Symmetries As a Guide to Physical Ideology

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Abstract

It is a widely-held belief that (the values of) physical quantities are part of a theory's ideology. For example, it seems that special relativity has an ontology of spacetime points and particles, and an ideology of mass and charge properties. But these intuitions cannot be reconciled with the logical structure of physical theories. From the mathematical details of a theory such as special relativity, it turns out that mass and charge properties exist in quite the same way that particles exist: the theory quantifies over them. However, there is a different distinction in physics that can carry the same load, namely that between internal and external quantities. Roughly, the internal quantities depend on the external ones; external quantities instantiate internal ones. In contemporary physics, the values of physical quantities are internal. In this sense, the latter distinction supersedes the former. But ideology has not become irrevelant: we can identify it with the structure of a theory's (external and internal) spaces. Although we can not read off a theory's ideology from the formalism in the same way that we can read off its ontology, we can use symmetries to discover this structure.

1 Introduction

Quine (1951) once distinguished between *ontology* and *ideology.* The former concerns what exists; the latter what we can say about that which exists. For instance, the Eiffel Tower exists; we can say *of* the Eiffel Tower that it is made of iron, or that it is impressive. Although Quine's terminology seems to suggest that ideology is concerned with subjective ideas, I side with Sider (2011) that it is as objective a subject matter as ontology. I thus am what Cowling (2020) calls an *ideological realist*: I believe that "theories place ideological demands on the world and seek to capture the metaphysical structure of reality in a different but no less objective or 'worldly' way than ontological commitments." For example, if a theory has 'being positively charged' as an ideological primitive, then it says that as part of the world's fundamental structure there is such a thing as being positively charged.

The vast majority of discussions around this distinction concern *metaphysical* ideology. To name just a few, there are lively debates around the ideology of theories of modality (Lewis, 1986; Cowling, 2013), theories of mereology (Cowling, 2013; Sider, 2013) and theories of tense (Van Cleve, 2016; Finocchiaro, 2019). The ideology of *physical* theories, on the other hand, has generated much less discussion. Perhaps this is so because the issue seem clear-cut. In particular, it is a widely-held belief that *(the values of) physical quantities are part of a theory's ideology.* It seems rather natural to say, for instance, that special relativity has an ontology of spacetime points and particles, and an ideology of spatiotemporal relations between these points as well as mass and charge properties of particles. The particles exist; we can say of the particles that they are massive, or that they have a certain charge. (Since the ontological reality of spacetime is a controversial issue, I will mostly discuss particles and their properties here). The magnitudes of quantities such as mass and charge are ideological posits.

The following quotes show that this belief holds sway in contemporary philosophy of physics. Firstly, Martens (2019b) states that mass magnitudes are ideological posits of Newtonian Gravitation:

> [W]e will assume standard [Newtonian Gravitation] to comprise an ontology of n particles in a flat Newtonian space and time, with an ideology of absolute distances, velocities and finite, positive, non-zero masses.

Secondly, Sider (2011) holds that fields (which are quantities insofar as they assign field values to spacetime points) are ideological:

> [T]he fundamental ideology of the special theory of relativity differs from the fundamental ideology of Newtonian physics: in place of electrical, magnetic, spatial, and temporal ideology, the special theory has unified ideology for electromagnetism and unified ideology for spatiotemporal metrical structure.

Finally, Duerr and Read (2019) more broadly claim that:

> [A] specification of the theory's ideology [consists of] the degrees of freedom that form the complete state of possible objects of that kind at a point in time.

Here, the 'degrees of freedom' of an object include the quantities whose values specify that object's state.

I claim that these intuitions cannot be reconciled with the mathematical structure of our best physical theories. From the details of a theory such as classical mechanics or special relativity, it follows that values for mass and charge exist in quite the same way that particles exist.[1] The theory of classical mechanics, for instance, quantifies over mass values just as it quantifies over particles. Paying careful attention to the mathematical structure of such theories thus necessitates a radical reconfiguration of the distinction between ontology and ideology. I will argue for this claim in §2. Although this conclusion is familiar from the literature on the metaphysics of quantities, the quotes above show that it is not yet learnt by philosophers of physics.

