Mathematical SETIbacks: Open Texture in Mathematics as a new challenge for Messaging Extra-Terrestrial Intelligence

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Abstract:

Beyond the obvious technical difficulties, human attempts to communicate with hypothetical Extra-Terrestrial Intelligences also present a number of philosophical puzzles. After all, an alien intelligence is likely the closest thing to a Wittgensteinian lion humanity could ever encounter. In this paper I advance a new challenge for the feasibility of communication with extra-terrestrials. The problem I raise is a practical problem that falls out of the history and philosophy of mathematics and the implementation of METI projects – specifically, the semiprime self-decryption schema of the Drake Pictures message strategy. The Drake Pictures strategy presumes that aliens share the concept 'prime number' with us, as understanding that concept is necessary to decrypt our message. However, if the concept 'prime number' exhibited open texture at any point in its history, it could have developed in a different direction than it did in our history. If that is so, an alien could have the concept 'prime number' and still not be capable of decrypting our messages. I argue that this new problem is more trenchant than previous arguments in both the philosophy and SETI literature, such as applications of the aforementioned Lion argument and other concerns about the possibility of long-range communication without the assumption of shared concepts.

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In the 2007 Doctor Who episode '42', the show's titular protagonist can only open a locked door on a crashing spaceship by successfully inputting a sequence of happy prime numbers. This rather implausible plot point is explained away within the story by the (human) crew of the ship setting the keys to their doors as the answers to trivia questions that only they have any reason to know. Thankfully, our hero just happens to know enough about human number theory to type in the correct sequence and open the door, and the day is eventually saved (Harper 2007).

More remarkable than this middling episode of family-friendly science fiction television is the fact that it closely mimics how attempts at communication with aliens are expected to proceed by the humans who attempt them. One of the most sophisticated and serious methods that humans have so far devised for making first contact with a potential universe of other species boils down to the same kind of number-theoretic guessing game. The use of mathematical trivia puzzles as keys to our radio transmissions into space is not an accident, and the technique of using them was developed as a response to real technical problems. However, just like the somewhat underwhelming contrivance that Doctor Who (also an alien) simply knew the answer to the human mathematical puzzles necessary to unlock the doors of the crashing spaceship, the technique of locking our messages to the stars behind specific mathematical conceptual barriers presumes that the intelligent alien life we hope to find on the other end has similar mathematical concepts to our own.

The problem of how to make our signals understood by a hypothetical alien interlocutor is not merely a technical one. Philosophers have been interested in the problems of alien conceptual structures since at least the time that the pre-critical Kant looked up at the starry skies above him (Kant 1981). However, the problem did not become practical until the first radio telescopes of the 20th century made it possible to send and receive messages from the stars. New philosophical considerations arising from these technical challenges have not yet received as much attention as they merit from the philosophical literature[1]. In this paper, I will present a new argument against the feasibility of making our signals understood by an extraterrestrial other. The new problem I present, the Problem from Open Texture in Mathematics, is a technological and practical problem that falls out of the philosophy of mathematics and the specific technological limitations of interstellar radio signals. Though there have been many arguments against the feasibility of interstellar and interspecies communication, I will argue that the one I present here is the most trenchant, for it would remain a problem even under the most idealized circumstances of similarity to our hypothetical interlocutors, where the only difference is their alienness itself. What starts as an engineering problem – how to send a readable message to a distant world – becomes a problem in philosophy of mathematics – whether or not aliens have the concept 'prime number' – and finally, I argue, becomes a problem in metaphilosophy – what it means to have the concept 'prime number' at all, in the face of the contingent history of mathematics. This is a problem context that brings philosophy of science, technology, mathematics, and language into the same conversation.

For the purposes of this paper, I will set aside the question of whether attempting to communicate with extraterrestrials is wise, and the question of whether there are any extraterrestrials with whom to communicate. There has been plenty of debate throughout the history of SETI and the canon of science fiction about whether or not the aliens we would hypothetically contact would be a force for good or ill in our society. The twin extremes of *War of the Worlds*-style alien invasion and a *2001*-style alien apotheosis are ripe subjects for debate, and no answer to those debates will be

[1] Though much SETI writing itself has a philosophical character, like (Shklovskii and Sagan 1966) and (Sagan 1973). When philosophers write about SETI, they typically do so in the context of arguments for and against evolution and design, or the ethics of possible contact, rather than in the context of philosophy of science. There are exceptions – see (Mash 1993; Weston 1988; Cowie 2024; Cleland 2019). Philosophers have also turned their attention to the recent astronomical controversy surrounding the object known as 'Oumuamua (Cowie 2023; Ćirković 2023).

attempted here. In any event, the decision on whether to attempt to make contact must necessarily be made in the absence of an answer to the question of whether those who are contacted will be friendly. So, for the remainder of this paper, let us assume that we have made the decision to send messages to the stars and that there is, at least in principle, some friendly interlocutor there to hear them.

