. "More precisely, it is assumed that effects from the small scale inhomogeneities onto the largest scales can be neglected, i.e. there is no substantial backreaction" Schander and Thiemann (2021). Thus, galaxies, clusters and other agglomerates of matter are treated as *test particles*.

broadcasting the measured value to the experimenter, via light signals. Apart from the experimental problem to construct a valuable experimental setting, and the need to take into account the galaxy's peculiar velocities, there remains the problem of defining an origin of such time measured by the clock-galaxy. One solution would be to conveniently place the origin as the 'zero time' of the formation event of such a cluster. Or also, analogous to the construction of GPS coordinates in Rovelli (2002a), as the time of the galaxy's encounter with another galaxy. In any case, the proper time of the galaxy will never be a global time. As Rovelli states: 'Our Galaxy and Andromeda are heading towards a collision: when they will meet, the times elapsed from the Big Bang will be different in the two galaxies. None of the two will have any claim of being more of a "true" time than the other.' (Rovelli 2024, p.18). It is practically impossible to have a clock measuring the proper time parametrising the Hubble flow.

In this context it is interesting to note that Brown and Read (2021) argue that the clock hypothesis is not strictly realised and is better understood as 'clock condition'. In a similar vein, one might view the clock hypothesis as constituting the *definition* of clocks as *objects that measure proper time*. The important point is that *if* the clock hypothesis holds, then we are able to treat the τ in the Friedmann equations as a measurable quantity just like the Hubble parameter. However, in the context of cosmology and the Hubble flow, there is a tension between applying the clock hypothesis and the idea of a clock as a real, experimentally accessible physical system. Real physical clocks do not seem to be the kind of things that can in fact measure proper time along the Hubble flow. In the following section we return to the ideas of partial and complete observable and **RRFs** to better understand both this challenge and the comparative merits of the approach of Thiemann described in the previous section.

5. Finding Cosmic Time

Let us recap. One the one hand, the widely used and accepted criterion for an observable in a theory with temporal diffeomorphism symmetry is that such observables should be Dirac observables and therefore have (weakly) vanishing Poisson bracket with all first class constraints. On the other hand, the widely used and empirically established Friedmann equations describing the dynamics of the scale factor can be understood to correspond to those generated by a first class constraint in a theory with temporal diffeomorphism symmetry. It seems like we must either give up on the Dirac criterion for observables or modify our understanding cosmological dynamics.

We have considered two alternative responses to this dilemma as follows. First, Thiemann argues that we should adopt the second option and demonstrates how we might explicitly reconstruct Friedmann cosmology as a de-parameterized theory based upon a phantom matter field acting as a physical clock that measures cosmic time. A deviation parameter then quantifies how much the dynamics of the universe, governed by the modified Friedmann equation, differs from the standard model of cosmology. Second, we have constructed an alternative approach that re-interprets Friedmann equations as evolution equations parameterized by proper time, rather than coordinate time. On this approach we understand the equations as describing the correlation between two independently measurable 'partial observables' given by the Hubble parameter and proper time along the Hubble flow. The Hubble parameter is a measurable quantity within modern cosmology. Furthermore, following the clock hypothesis, we have that since clocks measure proper time along world-lines, a clock following the Hubble flow will necessarily measure cosmic time. The problem, however, is that the proper time along the Hubble flow does not correspond to a physical quantity associated with a measuring procedure by a clock leading to a number, and so it seems we are no longer implementing the partial and complete observables procedure in the spirit of Rovelli (2002b). The crucial point is to define an experimental measure of cosmic time consistent with the clock hypothesis.

In this context, it is worth noting again that Thiemann holds a different understanding of the partial and complete observables approach to Rovelli. In particular, for Thiemann partial 'observables' are not observables at all and so there is not sense in which they need to be associate to a measuring procedure. We can thus understand the Thiemann approach to complete observables and cosmic time as built upon abandoning not one by two conventionally accepted aspects of the formalism, viz. the clock hypothesis and the distinction between partial and complete observability. An *observable simpliciter* is defined as a Dirac Observable and there is no requirement that such observables are built out of independently measurable functions, which however are not (Dirac) observables. On this way of thinking, $H(\tau)$ will *not* be an *observable simpliciter*, and should not be expected to commute with the Hamiltonian constraint.