However, there is a different distinction in physics which can carry the same load, namely that between *internal* and *external* quantities. Roughly, the internal quantities depend on the external ones; external quantities instantiate internal ones. In contemporary physics, the values of physical quantities are internal. There *is* a distinction between objects and quantities in physics; it is just not the distinction tracked by the ontology/ideology dichotomy. The latter is, in a sense, superseded by the internal/external distinction, which is discussed in §3. The internal/external distinction is of interest in itself. In particular, I will show that it is relational rather than intrinsic. This means that different formulations of the same theory could lead to different judgements on which quantities are internal and which ones are external; which entities 'have' and which ones 'are had'. For example, although typically field values are had by spacetime points, on certain (re)formulations of gravitational theory field values instead *have* a spacetime point as a location. Thus, the 'replacement' of the ontology/ideology distinction by the internal/external one is philosophically fruitful—and so far left unexplored.

This is not to say that ideology is of no importance in contemporary physics, or that ontology is all there is. Rather, in §4 I will show that ideology is closely related to the notion of *structure*, whose relevance to modern physics is well-known (see North (2021) and Dewar (2022) for two recent book-length treatments). The brief idea is that quantities form a structured *value space*, within which that quantity's values stand in certain relations to each other. The mass values 2 kg and 1 kg, for instance, stand in the relation

[1] In this paper I will only consider classical theories; it is a further question whether these claims generalise to quantum theories.

of one being twice as much as the other. These relations *are* properly considered part of the theory's ideology. The search for structures that 'match' a theory's dynamical equations is one of the most important questions in the philosophy of physics, so ideology remains of central importance.

Although it is impossible to 'read off' a theory's ideology—in this novel sense—from the formalism, I will suggest that *symmetries* function to indirectly probe this structure. As far as I am aware, this is the first time considerations of symmetries and structure are explicitly linked to the question of ideology. On my view, symmetries act as a guide towards the structure of physical quantities—as a guide towards a theory's ideology, analogous to Quine's syntactic criterion for a theory's ontology.

2 Reconfiguring the Ontology/Ideology Split

The aim of this section is to show that the values of physical quantities are ontology rather than ideology. Although this claim is contrary to popular belief, as the above quotes evidence, I take it to be the logical conclusion of a debate that took place in the 1980s. The debate concerns about what Field (1984) called the *problem of quantities*.[2] I will therefore be relatively brief here, and focus on the consequences for the traditional ontology/ideology distinction.

In brief, the problem of quantities is that a collection of predicates does not have any internal structure, unlike a collection of values of physical quantities. For example, mass has an order structure: one can order particles from less to more massive. This order relation, in order to *be* an order relation, has to satisfy certain axioms. These axioms quantify over the theory's mass predicates, thereby reducing ideology to ontology. The rest of this section elaborates on this chain of thought.

Quine famously argued that one can read off a theory's ontology from its formalism: "the ontology to which an (interpreted) theory is committed comprises all and only the objects over which the bound variables of the theory have to be construed as ranging in order that the statements affirmed in the theory be true" (Quine, 1951, 11). Of course, this criterion of ontological commitment is controversial, but that is not the point I wish to address here. The question is whether it is possible to formulate a similar criterion for a theory's ideology. Quine (1951) criticised a proposal due to Bergmann (1950), namely that the ideology to which a theory is committed comprises

[2] For the original debate, see Field (1984); Bigelow and Pargetter (1988); Mundy (1987); Armstrong (1988). For contemporary discussion, see Eddon (2013); Wolff (2020).

all and only the properties over which the bound *predicate variables* of the theory have to be construed as ranging in order that the statements affirmed in the theory be true. Quine's response is that the theory need not quantify into predicate position at all; as is well-known, Quine had a predilection for first-order theories. But it remains a question of interest which ideological commitment a theory carries. Quine's own suggestion was that a theory's ideology simply consists of the concepts expressed by the theory's predicates. Moreover, he pointed out that one can define said concepts either extensionally or intensionally; Quine himself of course preferred the former. I will not question this Quinean approach here.[3] Instead, I ask: how does this criterion fare for modern physical theories?