1. The Arecibo Stratagem

The profound problems weighed in the previous paragraph are, in truth, moot. Humanity has already attempted to make first contact with the stars. These efforts are collectively known as Active SETI (Search for Extra-Terrestrial Intelligence), CETI (Communication with Extra-Terrestrial Intelligence), or METI (Messaging Extra-Terrestrial Intelligence)[2]. The most famous METI projects are physical objects affixed to space probes, such as the plaques depicting humans on the Pioneer probes or the 'Golden Records' placed on the Voyager I and II space probes, which contain a sample of earth languages, music, and images, alongside a diagrammatic guide to playing the record. Most METI projects, however, are not physical objects, but signals broadcast in the direction of nearby stars. Some of these projects are clearly more for the benefit of humans than our galactic neighbours, like paying tribute to the 50th anniversary of NASA by broadcasting the Beatles' song 'Across the Universe' in the direction of Ursa Major or, in one notable case, advertising Doritos

[2] Though all three of these names for the practice are current in the literature, SETI and CETI are both also names of other related practices. SETI refers broadly to the attempt to detect, translate and send signals, and CETI can stand for both Communication with Extra-Terrestrial Intelligences and the Cetacean Translation Initiative, which seeks to establish translation protocols between humans and whales. Since METI is the only unambiguous acronym for the practice of sending signals, I will use it for the remainder of this paper. Practitioners recognize some subtle differences in aims for these practices (for instance, METI practitioners are motivated partially by the value for other species in receiving human messages even if they are not able to respond), but all practices still require some degree of mutual recognition between the message sender and receiver.

corn tortilla chips directly into space (University of Leicester 2008). Some, however, are designed with the explicit purpose of making real contact with real alien interlocutors. One of the earliest of these genuine attempts was the Arecibo Message, transmitted from the eponymous radio observatory in 1974. This message was designed to be a proof of concept for the use of terrestrial radio communications arrays to transmit messages to the stars, and a demonstration of how to overcome some of the more obvious difficulties in so doing (The Staff at the National Astronomy and Ionosphere Center 1975). Later in this paper I will argue that the specific structure of the Arecibo message gives rise to novel philosophical considerations, so it is worth discussing that structure in detail.

A message sent to the stars must presume as little commonality between the senders and recipients of the message as possible (how little is *necessary* will be discussed below). This requirement rules out conventional methods of radio communication. The most obvious problem is the lack of a shared language, but the more fundamental problem is the lack of a shared communications protocol to allow signals to be decoded when they are sent. Human radio communications are predicated on a set of shared conventions that transcribe the modulation of the radio wave into a coherent signal from which informational content can be deciphered. The designers of the Arecibo message had no such conventions upon which to draw, so they designed what they considered to be a message that would decode itself.

The Arecibo message is a series of 1679 binary characters, transmitted on two alternating frequencies. The binary string of characters has no obvious meaning on its own. However, if the alien receiver knows their number theory, they may notice that 1679 is no ordinary number. It is a semiprime number - the unique product of the prime numbers 23 and 73. If the message is arranged into 73 rows of 23 characters in the order in which they were received, the message forms an image:

Figure 1: The Arecibo Message, colourized. Public Domain. (SETI Institute 2023)

If instead the prime-number-savvy alien arranges the message in 23 rows of 73 characters, they would generate the nonsensical but not random-looking image:

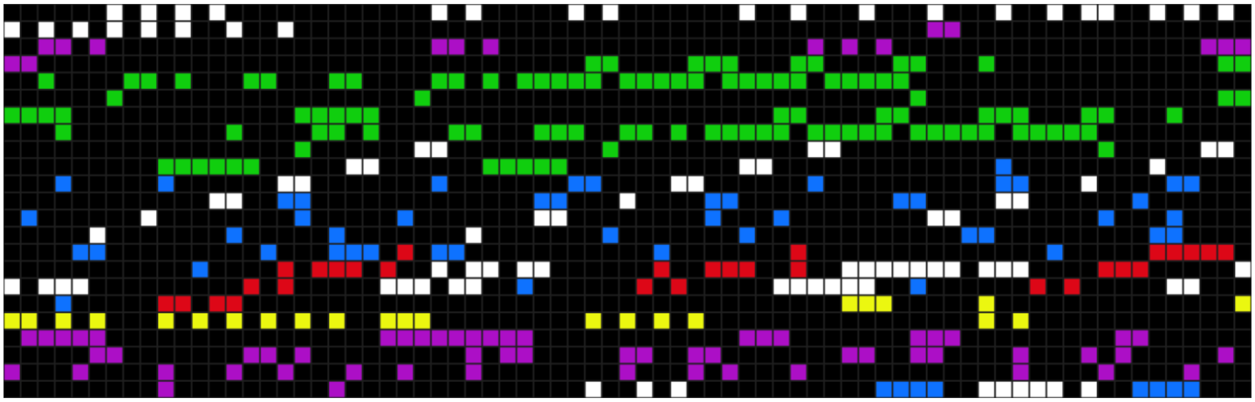

Figure 2: The Arecibo Message incorrectly decoded, colourized. Public Domain ("Arecibo Message" 2024)

And it is hoped that they would then try it the other way.

An anonymous paper authored by the Staff of the National Astronomy and Ionosphere Center – the group that designed and transmitted the message - claims that the fact "That 73 and 23 are prime numbers facilitates the discovery by any recipient that the above format [Figure 1] is the correct way to interpret the message."(The Staff at the National Astronomy and Ionosphere Center 1975) This brief explanation contains the two key elements of the Arecibo message's design: Prime factorization is putatively accessible to any potential recipient of the message, and anyone who decoded the message according to the prime factorization scheme would be left with no doubt that they had decoded it correctly. As the designers of the later Cosmic Call project[3] explain, the image format in general is also resilient to the effects of cosmic noise (Dutil and Dumas 2001). One flipped pixel in the sequence would not seriously compromise the comprehensibility of the message, so long as the recipient still recognizes the image structure of the message (though, as Carl Sagan notes, the

[3] The Cosmic Call messages sent from the Evpatoria Observatory in 1999 and 2003 used the same semiprime message structure as the Arecibo Message. They consisted of many 'pages' of 16129 characters (127x127). However, the authors note that they chose the semiprime message structure largely for aesthetic reasons, and their method also contains synchronization marks that can be used for decryption by either intuitive alien grasp of their meaning or the means of a Fourier analysis to pick up repeated message features. Discussion of the effects of the open texture argument I give below on a method of decryption that relies on Fourier Analysis is beyond the scope of this paper. See (Dumas and Dutil 2016) for documentation of the message and its contents.