Moreover, since the clock hypothesis is abandoned, τ is not observable, and for that $H(\tau)$ is not a (Dirac) observable too. Plausibly, it is precisely the abandonment of the clock hypothesis which led Thiemann to use a phantom scalar clock as the physical, observable clock of the theory. In any case, the crucial point is that adopting this perspective does not amount simply to an alternative interpretation of the theory. Rather, it is to reformulate the classical theory of cosmology such in a way that modifies the observable consequences. What modifications are made will depend upon the clock choice. However, there is no choice that corresponds to a strict preservation of the Friedmann equations: 'whatever matter is used for deparametrisation, there will be corrections [...] This should have observable consequences!' (Thiemann 2006, p.9). Non-standard observable consequences are of course a virtue in a physical theory. Furthermore it is worth noting that Thiemann's approach in the paper in question is connected to a specific research programme in terms of Phantom k-essence cosmology (Aguirregabiria et al. 2004) and thus the modifications in question could be independently motivated and in principle tested via the relevant modified matter or gravity theory. There are thus good methodological reasons to pursue more radical approach to identifying cosmic time.

Can we plot a plausible path towards a more conservative approach that preserves both the distinct notions of partial and complete observable and the clock hypothesis? Recall once more that on the original Rovelli definition a partial observable is a physical quantity associated with *measuring procedure* leading to a number. This definition fits well with the way the Hubble parameter features in our cosmological observational practice, even if its measurement is *indirect*. The problem was that proper time along the Hubble flow is not associated with a *direct* measuring procedure in any straightforward sense that involves a clock. Notwithstanding the clock hypothesis, there seems to be a tension between our intuitive notion of measurement of time (by a clock) and the role played by cosmic proper time in our scientific theories. It is worth considering at this juncture, however, that the intuitive notion of measurement has itself been disputed in the context of a practice orientated account of scientific measurement. This has lead to a reorientation of the philosophy of scientific measurement towards a model-based account. Furthermore, perhaps the most detailed and powerful example of the conceptual heavy lifting that a model-based account of measurement can do is in context of the measurement of time. We will see that the path, similarly to the case of H, will be to rely on *indirect* measurements and not measurements made by a clock, as is commonly done when dealing with time measurements.

Let us briefly consider key ideas from the pathbreaking work of Tal (2016) on the measurement of time via atomic clocks cf. Thébault (2021a). Following this account we recognise that the time 'measured' by atomic clocks does not correspond to a simple procedure of reading a number from a device. Rather, 'Coordinated Universal Time' (UTC) is based upon a *standardisation procedure* involving multiple atomic clocks distributed throughout the globe and systematic modelling at various stages. Caesium plays a particularly important role in modern time-keeping since it is transitions of an *idealised* caesium atom that are the basis for the definition of the second. However, as emphasised by Tal, this does not mean that one can simply read seconds from real caesium atoms. The caesium atom that defines the second is an idealised construct, at rest at zero degrees Kelvin and with no coupling to any external fields. Actual atomic clocks are then built to approximately realise the ideal caesium clock, with *known* sources of difference minimised and modelled. However, the 'primary' standards caesium clocks typically only operates for a few weeks at a time in order to calibrate 'secondary standards'. The secondary standards are a different class of atomic clocks that are less accurate but can be run continuously for a number of years. The secondary standard clocks also must be modelled. In particular, the 'readings' of the clocks are subject to quantitive adjustments relating to the known sources of difference between their ideal physical operation and their actual physical realisation. This allows the time that they read to be a close approximation to that read by their idealised counterpart. The crucial point is that UTC is not 'read' by either primary or secondary standards. Rather it is a product of a further abstraction based upon the readings of the different participant clocks throughout the world. Furthermore, not only are different clocks weighted differently in UTC, since some clocks are more noisy, but since the clocks are at different physical locations on the earth, one must also take account of their differing proper times, as determined by the relevant differences in gravitational field and four acceleration (Tal 2016, p. 302). The general implication for of a model-based account of measurement for the definition of a partial observable is thus to substantially liberalise the sense of 'procedure' in Rovelli's definition. In particular, on a model-based account of measurement, the procedure involved may include not just indirect measurement but various tears of modelling, calibration, and aggregation. Furthermore, and more importantly for our discussion, comparison between the status of UTC and cosmic time throws into relief our failure in the latter case to find an actual physical system that plays the role of the clock. *There are no clocks that measure UTC either.*

