Keynes, Wittgenstein, and Probability in the *Tractatus*

Matthew Coates*

Abstract

It has been questioned why Wittgenstein wrote a significant amount on the foundations of probability in the *Tractatus*. In this paper I answer this by claiming that the primary aim of Wittgenstein's account was to criticize a Keynesian theory of probability, and provide multiple pieces of evidence to demonstrate this. This then answers why Wittgenstein wrote such a significant amount on probability. He wrote it because it was salient at the time. Whilst Wittgenstein was at Cambridge there was significant discussion of probability by his philosophical interlocutors, particularly Keynes but also Russell, Moore and others. Wittgenstein thought he had the answers to the problems that were being discussed and set them out in the *Tractatus*.

1 Introduction

In 1921 John Maynard Keynes published *A Treatise on Probability* (Keynes, 1921/1978), the first major work in English on probability since Venn's *Logic of Chance* (Keynes, 1921/1978, 473). In this, Keynes set out a logical theory of probability, on the basis that between two propositions e and h there exists a logical relation, written as h/e. This relation is the probability relation, representing the probability of h given e. These relations are objective, with only a single probability relation existing between each two propositions.

Also published in 1921 was Wittgenstein's *Tractatus Logico-Philosophicus* (Wittgenstein, 1921/2001). A friend and philosophical interlocutor of Keynes, Wittgenstein himself made some brief remarks setting out a logical theory of probability in propositions 4.464 and 5.1 and between propositions 5.15-5.156. Though his remarks are compressed, he set out a logical theory of probability based on the number of rows in a truth table that come out true for the propositions involved.

Although Wittgenstein's remarks on probability had an impact on later theories of probability, serving a role in the development of probability for Waismann (1977) and then Carnap (1962, 299), they have mostly been ignored by Wittgenstein scholars and philosophers of probability. When they have been recognized, it

*Department of Logic and Philosophy of Science, UC Irvine. mncoates@uci.edu
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has been noted that "it is difficult to see why Wittgenstein devotes so much space to what must count as a relative side issue." (White, 2006, 86).

In this paper I answer the question of what Wittgenstein's purpose was in writing this section. Though Wittgenstein does not explicitly cite others in his treatment of probability, I claim that a primary aim of this treatment was to criticize a Keynesian theory of probability.

I do this in two separate ways. First I demonstrate that Wittgenstein very likely knew that Keynes was working on such a theory of probability prior to the writing of the *Tractatus*. I also demonstrate that that Wittgenstein's only definitely known major interactions (direct or indirect) with philosophers of probability were with Keynes and those around him with similar views (such as W.E. Johnson). However, I admit that there is room to doubt both that this means Wittgenstein was specifically targeting Keynes and that Wittgenstein knew the details of Keynes' theory.

Therefore, secondly I establish that Wittgenstein's remarks were targeting a theory of probability which had the features of a Keynesian theory of probability. This may have been Keynes' theory itself, or it may have been the collection of ideas that became Keynes' theory. The significant idea here is that the probability relation is an objective logical relation that exists separately from the propositions involved. Additionally, this relation has many of the other features of Platonic Atomism to which Keynes adhered (O'Donnell, 1992). Platonic Atomism consists of the philosophical views held by Russell and Moore taken by Hylton (1990) to be between 1898-1905, though this isn't a strict timespan and many relevant aspects of their beliefs continue after this period. I first demonstrate that the language that Wittgenstein uses to criticize this theory of probability is similar to that used against Platonic Atomism (such as a criticism of logical objects). I then show that Wittgenstein's remarks on probability need to be read in the context of his remarks on inference, which are critical of a Platonic Atomist (specifically Russell's) view and appear immediately before the remarks on probability.

This helps to answer the question of why Wittgenstein does include such a long discussion about probability in the *Tractatus*. By recognizing that Wittgenstein was responding to a Keynesian theory of probability, we can conclude that Wittgenstein may have devoted so much space to a relative side issue because he was engaging in debates that were occurring at Cambridge whilst he was also there. These debates involved people with whom he had engaged in discussion, such as Keynes and Russell. Wittgenstein thought he had the answers to the problems that were being discussed, and set them out in the *Tractatus*.

Keynes and Wittgenstein knowing each other, and being in the same circles at Cambridge, is well known. Despite this, the historical connections between their accounts of probability have mostly been ignored, even amongst philosophers who have paid attention to Wittgenstein's philosophy of probability. This has led to the contextual salience of Wittgenstein's account going unnoticed.

Before beginning, I set out two claims I am not making. I am not claiming that a Platonic Atomist conception of probability is Wittgenstein's only target for criticism. Often Wittgenstein is criticizing multiple views in different ways at the same time, such as Frege and Russell in 5.132. I am also not claiming one of Wittgenstein's or Keynes' theories of probability is better than the other, or that one actually satisfyingly solves the issues posed of the other. This has been discussed by Ongaro (2021), who discusses whether Wittgenstein's work would fall foul of the criticisms posed of other logical theories of probability. I am simply claiming that Wittgenstein wrote his remarks on probability at least in part to criticize a Keynesian conception of probability, and that explains Wittgenstein's purpose in writing this section.

The paper is structured as follows: in section 2 I set out some history of both *A Treatise on Probability* and the *Tractatus Logico-Philosophicus*, and the extent of Keynes and Wittgenstein's relationship during the period of 1912-1921, in order to demonstrate that Wittgenstein likely knew of Keynes' work. In section 3 I describe both Keynes' and Wittgenstein's conceptions of probability. In section 4 I set out relevant components of Platonic Atomism, so that I can later demonstrate Keynes held the same views, and describe both Moore and Russell's accounts of inference and logical consequence, as well as Wittgenstein's account and objections. In section 5 I demonstrate that Keynes did accept a Platonic Atomist philosophy like that of Russell and Moore that was previously described. Finally, in section 6, I use this background on Platonic Atomism to demonstrate that Wittgenstein's remarks on probability were targeting for criticism a Keynesian theory of probability.

2 Keynes and Wittgenstein

The main aim of this section is to establish that Wittgenstein very likely did know of Keynes' ideas on probability that went on to become the *Treatise*. I do this by briefly sketching a relevant history of *A Treatise on Probability* and the *Tractatus Logico-Philosophicus* and by providing details about the relationship between Keynes and Wittgenstein during the period of 1912 to 1921.

Keynes arrived as a student at Cambridge in 1902. Although enrolled in the mathematics tripos, he predominantly studied philosophy. In either 1904 or 1906 (The dating is disputed between Skidelsky (2003, 95) and Moggridge (1992, 131-6)) Keynes read a paper to the Cambridge society the Apostles titled "Ethics in Relation to Conduct". This paper foreshadows the ideas that would become A *Treatise* in Probability. In the paper Keynes criticizes Moore's use of probability in the Principia Ethica in the chapter also titled "Ethics in Relation to Conduct" (Moore, 1903/1993).

