

Variability and Substantiality. Kurd Lasswitz, the Marburg School and the Neo-Kantian Historiography of Science

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A trained physicist, Kurd Lasswitz (1848-1910) is best known as a novelist, the father of modern German science fiction, or as a historian of science, the initiator of the modern historiography of atomism. In the late 19th century, Lasswitz engaged in an intense dialogue with the emerging Marburg school of neo-Kantianism, contributing to shape most of its defining tenets. By the end of the decade, this research had grown into a two-volume *Geschichte der Atomistik* (1890), which remains the most successful example of neo-Kantian historiography of science. Lasswitz combined attention to historical detail with the search for the intellectual tools (*Denkmittel*) without which the ‘fact of science’ would be impossible. In particular, Lasswitz regarded Huygens’ kinetic atomism as a historical model of a successful scientific theory, shaped by the interplay of two conceptual tools: (a) substantiality, the requirement of identity of the subject of motion through time, which found its scientific expression in the extensive atom; (b) variability, the intensive tendency to continue in an instant, which found its conceptual fixation in the notion of differential. By raising the problem of individuality in physics, Lasswitz offers a unique perspective on the utilization of the history of science in 19th century neo-Kantian thought.

Introduction

Kurd Lasswitz studied mathematics and physics in Berlin and Breslau, where he also had the opportunity of attending the lectures of the philosopher Wilhelm Dilthey (Azzouni, 2009). After completing his dissertation in physics in 1873 (Lasswitz, 1873, 1874b), Lasswitz became a teacher at the Gymnasium Ernestinum in Gotha. During this time, he started cultivating his interests in the philosophy of physics, articulating a defense of kinetic theory of matter (Lasswitz, 1878a, 1879b) from a broadly Kantian point of view (Lasswitz, 1883). Lasswitz combined his theoretical support of atomism with a study of its history, publishing a series of historical essays (Lasswitz, 1874a, 1884a, 1879a, 1882). Furthermore, he carved out time to develop his literary pursuits, writing poems and short stories (see *e.g.*, Lasswitz, 1878b).

From 1883 to 1895, Lasswitz authored an impressive number of reviews of the latest scientific and philosophical works.¹ Through this activity, Lasswitz (1885a, 1887a, 1887b, 1884c, 1884b) established contact with a group of scholars centered around Hermann Cohen,² a Marburg professor of philosophy who had just published his controversial³ *Das Princip der Infinitesimal-Methode* (Cohen, 1883). Lasswitz (1885a, 1885b) realized that the interpretation of the infinitesimal calculus proposed in the book aligned with his previous work on the foundation of physics (Lasswitz, 1878a). The unexpected ally was welcomed with open arms. Cohen’s right-hand man, Paul Natorp, recently appointed as the editor of the *Philosophisches Monatshefte*, requested Lasswitz’s collaboration with the journal on a regular basis. Together with *Vierteljahrsschrift für wissenschaftliche Philosophie*, edited by Richard Avenarius, it became Lasswitz’s primary platform of publication.

Building upon Cohen’s ideas, Lasswitz began to forge his own philosophical perspective (Lasswitz, 1888a). The task of philosophy is to delve into the past of physics, searching for the intellectual tools or *Denkmittel*⁴ indispensable for detaching an objective nature from the flux of our subjective experiences. If ancient metaphysics was dominated by the thought-instrument of substantiality, as Cohen intuited,

¹For a comprehensive bibliography, see Roob, 1981.

²See Beiser, 2018.

³(Giovanelli, 2016, see).

⁴For the history of this expression, see Klein, 2021; following Klein, I translate ‘*Denkmittel*’ with ‘thought-instrument’.

modern science introduced a new thought-instrument of variability with the discovery of the infinitesimal method (Lasswitz, 1888a). Lasswitz incorporated this somewhat half-baked philosophy into a series of papers on early modern theories of matter (Lasswitz, 1888b, 1889a, 1889b). Lasswitz’s decade-long historiographical research was consolidated into a comprehensive two-volume *Geschichte der Atomistik* (Lasswitz, 1890), spanning close to a thousand pages, which was published in 1890. The book hailed Christiaan Huygens as the pivotal figure in modern science. In Huygens’ kinetic atomism, the two *Denkmittel* work together in an exemplary way. The atom is the scientific expression thought-instrument of substantiality, assuring the individual identity of the subject of motion over time; the thought-instrument of variability allows determining the continuous and reciprocal relation of causality among atoms as regulated by the principle of mechanics.

The *Geschichte der Atomistik* was well-received. In the 1890s, Dilthey recommended Lasswitz for philosophy chairs in Breslau, Bonn, and Würzburg (see Azzouni, 2009, 66). He was also in the final round for a professor position in Marburg, which Natorp eventually filled in 1893 (Cohen to Bosse, Jul. 22, 1893; CN, Vol. 2, Doc. 498). However, Lasswitz ultimately never managed to secure an academic position. As the general editor of Kant’s *Akademie Ausgabe*, Dilthey selected Lasswitz as the editor for the pre-critical writings. However, the editorial work conflicted with Lasswitz’s ambition as an author of future-themed novels (Lasswitz, 1897) and philosophical essays (Lasswitz, 1900). Lasswitz’s efforts to combine Fechner’s panpsychism with Kantianism progressively distanced him from the Marburg school. He passed away in 1910, when Natorp (1910) and Ernst Cassirer’s (1910) major monographs appeared in print.

Today, Lasswitz is remembered as a pioneering historian of atomism (Lüthy, 2003) and, most of all, as the father of German science fiction (Willmann, 2018). However, his role as a ‘non-resident member’ of the so-called Marburg school of neo-Kantianism has been largely forgotten. Lasswitz’s name is barely mentioned in the otherwise well-informed historical literature on neo-Kantianism (Koschnitzke, 1988) or, more specifically, on its Marburg variant (Dussort, 1963, Ferrari, 1988). By exploring the years from 1885 to 1895 in Lasswitz’s career, this paper aims to show that this neglect is unwarranted. Lasswitz was one of the few scholars who embraced Cohen’s controversial interpretation of the infinitesimal method and incorporated it into his own historical work. In doing so, Lasswitz produced the most successful example of neo-Kantian historiography of science. *Geschichte der Atomistik* remained the benchmark in atomism studies for decades (see Cassirer, 1929, 547ff. Bachelard, 1933, 9f.), in contrast to Cohen’s *Das Princip der Infinitesimal-Methode*, which never acquired a similar status in the historiography of mathematics.

Some readers may complain that a micro-historical study⁵ of this nature is of merely antiquarian value. However, this paper hopes to provide an example of the relevancy of ‘minor figures’ whose names are frequently overlooked in the conventional histories of philosophy (Beller, 1999). Indeed, Lasswitz’s neo-Kantian approach to the history of science might appear too ‘on the nose’; however, precisely for this reason, Lasswitz’s work unwittingly exposes the difficulties in which the neo-Kantian program of a historicized *a priori* remains entangled (Stump, 2015). As Cassirer (1929) pointed out, celebrating Lasswitz’ legacy in the late 1920s, in Lasswitz’s work, Huygens atomism is at the same time the ‘fact’ of historically given science and the ‘norm’ for all possible science. However, the transition from ‘fact’ to ‘norm’ is the stone against which neo-Kantian inevitably stumbles. Indeed, contrary to Lasswitz’s belief, the Huygensian requirement of particle individuality did not turn out to be a necessary condition of the possibility of physics.