Drake Pictures strategy does not have any features that shield against lost, rather than flipped, characters. This problem could be solved by simply repeating the message (Shklovskii and Sagan 1966, 424)).

To date, no response to any of humanity's attempts to reach out has been received[4]. The scale of the universe means that this fact is not especially in need of explanation – even the best possible way of communicating with the stars is very likely to fail. The universe is very empty. However, philosophers both before the sending of the Arecibo message and since have raised arguments that seem to make the likelihood of success even smaller. Let us turn to these arguments, and then add another to the sorry pile.

2. Lions In Space

There are many reasonable ways a critic could find fault with the content of both the original Arecibo message and the similarly structured Cosmic Call messages that followed. The iconography of a humanoid and radio telescope very likely would only appear as such to a being already familiar with iconic representations of humans and radio telescopes. The chemical representation of nucleotides (green in the colourized image) in the image are so light on structural details that they may not be meaningful to anyone, even other humans. And we are not merely proposing to speak to other humans.

The first, knee-jerk argument against the possibility of communication with alien species that may occur to the philosopher is an application of Ludwig Wittgenstein's famous remark in Part Two

[4] Even the so-called 'Wow! Signal' detected at the Big Ear telescope at Ohio State University in 1977, still the strongest candidate to date for an artificial signal received by earth, did not come from any location that had previously been contacted by a METI project.

of the *Philosophical Investigations* that "If a lion could talk, we could not understand him"(Wittgenstein 1968, 225).

A lion is a mammal like us with a most of the same evolutionary history. How much further, then, must a being with no such overlap be from us! In principle, we would have no reason to presume that a form of life totally historically disconnected from our own should give rise to any overlap in concepts or conceptual roles whatsoever[5]. If an alien could talk to us, we would not be able to understand it.

This is a simple argument, and if good, quite a hopeless one. The simple biological differences between our lives and those of the alien receivers of our signals would rule out communication before it had even begun – for how can one communicate without any shared concepts?

However, as (Sandis 2012) has argued, this glib argument misrepresents Wittgenstein's point by taking it out of context. Wittgenstein's point is not about lions, but about the connection between understanding speech and understanding behaviour. The worry is not that there is something *prima facie* incomprehensible about lions that talk, but that coming to understand the language of another requires coming to understand the connection between their behaviour and words, assuming sufficient regularity in the latter. Phylogeny is beside the point. Understanding behaviour is a prerequisite to understanding language, and some forms of behaviour, like that of animals, might be very difficult for us to understand.

[5] Barring, of course, the panspermia hypothesis, which proposes that life on earth is of extra-terrestrial origin (and thus, historically connected to other life throughout the universe). Even if this hypothesis were true, though, the evolutionary distance would remain far greater than the distance that separates us from fellow mammal *Panthera leo*.

In the context of Drake Pictures, this is a serious worry. The form of communication established by the pictures is necessarily minimal. Though serious attempts to send extraterrestrial messages are more complex than the proof-of-concept Arecibo message I described above, all are static messages. It is possible to add more information to the messages, but never the kind of behavioural back-and-forth that Wittgenstein thinks is so vital to 'find one's feet' with a previously unknown culture (Wittgenstein 1968, 225). Perhaps if the aliens were in front of us, we could find our feet, and they their pseudopods. Perhaps we could not. But it is certain that we cannot do such a thing via a decontextualized exchange of pictures[6].

A conventional answer to this kind of worry amongst METI researchers is that some concepts – mathematical concepts, usually - are sufficiently universal to transcend the problems of cultural and biological differences[7]. We do not find ourselves in the situation that Wittgenstein is concerned about because there is simply no barrier to overcome. These messages are not attempting to teach the aliens our language, nor to allow us to understand theirs. That can wait. The messages are simply attempting to point the recipient in the direction of Earth and indicate that somebody capable of complex thought lives there, using mathematics as a shared mode of communication in the place of language. At the 1971 international Communication with Extraterrestrial Intelligence conference later immortalized as (Sagan 1973), Francis Crick gave an explicit statement of this view

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[6] For a more thorough version of this argument, which takes its philosophical basis from Quine's radical translation rather than the Wittgensteinian lion, see (Jebari and Olsson-Yaouzis 2018)

[7] Exemplified by the following remarks by Crick and previously quoted statements by the NAIC, both of which are foundational in the SETI field. However, even in recent SETI work dedicated to readying humanity to decode an alien signal were we to receive one, we see the same sentiment. For instance, a 2019 report on the state of the art in technosignature detection released by the Keck Institute for Space Studies claims, when considering sources of anthropocentric bias in our searches for technosignatures, "Here, we strive to consider technosignatures differently, free from as many assumptions as possible. Strictly speaking, the only unbiased assumptions that we can make are that mathematics, physics, chemistry, and information theory are universal, and even then we recognize that there is likely more yet to be discovered in these domains—our knowledge of them is incomplete."(Lazio et al. 2023, 9). Though the authors recognize that our knowledge of math may be incomplete, no other variability in mathematical knowledge across the stars is considered.

in response to some skeptical remarks about our ability to comprehend an alien signal we receive voiced by historian William H. McNeill. Crick said: "I would like to make two comments on what you have said. I think you do not allow for the existence of mathematics. This is a natural language which it can be easily argued would be common to both parties. Moreover, it is easily put into a form, into a notation which is easily transmitted" (Sagan, 345-6). McNeill remained unconvinced by this argument (ibid, 346), but the bulk of conference-goers sided with Crick. If Crick is right to say that mathematics forms something like a shared communication basis between humanity and any other sapient species (even if we do not go so far as him in calling it a 'natural language'), then lion-like worries are not a significant cause for concern until in-person first contact requires more complex communication.