Keynes then developed these ideas into a larger fellowship dissertation at Kings College, Cambridge, which was submitted but rejected in 1908. He resubmitted successfully in 1909. Though Keynes had started

moving towards economics rather than philosophy as a result of the rejection, he was still developing his dissertation into a book. The first draft of this book was written in 1913, before undergoing substantial corrections. The process of correcting the draft was interrupted by the First World War, before being completed in 1920, and finally published as A *Treatise* in Probability in August 1921.

Wittgenstein arrived in Cambridge to study under Russell in 1911, and stayed there until late 1913, when he moved to Norway. Russell introduced him to Keynes in October 1912 and later Keynes wrote to Duncan Grant that Wittgenstein was "A most wonderful character... I like enormously to be with him." (Skidelsky, 2003, 159). Keynes, recognizing Wittgenstein's talent, had Wittgenstein elected to the Apostles, though Wittgenstein disliked the Apostles, tried to leave, and complained to Keynes about them (Skidelsky, 2003, 159). Despite this, there is epistolary evidence that Keynes and Wittgenstein continued to meet. A particular letter highlighting this states:

> "Thanks very much for the trouble you take over my business. My reason for not seeing you oftener last term was, that I did not wish our intercourse to continue without any sign that you wished to continue it." Wittgenstein to Keynes, July 1913 (Wittgenstein and McGuinness, 2012)

This letter suggests that they would typically meet, though in the term that this letter had been written their meetings had reduced. What they discussed in these meetings is not known, however, given Keynes was working on his theory of probability during this same period, they may have discussed this work.

Evidence also exists showing that Wittgenstein was aware that Keynes was working on a theory of probability. In a letter Wittgenstein sent to Keynes dated 12th of June 1919, Wittgenstein asks:

> "Thanks very much for the trouble you take over my business. My reason for not seeing you oftener last term was, that I did not wish our intercourse to continue without any sign that you wished to continue it." Wittgenstein to Keynes, July 1913 "Have you done any more work on probability? My M-S. contains a few lines about it which, I believe, solve the essential question.
>
> Yours ever, !! Ludwig Wittgenstein" Wittgenstein to Keynes, June 1919 (Wittgenstein and McGuinness, 2012)

With the !! added by Keynes, likely as an expression of amazement. Therefore, despite having been away from Cambridge since 1913, Wittgenstein knew enough about Keynes' project concerning probability to ask him how the work was going, and to note that he had been working on the same problems.

Therefore, we know that Wittgenstein and Keynes would meet, and we know that Wittgenstein knew Keynes was working on probability. However, there is still room to doubt he knew Keynes' ideas on proba-

bility. Wittgenstein would occasionally avoid philosophical or political issues when discussing with people, even when they were of shared interest, so there is some chance it was not discussed.

However, Keynes' work was also known around Cambridge, in the circles that Wittgenstein moved in, and Wittgenstein may have come to know it through this. For example, during the period between his two attempts at his fellowship dissertation, Keynes was in discussions with both Russell and Moore to work on corrections and improvements (O'Donnell, 1992, 15). Russell also made use of Keynes' work on probability in *Problems of Philosophy* (Russell, 1912/2001), thanking him in the published book as follows:

> "I have derived valuable assistance from unpublished writings of G. E. Moore and J. M. Keynes... and from the latter as regards probability and induction." (Russell, 1912/2001).

Russell also continued to discuss the work with Keynes during the preparations of *A Treatise on Probability*. Russell stated in a letter to Ottoline Morrell in July 1914:

> "I am doing a great deal of work... partly the proofs of Keynes's book on Probability, which I am reading carefully with Broad and then discussing with Keynes. It is an amazingly good book, really original, and with splendid comprehensive grasp-also very well written". (O'Donnell, 1992, 340)

Finally, W.E. Johnson also knew of Keynes' theory of probability. Johnson had been an examiner on Keynes' fellowship dissertation, and he was positive of Keynes' work. He also continued to help Keynes with the preparation of the *Treatise* for publication (O'Donnell, 1992), and he was developing his own theory of probability which shared commonalities with that of Keynes'. Johnson was also an interlocutor of Wittgenstein. He, unsuccessfully, attempted to tutor Wittgenstein in logic at Russell's request. Despite this, they remained close and Wittgenstein arranged to give Johnson a grant to help him reduce his teaching commitments and have more time for research.

Therefore, Wittgenstein had multiple ways he could have heard about Keynes' work. Even if Wittgenstein and Keynes had not directly discussed probability themselves, it is likely that Wittgenstein would have come to learn of Keynes' views from others around Cambridge, such as Russell, Moore and Johnson, who were in philosophical discussions with both Keynes and Wittgenstein. Hence, we can conclude that it is very likely that Wittgenstein knew Keynes' ideas on probability.

Keynes, and those around Keynes, are the only probability theorists that Wittgenstein is known to have interacted with, either directly or indirectly through other works, prior to the writing of the *Tractatus*. However, there are suggestions he was familiar with the work of Johann von Kries, or Bolzano. Both of these were suggested by von Wright (1969), with the former advanced by Heidelberger (2001). The Bolzano

suggestion is unlikely, and is only based on a similarity between their theories. Bolzano's writings on probability were not very well known, having been mostly forgotten until the work of Waisman in 1930, and it is unlikely that Wittgenstein would have known of them whilst writing the *Tractatus*. Therefore I will focus on the von Kries claim.

The main reason Heidelberger gives to suggest that Wittgenstein knew of von Kries' work is the use of *spielraum* by both Wittgenstein and von Kries. Von Kries used *spielraum* in his definition of the probability of an event: relative to a set of natural laws, the probability is given by the ratio of the range (*spielraum*) of objective possibilities that would make the event occur to the range (*spielraum*) of all the objective possibilities (Ongaro, 2021). Wittgenstein also uses the word *spielraum*, in propositions 4.463 and 5.5262 in connection with the truth-grounds of a proposition, though not in connection with probability directly, which led McGuinness (1982) to doubt that it was connected to the work of von Kries.

Even if we accept that Wittgenstein did get *spielraum* from von Kries, as suggested by Heidelberger, how would he have come to know it? Heidelberger suggests that it is via Boltzmann. Von Kries' ideas are referred to by Boltzmann in an address which was published in his Popular Writings, where he praises Von Kries for producing a "logical justification" of the calculation of probability (Heidelberger, 2001, 186). It is known that Wittgenstein did possess this book soon after its publication (Wilson, 1989, 257), and he may have to come to know of Von Kries' work via Boltzmann. However, this is a very indirect way that he could have known of Von Kries, assuming that Wittgenstein both noticed this remark in Boltzmann and then found von Kries' work to study. There is no evidence for either of these claims, and the latter particularly requires significant leaps.