Consider, then, that we can 'measure' cosmic time via various cosmological phenomena. For example, we can use the analysis of the power spectrum of temperature fluctuations of the CMB (TT-power spectrum) measured by the PLANCK satellite. In particular, the CMB power spectrum represents the power distribution of the temperature anisotropies as a function of angular scale. The different angular scales correspond to the scales of the baryonic acoustic oscillations (BAOs) which are pressure waves generated by interactions between radiation (photons) and matter (baryons) in the primordial plasma before decoupling era. Anisotropies reflect primordial density differences, which led to the formation of galaxies and other structures (Kolb 2018, ch.9). The power spectrum contains peaks and valleys at different scales and their position and amplitude are sensitive to cosmological parameters. For example, a higher density of matter leads to higher peaks that are closer together, and a flat universe tends to have a different distribution of peaks than a curved universe. The Hubble constant H_0 also affects the scale of the fluctuations and the position of the peaks.[16] See Figure 1.

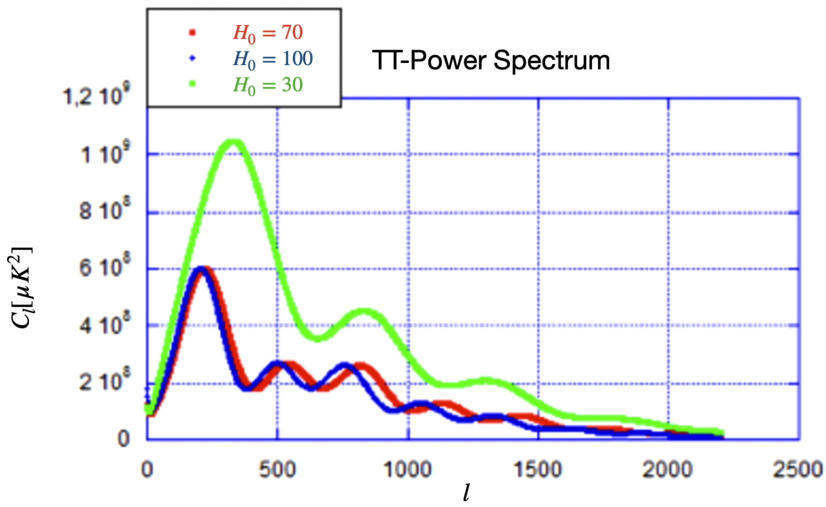

FIGURE 1. Temperature power spectra for three different Hubble constants. The red curve represents the model with $H_0 = 70$ Km/s/MPc. The blue curve represents the model with $H_0 = 100$ Km/s/MPc. The green curve represents the model with $H_0 = 30$ Km/s/MPc. For the acquisition of the data to be plotted, we used the CAMB web interface https://lambda.gsfc.nasa.gov/toolbox/camb_online.html.

By reconstructing the power spectrum it is possible to determine cosmological time. In particular, cosmological time is determined by integrating the equations of the expansion of the universe:

$$\tau(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')},$$

where $H(z)$ is obtained from the cosmological parameters Ω_I, H_0 which are determined by the experimental power spectrum. To be precise, $\tau(z)$ above does not correspond to the proper time of a real observer comoving with the Hubble Flow, since in observational practice, we know the Universe is not *perfectly* homogeneous

[16]Formally, the TT-power spectrum is the expansion in Legendre polynomials $\mathfrak{P}$ of the variance of temperature fluctuations:

$$\langle \frac{\Delta T}{T}(\gamma_1)\frac{\Delta T}{T}(\gamma_2)\rangle = 1/2\pi \sum_l l(l+1)C_l\mathfrak{P}_l(cos\theta),$$

where γ represents the direction in which temperature fluctuations ΔT are measured and C_l is the coefficient of the l-th multipole, representing the amplitude of temperature fluctuations at a certain angular scale $\theta \propto 1/l$. There is a privileged angular scale for these oscillations, which corresponds to the size of the sound horizon at the time of recombination. This angular scale corresponds approximately to the multipole $l = 200$ with a dependence on cosmological parameters.