This attitude of mathematical universality is common both implicitly and explicitly in most METI literature. However, this assumption it is not without its problems. Let us move on to another Wittgensteinian rejoinder.

3. Mathematics in Space

It is hard to doubt that of all complex human concepts, mathematical concepts have the greatest claim on possible universality. Aliens may not have ten fingers upon which to count, they may not think Bach sounds beautiful, and they certainly would not know what to do with a friendly 'Hello there!', but the Pythagorean theorem would not suddenly become false beneath another sun.

However, as Boudewijn de Bruin argues in his 2001 paper on Wittgenstein and alien communication, the universality of the truth of mathematical propositions does not guarantee their usefulness as interspecies communication tools. Mathematical propositions are (putatively) true or false independent of any experience, but it is not the mathematical propositions themselves that we are proposing to communicate with the aliens – they are a means to a different communicative end. In order to use mathematical propositions to communicate with aliens it needs to be the case that the aliens have cognitive access to the same mathematical concepts that we do. Even if the same facts are true everywhere, contingent features of our history and biology caused us to instantiate the mathematical concepts that we have instantiated within our mathematical practice[8]. A different history could produce beings that had none of the same mathematical concepts, regardless of the universal truth of the mathematics itself[9]. So, while mathematics is true everywhere, our attempts to use it to communicate could be fruitless if the concepts upon which our communication relies are not the ones that the aliens know. Since we do not know the first thing about the history of their species, we have no way of knowing which concepts, if any, are shared. Mathematics may or may not be universal, but it is not a universal language. Thus, even mathematics cannot save us from the difficulty of communication with aliens. (de Bruin 2001)

However, that story is not quite right. As METI researchers have themselves argued, we aren't completely devoid of information about the cognitive world of our possible interstellar interlocutors. The universe is doubtless full of life of all kinds, and we can say very little in principle about it as a whole. But we do know that any alien to whom we successfully transmit a radio message must necessarily have a device capable of receiving a radio signal like the one we sent. Our signals may harmlessly graze dozens of inhabited planets for which this is not true, because only the planets with radio telescopes will ever be able to hear our signals. Jill Tarter, the longtime director of the SETI institute, claims that this constraint changes the operational definition of intelligence itself for the

[8] Note that this point is independent of the debate within the philosophy of mathematics about either the real existence of mathematical objects or the universal truth of mathematics. However, if mathematics is not universally true, the point follows trivially. I will set aside concerns about relativism for the sake of the paper, since no participant in the debate seriously entertains the theory.

[9] "No experiments will ever be able to falsify mathematical statements, and hence, in every out of the way corner of the cosmos, they will be true. But, that corner may be so different from our world, that our mathematics would not arise there. If the world is too different, the math will be different" (de Bruin 2001)

whole SETI project. She writes, "Thus, SETI has a very pragmatic definition of intelligence, that is, the ability to build a transmitter. This is the practical definition because this is what can be detected." (Tarter 2001). This is no trivial similarity – the constraints on effective radio transmission are physical facts about the universe, and those are shared throughout space. Lev M. Gindilis gave this argument at the aforementioned CETI conference:

> To be more definite, let us assume we are using an electromagnetic channel. The specific character of a CETI system is such that no correspondent knows in advance what the other is up to and he can only assume what strategy the other follows; on this basis he seeks to coordinate his actions with the actions of his correspondent.
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> For example, the recipient may make certain assumptions about the system of transmission employed by the sender and on the basis of these assumptions he will use a certain mode of reception. In turn, the sender must take into account the methods of reception that are to be used by the recipient and he must do so on the basis of his assumptions concerning his sender's actions. What this amounts to is a game situation, a typical game situation. The specific feature of this interstellar communication game – unlike a game between the communications experts of hostile armies – is that the correspondents, instead of seeking to upset one another's schemes, are trying jointly to find a solution to the problem that will enable them to make the game a success.
>
> *A solution of this problem is facilitated by the fact that there are certain common elements, designed neither by the sender nor recipient.* Such an element is the communication link itself. By this link I mean the region of cosmic space between the sender and the recipient, between their antennas – the interstellar and interplanetary medium, and the planetary atmospheres.

> A study of the parameters of this link makes it possible to draw certain conclusions as to how such a system should be arranged, or at any rate as to how it should not be arranged. For one thing, we may make definite conclusions about the optimal wavelength in CETI. This being so, we are guided by certain objective laws and are endeavoring, on the basis of such laws, to formulate certain rules of the game.
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> -Lev M. Gindilis, in (Sagan 1973) (trans. Boris Belitsky, viva voce) Emphasis mine.

Gindilis argues that the mere existence of a communication link between Earth and another world presupposes that both planets have radio telescopes, radio engineers to work them, and some general knowledge about the kinds of astronomical situations that can impede or allow radio transmission. Even if the radio technology of the two worlds is very different, the same natural constraints apply to both. The mathematics of the aliens could be very different from our own, but mathematics is not merely theoretical. We could not have built our radio telescopes without the aid of our geometrical knowledge. It seems equally implausible that the denizens of another world could build their telescopes without some mathematical knowledge of their own[10]. Surely that guarantee of some similar mathematical concepts is sufficient if, as previously established, we are using mathematical concepts to communicate?