However, there is another way Wittgenstein could have known of von Kries, not mentioned by von Wright or Heidelbeger, and that is through discussions on probability with Keynes. Keynes had studied the works of von Kries and mentions them often as an influence in the *Treatise*. Wittgenstein may have only come to know of von Kries' ideas after these discussions, and that is what led to his adoption of *spielraum*. However, given this this assumes that Wittgenstein knew of Keynes' work and ideas, it further supports my claim in this section.

Whilst Wittgenstein's knowledge of von Kries is only speculation, with indirect evidence, it is known that Wittgenstein did directly interact with Keynes. This gives justification to my claim Wittgenstein was, in part, responding to Keynes rather than some other theorist of probability with his remarks on probability in the *Tractatus*. However, even if Wittgenstein did know of von Kries' work, it does not affect my argument. He could still have been critiquing Keynes using ideas on probability that had been influenced by those of von Kries. I only note this because if Wittgenstein knew of the work of von Kries, a likely conduit would have been via discussions about probability with Keynes, supporting my claim that they likely discussed

probability together.

However, it could still be doubted in two ways that Wittgenstein was responding to Keynes. Firstly, Wittgenstein may have known about Keynes' work but not have been responding to it with his remarks. Alternatively, Wittgenstein may have known Keynes was working on topics related to probability, but not known the details of his theory. Therefore, from this alone we cannot assume that Wittgenstein was writing to criticize Keynes.

The rest of this paper is dedicated to showing, through other evidence, that Wittgenstein was likely writing his remarks on probability to criticize a Keynesian theory of probability.

3 Keynes and Wittgenstein on Probability

In this section, I describe Keynes and Wittgenstein's conceptions of probability, and their similarities and differences. Keynes' theory was a logical theory of probability. His aim was to generalize logic by making partial entailment its new foundation, whilst the standard deductive logical consequence became a special limiting case. Probability is the new logical relation between two sets of propositions which expresses partial entailment. He describes probability as follows: Take e and h to be two sets of propositions. We can then state that e stands in relation with h, written as h/e. This relation is the probability relation, representing the probability of h given e. (Keynes, 1921/1978, 4-5, 12).

These probability relations, whilst being associated with (partial) entailment, are also associated with a degree of rational belief. If one set of propositions partially entails another set of propositions to some particular degree α, then, given the first set of propositions it is rational to believe the second set to an equivalent degree. Similarly, if the set of propositions e gives a rational degree of belief in h of degree α, there is a probability relation of degree α between e and h. This is then written $h/e = \alpha$. These degrees of belief are the "degree of belief which it is rational to entertain in given conditions, and not merely with the actual beliefs of particular individuals" (Keynes, 1921/1978, 4-5).

Not all probability relations are capable of numerical expression. Nor are all probabilities comparable to one another. The numerical probabilities are a special case that can be measured and compared to each other. However, the majority of probabilities cannot be measured numerically, and whilst some pairs of non-numerical probabilities can have their likelihoods compared via ordinal comparison, there exists pairs of non-numerical probabilities that cannot be compared at all (Keynes, 1921/1978, 32-37).

For Keynes, these probability relations are what probability just is. Probability always refers to a relation between a pair of sets of propositions, rather than being a property of a proposition in of itself. It is "without significance to call a proposition probable unless we specify the knowledge to which we are relating

it" (Keynes, 1921/1978, 4). Between any two sets of propositions there is only one probability relation, "any conclusion a bears to any consistent premiss h one and only one relation of probability" (Keynes, 1921/1978, 146). This relation is "objective" and has been fixed "independent of our opinion" (Keynes, 1921/1978, 4). However, the same conclusion can have different probability relations with different sets of premises (Keynes, 1921/1978, 4).

It is not possible to define probability and the probability relation further, and it cannot be decomposed into simpler notions. Instead, the probability relation forms "a new logical relation" which "cannot be explained or defined in terms of our previous notions." (Keynes, 1921/1978, 8) such as the other indefinables of formal logic. Keynes gives two reasons for this indefinability. The first is because of a "failure to find a definition" in past attempts, and the second is "because the notion presents itself to the mind as something new and independent." (Keynes, 1921/1978, 8). This highlights another component of Keynes' theory: that the probability relations exist independently from us.

Keynes claims that "we possess some power of direct inspection" that allows us to come to know these probability relations. Keynes similarly considered other logical relations to be known in the same way. He made this clear in his original doctoral dissertation, stating "relations of probability are things that can be directly perceived, just as many other logical relations are by general admission objects of intuition ..." (O'Donnell, 1992, 81). This view continues into the *Treatise* (Keynes, 1921/1978, 56-57).

However, "although we cannot exclude every element of direct judgment", there are general logical rules and principles which can help us to recognize the probability relations which may be more difficult to recognize by intuition alone. Whilst we may not be able to immediately intuit a specific relation, we may be able to put it into a form that we can intuit through the use of these rules. He claims that "The object of a logical system of probability is to enable us to know the relations, which cannot be easily perceived, by means of other relations which we can recognise more distinctly (Keynes, 1921/1978, 57).

An example of such a rule for Keynes is the Principle of Indifference (PIE). In its original form, PIE (or principle of insufficient reason), stated that "equal probabilities must be assigned to each of several arguments, if there is an absence of positive ground for assigning unequal ones" (Keynes, 1921/1978, 45). For example, if there is no known reason for saying a book is red instead of blue then, relative to this knowledge, equal probability should be assigned to each possibility.

However, this leads to contradictions. For example, assume there is a book but you have no knowledge at all about the color of that book. By PIE you should assign probability $\frac{1}{2}$ to both the proposition "The book is red" and "The book is not red". However, you should similarly assign probability $\frac{1}{2}$ to the proposition "The book is blue" and probability $\frac{1}{2}$ to the proposition "The book is not blue" and so on.

Despite the contradictions, Keynes believed that PIE was an important principle for discovering prob-

ability relations, particularly for perceiving numerical probabilities. Keynes claims that "In order that numerical measurement may be possible, we must be given a number of equally probable alternatives" (Keynes, 1921/1978, 44), a judgment made through PIE. Therefore, he attempts to provide logical criteria for when it could be used to avoid these contradictions (Keynes, 1921/1978, 44-70). After setting out these criteria, he allows PIE as a logical rule that we can use to know when a pair of probabilities are equal or unequal, instead of by just intuiting them directly. However, the judgement of when PIE can and cannot be used must itself be made through intuition (Keynes, 1921/1978, 57-58).

Keynes presented his view of logical probability in opposition to the frequentist theory of probability, such as Moore's naïve account in the *Principia Ethica*, and Venn's more substantial account. Frequentists defined probability in reference to events that occur repeatedly, with probability the probability of an event being the long run frequency of the repeated event, rather than relations between propositions.