and isotropic: not even on cosmologically large scales. The FLRW model is an idealisation to describe the average behaviour of the Universe. The Hubble parameter $H(z)$ in the formula depends on parameters such as Ω_{CDM} (dark matter density), which are in part determined using perturbative methods, which take into account small inhomogeneities. Therefore, in measuring the parameter $H(z)$, contributions of inhomogeneity (such as galaxy clusters, voids) and anisotropies are taken into account. Nonetheless, the value of $H(z)$ used in the theoretical calculation of $\tau(z)$ refers to the 'average' behaviour of the Universe, described by a homogeneous and isotropic model. We mean that the formula $\tau(z)$ calculating cosmic time is based on a model that idealises the Universe as perfectly homogeneous and isotropic, using an 'average value' of $H(z)$. This means that the cosmic time $\tau(z)$ is a global average, which does not reflect local fluctuations or deviations from isotropy, but rather provides an approximated estimate. One could say that, experimentally, the contributions from inhomogeneities and anisotropies of the actual Universe are '*averaged out to zero*', in the sense that these contributions are present in the data but, when calculating the Hubble parameter $H(z)$, local fluctuations are essentially 'smoothed out' and do not significantly influence the overall result. Consequently, experimental measurements *approximate* the theoretical ideal cosmic time, which refers to an ideally homogeneous and isotropic Universe, analogously to what happens with primary and secondary clocks for measuring UTC. It is the case that $\tau(z)$ that is used to make predictions in cosmology, and it is a partial observable, pace Thiemann (2006).

We thus have that in a more liberalised sense of 'measurement procedure', it *is* the case that we can treat proper time along the Hubble flow as a partial observable in the context of cosmology. This has not involved simply stipulating proper time as partial observable via the clock hypothesis. However, it also has not involved the abandonment of the clock hypothesis altogether. Rather, the clock hypothesis reemerges within cosmology as something like a 'coordinative definition' in sense of Reichenbach (1928). That is, the clock hypothesis allows for the *coordination* of a concept (an ideal clock measuring the Hubble flow temporal parametrisation) with an empirical phenomenon (cosmic time). This broadly logical empiricist understanding accords with other discussions of the clock hypothesis in the recent philosophical literature (Adlam et al. 2022) and makes sense of the physically nontrivial but partially definitional role.[17] We recover the Friedmann equations and cosmic time whilst keeping both the clock hypothesis and the partial/complete observable distinction, albeit each in modified form. The conservative option for finding cosmic time is thus a live possibility.

[17]See also Fletcher (2025) who classifies the clock hypothesis as a *representational principle*.

6. Summary

In this paper we first provided a brief overview of the complete and partial observables programme as a response to the problem of time that seeks to preserve the Dirac criterion for observables. We then applied recent work on reference frames to disambiguate two important details in the definition of a partial observable and better understand what it means for a physical variable to play the role of a clock in the context of a complete observable. We then considered the status of time in the Friedmann equations. In particular, we reviewed Thiemann's challenge to the standard interpretation of these equations as dynamical equations. This led to a discussion of his solution in terms of a de-parameterization via a phantom field through the well-known Brown and Kuchař (1995) mechanism. We next considered the idea of the Hubble flow and sought to re-frame comic time as a proper time parameter along the Hubble flow. In this context, we considered the question of the observability and measurability of Hubble parameter and the question of whether we can consider to be a complete observable whose dynamics is described by the unmodified Friedman equations. We then articulated the specific problem of there being no physical system which can be understood to even approximately measure proper time along the Hubble flow. This leads to the final step in the articulation of our proposed solution: the introduction of a more liberalised sense of 'measurement procedure' in the context of cosmic time. This more liberalised notion draws upon the model-based account of time measurement developed in the context of atomic clocks and the measurement of Coordinated Universal Time due to Tal (2016). Provided one is willing to reinterpret our criteria of observability in light of the clock hypothesis and the model-based account of measurement one can and preserve the Friedmann equations and find time in cosmology.

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Appendix A. Gauge Freedom and Gauge Fixings

The framework of Dirac observables has as its intended goal the removal of underdetermination in phase space dynamics due to the presence of gauge freedom. This is clearly the important point and not whether or not the letter of the Dirac criterion or the spirit of general covariance have been respected. The observables of the theory are required to be gauge-invariant quantities. Here we briefly summarise some formal aspects relating gauge invariance and gauge fixings in order to clarify the formal status of cosmic proper time within the foundations of general relativity. See Gomes (2024) for further details.