1 Lasswitz and Early Cohen’s Group

1.1 Lasswitz and Cohen’s Historical Work on the Infinitesimal Method

At the close of 1883, Georg Cantor (the father of modern set theory) began a correspondence with Lasswitz regarding a paper of the latter on Giordano Bruno’s atomism (Lasswitz, 1884a). In passing, Cantor revealed that he had just submitted a critical review of Cohen’s work Cohen to the journal *Deutsche Literaturzeitung* (Cantor, 1884). Cantor’s criticisms unintentionally drew Lasswitz’s attention to Cohen’s booklet. Although Lasswitz’s response has not been preserved, based on Cantor’s impatient reactions, it can be inferred that Lasswitz tried to defend Cohen’s stance Eccarius, 1985. Lasswitz had already reviewed works by other members of the Marburg group (Lasswitz, 1884b, 1884c). After his correspondence with Cantor, he quickly wrote a review of *Das Princip der Infinitesimal-Methode* (Lasswitz, 1885a). Lasswitz did express frustration about “the difficulties of the subject matter and obscure writing style” (494).

⁵See e.g., Levi, 2001.

However, unlike Cantor, Lasswitz appreciated Cohen’s effort to link the discovery of the concept of the differential dx to Kant’s rather obscure claim that ‘reality as intensive magnitude’.

Causality and substantiality establish a relation between something that is already ‘given’ (something that it is conserved, that changes, etc.). In order to define this ‘something’ itself—so Lasswitz summarized Cohen’s point of view—one needs a different *Denkmittel*: “This thought-instrument is called *reality*” (Lasswitz, 1885a, 498). In Kantian parlance, the category of ‘reality’ (*Realität*) does not refer to existence (*Wirklichkeit*); rather, it indicates the quality that defines ‘something’ as opposed to something else. If space and time are extensive magnitudes, the physical ‘reality’ extended in space and time is characterized by its intensive magnitude. In Cohen’s reading, the ‘differential’ was invented to provide a mathematical expression to this ‘intensive reality.’ The ‘differential’ expresses the intensive tendency to generate extension. Applying this reasoning to the problem of the constitution of matter, Cohen claimed that the “atomic hypothesis” has become unnecessary (Cohen, 1883, §93). In dynamic theories of matter, extended bulk matter is grounded in inextended points that have the intensive tendency to expand: “the differential takes the place of the atom” (Lasswitz, 1885a, 500).

As we shall see, Lasswitz found this last point contentious (section 1.2). However, he was broadly sympathetic to Cohen’s attempt to present “the differential (dx) as an example of intensive magnitude” (501). Still, Lasswitz expressed some concerns about Cohen’s use of the notion of ‘differential’: “One may recognize the infinitesimal reality as a most fruitful means of thought, and yet be in doubt as to its relation to that of the mathematician” (502). Lasswitz, in fact, wondered whether the intensive reality “can really be identified with the mathematical differential or rather with the functional connection with the differential of a second variable,” that is, in the relation between dx and dy in the differential quotient (502). This is particularly evident in the case of the differentials of higher orders (502f.). The ‘quality’ or ‘reality’ “only appears in the differential equation” (503) not in the isolated ‘differential’.

Cohen was pleased by Lasswitz’s positive review, one of the few he had received (Cohen to Lasswitz, Apr. 7, 1886; CN, Vol. 2, Doc. 8). However, the review did not address a more contentious issue. Lasswitz could not agree with Cohen’s arguments against of atomism. By reading Cohen’s book, one could be led to the conclusion that “atomism could no longer be justified as soon as one agrees to a theory of knowledge that rests on the ground of Kant’s critique” (Lasswitz, 1885b, 137f.). Thus, Lasswitz wrote an extensive rebuttal (139). Lasswitz could draw upon his previous work, in which he insisted on the compatibility between atomism and criticism (Lasswitz, 1878a) and supported kinetic atomism against the vortex theory of atoms Lasswitz, 1879b, 207.

1.2 Lasswitz’s ‘Justification’ of Kinetic Atomism

Lasswitz’s philosophical outlook is quite conventional. The task of physics involves reducing any change of the different types of sensation to the single type of change that allows for a mathematical representation, that is, ‘motion’ or change of position in time. Motion is characterized empirically by visual and muscular sensations, which correspond to two different aspects of motion. (a) the sensation of sight leads to the ‘phoronomic’ motion, *i.e.*, the displacement of individualized parts against a background (*Ortsveränderung*)⁶ (b) the capacity of the body in motion to overcome obstacles in virtue of its motion gives rise to the sensation of impetus (*Andrangsempfindung*), which distinguishes physical motion from a purely geometrical change of position.

The task of science is to fix conceptually (*begrifflich fixieren*) these ‘sensible’ facts, transforming them into measurable quantities and entering into a functional relationship with other measurable quantities. Phoronomy (or kinematics) defines ‘motion’ as the relation among *extensive quantities*, distances traveled in a time interval as measured from a particular reference frame. However, phoronomic motion has no physical reality; past motion does not exist anymore, future motion does not exist yet, and in an instant, there is no change in position and, therefore, no motion. Thus, Lasswitz argued, “[t]he fact of dynamics requires [that] an intensive magnitude is added to the extensive magnitudes of phoronomy” (Lasswitz, 1885b, 140f.). In an instant, where there is no change of position, one can still define the intensive tendency to continue with the same velocity. This tendency represents the physical ‘reality’ of motion, which manifests itself subjectively in the sensation of impetus:

The ‘sensation of impetus’ to which I sought to trace the formation of the concept of atom is, in fact, the starting point for the solution of the basic mechanical problem. However, the solution consists in the conceptual formulation of it as an intensive quantity that indicates a tendency to motion. Cohen has convincingly

⁶E.g., the motion of a blue region of a blue background cannot be perceived.

demonstrated that the category of reality can express this tendency in connection with the principle of intensive magnitude. I welcome this insight. This intensive magnitude, which is objectified by the thought-instrument [*Denkmittel*] of reality, is what I understood with the sensation of impetus [*Andrangsempfindung*] as ‘the real of the motion’ [*dem Realen der Bewegung*]. I recognize Cohen’s expression as the more appropriate term for what I imperfectly called the ‘sensation of impetus’ [*Andrangsempfindung*]. (Lasswitz, 1885b, 142)

This excerpt outlines the reasons Lasswitz was drawn to Cohen’s approach. In his first monograph, Lasswitz (1878a) gave a *psychological* interpretation of the physical content of motion, using the concept of *Andrangsempfung*. Cohen provided a *conceptual* account of the same issue by resorting to Kant’s category of reality and the related notion of intensive magnitude. The notion of intensive reality expresses the problem that Galileo was trying to grasp when he resorted to expressions like ‘the impetus, the moment of descent, the tendency to motion, etc.’ (see Cohen, 1883, 51). Leibniz and Newton provided the mathematical solution to the problem with the discovery of the ‘differential’.

According to Lasswitz, however, Cohen made the mistake of applying the same reasoning not only to motion, but to matter itself. As a consequence, Cohen rejected atomism and expressed some sympathy for dynamic theories of matter *à la* Boscovich or *à la* Kant (94). However, according to Lasswitz, dynamical theories of matters are doomed to fail. The category of ‘reality’ and intensive magnitude is necessary to define physical motion; however, it is not the proper conceptual tool to define the ‘subject’ of motion, the ‘something’ that moves. According to Lasswitz, this problem “cannot be solved through the concept of intensive reality, but only via the concept of substance” (Lasswitz, 1885b, 144).

Kant’s theory of matter shares the same shortcomings of all ‘plethoric’ theories of matter, which conceive matter as a continuum (Lasswitz, 1879b, see). Such theories are incapable “of separating out from the general matter a certain part as a body, which acts as a finite quantum ‘as a whole’ ” (Lasswitz, 1885b, 143). The notion of the material ‘particle’ (that is, a physically individuated parcel of space) was motivated by the necessity of being able to identify the trajectory of the parts of a homogeneous medium. Particles are supposed to be impenetrable since the individuality of identical particles would be lost if they overlap. If they are impenetrable, they must be extended, since inextended particles cannot touch without overlapping. The parts of such extended particles must cohere, so that they can be transferred as a whole; otherwise, the problem of individuation would be shifted from the whole to the parts.