I now introduce Wittgenstein's views on probability occurring in propositions 4.464, 5.1 and 5.15- 5.156 of the *Tractatus*. He later briefly discussed it in *Philosophical Remarks* (1975), as well as in discussions with Friedrich Waismann and other members of the Vienna Circle. However, I only focus on the work published in the *Tractatus*, given the others are later than the time period being focused on. The basic elements of Wittgenstein's definition are expressed between 5.15-5.153:

> 5.15: If T_r is the number of the truth-grounds of a proposition 'r', and if T_{rs} is the number of the truth-grounds of a proposition 's' that are at the same time truth-grounds of 'r', then we call the ratio $T_{rs} : T_r$ the degree of probability that the proposition 'r' gives to the proposition 's'. . . .
>
> 5.1511: There is no special object peculiar to probability propositions.
>
> 5.152: When propositions have no truth-arguments in common with one another, we call them independent of one another. Two elementary propositions give one another the probability 1/2. If p follows from q, then the proposition 'q' gives to the proposition 'p' the probability 1. The certainty of logical inference is a limiting case of probability. (Application of this to tautology and contradiction.)
>
> 5.153: In itself, a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way.

For example, if a proposition P is true in 7 rows of a truth table, and in those 7 rows proposition Q is also true in 3 of them, then we can see we get a ratio $T_{PQ} : T_P = 3 : 7$, so the probability of Q given P is 3/7.

There are similarities between Keynes' and Wittgenstein's accounts. Both accounts are logical theories of probability, with Wittgenstein's expressing a logical relation between the truth grounds of two proposi-

tions. Wittgenstein's, like Keynes' is also an objective probability. For Wittgenstein, the relation is entirely and uniquely determined by the truth-grounds of the propositions involved. Finally, both take conditional probabilities to be the fundamental notion of probability, with each claiming that we could not talk about how probable a certain proposition is in isolation. However, it is possible to effectively define nonconditional probabilities in Wittgenstein's system by taking the conditional probability of a proposition given a tautology (Von Wright, 1969).

Nonetheless, despite both being objective logical theories of probability, they differ in significant ways. The biggest difference is Wittgenstein's claim that "There is no special object peculiar to probability propositions." (5.1511). Unlike Keynes, Wittgenstein does not need to introduce a new logical object, external to the propositions, for the probability relation (nor does he need to for any logical relation). For Wittgenstein, the truth grounds of a proposition are the sense of that proposition. The probability relation then simply follows from the senses of the propositions involved, rather than requiring some new object.

Wittgenstein also does not require the use of what Keynes claims are intuited rules, such as the controversial Principle of Indifference, for finding numerical probabilities. Keynes required the determination of equipossibility through intuition and the use of PIE, separate from the propositions involved, in order to assign numerical probabilities. Instead, for Wittgenstein, numerical probabilities can be assigned by complete analysis of the propositions into their elementary propositions. Wittgenstein explains that any two elementary propositions give each other probability 1/2 (5.152) and every proposition has a unique final analysis which reveals it to be a truth-function of elementary propositions (e.g. 3.25, 4.221, 4.51, 5). From this, further probability values to be calculated, and it plays the same role as equipossibility through PIE for Keynes in determining unique numerical probabilities. This allows for judgments of numerical probability values solely through what is included in the propositions themselves – by complete analysis of the propositions into their elementary propositions – without requiring the application of a separate intuited logical rule. Additionally, following as a result of these elementary propositions, for Wittgenstein all probabilities are numerical, and this means all probabilities are comparable.

Although Wittgenstein removed the Principle of Indifference, it does not solve all the issues of numerical probabilities for logical probabilities. Wittgenstein is not able to say what the elementary propositions are or how to determine them, and without knowing what they are we cannot reach these unique final analyses that allow the determination of numerical probabilities. As Black (1964, 254) notes, it may be possible in the ideal language of the *Tractatus* to perform the complete analysis required, but "applied to any language actually available, it would compel us to treat all probability measures as unknown."

4 Platonic Atomism

I now introduce Platonic Atomism, and Moore, Russell and Wittgenstein's views on inference, implication and logical consequence. I demonstrate how Wittgenstein's views were written, in part, as a criticism of Platonic Atomism. I do this so that I can show both that Keynes was a Platonic Atomist, adhering to the commitments of Moore and Russell, and that Wittgenstein's criticisms of a Platonic Atomist conception of logical relations, can also be applied to Keynes' Platonic Atomist view of probability, to show that Wittgenstein intended to do that.

Platonic Atomism is a name given by Peter Hylton (1990) to the philosophical commitments that G.E. Moore and Bertrand Russell subscribed to in the late 1890's and early 1900's, in opposition to the idealism of T.H. Green and F.H. Bradley. I focus on the Platonic Atomist period of Russell and Moore because that was the most influential on Keynes himself. It is also the period of the *Principles of Mathematics* (Russell, 1903/2010), where the relevant ideas being criticized by Wittgenstein occur. However, many of the key commitments continue, in sometimes slightly altered form, in Moore and Russell's thought after this period.

I explain the commitments which are relevant for understanding the views of Moore and Russell on logical relations. These are that the objects of thought are objective and mind-independent entities, that some objects of thought are indefinable, and that these indefinable objects of thought are known through intuition. Some of these features can be seen in Russell's preface to the *Principles of Mathematics*, though it specifically provides Russell's understanding of Moore's Platonic Atomism project, rather than Moore's understanding of his own work:

> "On fundamental questions of philosophy, my position, in all its chief features, is derived from Mr G. E. Moore. I have accepted from him the non-existential nature of propositions (except such as happen to assert existence) and their independence of any knowing mind; also the pluralism which regards the world, both that of existents and that of entities, as composed of an infinite number of mutually independent entities, with relations which are ultimate, and not reducible to adjectives of their terms or of the whole which those compose."(Russell, 1903/2010, xviii)

I first discuss the idea of the independence of objects of thought from any knowing mind, which is part of the anti-psychologism of the Platonic Atomist view. Moore provided this position in "The Nature of Judgement" (1899), where he gave an argument for the claim that the nature of the objects of thought is independent of the mind, concluding that "The concept is not a mental fact, nor any part of a mental fact." (Moore, 1899, 179).

A concept, for Moore, is a "possible object of thought" (Moore, 1899, 79) that is mind-independent. Propositions are composed of any number of concepts, along with a relation between them. These proposi-

tions, as well as the relations, are also possible objects of thought and are also mind-independent. After 1899, Moore drops the use of the word concept, switching to using words like "idea", "object", "object of thought" and "notion" (all used to describe goodness in *Principia Ethica* (1903/1993)). He made this change at the same time as distinguishing universals from particulars. Whilst not all of Moore's views of concepts survive this change, for my purposes these differences do not matter. In both cases they are mind-independent possible objects of thought that make up propositions.