In the context of general relativity the desire is to formulate the representations of observables such that they are invariant under diffeomorphisms. A space of models Φ within GR can be seen as a principal bundle with $Diff(\mathcal{M})$ as its structure group and $[\Phi] := \{[\varphi], \varphi \in \Phi)\}$ its base space. Selecting a reference frame amounts to defineing a *unique* section map $\sigma : [\varphi] \to \sigma([\varphi]) \in \Phi$, where the choice of a section is a smooth injection from the space of equivalence classes of models to the space of models, and corresponds to a choice of a submanifold on the fibre-bundle that intersect each fibre $\mathbb{F}_\varphi := \mathbf{pr}^{-1}([\varphi])$ exactly once (with $\mathbf{pr} : \varphi \to [\varphi]$).

Equivalently we can make use of the so-called projection operator $f_\sigma : \varphi \to f_\sigma^* \varphi$, an equivalent of the section map, which takes any element of a given fibre to the *unique* image of the section. It is an embedding map, *acting within a fibre* and it is characterised by the auxiliary condition $F_\sigma(\varphi) = 0$, making the choice of a reference frame (or a section) analogous to a gauge-fixing procedure. The use of the projection operator $f_\sigma : \Phi \to \Phi$ instead of the section map $\sigma : [\Phi] \to \Phi$ codifies the *external sophistication* demand of the unnecessity to represent elements $[\varphi]$ of $[\Phi]$ *intrinsically* by some parameterization of $[\Phi]$. Thus the discard of the *structure-first, or internal approach* (Dewar 2019; Jacobs 2021). The quantity resulting from the choice of a section is the relational, gauge-invariant observable $f_\sigma^* \varphi \equiv (\varphi)_F$. See Figure 2.

The important point is that the transformation that changes the reference frame corresponds to a *change of section*. So, it should *not* be understood as something that acts on the fields configuration: it does not act on the dynamically possible models φ, but acts *directly* on the already constructed gauge-invariant observables, changing frames (section) and getting us to a different and *new* observable, i.e. a new representative of a fibre. This substantiates Thiemann's claim that a change of reference frame has observable consequences for the dynamics (see section 5).

In order for the cosmic proper time τ to be considered a physical clock dynamically coupled with the Hubble parameter (and thus with the metric), the following condition must be met: If (g_{ab}, τ) is a dynamically possible model of the theory, then neither $([d^* g]_{ab}, \tau)$ nor $(g_{ab}, d^* \tau)$ is, $\forall d \in Diff(\mathcal{M})$. Thus, the choice of τ as the **RRF** clock (rather than one of its diffeomorphic copies) provide a *unique* representation of $H(\tau) := H \circ \tau^{-1}$ for some initial data, which is thus a *bona-fide* gauge-invariant, complete observable. For this reason, τ fixes the gauge for the FLRW metric and by definition is such that there is no longer gauge freedom in the theory, and the potential for underdetermination. However, given the correspondence between the choice of a reference frame and the choice of a gauge (Bamonti 2023; Gomes 2024), we always have the possibility of changing reference frames and obtaining new relational observables.

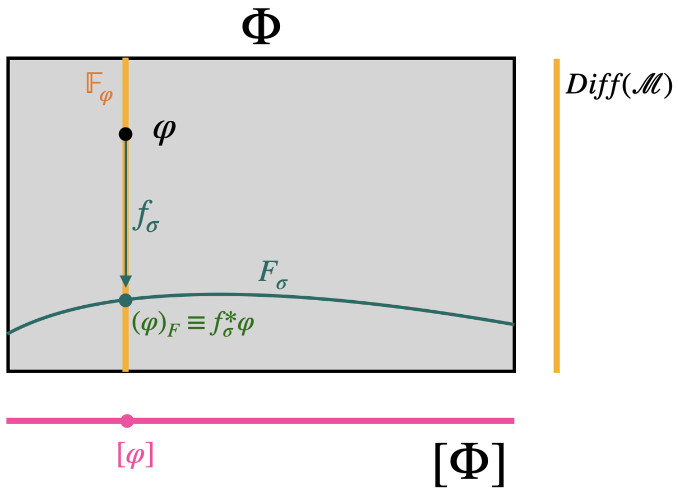

FIGURE 2. The space of models Φ. Each point corresponds to a particular configuration φ. A reference frame σ picks out a *unique* representative $(\varphi)_F$ for each fibre $\mathbb{F}_\varphi$. Models belonging to the same fibre are taken to be physically equivalent, since a fibre corresponds to a gauge orbit.

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