As one can see, the conditions that the parts of matter must satisfy to serve as the individual subject of motion seems indeed to describe what is traditionally called an ‘atom’: “as soon as substance appears as a principle or means of individuating matter, we have atomism” (144). The impenetrability, extension, cohesion, etc. of atoms should not be confused with the homonymous sensible properties of macroscopic bodies; they are conceptual conditions necessary for the individuation of matter. Lasswitz concluded that, contrary to Cohen’s claim, “the concept of the differential does not exhaust the thinking tools of natural science” (146); the concept of the atom is equally essential: “The differential serves to describe motions, but the moving object, as soon as it appears as an independent whole, requires the concept of the atom” (177f.)

In this way, Lasswitz claimed to have provided a ‘justification’ of the kinetic theory of matter, in which all phenomena are reduced to the collision between strictly impenetrable and rigid atoms. ‘Collision’ is not to be mistaken with the clash of macroscopic bodies; it is only the statement that the motion of two atoms after their encounter is determined by their motion before. When two atoms approach each other within a defined distance (known as the radius of the ‘sphere of action’), their direction and velocity change instead of proceeding in a straight line. The collision rules are the conditions of the univocal determinability of the velocities after the collision, if the velocities before the collision are known. Relying on a paper by Gustav Lübeck (1877), Lasswitz could show that conservation of momentum $\sum mv$ and kinetic energy $\sum \frac{1}{2}mv^2$ are necessary and sufficient for this task (Meyer, 1877, 239f. 1874). These are, of course, the laws of elastic collisions; however, this is only an accident; atoms satisfy those laws not because they are elastic but because their velocities are supposed to be fixed unambiguously.

2 The Development of Lasswitz’s Philosophy

Lasswitz’s positive review of Cohen’s book was the exception rather than the rule. The young Edmund Husserl was baffled by Lasswitz’s “enthusiastic reception” of Cohen’s “nonsensical profundity or profound nonsense” (Husserl to Brentano, Dec. 29, 1886; Husserl, 1994–, 5). Inevitably, Lasswitz’s work attracted the attention of the Cohen’s circle, as Adolf Elsas, a physicist close to Cohen (1895) confirmed to Lasswitz in private correspondence (Elsas to Lasswitz, Jan. 7, 1887; CN, Vol. 2, Doc. 11). Indeed, at about the

same time, Natorp, who has just become the editor of the *Philosophischen Monatshefte*, asked Lasswitz to start to collaborate with the journal as a book reviewer (Natorp to Lasswitz, Sep. 24, 1886; CN, Vol. 2, Doc. 10). As a first assignment, Natorp asked Lasswitz to review a monograph by Ferdinand August Müller (1886), a former doctoral student of Cohen's, who was however critical of his approach to the 'infinitesimal method'. In 1888, the first volume of *Philosophische Monatshefte* edited by Natorp was published. In the introductory editorial (Natorp, 1888), Natorp emphasized the importance of maintaining a lively relationship between philosophy and the sciences. It was probably no coincidence that the issue opened with Lasswitz's review, that he had expanded into a comprehensive article, "Das Problem der Continuität" (Lasswitz, 1888a), outlining his philosophical views.

2.1 Substantiality and Variability

According to Lasswitz (as one might expect from 19th century neo-Kantian), the task of philosophy is to discover the conditions that are necessary to transform subjective sensations into an objective 'nature' (8f.). Lasswitz preferred to call those conditions *Denkmittel* (or thought-instrument), in order to indicate that they are intellectual tools used in the 'process' of construction of nature. If one analyzes the logical structure of scientific theories as the 'result' of this process, one can call these tools *Grundsätze* (or principles), as it was more common in neo-Kantian literature (Lasswitz, 1888b, 460). The *Denkmittel* of substantiality dominated ancient metaphysics; attempts to implement the *Denkmittel* of causality, however, failed: "Until the seventeenth century, it was not possible to combine substantiality and causality in such a way to allow the explanation of nature; there was no possibility of representing causal events *mathematically*" (Lasswitz, 1888a, 16). The thought-instrument of causality presupposes the changeability of things. However, the thought-instrument of substantiality cannot understand the 'becoming' as Eleatic philosophy clearly shows. "The thing either remains unchanged, or it is no longer the thing" (17). The 'transition' itself is not comprehensible. Thus, there is no mediation between substantiality and causality, between what stays the same and what it becomes different. A new thought-instrument is necessary to understand the moment of passage. Lasswitz called it the *Denkmittel* of variability.

The *Denkmittel* of variability was discovered in the attempt to overcome the Eleatic objection against the possibility of motion, the claim that the arrow is at rest during each instant. As we have seen, the intensive notion of the infinitesimal came to the rescue. In the instant where there is no motion, one can still define a tendency to continue with the same velocity. Nevertheless, "[t]he formulation of the principle of intensive magnitude and its connection with the problem of continuity and the infinitesimal method contains, despite Cohen's great merit, still something problematic" (29). Cohen should not have identified the dx with the 'intensive reality'; it would have been preferable to identify the intensive reality with the dy , that is, with the the derivative function $dy = f'(x)dx$. In Lasswitz's view, the connection of the intensive magnitude with the category of reality lies in the 'concept of function' (*Functionalbegriff*). Functional concepts presuppose 'variables'. Thus, Lasswitz preferred to speak of the thought-instrument of variability rather than of the category of 'reality':

The category of reality is thus contained in what we have called the thought-instrument of variability, something that is a unitary element in itself but has a tendency to change. [...] Without the thought-instrument of variability, the flying arrow would rest at every point of its trajectory. This thought-instrument permits the abstraction of extension without eliminating the tendency. [...] The latter is denoted mathematically by a differential, and the sign dy should be suitable for this, because according to mathematical school usage, dx means the differential of the independent variable, dy that of the function. The connection between the principle of intensive magnitude and the category of reality with the infinitesimal method only becomes clear through the reference to the concept of function [...] In order to maintain this distinction between dy and dx [...] we have chosen the neutral expression '*Denkmittel* of variability' [...] for the principle of the intensive magnitude. (29)

Lasswitz was confident that the choice of the derivative function dy instead of the differential dx did not contradict Cohen's claim that reality designates the thought-instrument that takes something as given independent of everything else (without relation) (30). The function expresses the law of development for something, thereby capturing its quality or reality (Lasswitz, 1888a). With this clarification, Lasswitz claimed to have addressed the issue that mathematicians had with Cohen's interpretation of dx (30).

The *Denkmittel* of variability provides a means of reconciling substantiality and causality, which ancient philosophy had been unable to achieve. The *Denkmittel* of substantiality requires the identity of the subject of motion over time, and this was expressed in the scientific concept of the 'atom'; on the other hand, the *Denkmittel* of variability required the possibility to define motion in the instant, and this

was achieved by the concept of the ‘differential’. By combing these two *Denkmittel* science was ultimately capable of accounting for the causal action of one atom to another. The importance of continuity lies not in matter distribution in space but in the distribution of motion over time.: “the world is no less continuous because it consists of atoms, as long as world events are continuous” (Lasswitz, 1888a, 21).