Both Moore and Russell take every possible object of thought to have being, i.e. they are real. This includes relations, so relations are mind independent and real. However not every possible object of thought has existence. This is stated by Russell in the *Principles of Mathematics*:

> "Being is that which belongs to every conceivable term, to every possible object of thought—in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves. Being belongs to whatever can be counted. . . . Numbers, the Homeric gods, relations, chimeras and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them." (Russell, 1903/2010, 455)

Next, I discuss Moore and Russell's use of indefinable objects, introduced in Moore's "The Nature of Judgement" (1899), when discussing propositions. A proposition consists of any number of concepts, along with a relation between these concepts. It is as a result of this relation that the proposition may be true, or it may be false. However, exactly what kind of relation makes a proposition true or false is indefinable.

> "A proposition is constituted by any number of concepts, together with a specific relation between them; and according to the nature of this relation the proposition may be either true or false. What kind of relation makes a proposition true, what false, cannot be further defined, but must be immediately recognised." (Moore, 1899, 180)

For Moore, definitions "are only possible when the object or notion is something complex" (Moore, 1903/1993, 7), i.e. when the objects or notions can be reduced to simpler terms. For example, when we define horse we "may mean that a certain object, which all of us know, is composed in a certain manner: that it has four legs, a head, a heart, a liver, etc., etc., all of them arranged in definite relations to one another" (Moore, 1903/1993, 8). Then, we "might think just as clearly and correctly about a horse, if we thought of all its parts and their arrangement instead of thinking of the whole" (Moore, 1903/1993, 8).

In contrast, "indefinable" objects cannot be decomposed into simpler terms. An understanding of these indefinable terms is needed, because without it we could not understand any definable term. Russell takes a similar approach to definitions and indefinables. In "The Axioms of Geometry" he states "Unless, then,

some terms can be understood without a definition, no term can be understood by the help of a definition. . . . The meaning of the fundamental terms cannot be given, but can only be suggested." (Russell, 1899, 411-412).

For Russell, the discussion of indefinables "forms the chief part of philosophical logic" (Russell, 1903/2010, xliii). He introduces logical constants, which are indefinable. There are only a small number of these logical constants, suggested by Russell to be eight or nine (Russell, 1903/2010, 11). These logical constants are "the particular notions which appear in the propositions of symbolic logic, and all others definable in terms of these notions" (Russell, 1903/2010, 11).

The indefinables come to be known by intuition or acquaintance. However, philosophers can help other people to intuit indefinables. According to Russell, one of the aims of philosophical logic is to "make others see clearly, the entities concerned, in order that the mind may have that kind of acquaintance with them which it has with redness or the taste of a pineapple" (Russell, 1903/2010, xliii). However, if this does not help, and somebody still cannot intuit these entities "there is nothing to be done" (Russell, 1899,412).

These notions provide context for Moore and Russell's views on inference and logical consequence, the relation which holds between p and q when the former follows from the latter, as well as the related concept of implication. Moore discusses his views in "The Nature of Judgement" (Moore, 1899), where he recognized that "it could not be maintained that the conclusion was only connected with the premisses in my thoughts, and that an inference was nothing, if nobody was making it" (Moore, 1899, 183). Instead, Moore claims that "the relation of premises to conclusion is an objective relation" (Moore, 1899, 183). So for Moore, logical consequence is an objective relation that exists independently of our minds.

Russell also discusses inference, and the related concept of implication, in *The Principles of Mathematics*. In this, Russell clarifies the difference between formal and material implication. Formal implication involves propositions of the general type "ϕ(x) implies ψ(x) for all values of x' where ϕ(x), ψ(x), for all values of x, are propositions." (Russell, 1903/2010, 11). In contrast, material implication is the relation "between propositions not containing variables" (Russell, 1903/2010, 11) and is typically just called implication by Russell. Material implication is the relation through which "it is possible for us validly to infer" (Russell, 1903/2010, 34).

Both formal and material implication are taken to be indefinable logical constants. For example, material implication is indefinable for Russell because:

> If p implies q, then if p is true q is true, i.e. p's truth implies q's truth; also if q is false p is false, i.e. q's falsehood implies p's falsehood. Thus truth and falsehood give us merely new implications, not a definition of implication. (Russell, 1903/2010, 14)

There are also rules of inference. These are the rules according to which the inference proceeds. To prevent an infinite regress, where a rule of inference is just another statement within the theory, meaning another rule of inference is required to show that the conclusion follows from the premisses together with this statement; and the new rule is in turn subject to just the same argument, Russell again appeals to intuition (Hylton, 2005 85). For Russell, we know these rules through intuition, and in the end the validity of an inference is not a matter of rules and instead:

> "it remains the case that the fact that our rule does imply the said implication, if introduced at all, must be simply perceived, and is not guaranteed by any formal deduction". (Russell, 1903/2010, 42)

This view continues into the *Principia Mathematica*, where the rules of inference are again simply included in the primitive propositions of logic (Whitehead and Russell, 1910/2019, 94, 98, 106).

Of course, Russell's material and formal implications are not what is typically considered as logical consequence, something Russell recognizes in "Necessity and Possibility" (1905/1994b, 514-515). Any true proposition would be a logical consequence of any false proposition if it was. However, as will be seen, Wittgenstein explicitly criticizes Russell's views on inference, and Russell does consider implication to be the relation through which "it is possible for us validly to infer" (Russell, 1903/2010, 34). Additionally, in "Necessity and Possibility" logical consequence is a case of material implication where the inference has proceeded via certain specific rules. Whilst this makes logical consequence definable, it is still a case of an indefinable material implication, but where the intuited general rules of inference have been replaced by a number of intuited specific rules. Therefore, there is justification for considering Russell's views on implication rather than what is typically considered logical consequence in this treatment, and my comparisons with Keynes' probability relation will still apply to both.

In summary, Moore considers logical consequence to be an objective relation that exists independently of our minds (Moore, 2019, 149). Russell, instead of considering logical consequence, introduces the related notion of material implication as an indefinable, objective, relation by which inferences occur. This relation can either simply be perceived, or we can perceive certain rules of inference which imply the implication.

Wittgenstein describes his account of logical consequence and inference between propositions 5.1-5.143. Wittgenstein takes the sense of a proposition to be the truth grounds of that proposition. The truth grounds of a proposition are the combination of truth values of its elementary components which makes the proposition come out as true. Wittgenstein uses this to define logical relations, such as logical consequence. The truth of one proposition P is a logical consequence of another proposition Q if all the truth grounds of Q are also truth grounds of P (5.12). The truth grounds of one are contained in the other.

These remarks by Wittgenstein on logical consequence and the nature of inference directly target Russell's (and Frege's) views. For example, in proposition 5.132 Wittgenstein states:

> "If p follows from q, I can make an inference from q to p, deduce p from q. The nature of the inference can be gathered only from the two propositions. They themselves are the only possible justification of the inference. 'Laws of inference', which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous."