On the semantic view of theories, a theory is presented as a class of models. In the language of model theory, a *model* (or interpretation) of a theory T in a signature Σ consists of a domain D of objects and a function that assigns an extension to each of Σ's names and predicate symbols. We will assume without loss of generality that the theory does not contain names, since one can always replace a name with a unary predicate (Quine, 1960; Lewis, 1970). The models of an arbitrary theory T are then of the form:

$$\langle D, P_1, P_2, ..., P_n \rangle$$

where n ranges over the theory's predicate symbols, broadly understood to cover both monadic and polyadic predicates (i.e. relations). Following Quine, the elements of D over which T's quantifiers range are the theory's ontology; the concepts expressed by the predicate symbols P_i are its ideology.

For illustrative purposes, consider a theory of massive particles located in space. This is an unrealistic and incomplete toy theory. In particular, no dynamics are specified as of yet. But this simple set-up is enough to get the problem of quantities off the ground, and the argument straightforwardly generalises to more realistic theories. I will discuss some of the latter at the end of this section.

Let the ontology of this toy theory consists of

(i) a collection $\mathcal{B}$ of massive bodies; and

(ii) a collection P of spacetime points,

[3] For what it's worth, I believe that Quine's criterion for ideological commitment is too restrictive. I think that there is good reason to believe that the properties and relations that one can *implicitly define* from a theory's primitive predicates are also part of its ideology. But I won't argue this point here, as it doesn't matter to the question at hand; see Jacobs (2022) for further discussion.

while the ideology consists of

(i) a collection **D** of relations between spacetime points;

(ii) a collection **M** of mass magnitudes; and

(iii) a position function $x(i)$ from particles to points.

I will leave the relations contained in **D** aside, except to note that they must satisfy certain axioms, such as the triangle inequality, in order to represent a classical spacetime.

Even before the introduction of a dynamics, this theory faces a serious issue. Consider a universe that contains two particles: one's mass is 1 kg, and the other's mass is 10 kg. We would like to say that the second particle is more massive than the first. One reason we would like to draw mass comparisons is to formulate certain law-like generalisations: for example, that the more massive a particle is, the higher the gravitational force it exerts on other massive particles. The formulation of a proper dynamics thus depends on this prior task. However, none of the theory's resources so far allow us to say this: the relation 'more massive than' is not within the theory's ideology. In particular, note that although one can associate mass properties with real numbers (by the kind of representation theorems found in Krantz et al. (1971)), this association is merely conventional: the elements of **M** are primitive predicates. The numbers here would function merely as labels. Therefore, we cannot say that the 1 kg particle is less massive than the 10 kg particle simply because 1 is less than 10. Intuitively, the first particle is less massive than the second particle in virtue of the monadic mass property that each particle has, but the theory's structure does not reflect this fact.

It would seem that there is an easy fix: introduce a new symbol $\leqslant$ which expresses the relation 'less than or equally massive as'. But the introduction of a new relation by itself does not suffice, since we must also introduce axioms to ensure that $\leqslant$ is well-behaved. For instance, we need to stipulate that if particle a is 1 kg and particle b is 2 kg, then $a \leqslant b$; and likewise that if particle a is 2 kg and particle b is 1 kg, then $b \leqslant a$; and so on. Clearly, there is no end to this: since there are uncountably many mass magnitudes, we will also need uncountably many axioms for $\leqslant$. In fact, a new axiom is required for any possible *pair* of mass values! But this means that our theory, once it includes $\leqslant$, is not recursively axiomatisable. It is a basic requirement on any successful theory that it can be contained within the front and back cover of a textbook, but that is not the case even for the toy theory we have formulated here.

This is what Field (1984) calls the 'problem of quantities'. Field also notes that one can solve the problem via an appeal to 'heavy duty Platonism': *identify* the mass magnitudes with positive real numbers. We can then simply say that one mass is smaller than another iff it is associated to a smaller real number. But this solution is unsatisfactory. While I don't share Field's commitment to nominalism, I agree that magnitudes such as mass are not real numbers. For example, it makes sense to subtract numbers but not to subtract masses. One way to see this is to note that the numerical value of the difference $m(a) - m(b)$ between the masses of particle a and particle b depends on a choice of unit, so it does not express an objective relation between them (Mundy, 1986; Baker, 2020).