2.2 Galilei and the *Denkmittel* of Variability

Natorp was delighted by Lasswitz’s paper and looked forward to reading Lasswitz’s “research on the genesis of modern science” (Natorp to Lasswitz, Oct. 8, 1888; CN, Vol. 2, Doc. 15). Indeed, Lasswitz’s philosophical utterances were not for their own sake. On the one hand, Lasswitz claimed to have ‘found’ the thought-instrument of substantiality of variability, causality, etc. by investigating the history of science. On the other hand, Lasswitz used those *Denkmittel* as interpretive tools in his work as a historian. Indeed, Lasswitz, not without some clumsiness, tended to present figures in the history of science for their contribution to the definition of a particular *Denkmittel*. A two-part paper on Galilei’s theory of matter (Lasswitz, 1888b) published in 1888 is a typical example of this Lasswitz’s style of making history of science.

According to Lasswitz, Galilei’s outstanding achievement was the scientific expression of the *Denkmittel* of variability in the history of modern science. Galilei is often credited for having formulated the ‘principle of virtual velocities.’ When bodies are in equilibrium, ‘there is already the moment as a tendency to fall’ so gilt ihm doch auch hier bereits das Moment als die Neigung zum Sinken (470). Bodies are trying to descend but are mutually hindered and hence have not achieved any actual motion yet. Two bodies are in equilibrium with each other if they are in inverse ratio to the velocity that the weights would acquire simultaneously by motion compatible with the constraints. The product mv is called ‘moment’. The general cause of equilibrium is the equality of *moments*, not the equality of absolute weights. Galilei discovered that ‘the concept of velocity is already present here, even if only in a virtual sense’ es liegt also schon der Begriff der Geschwindigkeit, wenn auch im virtuellen Sinne (470).

Galilei was then able to transform the ‘static’ concept of ‘moment’ into its ‘dynamic’ counterpart by postulating that one can define velocity in every arbitrarily (*beliebig klein*) small part of time. In this way, a pure rational element is substituted for a sensible and intuitive one. Galilei assumed that the “quality of velocity is not eliminated with the quantity of time” but remains as what “characterizes the process of motion” as such (473). In this way, “through the conceptual characterization of the intensity of motion” Galilei was able to explain how “in the unit of time the still remains tendency [to motion]” even if there is no change of position, i.e., no motion (473). The Eleatic objection to the motion was overcome. By modeling his dynamical concept of ‘moments of velocity’ upon the static concept of ‘moments of weight,’ Galilei was able to understand acceleration; the motion of a free falling body can be regarded as an aggregate of infinitely many momenta.

2.3 Gassendi and the *Denkmittel* of Substantiality

As Cohen himself conceded (Cohen to Lasswitz, Dec. 6, 1888; CN, Vol. 2, Doc. 16), Lasswitz was able to incorporate Cohen’s insight into his historiographical work successfully. However, in the second part of the paper, published a year later (Lasswitz, 1889a), Lasswitz articulated his objection against Cohen’s support of dynamical theories of matter. The thought-instrument of variability, which was applied successfully to *Zeiterfüllung*, ‘the filling of time,’ cannot be expected to apply with equal success to the *Raumerfüllung*, the ‘filling of space’ (Lasswitz, 1889a). Galilei attempted to solve the problem of the rarefaction and condensation of matter by resorting to the *rota Aristotelis*: a body could be composed of an infinite number of unquantifiable atoms, or non-quanta just as the total speed of a body is the sum of an infinite number of indivisibles of speed. The transition from the solid to the liquid state is comparable to the transition from rest to motion (Palmerino, 2001). According to Lasswitz, endeavors of this nature are doomed to failure.

If Galilei’s approach were pursued systematically, it would lead to the “theory of intensive points” formulated by Boscovich in a Newtonian setting or to its Kantian plethoric version (Lasswitz, 1889a, 45). Dynamical theories of this kind, however, fail in the “individualization of matter into closed bodies” (143). Therefore, the sole thought-instrument of variability is not sufficient for developing a proper theory of matter. The predicate of *common* motion of the parts of a space quantum can only be attributed to it by thought-instrument of substantiality [Das Prädikat gemeinsamer Bewegung der Theile eines Raumquantums kann nur durch das *Denkmittel* der Substantialität ihm beigelegt werden, so dass ein

einbeitliches Mossen] (Lasswitz, 1889b, 46). As we have seen, the latter leads to the corpuscle, and ultimately to the atom.

In a paper published the same year, Lasswitz credited Pierre Gassendi, Galileo’s contemporary, with grasping the issue at stake. Gassendi introduced the property of ‘solidity’, which includes impenetrability and rigidity, as the basic features of atoms. According to Lasswitz “one would not grasp the concept of solidity adequately if one wanted to understand it as the idealization of a sensual property, hardness” (460). For Gassendi, solidity is the expression for the property of the parts of space, through which they are “space-asserting individuals [*raumbehauptende Individuen*]” (461). The solidity is the instantiation of the requirement of the *Denkmittel* of substantiality, it is the conditions without which the individualization of matter would not be possible.

Gassendi understood that solid atoms were necessary as the proper ‘subjects’ of motion; however, he failed to grasp the reciprocal actions of atoms and how they exchange motion. According to Gassendi, atoms possess a *vis motrix* as an intrinsic and essential property, which they can never gain or lose (see, e.g. LoLordo, 2008). An atom may be temporarily impeded by an obstacle, but as soon as it is released, it resumes its natural velocity. The different velocities observed in nature are due to the alteration of rest and movement ratios, much like how varying densities occur from the combination of emptiness and fullness. In Lasswitz’s parlance, Gassendi did not have the *Denkmittel* of variability; thus, he could not grasp how the *vis motrix*, however, measured, could be distributed among atoms. Consequently, Gassendi had to attribute different shapes to atoms to account for the variety of observed phenomena. As Lasswitz noted, “[h]e unequivocally established the substantial independence of the atom; but he failed at the concept of velocity” (Lasswitz, 1889b, 468).

3 Lasswitz on the History of Kinetic Atomistic

The contours of Lasswitz’s somewhat simplistic historical scheme were starting to become apparent. Galileo successfully applied the *Denkmittel* of variability to motion, but he failed to appreciate the importance *Denkmittel* of substantiality to define the subject of motion; Gassendi was guilty of the opposite mistake: “the cross-pollination could only take place in the future; Huygens made it possible by exhibiting the principles of mechanics” (470). By that time, Lasswitz had already concluded to collect the results of these and previous historical investigations (Dilthey to Lasswitz, Dec. 1, 1888; Dilthey, 2011–22, Vol. 2, Doc. 718) in a monumental two-volume *Geschichte der Atomistik*. The *Vorwort* is dated October 1889 (Lasswitz, 1890). A few weeks later, Natorp wrote to Lasswitz that he had seen the first volume on Cohen’s desk and was already planning to write a review (Natorp to Lasswitz, Dec. 22, 1889; CN, Vol. 2, Doc. 18).

Geschichte der Atomistik represents the crowning moment of Lasswitz’s decades-long studies on atomism. The present paper cannot adequately convey the wealth of information contained in a thousand-page account of the history of atomism, stretching from the church fathers to Newton. However, the book has a recognizable protagonist. Between Galileo and Descartes at the beginning of 1600, on the one hand, and on the other hand, Leibniz and Newton at the turn of 1700, Lasswitz indicated in Huygens as the central figure of modern science. Lasswitz’s choice is far from being obvious. Even today Huygens remained surprisingly under-researched. Huygens appeared to Lasswitz as the last stage was of a three-step historical-philosophical scheme: “The objectification of the sensation to the law-like moving atomic world takes place in the development, which is designated by the three names: Gassendi, Galileo, Huygens ” (Lasswitz, 1890, 2:376).