What Wittgenstein means by this paragraph has been subject to debate, for example by Ricketts (1985), Proops (2002). At the very least it appears as if Wittgenstein is targeting Russell and Frege separately. For the former, he is criticizing how Russell justifies inference, and it appears it is because Russell allows for something beyond what is included in the premises and conclusion of the inference to justify the inference. One reason for this is because Russell allows for the rules of inference to "imply the said implication", and how they do this is something that must be "simply perceived." So rules of inference can justify inferences. For Wittgenstein, such rules would not be needed, as everything needed to justify the inference is already included in the propositions.

An alternative view, from Ricketts (1985), is that Wittgenstein is targeting Russell's views on the rules of inference as justifying inference only by serving in the premises of an argument, and Wittgenstein is criticizing this through an appeal to Lewis Carroll's regress argument. However, Russell explicitly claims in the *Principles of Mathematics* that inference rules are not required as premises, they are intuited principles that help to make inferences. Given Wittgenstein had read the *Principles of Mathematics* (5.5351), it seems likely that he would know this, making this view unlikely.

This criticism of Russell is related to the debate over internal and external relations. Wittgenstein defines an internal relation as:

> "A property is internal if it is unthinkable that its object should not possess it (This shade of blue and that one stand, eo ipso, in the internal relation of lighter to darker. It is unthinkable that these two objects should not stand in this relation.) (Here the shifting use of the word 'object' corresponds to the shifting use of the words 'property' and 'relation'.)." (4.123)

For Wittgenstein, the logical consequence relation is an internal relation. Logical consequence is a result of the sense of the propositions involved and it is impossible to think of that proposition without that sense. Wittgenstein also explicitly states that it is internal, stating:

> "If the truth of one proposition follows from the truth of others, this finds expression in relations in which the forms of the propositions stand to one another: nor is it necessary for us to set up

these relations between them, by combining them with one another in a single proposition; on the contrary, the relations are internal, and their existence is an immediate result of the existence of the propositions." (5.131)

In contrast, whether Russell (and Moore) actually considered implication and logical consequence to be external relations is up for debate. At times they seem to claim so, for example Russell stated:

> "Mr. Bradley has argued much and hotly against the view that relations are ever purely "external". I am not certain whether I understand what he means by this expression, but I think I should be retaining his phraseology if I described my view as the view that all relations are external." (Russell, 1899, 143)

And as late as 1905 Russell mentions the problems inherent in "the view that relations are not purely external." (1095/1994a). However, it is never explicitly discussed in the context of inference, implication, and logical consequence. Russell is also not completely clear what the internal relations of Bradley that he is responding to are, nor what he himself means by the relation being external. Finally, attempts to make logical consequence an external relation in a way that Moore and Russell would support causes problems (Hylton, 1990, 149).

Nonetheless, it appears that Russell takes all relations, including logical ones, to have being themselves, separate in some way from the propositions that the relation is between. The propositions are placed in relation to each other by combining them with one another using the relation, so they would not be internal for Wittgenstein, unlike his own view that the relation simply follows from the senses of the two propositions and do not need to be placed in relation to each other.

5 Keynes the Platonic Atomist

In this section I demonstrate the influence of Moore and Russell on Keynes' views, and show Keynes adhered to the views of Platonic Atomism. When Keynes arrived in Cambridge in 1902, he predominantly came under the influence of Moore and Russell. He attended Moore's lectures on ethics and joined The Apostles society, to which Russell and Moore also belonged. As Keynes himself noted, *A Treatise on Probability* was written under the influence of the works of Russell and Moore. For example Keynes claims he has "been much influenced by W. E. Johnson, G. E. Moore, and Bertrand Russell, that is to say by Cambridge" (Keynes, 1921/1978, Preface)[1].

[1] Also see Keynes, 1921/1978, 125

Further in the *Treatise*, when writing about the possibility of investigating the "nature and reality of objects of perception" (Keynes, 1921/1978, 266), Keynes also recommends Moore's paper on that topic as being similar to his own view.

When reflecting on the work in "My Early Beliefs" Keynes again notes the influence of both Russell and Moore. He claimed that his philosophical work occurred "under the influence of Moore's method" (Keynes, 1978, 440), and that he was "writing under the joint influence of Moore's *Principia Ethica* and Russell's *Principia Mathematica*." (Keynes, 1978, 445). However, it would be more accurate if Keynes had acknowledged Russell's 1903 *Principles of Mathematics*. Most of Keynes' philosophical work occurred prior to the publication of the *Principia Mathematica*, and Keynes adheres to certain beliefs expressed in the *Principles of Mathematics* which Russell later gave up on (O'Donnell, 1992, 47).

There is recognition going the other way too. Russell thanked Keynes in *Problems of Philosophy* (1912/2001) for his help with ideas on probability and induction. Russell also stated that the *Treatise* is "undoubtedly the most important work on probability that has appeared for a very long time", (Russell, 1922, 119) in his book review.

Keynes did criticize some views of Russell and Moore, for example Moore's use of probability in the *Principia Ethica*. He also suggested that Russell's logic, whilst possessing "beauty, interdependence, and completeness"(Keynes, 1921/1978, 128), was not suitable for capturing the connection between logic and ordinary reasoning. However, Keynes was not critical of the general claims of Platonic Atomism, and saw himself as correcting those issues of Moore and Russell within a Platonic Atomist framework.

This can be seen by the fact that a Keynesian theory of probability does adhere to the tenets Platonic Atomism, which I now demonstrate. I focus on the same attributes that I did in Section 4. The probability relation is a relation separate to the propositions involved, it is an objective and mind-independent entity, it is indefinable, and is known to us through intuition.

For Keynes, the probability relation is separate to the propositions involved. There exists two propositions (or sets of propositions), and then there exists a new relation between them, the probability relation, which is not based in the propositions themselves. These probability relations are objective and mind-independent entities. There is one single probability relation between two propositions. This relation "is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion." (Keynes, 1921/1978, 4).

Keynes claims that this probability relation is indefinable using the same reasoning as Moore and Russell. Keynes claims to define the probability relation, he would need to be able to reduce the notion to simpler terms. However, he claims that "we cannot analyse the probability-relation in terms of simpler ideas" so it

"cannot be explained or defined in terms of our previous notions" such as other logical relations or what is included in the propositions, it is something new. Therefore, he claims that "a definition of probability is not possible" (Keynes, 1921/1978, 8).

Finally, these probability relations are known to us through intuition, or direct judgment. Keynes states that "the failure to explain or define 'probability' in terms of other logical notions, creates a presumption that particular relations of probability must be, in the first instance, directly recognised" (Keynes, 1921/1978, 56) and that "there is little likelihood of our discovering a method of recognising particular probabilities, without any assistance whatever from intuition or direct judgment" (Keynes, 1921/1978, 56).