The Drake Pictures messaging technique was designed with arguments like this in mind. The only human mathematical concepts that our hypothetical interplanetary pen-pals need share with us to read our message once they have received it is 'semiprime number', and its algebraic components,

[10] As an anonymous reviewer has pointed out, it is not strictly true that the ability to receive radio signals implies the possession of a radio telescope. Perhaps some species could evolve in such a way that they naturally perceived in the radio band of the electromagnetic spectrum, in the same way that we perceive in the so-called visible band. Whether evolution could produce a creature to see such a noisy world at enough fidelity to scan the sky is beyond the scope of this paper. However, were this to be the case, we would merely have to hope that the Radio Vision Aliens were also interested in number theory, since we would have no basis upon which to assert that fact.

'prime number' and 'number'. In the history of human mathematics, the concept of a prime number is so basic and ancient that we do not even know how ancient it is. So, the argument goes, any civilization capable of building a powerful radio telescope must know at least that much[11]. By limiting the possible recipients of our message to only those who can build radio telescopes we are, of course, limiting the probability of our success. We have effectively added another factor to the Drake Equation. Any approach to METI that hopes to succeed should try to do this as sparingly as possible, and if we must do it, to be as sure as we are able that any message that reaches one of those radio telescopes can be understood.

4. Open Texture in Space

The presumption amongst the NAIC scientists at the Arecibo Telescope that mathematics would form the most universal basis for communication of any on earth is reasonable, and likely correct. And, in the previous section, we established that there are good reasons to believe that our interstellar interlocutors may share some mathematical conceptual roles or even concepts with us. However, it does not follow from mathematics' status as the *best* form of communication with the stars that the Drake Pictures method is an *effective* method of communication. In this section I will advance a new problem for the plausibility of communication with the Drake Pictures strategy. I will argue that the concept 'prime number' itself is historically contingent due to its possession of a property called Open Texture either now or in its past. It is my hope that by illuminating this

[11] Indeed, the designers of the recent BITG message, a descendant of the previously mentioned Cosmic Call, specifically add both a list of prime numbers and a representation of the largest known prime number, $2^{82589933}-1$, to their formulated message. This is because, as they say, "prime numbers are a clear indicator of life" (Jiang et al. 2022, 15)

problem I can open a door to potential solutions of it, and useful considerations for those tasked with the inverse problem of interpreting any future messages we may receive from somewhere else.

The idea that some concepts have a property called open texture was introduced by Friedrich Waismann in his 1945 critique of verificationism (Waismann 1945). A concept is open-textured if and only if there exists some possible context in which whether the concept applies or does not apply would be undecided by the content of the concept. Open texture is not the same as vagueness. Vague concepts are undecided or partially undecided in the contexts of their typical use – when I say a salmon-coloured object is pink and you say it is orange, our disagreement is down to the vagueness of 'pink' and 'orange'. Open texture occurs in concepts that are typically precise, but whose precision may fail us suddenly and without warning if unanticipated scenarios occur[12]. Waismann's classic statement of the meaning of open texture concerns the properties of cats:

> Suppose I have to verify a statement such as 'There is a cat next door'; suppose I go over to the next room, open the door, look into it and actually see a cat. Is this enough to prove my statement? Or must I, in addition to it, touch the cat, pat him and induce him to purr? And supposing that I had done all these things, can I then be absolutely certain that my statement was true? Instantly we come up against the well-known battery of sceptical arguments mustered since ancient times. What, for instance, should I say when that creature later on grew to a gigantic size? Or if it showed some queer behaviour usually not to be found with cats, say, if under certain conditions, it could be revived from death whereas normal cats could not? **Shall I, in such case say that a new species has come into being? Or that it was a cat with extraordinary properties?** (Waismann 1945, emphasis mine)

[12] The distinction between vagueness and open texture in Waismann is delicate and may or may not be sustainable. See (Shapiro and Roberts 2019) for further discussion.

As Waismann emphasizes, closing a patch of open texture in a concept is never merely a matter of reading the content of the concept off of its typical use – it is a decision[13]. The content of the concept cannot close the concept for you. And so, in courts of law, medical handbooks, cat-fanciers associations, and ever-evolving oceans of discourse, speakers constantly consciously and unconsciously choose to seal up the gaps in their concepts.

Waismann argued that all empirical concepts exhibit open texture, but that mathematical language was typically closed and precise (Waismann 1945, 45). Stewart Shapiro and Craig Roberts, amongst others (c.f. F. S. Tanswell 2018; Vecht 2023), have argued that this is not true. Mathematical concepts often do exhibit open texture, though some concepts are more open than others (Shapiro and Roberts 2021), and mathematicians are more concerned than most with finding and closing the gaps. Indeed, Imre Lakatos' account of the evolution of mathematical concepts, the celebrated *Proofs and Refutations*, is nothing but a long history of the closing of the concept 'Polyhedron', though not in those terms[14]. 'Polyhedron' was not initially a *vague* concept – there is nothing ambiguous about calling a cube a cube, and no sorities series exists between, say, flat polygons and solid polyhedra – but certain possible extensions of the domain of the concept did not admit of such a neat closure. Is a picture frame with a hollow center a polyhedron? What about a Keplerian urchin? Lakatos chronicles and condenses a century of protracted debate over what the extension of the domain of 'polyhedron' ought to be (Lakatos 1976). If the debate could have been settled by consulting Euclid or Euler, it would have been settled immediately. But as Lakatos

[13] Waismann, of course, was a close collaborator with Wittgenstein, both at Vienna and at Cambridge, and was deeply influenced by his work. Notably, Waismann's example of the cat, quoted above, bears a strong resemblance to Wittgenstein's example of the chair, from section 80 of the *Philosophical Investigations.* There is some scholarly debate over whether Waismann's notion of Open Texture is just a different formulation of the later Wittgenstein's notion of either Family Resemblance or Rule Following. See (Bix 2012) and (Makovec 2025) for the case for the prosecution and defense, respectively.