Huygens’ great achievement was to show how “[t]he *Denkmittel* of variability does not apply only to the change of the velocity of a single body”, as in the case of Galileo, but to the “distribution of velocities” (2:378) among invariable atoms. In particular, Huygens must be credited for having established laws of motion of the atoms. Atoms act upon one another by ‘collision’ (*Stoss*). According to Lasswitz, for Huygens ‘collision’ meant nothing but the fact that the motion of two atoms after their encounter is determined ‘univocally’ (*eindeutig*) by their motion before their encounter. Huygens was able to prove that the principle of conservation of the algebraic sum of momenta $\sum mv$ and of *vis viva* $\sum mv^2$ are the necessary conditions of the possibility of determining the motion uniquely. The rigid atom provides the subject of motion, and the laws of mechanics regulate the continuous exchange of velocities among atoms. Here, for the first time, the essential conceptual tools necessary to separate objective reality from the ever-changing flux nature of our experiences are brought together in a coherent form.

3.1 Huygens and the Peak of Kinetic Atomism

It is well-known that in his 1692-94 correspondence with Huygens, Leibniz pointed out the elephant in the room of Huygens' atomism (see Lange, 1873, 2:202). If one assumes that Huygens' collision laws apply to atoms, one is in front of a dilemma (Lasswitz, 1890, 2:367): (a) the atoms must either themselves be elastic (b) *vis viva* must be at least partly lost in their collision. However, (a) is not possible since the atoms per definition do not have movable parts; (b) contradicts the principle of the conservation of *vis viva* from which Huygens derived his collision rules: "Therefore, says Leibniz and it is said in general, absolutely unchangeable atoms are an absurdity" (2:367). However, Huygens denied that the alternative between (a) and (b) is exhaustive. He insisted that atoms must be absolutely hard and still not lose none of their motion in their impact: Huygens promises to explain his view *un jour*. "Has that day appeared? Or have these reasons remained hidden from us forever?" (2:367).

Huygens never explained hard atoms' rebound in any letters to Leibniz or other published works during his lifetime. However, Lasswitz claimed that Huygens did provide an 'explanation' in his posthumous book on impact. Huygens' 'explanation' was however nothing but the 'proof' (*Beweis*) of his rules of collision. According to Lasswitz, Leibniz saw that Huygens' collision rules applied to the behavior of perfectly elastic macroscopic bodies; thus, he inferred that atoms must be elastic (Blackwell, 1977, see). On the contrary, Huygens derived the rules of collision from the principles of mechanics; thus, he inferred that atoms must satisfy those rules: "These principles of mechanics are the conditions for the possibility of atomism and that the laws of collision are derived from them, not the other way round, that the movement of matter presupposes the laws of collision" (Lasswitz, 1890, 2:368). The two principles of mechanics are justified by the fact that they are necessary conditions that allow the unambiguous determination of the velocity of atoms after the impact if now their velocities before the impact:

Huygens' assumptions are therefore equivalent to these two principles of mechanics, the law of the conservation of the center of gravity and that of the conservation of energy. Even if they first appear here in the form of theorems [*Lehrsätze*], this is only an accidental circumstance of the formulation. What is essential and decisive in Huygens is that he does not start from sensible intuitions or anthropomorphic representations but from mechanical facts. The principles of mechanics are fundamental because they are necessary and sufficient to univocally [*eindeutig*] determine the motions of bodies, to calculate their velocities and directions, if those before the collision are given. It is not because bodies are elastic that their *vis viva* is conserved after the impact; but because *vis viva* must be conserved, the impact occurs in the way observed in bodies which we call elastic. The elastic displaceability of the parts, this sensuous fact, is not a condition of the laws of impact. Huygens does not call the bodies he is dealing with elastic but hard; and this does not mean a sensuous property, but the same concept of solidity, the property of the substance to assert its space unchangeably. The space-assertion [*Raumbehauptung*] of individual substances and the principles of mechanics are therefore made by Huygens the basis of the theory of matter; from them, he derived the laws for the modification of the motion of atoms. The fact that we find the same laws in the sensuous impact of elastic bodies is entirely irrelevant.

Huygens did not transfer empirical laws of collision to elastic bodies to the atoms; on the contrary, Huygens' fixed the condition that any empirical law of collision among atoms must satisfy if their velocity after the impact must be univocally determined. In this sense, Huygens neither considers 'hardness' as a sensory property of the macroscopic body; nor he attempted to construct 'hardness' by analyzing the mutual forces that keep the shape and volume of the single atoms in equilibrium. According to Lasswitz, the meaning of the term 'hard' is *implicitly defined* by the two conservation principles (Mormino, 1996, see, e.g.):

But the exchange of velocities is mathematically defined by the principles of mechanics, since $\sum mv$ and $\sum mv^2$ are constant quantities. What happens to the atoms when they meet is a question that should be dismissed as entirely impermissible, as it presupposes a sensible intuition of the atoms. The latter are represented as small, hard bodies that collide, as we observe in the case of sensible bodies. However, this is precisely where Huygens' made progress in turning the corpuscular theory into a science, as he overcame this sensible conception and replaced it with rational and mathematically formulated concepts. The absolute atom and the totality of atoms in motion are theoretical entities (*[begriffliche Gebilde]*), and their encounter in space no longer signifies the anthropomorphism of their clash, but rather the geometric determination of their position at a given time. Moreover, their behavior after the so-called collision is not deduced from the analogy of the rebounding of macroscopic bodies but rather determined by the mathematical formula that regulates the distribution of velocities. As long as one ponders what must happen when immutable bodies meet in the analogy of sensible bodies, one is lost in utterly sterile speculation.

In Huygens' view, the laws of collisions determine the values of the atoms' velocities *just before and after* the collision. Any attempt to investigate what happens *during* the collision of atoms itself is based on

the confusion with the collision of atoms and that between small macroscopic bodies. Atoms are not the ultimate parts of macroscopic matter but the necessary conditions of any mechanical explanation of the behavior of macroscopic matter. Gassendi's 'atoms' were introduced as an expression of the *Denkmittel* of substantiality, to assure the individuality of the moving parcels of space (Lasswitz, 1890, 2:379). Galileo's 'moment' was introduced to satisfy the requirement of the *Denkmittel* of variability so that the velocity of a falling body is defined at each instant (2:379). Huygens' merit was to have combined these two *Denkmittel* by formulating the laws of the distribution of velocities among atoms. In this way, for the first time, he allowed for the mathematical representation of the continuous causality and reciprocal action among atoms (2:380).

3.2 *The Decline of Kinetic Atomism. Leibniz and Newton*

Lasswitz credited Huygens transformed ancient philosophical atomism into a modern scientific theory. However, contemporaries fundamentally misunderstood Huygens' result. If atoms are perfectly hard, it was argued, then all changes in velocity would have to be discontinuous, as two atoms colliding would have to instantly alter their velocities if they did not experience any deformation from the collision. Thus, the velocity would not be defined univocally at the moment of the impact, contrary to Huygens' claim. With some simplification, one can argue that, according to Lasswitz, two distinct approaches have been taken to address this issue. The first approach, as championed by Leibniz, relied on contact action. The second, drawing from Newton's work, relied on distant action.

Leibnizian program. Leibniz famously argued that the notion of perfectly hard atoms colliding is inconsistent with the principle of continuity. The latter requires elasticity, which in turn presupposes movable parts. Leibniz concluded that matter must be an elastic plenum, infinitely divided by the different motions of its parts. As a result, however small, no part of matter remains the same for no longer than a moment. Thus, it becomes impossible to determine the identity and individuality of parcels of matter: "What distinguishes one particle from another in motion, what gives unity to the moving part of space?" (2:239). Since he could not define the identity of the subject of motion by applying the thought-instrument of substantiality to space elements, Leibniz was compelled to refer to the time element. Leibniz was left with the sole thought-instrument of variability and identified the substance with the 'force', the 'constant tendency' to continue from state to state (Willmann, 2012).