Similarly to how Russell allows for inferences to be known in two ways, through immediate intuition or through certain rules of inference, Keynes does the same for probability relations. A probability relation can be known immediately through intuition, but there are also certain logical rules through which we can know it, such as the Principle of Indifference. However, like Russell, to know that applying this logical rule is actually correct, we must have made a number of intuitions. (Keynes, 1921/1978, 57-58).

I can now define the key features of a Keynesian theory of probability. This is a theory of probability with the properties 1. probability is a logical notion, and the probability relation is a logical relation, 2. Probability relations are objective, 3. Conditional probabilities are the primitive kinds of probabilities and 4. The probability relation is more than simply an excerpt ("Auszug") (5.156) from the propositions, they are something additional to the propositions involved. Finally, these relations have the other properties Platonic Atomism attributes to them. Obviously the theory put forth by Keynes in the *Treatise* is such a theory, however, it could also just refer to any collection of these ideas together.

6 Keynes as a Target of Wittgenstein's Remarks

Having demonstrated that a Keynesian theory of probability follows the commitments of Platonic Atomism, I demonstrate that one of Wittgenstein's targets for criticism in his remarks on probability was a Keynesian theory of probability. I first establish that a target of Wittgenstein's remarks was some logical view of probability. Firstly, it could be argued that Wittgenstein was not responding to any previous view in particular, he was just deducing answers from the broader Tractarian framework and probability was one of those. However, this seems incorrect, as Wittgenstein appears aware of issues in particularly logical probability related to the Principle of Indifference and numerical probabilities enough to develop an alternative solution by using a uniform distribution on partitions of elementary propositions.

Secondly, 5.1511 states "There is no special object peculiar to probability propositions", where a probability proposition is a proposition along the lines of 'Proposition A gives probability m/n to Proposition B'.

Although Wittgenstein himself does not consider probability relations to be objects in a Tractarian sense, he appears to be implying that there is an existing view of probability which does posit a special object for the probability relation. He is making clear that his view does not require this, just as it does not require special objects for other logical relations (4.441). Instead, probability is an internal relation between the two propositions. It follows from, and is between, the senses of the propositions (their truth grounds) and it is not "necessary for us to set up these relations between them, by combining them with one another in a single proposition; on the contrary, [...] their existence is an immediate result of the existence of the propositions" (5.131). This is the interpretation typically given, for example by Black (1966, 248), McGuinness (1982, 159-160), Fogelin (1987, 51), and Frascolla (2010, 201).

It is typically logical theories of probability which regard conditional probabilities as primitive, positing a new relation between propositions to do this. This is in contrast to other interpretations of probability which regard probabilities of single events as primitive. It is therefore likely that at least one of Wittgenstein's targets for criticism was some logical theory of probability which does posit a new object for the relation between propositions.

The use of "object" in this proposition provides stronger evidence that the logical theory of probability being criticized is a Keynesian theory of probability. As made clear in 5.4, the immediate targets for remarks criticizing "logical objects" and "logical constants" are Frege and Russell. Russell conceived of his logical constants as objects, which Keynes built on to similarly conceive of his probability relation. Though Keynes does not himself use the term logical constant, he is positing a new logical constant in the same way as Russell did. Given that logical constants as so conceived by Russell are one of the targets of the Wittgenstein's other criticisms of "logical objects" and "logical constants", it seems likely that the target is similar in 5.1511. This would likely indicate that a Keynesian theory of probability is the target for criticism, as that is an interpretation which does take this view.

Furthermore, I provide additional evidence for the claim that he was targeting a Keynesian theory of probability. As seen previously, between propositions 5.1-5.143 Wittgenstein provides an account of logical consequence and inference. Proposition 5.1 is also the first significant mention of probability by Wittgenstein in the *Tractatus*, where he states "Truth-functions can be arranged in series. That is the foundation of the theory of probability." (5.1). This suggests a close connection between Wittgenstein's remarks on logical consequence and inference and those that were then made on probability, as they both begin with the same proposition. Therefore, the account of probability should be interpreted together with, and in light of, the propositions on logical consequence and inference to be fully understood.

As seen, Wittgenstein's comments on logical consequence and inference were, in part, targeting an account provided by Russell during his Platonic Atomist period. Russell proposed implication to be a

relation which had being separate from the propositions that it was between, and that implication was the relation through which it is possible to validly infer. Russell also allowed for something beyond what is included in the premises and conclusion of the inference to justify the inference. These are the rules of inference. Russell allows for the rules of inference to "imply the said implication", and how it does this is something that must be "simply perceived." Wittgenstein disagreed with this. He did not consider it to be a relation which had being separate from the propositions that it was between. Nor did he believe that its justification should depend on something beyond what is included in the premises and conclusion of the inference. Instead, it simply depends on the senses of the premises and conclusion themselves.

It is immediately following these comments that Wittgenstein provides his account of probability. As with logical consequence and inference, Wittgenstein attempts to provide an account where everything needed to justify probability is included in the propositions involved. He does this the same way, where the only thing needed is the senses of those propositions. He similarly attempts to prevent the need for logical rules which would provide justification for probability through the use of something beyond what is included in the propositions involved, such as intuition of their application. For example, he replaces the need for intuited judgements of equipossiblity for providing numerical probabilities, by allowing for objective judgments solely through what is included in the propositions themselves – by complete decomposition into their elementary propositions (5.152). Given the close connection between the remarks on logical consequence and inference, and those on probability, and that he was targeting for criticism Russell's Platonic Atomist view in the former, it seems likely that he was targeting a similar view for criticism in the latter.

As previously seen, a Keynesian theory of probability is such a similar view. Keynes proposed the probability relation to be a relation separate from the propositions that it was between. He also allowed for something beyond what is included in the premises and conclusion of the probability relation to justify the probability. These are certain logical rules such as the Principle of Indifference for the judgment of equipossibility, whose application is made through intuition. Therefore, one of Wittgenstein's targets for criticism is likely to be a Keynesian theory of probability.

Wittgenstein's removal of Keynes' separate probability relation, with probability only depending on the senses of the propositions, anticipates one of Ramsey's (1926/1990) criticisms of Keynes' logical probability. Ramsey states that "there really do not seem to be any such things as the probability relations he describes" (1926/1990, 161) as he does not perceive them and it does not appear that others do either, given disagreements about probability values. Wittgenstein's system does not have separate probability relations that need perceiving, as probability is just a result of the internal properties of the propositions themselves. In fact, Ramsey goes on to praise Wittgenstein's definable view of "necessity" and "system of formal logic" (1926/1990, 165) which are based on these same internal properties, though he still does not think it is

adequate for probability "as this gives no justification for induction" (1926/1990, 193).