[14] It is not known whether Lakatos was aware of Waismann's work on the subject of open texture, though he was aware of Waismann's work more generally. There are possible avenues by which he could have heard about Open Texture. See (F. Tanswell, Larvor, and Rittberg Forthcoming) for one.

demonstrates, the matter of locking down the new, more closed concept was no mean feat. The development of mathematical concepts is historically contingent upon the decisions of hundreds of mathematicians over hundreds of years.

Shapiro and Roberts argue (Shapiro and Roberts 2021) that there is good reason to suspect that at least some mathematical concepts are open textured (even if some or most seem, as Shapiro puts it elsewhere, borrowing a phrase from Wittgenstein, 'hard as rails' (Shapiro 2006, 434)). The important concept for our purposes, 'prime number' may or may not be among that set. 'Prime Number' seems quite well-defined as a concept, after all, and proofs involving the prime numbers typically do not run into any unforeseen failures of definition. However, outside of very circumscribed domains of application (Waismann's example is a complete description of a particular game of chess, with each move noted in turn (Waismann 1945)), it is not in principle possible to prove that a given concept is *not* open textured[15]. The force of open texture derives from the fact that it is unforeseen and unforeseeable.

In the next section, I will argue that 'Prime number' has at been at least trivially open-textured in its history, since one can construct a nearly identical notion of prime number in which 1 is also a prime number. Further, I will argue that if 'prime number' was ever open textured in our history, then we have good reason to believe that an alien mathematics could have a concept of 'prime number' with a different extension and intension from ours simply by making different choices in its closure at a historical crisis point. Then, I will motivate this point by constructing two fables about alien civilizations who have different versions of the concept of prime number because they made

[15] *Pace* (Vecht 2023), whose counterargument will be considered later. I am indebted to Dejan Makovec for this point, and the many hours of discussion that led to it.

different decisions about how to close the open texture of the concept than we did. Through that decision alone, I will argue, they could fail to read our pictures.

This concern over alien intelligence possibly having a subtly different concept of 'prime number' is not well-reflected in the literature on METI. Even recent publications, like Jiang et al.'s summary of the state of the art in Drake Picture schemes, has this to say about the subject of the prime numbers: "Prime numbers can be safely assumed to be unique to intelligent construction given there being no known naturally occurring phenomena in the cosmos which generates that particular series and would thus signal any ETI that something of intelligent origin is contained within this signal" (Jiang et al. 2022). Though the authors are right to note that no known natural phenomenon produces prime number signals, it is not enough to merely be unique. If the receivers of the signal do not recognize our prime numbers, their status as a signifier of intelligence is not guaranteed.

Given the way the Arecibo self-decrypting message works, if the concept 'prime number' was open textured at any point in its history, even an alien interlocutor who possessed the concept 'prime number' might be unable to decrypt the message. The history of their concept that fills the role 'prime number' could be sufficiently different from ours that the closure of the openness of the 'prime number' concept gave it a different domain from our own, possibly causing them to miss the significance of the semiprimeness of our signal. And thus, even if their civilization was arbitrarily similar to ours in their goals, technology, and conceptual structure, we could still fail to communicate on the basis of our lack of shared conceptual history alone. If 'prime number' has been open-textured in its history, then even our best method of communication can fail. Has it?

5. As Open-Texured as Eratosthenes' Sieve

There is one obvious context in which the concept 'prime number' has exhibited open texture, and that context is the aforementioned number 1. Euclid's definition of a prime number as '…that which is measured by an unit alone' (Euclid 1908, Bk. VII Definition 11) is silent on the question of whether 1 is itself a prime number. Certainly 1 is not a composite number – but is it a prime with an unusual property; or something else entirely, in a class of its own? Doubtless, any reader who has read this far knows the 'right answer' to this question. 1 is not prime. If 1 were prime, mathematicians would be forced to rephrase the Fundamental Theorem of Arithmetic in a more cumbersome way, as well as the many other theorems that depend upon it[16]. Though it is not difficult to see why mathematicians decided to exclude 1, that decision was not made because of the content of the concept, it was a decision made for the sake of a more elegant FTA. Aesthetics and pragmatics sealed the deal, not conceptual analysis into the True Meaning of Primeness.

The primeness or not-primeness of the number 1 should not be expected to be relevant for the feasibility of Drake Pictures, since the method at no point involves the number 1. It is here for illustrative purposes, to show how open texture can manifest in the prime number concept. The claim I want to motivate is that the concept prime number could exhibit more troublesome open texture if the history of mathematics were just a little bit different – as it would be on any alien world. Any point of open texture between Euclid and the present is a point at which the concept 'prime number' could have evolved differently.

It is not necessary for my argument that 'prime number' currently exhibits open texture, only that it did so at some point. Joost Jacob Vecht has proposed a 'clarification' of the notion of Open Texture that defines it in terms of either the absence of full analyticity or algebraicity (and argues that these definitions are coextensive). On this account of open texture[17], it's clear that 'prime

[16] See (Caldwell and Xiong 2012; Caldwell et al. 2012) for a history of the long debate over the primeness of the numbers 1 (and 2). As the authors point out, whether 1 is considered prime is even now a matter of definition, and thus of "choice, context, and tradition" rather than proof.