Newtonian program. After the success of his theory of gravitation based on the notion of attraction at a distance, Newton attributed to atoms forces of attraction and repulsion (Lasswitz, 1890, 2:480). Boscovich carried out the full implications of Newton's program. Like Leibniz, he recognized the conflict between continuity and atomism. However, he avoided the idea of an infinite regression of elastic parts by conjecturing that the basic elements of matter must be just simple points (2:563) endowed with repulsive forces. In this way, the possibility of applying the *Denkmittel* of substantiality is lost, and one is left with the sole variability. For this reason, as Lasswitz had repeatedly argued, dynamic theories of matter *à la* Boscovich are unable to explain how "a unitary mass particle is formed [...] how a sum of such points could achieve unitary movement" (2:52)

Lasswitz concluded that "the great physicist and mathematician Newton arrived at the same result as the great philosopher and mathematician Leibniz" (2:580). Whereas the Leibnizian program inevitably leads 'metaphysical' monadology, the Newtonian program leads to a physical 'monadology'.

Both programs appeared to Lasswitz as an epistemological regress with respect to Huygens' kinetic atomism. Both were the consequence of an attempt to reply to an apparently cogent objection against atomism. If atoms were hard, their velocity at the moment of impact would be undefined. However, Lasswitz argued, "this objection rests only on the old mistake of bringing sensible intuition back into the motion of those theoretical entities (*rationalen Gebilde*) that we call atoms" (2:380). Interaction between substances in space "is not at all sensually representable; it is a transcendental principle of experience" (2:380). If one tries to describe two atoms at the moment of their collision, one proceeds in the same way as if one tries to imagine the flying arrow at a point on its path. Discontinuity lies in the subject of the motion, not in the motion itself (2:379).

3.3 Condition and Ideal. Critical Philosophy and the History of Science

Lasswitz made no secret that his work was a textbook application of what (Cohen, 1885) had called the ‘transcendental method’. Critical philosophy starts with the “fact of science [*Faktum der Wissenschaft*]” (Lasswitz, 1890, 2:383) and searches for the conditions of its possibility. According to Lasswitz, the history of atomism offers the “suitable material for studying the thought-instruments on which mechanical natural science is based and for learning about the transcendental conditions for the possibility of an objective nature” (2:384).

Those thought-instruments are not derived *a priori* from some higher instance; they are found *a posteriori* in the history of science: “The discovery of the principles of mechanics is the *empirical fact* on which critical idealism could be based” (2:384). As a consequence, the critical philosophers must always be aware that their conclusions are provisional:

Critical philosophy cannot define the conditions of experience and the principles of physics *a priori*. Instead, it can do so only through the historical process. Just as physical knowledge changes, the theory of transcendental conditions of experience will also change over time. The essential difference between the transcendental principles and the changing theories is not in how the principles of scientific knowledge are formulated in the consciousness of a given epoch, but in the fact that they must be formulated. There is an eternal determination for the direction of consciousness, a supreme law of objectivization. It is an insoluble problem to predict which intellectual instruments will be discovered and which ones will vanish from human consciousness. However, each cultural epoch becomes conscious of its own intellectual instruments as the synthetic unities that guarantee the possibility of scientific experience amid the vacillations and gropings of special investigations and hypotheses. This is achieved by showing the shifting theoretical content to be dependent not only on empirical accident, but also on an enduring trend of consciousness. (2:393)

Lasswitz was aware that his ‘transcendental deduction’ crucially depended on his hypothesis that “the epistemological foundations of physics are complete with Huygens ” (2:384). Indeed, as Lasswitz expressly pointed out, Huygens’ kinetic atomism plays a double role in his system: (1) it is a *condition* (*Bedingung*) of physics, it encapsulates the conditions of possibility of physics in their uncontaminated form; (2) it represents the *ideal* (*Ideal*) of physics that any physical theories must ultimately converge.

Kinetic atomism aims to describe all physical processes without assuming either force or potential energy. Lasswitz conceded that, in this form, kinetic atomism was at most able to account for the behavior of monatomic gases. For more complex cases, 19th century physics could not avoid falling back on the hybrid Newtonian atomism, which introduces potential energy and central forces acting at a distance between material particles. For Lasswitz, this was, however, only a provisional compromise. Unfortunately, modern physics was still “far away” from the Huygensian ideal (2:394). At the same time, Lasswitz saw some encouraging signs that physics was “was edging closer” to its aim (2:394). The development of modern energetics⁷ showed the tendency of replacing central forces with the spatial distribution of different types of energy (Lasswitz, 1893). This conclusion is somewhat surprising given the anti-atomistic bent of energetics (Ostwald, 1895). As far as I can see, Hertz’s forceless mechanics (Hertz, 1894) came closer to Lasswitz’s ideal of a physical theory. However, what is more surprising is Lasswitz’s resistance to update that ideal in light of the changes that the ‘fact of science’ underwent in the 19th century.

4 The Reception of the *Geschichte der Atomistik* within the Marburg School

In Marburg, the *Geschichte der Atomistik* was perceived as the most convincing attempt to develop further the spirit of “Cohen’s path-breaking book about the principle of the infinitesimal method” (Elsas, 1891, 301). Nevertheless, Lasswitz translation of Cohen’s category of reality into the thought-instrument of variability was not considered fully convincing. The strict opposition between the *Denkmittel* of variability and *Denkmittel* of substantiality should have been replaced with a reciprocal correlation. There was, however, disagreement on the specific form this correlation should take. Even though Natorp and Cohen went to the trouble to address these issues publicly and privately, it appears that both scholars delegated the task of formulating a more detailed rebuttal to Lasswitz to their doctoral students. Notably, the young Cassirer expanded Natorp’s line of reasoning, while Cohen entrusted one of his favored pupils, Otto Buek, with the task. However, the two lines of argument diverged markedly, foreshadowing a widening rift between Cohen’s and Natorp’s philosophical stances (see CN).

⁷See Lasswitz to Ostwald, Apr. 9, 1892; Ostwald, 1961, Doc. 117.

4.1 Natorp and Cassirer

Natorp (1891a) wrote an extended review of the *Geschichte der Atomistik*. The review is especially noteworthy because it was based on Natorp's paper 'Quantität und Qualität' (Natorp, 1891b), his first attempt to a systematic contribution to philosophy, which appeared in the same issue of the *Philosophische Monatshefte*. Natorp agreed with Lasswitz that the central concept in calculus is the 'differential quotient' dy/dx not the isolate differential dx , as in Cohen's original controversial formulation (153). Just like Lasswitz, Natorp insisted that the differential quotient is not at all a quotient in which dy and dx have any independent meaning; it is actually a new function, the derivative function, $y = f'(x)dx$. This functional relationship is "the 'internal source' law according to which the creation of quantity is thought to happen" (Natorp, 1891a, 345). The concept of 'law' embodies the idea that quality takes precedence over quantity.

In this sense, according to Natorp, the contrast between quantity and quality was more fundamental than Lasswitz's opposition between variability and substantiality. In particular, Natorp argued that there seems to be no reason why 'variability' should be related to time, while 'substantiality' is related to the extension of space (347). Indeed, the notion of point mass was sufficient for the individuality of the subject of motion without the need to introduce the notion of the extended atom. The notion of substance requires only the constancy of something in general; this 'something' might be the 'mass'. However, Natorp could resort to the authority of Wilhelm Weber⁸ who admitted that the idea of spatial expansion is not necessarily attached to the concept of mass (Natorp, 1891a, 348).