However, if Wittgenstein was criticizing Keynes, why did Wittgenstein not mention him by name like he did with Frege and Russell? First, it should be noted that Wittgenstein does not typically give any sources by name in the *Tractatus*, as he makes clear in the preface. Only Frege, Russell, Hertz, (4.04) and Fritz Mauthner (4.0031) mentioned by name. Therefore, Frege and Russell are unusual in being mentioned, rather than it being unusual that Keynes was not. As a result, other works on Wittgenstein's theory of probability have also made an assumption that he was working with some other view of probability in mind when writing his views in the *Tractatus*, either to criticize it or build on it. As previously noted, this is often taken to be von Kries (von Wright, 1969; Heidelberger, 2001).

Secondly, it could be that Wittgenstein was not just responding to Keynes, but instead was responding to the general ideas on probability that were around Cambridge during this period. Keynes was also in discussions with others, such as Russell, Johnson and CD Broad, who shared and helped build on these same ideas. This is why I say Wittgenstein was responding to a Keynesian theory of probability, rather than just saying Keynes' theory of probability.

For example, Wittgenstein could also be critiquing W.E. Johnson. Johnson also had philosophical discussions with Wittgenstein in Cambridge at the same time as Keynes, and was also working on problems related to probability. Nonetheless, this is not is at odds with my claim that Wittgenstein was critiquing a Keynesian account of probability. Those taking Keynesian accounts of probability, such a Keynes and Broad, as well as those studying it, typically consider Johnson to have also been part of it, as well as a major influence on the other people working in it. Braithwaite, for example, refers to the tradition as the "Johnson-Keynes theory" (1925, 484).

The problem with stating definitively that Johnson was being responded to, is that Johnson rarely published and we have little other evidence for his views. Though Johnson, like Keynes, posits a logical, objective, probability relation separate to the propositions and relative to the evidence given, Johnson does not go as in depth about the philosophical nature of this probability relation. His little published work on the topic mostly focused on mathematical aspects of probability, or brief remarks on probability in early volumes of his *Logic* (1921), and he did not finish the probability focused 4th volume before he died. There is also little evidence about when he produced which of his views on probability. The most we have is others, such as Keynes and Broad, indicating that Johnson had been working on probability in the decades prior to the publication of Logic. So it's not clear what Wittgenstein would have been aware of, unlike with Keynes whose views are made clear in his 1908 dissertation.

Finally, whilst I have shown that Wittgenstein was very likely targeting a Keynesian theory of probability, this does not mean that Wittgenstein was not targeting multiple views at the same time. As seen with

proposition 5.132, there are times when Wittgenstein is targeting multiple different views for criticism with his remarks (in that case Frege and Russell) and sometimes in different ways in each case. The same could be true for probability.

A plausible second target for Wittgenstein's criticism is a frequentist theory of probability. It seems plausible that Wittgenstein would know of such a theory, as there are multiple ways he could have been introduced to it. He could have heard of it from Moore, Russell, Johnson or Keynes, who were familiar – for example Keynes' logical interpretation of probability was explicitly presented in opposition to the frequentist view. Wittgenstein may also have been introduced to it by his college tutor James Glaisher, who had assisted Venn with his frequentist *The Logic of Chance*. Whilst the role of college tutor was not academic and did not require intellectual discussions, and no correspondence between Wittgenstein or Glaisher nor any record of their conversations is known to survive, they may have had conversations about it.

It was typical for those presenting new interpretations of probability during this time period at Cambridge to challenge the frequentist view, where it was seen as the orthodox view to be critiqued. This is what Keynes did, whilst Ramsey, in "Truth and Probability" (1926/1990), also first critiqued the frequentist interpretation, before critiquing Keynes' logical interpretation and finally providing his own interpretation. Therefore, it could be the case that Wittgenstein was doing the same.

A second reason is Wittgenstein's claim that propositions cannot be probable in of itself (5.153). This cannot be written in opposition to Keynes, as Keynes agreed that in itself a proposition is neither probable nor improbable, it must always be in a relation with another proposition. Whilst it may be the case this was simply an independent thought of Wittgenstein, it could be something he wrote explicitly to criticize empirical views of probability, such as the frequentist view, which do take events to have probability in of themselves.

Therefore, it may be the case that Wittgenstein presented a logical theory of probability in opposition to the frequentist theory. The specific logical theory of probability that Wittgenstein then presented was provided in opposition to the one that Keynes provided for the reasons that I have previously established. However, there is much less evidence for the possibility that Wittgenstein was criticizing the frequentist interpretation, with no record existing of his interactions with that view, so it remains speculation

7 Conclusion

In this paper I have provided context to the remarks on probability made by Wittgenstein in the *Tractatus*, to help explain why he wrote these remarks. I have claimed that these remarks were made, at least partially, in response to, and to criticize, a Keynesian theory of probability. Not only did Wittgenstein very likely

know of Keynes' views on probability, but also Wittgenstein's remarks on probability are an extension of his critiques of a Platonic Atomist account of inference in logic similar to Russell's. Keynes' theory of probability was seen by both its author as well as others as an attempt to extend such a Platonic Atomist account of inference to include logical probability. This then makes it likely that Wittgenstein's account of probability is an attempt to critique a Platonic Atomist view of probability, such as Keynes', in the same way.

This responds to White's (2006) claim that "it is difficult to see why Wittgenstein devotes so much space to what must count as a relative side issue." Wittgenstein devoted so much space to this issue because he started developing his ideas in Cambridge, where the foundations of probability were being worked on by Keynes with help from Russell, Moore and Johnson, all of whom where philosophical interlocutors with Wittgenstein. He was taking part in this Cambridge debate and thought he had the answers to the problems that were being discussed. This then explains Wittgenstein's comment to Keynes that his remarks on probability "solve the essential question." (Wittgenstein and McGuinness, 2012).

Following the publication of both books, there is little evidence of further discussion on probability between Keynes and Wittgenstein, with Keynes moving completely into economics research. However, Keynes read the *Tractatus*, from a letter that Keynes sent Wittgenstein alongside a copy of *A Treatise on Probability*:

> "The reason was that I wanted to try to understand your book thoroughly before writing to you; yet my mind is now so far from fundamental questions that it is impossible for me to get clear about such matters. I still do not know what to say about your book, except that I feel certain that it is a work of extraordinary importance and genius. Right or wrong, it dominates all fundamental discussions at Cambridge since it was written.
>
> I have sent you in a separate package copies of the various books which I have written since the war. Probability is the completion of what I was doing before the war, I fear you will not like it. (29 March 1924, Keynes to Wittgenstein, Wittgenstein and McGuiness, 2012)

Though he mentions reading it, there is no evidence of Keynes' further thoughts on the *Tractatus*, and likewise we have no evidence for Wittgenstein's further thoughts on *A Treatise on Probability*. Keynes does recognize that Wittgenstein may not have liked it. This may just be Keynes recognizing the differences between their views on probability. However, if my thesis is correct, it may indicate Keynes recognizing that Wittgenstein was explicitly targeting his views on probability.

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