[17] Notably, this account is a *precisification* of Open Texture, and thus strictly narrower than Waismann's more open-ended definition.

number' is currently closed, since has an algebraic, rather than assertory, definition. However, Vecht is aware of the potential problems this account has in the face of a concept's history. It seems to be the case that mathematical concepts that exhibit open texture – such as Lakatos' 'polyhedron' – can eventually evolve into closed concepts, as happens to 'polyhedron' at the end of *Proofs and Refutations*. But it also seems to be the case that Open Texture is a persistent property of concepts, or in Waismann's metaphor, a descriptive horizon that we carry with ourselves. Vecht solves this problem by positing a special move available to us in our pursuit of clear concepts: the wholesale replacement of our old open concept with a new, formally-defined closed one (Vecht 2023)[18]. Even if 'prime number' is currently closed, it is clear that it was not always so. At some point, we played our special move. Under Euclid's definition, a prime is simply "that which is measured by an unit alone" (Euclid 1908). The scope of this definition is not sufficiently locked-down to meet Vecht's standard for algebraicity. Implicitly Euclid's definition spans the space of numbers (earlier defined as "a multitude composed of units" (ibid. Definition 2). But though it is true that Euclid's numbers read most naturally to a modern reader as the natural numbers, this is certainly not because Euclid saw the multitude of ways we can now speak of number and magnitude and carefully selected his modern number field of choice. Rather, all of the terms that go into the Euclidean definition of 'prime number' exhibit open texture, including 'number', 'measure', and 'unit'. As Caldwell and Xiong show, the modern rigorous definition of Prime Number, excluding 1 and including 2, only became universal in the early 20th Century (Caldwell and Xiong 2012). For most of the history of mathematics, I claim, 'prime number' has carried a descriptive horizon along with it.

If it did so, then even an alien interlocutor who has a concept of primeness may not have ours. They could have started from the work of their own Xeno-Euclid, evolved 'prime number' in a

[18] My thanks to an anonymous reviewer for bringing this argument to my attention.

different direction, and then formalized their concept into rigid algebraicity in a different manner than the one we selected. That might be sufficient to sink our strategy. Allow me to motivate this argument with two brief thought experiments.

6. Prime Conceptual Real Estate

Up to this point in the paper, I have kept speculation about what other planets might or might not be like to a minimum. The activity of imagining different ways in which alien civilizations could develop is both gloriously boundless in possible scope and cruelly bounded by the limited frame of the human imagination. However, in order to make concrete the possibility of miscommunication due to open texture, I can no longer avoid at least a brief visit to a few hypothetical planets. I will try to limit the fanciful nature of these fables as much as I am able. After all, my goal is to conjure an image of a communication scenario that is, beyond the problems caused by the open texture of mathematical concepts, ideal for interaction. We can imagine the inhabitants of this world as arbitrarily biologically, socially, and intellectually similar to ourselves. They have two legs and two arms with five fingers per hand, they have science and mathematics comparable to our own, they have radio telescopes that receive in the kinds of frequencies we are transmitting, and they have access to the concept 'prime number'. The only variable I will change is the history of mathematics, and with it, the development of their 'prime number' concept.

Let us consider two worlds, conveniently situated in the direction of the M3 globular cluster to which the Arecibo message was sent. These worlds are just like our own in all respects save the history of their inhabitants. The denizens of these planets look, act, and think like human beings. They have similar concepts, similar institutions, and similar reverence for mathematics to us. However, they did not have a Euclid, or an Euler, or any other specific Earthling mathematician you care to name. They had their own history of mathematics. I will argue that it is at least conceivable that a different history alone would produce a society unable to recognize the semiprime structure of a Drake picture.

Fable 1: Twin-Prime Earth

Our first world plays host to a mathematics that is very much like our own. However, the weight that our Ancient Greeks placed on geometry, their ancient mathematicians placed on number theory. This advanced understanding of numbers and their relations (by our standards) led them to discover and prove[19] the Twin Prime conjecture before developing their notion of a prime number. They noticed something interesting about 11 and 13, 17 and 19, and all the other pairs before they had a name for that kind of number more generally. The proof of the infinity of these primes became a celebrated and oft-taught result, known to even the least mathematically curious engineer. Eventually it would be discovered that there are some non-twin numbers that also share some of the properties of the twin primes – call them the singleton primes. These numbers proved a conceptual challenge for the denizens of Twin-Prime Earth – were they prime numbers with an unusual property – singleness – or did their lack of the crucial property 'twinness' disqualify them from the proper moniker 'prime'? Were they an expansion of the concept 'prime number', or a deviation from it? Eventually the mathematicians voted – they would remain with their original concept. A few recalcitrant number theorists continued to study singleton primes, but their results garnered little further attention.

[19] For the purposes of this example, assume the Twin Prime Conjecture is true. However, the same argument could be made without loss of generality for any other proper subset of the prime numbers.

When the Twin-Prime Earthlings took their mandatory mathematics classes to get the physics degrees necessary to play around with radio telescopes, they learned the Twin Prime theory and how to prove it. They also learned about the various concepts derived from their notion of primeness, such as semiprime numbers – numbers that are the unique product of two Twin Primes. Our target number, 1679, is not a semiprime number on this notion of prime. 73 is a twin prime, but 23 is not. Thus, in Twin Prime Earth, the numerical properties of our signal could easily pass unnoticed. They would be capable of factoring the number, but the force that the semiprimeness of 1679 is supposed to impress upon them, the call to perform the necessary factorization, would be absent. There are indefinitely many ways to analyze a signal, and only so much time to devote to what may just be noise. Without the recognition of the semiprime structure of the message, that the message had structure at all could be missed.

This kind of example would be familiar to a reader of Lakatos' Proofs and Refutations. It is the number theory version of the world that the student Beta is arguing for when he proposes that only Eulerian polyhedra be considered polyhedra at all. When mathematics discovers a putative property of a concept under consideration that applies to some of the examples of the concept but not all, mathematicians are faced with a kind of choice – either they must reject that property or alter the extension of the concept to fit it. Eventually, things that seemed like results of a proof become definitive of the concept itself. Perhaps we would be justified in calling the mathematicians of Twin-Prime Earth too conceptually conservative. But we cannot call them impossible.