Aside from Platonism, there are broadly two options. The first is to start over from scratch and see if we can present our theory in terms of a wholly different ideology. This is the nominalist project of Field (1980). The term 'nominalism' here refers not just to a denunciation of numbers and other abstracta, but also of physical universals. On this option we are not allowed to quantify over properties, whether that quantification is first- or second-order. Instead of the set $\mathbf{M}$ of mass magnitudes, then, Field introduces a pair of mass relations: a ternary *betweenness* relation, $zBxy$, and a quaternary *congruence* relation, $xyCvw$. Field furthermore proves a *representation theorem* to show that it is possible to present a finite set of axioms for these relations such that one can associate each particle with a positive real number by a well-behaved function f. For instance: if $f(a) = 1$, $f(b) = 2$ and $f(c) = 3$, then b is between a and c. The numbers here seem to play the role of monadic mass properties, but unlike the predicates in $\mathbf{M}$ they are not part of the theory's ideology, which consists only of the relations B and C. This is reflected in the fact that the association between particles and numbers is not unique. In particular, Field also proves a *uniqueness theorem*, which shows that if f is a well-behaved representation function then so is αf, where α is any positive real number. This non-uniqueness corresponds to the freedom to choose a mass unit.

However, Field's axiomatisation is successful only when a certain 'richness' assumption is satisfied, namely that for any pair of particles a and c there exists another particle b whose mass is between a and c. Field in effect defines quantitative relations—twice as massive as, equally massive to—in terms of quantification over particles. For instance, the mass ratio between any pair of particles a and b is twice that between any pair of particles c and d iff there exists a particle z such that (i) $zBab$, (ii) $azCzb$ and (iii) $zbCcd$. If there is no such particle z, then this definition is empty. But the assumption that there always exist particles that satisfy these constraints is unreasonable, since it rules out worlds that contain only a finite number of particles, or worlds in which quantities such as mass or charge come in discrete quanta.[4] It is telling that Arntzenius and Dorr (2012), whose aim it is to continue the spirit of Field's project, explicitly embrace quantification over mass magnitudes. I will therefore not follow Field's nominalism, but consider instead the alternative route.

The second option is to relinquish our commitment to nominalism. If we allow ourselves to quantify over the elements of $\mathbf{M}$, we can finitely axiomatise structural relations like $\leqslant$. For our requirement is just that $\leqslant$ defines a (total) order on the set $\mathbf{M}$, which means that it must satisfy a total of three axioms: antisymmetry, transitivity and connexity. For instance, antisymmetry says that

$$\forall m_1, m_{2 \in \mathbf{M}}(m_1 \leqslant m_2 \rightarrow m_2 \nleqslant m_1)$$

On this view, then, the explanation of the fact that a 1 kg particle is less massive than a 10 kg particle is simply that the 'less than' relation holds between their respective mass magnitudes. This does not require the existence of any further particles, as on Field's approach. It does not matter here whether the quantification is first- or second-order, because either way mass magnitudes now fall within the range of an existential quantifier and hence have become part of the theory's ontology.

Once we quantify over the elements of $\mathbf{M}$ in this manner, by Quine's own criterion they become part of the theory's ontology rather than ideology. In addition to massive particles and spacetime points, our theory is now committed to the existence of an infinitude of mass magnitudes. This inflation of the theory's ontology may seem unparsimonious, but I suggest that we embrace it. The inflation is not gratuitous, since it allows us to express the relations between mass values, which are necessary for an adequate formulation of the theory's dynamics. The enlarged ontology thus expands our expressive resources. I say this in a Quinean spirit: if our best theory quantifies over mass magnitudes, then there *are* mass magnitudes.

Notice in particular that this is not a case of a trade-off between ontology and ideology (Oliver, 1996). It is often true that one can rid a theory of ontological bloat at the cost of an expanded ideology, or vice versa. For instance, one can trade in primitive modal ideology ('necessarily', 'possibly') for an ontology of possible worlds. But as Bennett (2009) notes, such an

[4] Interestingly, a similar problem occurs for relationist definitions of acceleration in terms of a 'colinearity' relation between triples of points at different times (Pooley, 2013, §6.1.1).

exercise often feels pointless: "small ontology, larger ideology; larger ontology, smaller ideology [...] it starts to feel as though we are just pushing a bump around under the carpet" (65). This is not the case here. True, ideology replaces ontology. But our concern here is not parsimony. It is the expressive power of the theory, and on this score "larger ontology, smaller ideology" clearly pays off. The bump is pushed into a more convenient spot of the carpet, so to speak.