Despite their differences, Natorp appreciated, that he could discuss with Lasswitz on a common ground, the 'fertile bathos of experience' (Natorp to Lasswitz, Jun. 3, 1891; CN, Vol. 2, Doc. 19). Ultimately, these divergences could only be reconciled through a thorough examination of their respective underlying systematic assumptions. Indeed, by sending his review to Lasswitz, Natorp for the first time laid down his project of extending his 'deduction' of the fundamental categories beyond quantity and quality to derive the notion of substance and causality (Natorp to Lasswitz, Jun. 3, 1891; Vol. 2, Doc. 19; Lasswitz to Natorp, Jun. 27, 1891; Vol. 2, Doc. 20). To some extent, Natorp's response to Lasswitz was further developed by Cassirer, who had relocated to Marburg in the winter of 1896. At around, Cohen started to speak of a "little school" that was gathering in Marburg (Cohen to Natorp, Apr. 19, 1897; Vol. 2, Doc. 42). Cohen and Natorp utilized a series of philosophical prizes (*Preisaufgaben*) to support their students, and it was through this mechanism that Cassirer received his first recognition, winning the award for the 1898-1899 academic year (1:382).

Cassirer used the first part of the prize essay as the basis for his dissertation on Descartes (Cassirer, 1899), which became the first chapter of Cassirer's book on Leibniz, *Leibniz' System in seinen wissenschaftlichen Grundlagen*, published in 1902 (Cassirer, 1902). The young Cassirer's pages on Leibniz's dynamics confirms that he aligned with Lasswitz's interpretation of Cohen's 1883 book (**Giovanelli2022b**). The reality of motion that is lost when motion is only a geometrical change of position "is obtained again by fixing the overall process in the single temporal element [*im einheitlichen Zeitmoment*]" (291). It was Lasswitz's merit to have understood the connection between this 'concept of tendency' [*Tendenzbegriff*] and the modern concept of energy (336). However, Lasswitz restricts this connection to Huygens' foundation of kinetic atomism, while he does not attribute any original part to the dynamics of Leibniz in the preparation of the new ideas. Indeed, as we have seen, Lasswitz accused Leibniz of having "*substantialized* the force as the cause of motion" (336).

However, in Cassirer's view, this criticism was unjustified. In supporting Huygens' atomism against Leibniz, Lasswitz misunderstood "the proper meaning of Leibniz's concept of substance" (336–337). The reason should be searched in the opposition between Lasswitz as a 'systematic philosopher' and Lasswitz as an 'historian of science':

Lasswitz' critique of Leibniz's concept of substance is based on Lasswitz's own systematic. According to the latter, 'substantiality' and 'variability' are two separate thought-instrument [*Denkmittel*], which in their application serve essentially different problems. [...] Lasswitz' thought-instrument of substantiality essentially exhausts itself in the function of the spatially constant 'thing', while the concept of the law only comes into its own in the method of variability. However, this immediately calls into question the justification for coordinating the two basic ideas. Because the thing and the law are not aligned, but belong to different levels of epistemological consideration [...] Lasswitz himself carried out this general thought in the development of the concept of nature in an excellent way: Here in the philosophical foundation he is in closer agreement with Leibniz's principles than in his special criticism, which sees the fulfillment of the idea of substance in the concept of the atom. One is justified in requiring that a factor within the process of motion is single out

⁸Natorp refers to a letter of Wilhelm Weber's cited in (Fechner, 1864, 89).

that serves as a ‘subject of the motion’ in addition to the intensive magnitude of the speed. This requirement can, however, be fulfilled without the extensive quantity already being assumed in this subject. Leibniz can do without the concept of the atom because he has another logical tool at his disposal for expressing the constancy of the mobile and for the problem of matter. (Cassirer, 1902, 337f.)

This logical tool is the concept of ‘mass’. In Cassirer’s anachronistic reading, Leibniz recognized that different bodies might require different amounts of work to attain the same velocity, and he introduced ‘mass’ as the factor of proportionality between work and velocity: “The constancy of this factor is the objective-scientific expression of the problem of the constancy of the body” (340). Since mass was now understood as a purely dynamic relation, Cassirer, like Natorp, concluded, the concept of extension became irrelevant, and the concept of mass was reduced to that of a mass point (342). There where Lasswitz saw the opposition between variability and substantiality, Cassirer glimpsed the emergence of the general tendency to substitute the “concept of being [*Seinsbegriff*] with with the *concept of function* [*Funktionsbegriff*]” (539).

4.2 Cohen and Buek

According to Natorp, Cohen too initially planned to write a review of *Geschichte der Atomistik* (Natorp to Lasswitz, Dec. 22, 1889; CN, Vol. 2, Doc. 18). However, he must have ultimately left the task to Natorp himself. Still, Cohen held a genuine appreciation for Lasswitz’s creative ‘appropriation’ of his interpretation of the concept of infinitesimals: “Your work not only instructed and encouraged me a lot, also in the systematic; I was also deeply pleased that my basic views were not shaken” (Cohen to Lasswitz, Jul. 27, 1891)²¹. Indeed, Cohen recognized that Lasswitz had successfully made his view of the intensive infinitesimal more accessible outside the Marburg circle. However, Cohen was quick to point out that Lasswitz had watered it down in the process: “I see more clearly than the others that we are fundamentally in agreement. As a result, I also believe that I can understand where your point of view differs from mine” (Cohen to Lasswitz, Jul. 27, 1891; Vol. 2, Doc. 21)

In particular, Cohen did not fully agree with Lasswitz’s use of the term ‘variability’: “I know that your ‘variability’ wants to and can lead to an effective improvement, but I could not truly convince myself of the accuracy of its formulation” (Cohen to Lasswitz, Jul. 27, 1891)²¹. As we have seen, Lasswitz had suggested substituting the dx , with the dy , with derivative function. However, for Cohen the issue at stake was more fundamental. As Lasswitz himself had sensed, Cohen aimed to ground the ‘reality’ of x itself and not merely the relation between two variables x and y as they are already given (see also Elsas, 1891). Cohen aimed to elucidate this point more thoroughly in a future publication: “I hope that you will soon encourage me to improve the formulation and justification of the intensive reality” (Cohen to Lasswitz, Jul. 27, 1891; CN, Vol. 2, Doc. 21).

Cohen addressed Lasswitz’s work in public writings in a long “Einleitung mit kritischem Nachtrag” to the 5th edition of Lange’s *Geschichte des Materialismus* (Cohen, 1896) published in 1896. Cohen expressed again appreciation for Lasswitz’s attempts to present his interpretation of the infinitesimal “in his excellent work *Die Geschichte der Atomistik*” (XLVII). However, he considered Lasswitz’s presentation as a “weakening” where the “agreement with which he has supported my theory, has at the same time compromised it again” (XLVII). Cohen believed that in Lasswitz’s work, “the concept of reality [is reduced] to a *concept of relation of variability*” (XLVII). The issue at stake is not simply the functional relation between the variables x and y , but rather the constitution of x itself before it enters into a relation with y . The x as the extensive ‘real’ cannot be taken as given; it must be produced from its *origin*, its intensive ‘reality’, the dx . The notion of intensive reality is presented here for the first time as a special case of the more fundamental concept of ‘origin’ (*Ursprung*) (XLVI) that will become a trademark of Cohen’s philosophy. It was to this concept that Cohen was alluding to when he promised to Lasswitz an improved formulation of the notion of ‘intensive reality’ (Cohen to Lasswitz, Jul. 27, 1891; CN, Vol. 2, Doc. 21).