Fable 2: A World of Pure Imagination

For our second fable, let us journey to an even stranger world. The inhabitants of the World of Pure Imagination like their mathematics abstract. This love of the intangible led them to develop very general concepts of number early in their history. Natural numbers quickly gave way to integers, complex numbers, and even quaternions, well out-pacing the development of other branches of mathematics. When this species developed a notion of prime number, it was a more general notion than most non-mathematician Earthlings would imagine. Our familiar prime number in the ring of the integers remains, but it is joined by many others. All the multifarious varieties of primeness that on Earth go by different names – Gaussian Primes, Eisenstein Primes, prime ideals, and all the others – in the world of Pure Imagination they are all simply Prime Numbers.

The entire extension of our concept of Prime Number is contained within theirs, but theirs is many times broader than ours. This pattern of conceptual expansion is familiar from our own history. What was once 'geometry' simpliciter became 'Euclidean geometry' – a mere subset within a broadened concept. So too the Pure Imaginationers' notion of our natural primes. When human mathematicians developed notions of primeness suitable for use in the complex plane, we gave those primes new names. We were faced with the decision to either call these new numbers something other than prime, or to call them primes with a new property. We chose the former. The denizens of the World of Pure Imagination chose the latter.

After all these additions, the Pure Imagionationers' notion of primeness is so broad that noticing patterns of primeness is commonplace and not remarkable. We may fear that the radio telescope operators in this world would react to the natural prime factors in our signal as a mere triviality, no more notable than seeing the word 'carthorse' and remembering that it is an anagram of 'orchestra'. Sure, they would recognize that 23 and 73 were prime factors of 1679, but they would also recognize many other prime factors, from their many different ways of cashing out multiplication. 1679 only has two unique prime factors using our concept of prime, after all. A broader notion would render the uniqueness of prime factors much less likely. In this way again could our message fall on eyes that could not recognize them, by reason of the history of mathematics alone. Perhaps we would be justified in calling the denizens of the World of Pure Imagination too conceptually liberal. But we cannot call them impossible.

A signal-sender could argue that both these scenarios just represent possible fail states – that, in the same way that we can only hope to communicate with species who have the ability to detect radio signals, we can only hope to communicate with species who share our notion of 'prime number'. Though that argument is open to the signal-sender, it represents a retreat from the fundamental goal of METI. Moreover, though the chance of intelligent life being out there somewhere seems high to most believers in the METI project, there is no in-principle reason to believe that our formulation of the prime numbers exists anywhere but here. If we can do better, we should try to do so.

6. Conclusions

Let us review the arguments up to this point. The first Wittgensteinian argument was an extra-terrestrial generalization of the Lion argument. Even more than lions, aliens have a form of life very different from our own. It would be implausible to assume that they would even recognize our conceptual roles, let alone our concepts themselves – and without either shared concepts or the back-and-forth necessary to acquire them, communication is impossible. This is a general argument about alien communication that seems to throw cold water on the whole enterprise. However, the xenophiles amongst us have a rejoinder for this general argument: if mathematics is universally true, we can use mathematical concepts to communicate. This rejoinder is met with another Wittgensteinian argument, this time from De Bruin. The universality of mathematical *truth* does not guarantee the universality of mathematical *concepts*. The aliens could have a totally different set of mathematical concepts in their conceptual store because their historical goals, lifestyle, and activities were different from our own. Thus, mathematics cannot be a shared language. The METI practitioners have a rejoinder for this as well: we may assume a certain degree of similarity between the goals of humanity and the aliens with which we hope to communicate solely on the grounds that building a radio telescope is a necessary condition on any communication between us and them. So, we can assume that they are in possession of at least the mathematics necessary to build a radio telescope. My final argument, the argument from open texture, claims that even significant conceptual overlap might not be enough to guarantee the possibility of communication. The Drake Pictures method requires that our alien interlocutors recognize one specific concept: 'prime number'. But even if the aliens and humanity are sufficiently similar to even have both developed the concept 'prime number', which is necessary for the messaging strategy to work, the different history of the prime number concept as its open texture was closed over time could still leave them unable to decipher our messages. In my two fables, I showed that these alternate histories are possible, though it need not be true that they are likely. These stories represent the theoretical maximum of our hopes for similarity. Aliens cannot be more similar to us than the aliens in these fables, for the aliens in these fables are differentiated from us only by their different history of mathematics, which is guaranteed by their alienness alone. These are the best circumstances we could possibly desire, and even they are not enough.

The arguments considered for and against the possibility of alien radio communication began very broad and grew ever more specific. The first argument, the adapted Lion argument, is a general argument about communication over extreme cultural differences. My final argument, the novel argument from Open Texture, is born of the specific concrete practices used in METI. This context, then, provides a rare opportunity to see a concrete consequence of what seems to be a very abstract problem in the semantics of mathematics. We have plucked a problem from Plato's heaven and placed it in our own. If, when we meet the aliens they do have precisely our notion of prime number, my argument for the open texture of 'prime number' may be falsified. Falsification is a rare thing to find in the philosophy of mathematics. I certainly would not begrudge it under these circumstances.

I do not profess to offer a solution, but it is also not my intention to be an Eeyore. Use of the prime numbers as a technosignature are common in many contexts both in METI – the sending of signals – and in SETI – the reception of putative signals. Moreover, more recent and sophisticated message strategies rely on more complicated mathematical concepts, which can be subjected to the same analysis. It is my hope that more philosophical attention to this previously unmarked axis of possible variation in these possible signals will allow us to construct better signals, and better prepare us for the hopeful future in which we receive them.

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