From this point of view, Cohen found Lasswitz’s separation between ‘substantiality’ and ‘variability,’ between the atom as the subject of motion and the process of motion unacceptable. According to Cohen, just as motion, matter itself must be ‘produced’ from its ‘origin’, the unextended point. Cohen believed to have found in Michael Faraday’s (1844) theory of matter the scientific counterpart to this idea (Cohen, 1896, XXIX). Cohen’s reference to the field concept was brief but not isolated. In 1902, another philosophical *Preisaufgabe* was announced, in which candidates were asked to explicitly deal with Faraday’s relation to Boscovich’s theory of matter (Sieg, 1994, 501). The prize went to the ‘cand. phil’

Otto Buek (1873-1956), a German-Russian scholar born who had landed at Marburg for his doctorate (Sieg, 1994, 207ff.). Responding to the *Preisaufrage*, Buek developed Cohen's insights about the field concept and the Maxwell-Faraday theory of electricity, in what would later become his dissertation, *Die Atomistik und Faradays Begriff der Materie*, to appear in the in 1905.

In reconstructing the origin of the field concept in Faraday's work, Buek homed in on the fact that Faraday—who in the 1830s had employed the concept of lines of force merely to represent the alignment of polarized particles—later tried to overcome the problem of explaining the mode of transmission of the force between contiguous particles by assuming that matter will be continuous throughout, thus eliminating the difference between particles and any intervening space (Faraday, 1844). Historically, Buek attempted to trace Faraday's treatment of material particles as nothing but point-centers of converging lines of force back to Boscovich's dynamic theory of point-atomism, as it was developed in the mid-eighteenth century.⁹

Even if Buek was careful not to burden his historical reconstruction with philosophical debate internal to Neo-Kantianism, one of his polemical targets was clearly Lasswitz's 'critical atomism.' Buek explicitly opposed Lasswitz's separation of the subject of motion from motion itself:

However, Lasswitz attempted to provide a deduction for the application of a fixed piece of space to the subject of motion, an undertaking that originated from an incorrect abstraction. The atom is supposed to represent substantiality for itself, while motion is solely subject to the means of variability. However, according to this view, substance is not understood as a functional relationship to motion, but has been given the power to produce a tangible structure, such as the small body of the atom, solely from itself. Also, the rebounding of the hard corpuscles in the collision is simply stated as a principle. This is, however, not permissible, because the principles of mechanics cannot be overridden, even if the dividing of matter descends below the threshold of sensory perception. Atoms, as elementary particles, must not be subject to different laws than large bodies. (Buek, 1904)

Lasswitz attempted to condense Cohen's connection between intensity, reality and the infinitesimal in his *Denkmittel* of variability, and applied it to the process of motion alone; however, in doing so he hypostatized in the extended atom the subject of motion as what remains identical in the process of change. Huygens' 'kinetic atomism' was the historical realization of this epistemological model. Faraday's critique of atomism could be presented by Buek as a historical example of an alternative theory of matter, in which Cohen's connection between intensity, reality and the infinitesimal could find universal application. In Faraday's view, there was no reason to suppose that hard particles exist over and above fields of force. Instead, particles are merely regions of the field, and a moving particle is simply a changing field.

Conclusion

As we have shown, Natorp was compelled to further develop his opposition between quantity and quality while corresponding with Lasswitz, foreshadowing his later deduction of the categories of substance and causality that appeared in his published work nearly a decade later (Natorp, 1900). Similarly, it was in reaction to Lasswitz that Cohen introduced the notion of 'origin' for the first time, which became the cornerstone of his system (Cohen, 1902). To the best of my knowledge, Lasswitz never responded in any of his published works. However, the relationships with Marburg group remain friendly. As late as 1910, Natorp wrote to Lasswitz informing him of the release of *Die logischen Grundlagen der exakten Wissenschaften* (Natorp, 1910). He recounted that the Marburg school was increasingly becoming a significant player in the German philosophical landscape (Natorp to Lasswitz, Feb. 2, 1910; CN, Vol. 2, Doc. 121). Regrettably, Lasswitz did not live long enough to witness the golden days of Marburg neo-Kantian. He died in October 1910.

Cassirer published his celebrated *Substanzbegriff und Funktionsbegriff* (Cassirer, 1910) around the same time. In this book, Cassirer, with his customary conciliatory approach, attempted to settle the debate with Lasswitz. The history of atomism is presented not as the result of an opposition between 'variability' and 'substantiality', but rather as another example of the transition from 'substance concept' to 'functional concept' in the history of science (208f.; tr. 1923, 156f.). The size and shape of atoms progressively disappeared, and ultimately, the extensive atom was replaced by the unextended material points, particles whose size can be neglected when, e.g., rotation is not taken into account: "The postulate of *identity*, which is of course inevitable, is here satisfied not by any kind of material substratum, but but

⁹Boscovich's influence on Faraday is usually downplayed in recent literature cf. Spencer, 1967.

by permanent *forms of motion*” (Cassirer, 1910, 24; tr. 1923, 161). Although Cassirer only briefly touches upon this postulate, its importance reemerged in a different form in the 1920s when relativity theory imposed the preeminence of the concept of the field over that of the particle (Cassirer, 1921).

In the third volume of the *Philosophie der symbolischen Formen*, completed in 1927, Cassirer made reference to Einstein’s 1920 Leiden lecture (Einstein, 1920). In relativity theory, Einstein argued, unlike the parts of matter, the parts of the field lack individual identities that can be tracked over time (Cassirer, 1929, 547; tr. 1957, 467). It is worth noting that Cassirer underscored the novelty of this perspective by drawing a comparison with Lasswitz’s portrayal of Huygens’ atomism:

If we approach modern relativistic with the general philosophical attitude expressed in these lines and compare it with Lasswitz’s picture of kinetic atomism, the salient features of the theoretical change undergone by physics in the last decades stand out in a peculiarly incisive and instructive manner. Modern physics cannot dispense with Lasswitz’ two basic intellectual instruments, ‘substantiality’ and ‘variability.’ But in making use of these instruments, it moves them into a new systematic relationship. It can no longer separate them by relating substance essentially and primarily to space and change essentially to time. For this separation would imply that space and time themselves can be sharply differentiated in the construction of physics, that they confront each other as independent forms. It is the very questioning of this assumption that forms the beginning of relativistic physics. (Cassirer, 1929, 550; tr. 1957, 471)

From this perspective, the electromagnetic field is no aggregate of material points. We may speak of the parts of the field, but their individuality cannot be followed through time (see Reichenbach, 1928, 301f.). It is of course still possible to follow the individuality of material particles. However, in a field theory of matter, in which particles are nothing but high intensity field regions, it would be “no longer any meaning in speaking of one and the same matter at different times” (Cassirer, 1929, 552; tr. 1957, 473).

When the third volume of the *Philosophie der symbolischen Formen* was published in 1929, the dream of a field theory of matter had been superseded by the new quantum theory. However, as Cassirer noted a few years later, the renunciation of particle individuality “could not be retracted even in the quantum theory for which the problem presented itself in a new and more general sense” (Cassirer, 1936, 222; tr. 1956, 177). Once again, Cassirer contrasted this result with Lasswitz’s “excellent presentation of Huygens’ atomism” (Cassirer, 1936, 182; tr. 1956, 146). The real philosophical challenge of quantum theory was not the threat to the universal validity of the causality principle but to the requirement of ‘individuality’ of the subject of motion that Lasswitz identified as a necessary condition for the possibility of physics.

Abbreviations

- CN Holzhey, Helmut. 1986. *Cohen und Natorp*. Vol. 2. Basel: Schwabe.
- CPAE Einstein, Albert. 1987–. *The Collected Papers of Albert Einstein*. Edited by John Stachel et al. 15 vols. Princeton: Princeton University Press.
- ECW Cassirer, Ernst. 1998–. *Gesammelte Werke: Hamburger Ausgabe*. Edited by Birgit Recki. 26 vols. Hamburg: Meiner.

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