In sum, we have arrived at a revised toy theory whose ontology consists of

(i) a collection $\mathcal{B}$ of massive particles;

(ii) a collection P of spacetime points; and

(iii) a collection $\mathbf{M}$ of mass magnitudes;

and whose ideology consists of

(i) a collection $\mathbf{D}$ of relations between spacetime points;

(ii) a collection $\mathbf{R}$ of relations between mass magnitudes (e.g. $\leqslant$);

(iii) a position function $x(i)$ from particles into points; and

(iv) a mass function $m(i)$ from particles into mass magnitudes.

The introduction of a mass function in place of mass predication is a matter of convenience which incurs no loss of generality; see Lewis (1970, 429) for a similar move. It allows us to avoid quantification into predicate position, further stressing the parallels between spacetime points and mass magnitudes. But even if the quantification were second-order, the key point that mass magnitudes fall within the range of an existential quantifier—and so *exist*—remains. Given the intertranslatability of these quantificational structures, any distinction between ontology and ideology based on the difference between first- and second-order quantification seems to me untenable. Quine thought much the same of Bergmann's initial suggestion: "in an effort not to omit important issues of ideology [Bergmann] would warp ontology around to include them" (1951, 14).

To clear up our notation, I will call the structured set $\mathcal{M} := \langle \mathrm{P}, \mathbf{D} \rangle$ *spacetime*, and $\mathcal{V} := \langle \mathbf{M}, \mathbf{R} \rangle$ *mass value space*.[5] With respect to the latter, it

[5] The idea of a structured value space for physical quantities is found, in different forms, in Stalnaker (1979); Gärdenfors (2000); Denby (2001); Funkhouser (2006); Jacobs (2021a); Caulton (2015).

is natural to think of $\mathcal{V}$ as representative of the *determinable* quantity 'mass', while the elements of $\mathbf{M}$ represent *determinate* mass magnitudes.[6] The relations defined over a value space allow us to capture certain characteristic features of determinables, such as the existence of a notion of 'closeness' between their determinates. Another way to phrase the point of this section, then, is to say that *determinates* are part of a theory's ontology, whereas *determinables* are part of the ideology.

This type of view has been defended by Mundy (1987) and more recently by Eddon (2013) and Wolff (2020). Arntzenius and Dorr's (2012) 'locationism' is a closely related position. The latter stress the similarity between physical magnitudes and spatiotemporal locations: in each case, particles are said to 'occupy' or 'instantiate' a point or magnitude respectively, and both points and magnitudes stand in certain relations to each other. Conversely, Teller (1987) argues that one can interpret location in spacetime itself as a physical quantity: just as particles are mapped into a space of mass values, so they are mapped into spacetime. Whether quantities are locations or locations are quantities, then, magnitudes are on a par with spacetime points as ontological posits of the theory.[7]

Let's return to the question at hand: are the values of physical quantities ideology? We have seen that the desire to compare (in a broad sense of which order comparisons are a specific instance) mass magnitudes has led to a formalism that blurs the traditional distinction between ontology and ideology in classical particle mechanics. In particular, mass magnitudes are typically thought of as part of classical mechanics' ideology, as evinced by the quotes in the introduction. But since mass value space has an internal structure, mass magnitudes are better thought of as part of the theory's ontology. The same is true for other quantities, like charge. In order to say that one particle is oppositely charged from another particle we must introduce a relation 'oppositely charged'. This relation is well-behaved only if it partitions the charged particles into a pair of disjoint sets: the positive and the negative ones. We can finitely axiomatise such an equivalence relation only if we allow ourselves to quantify over charge properties. Relatedly, Sider (2011, Ch. 2) discusses the shift from distinct electric and magnetic fields to a unified electromagnetic field as a matter of ideology. On the natural assumption that fields are represented by functions from spacetime into

[6] For an introduction to the determinable/determinate distinction, see Wilson (2017).

[7] This need not mean that magnitudes are *locations*, as proposed by Arntzenius and Dorr (2012). In Jacobs (2023a), I defend a version of 'fibre bundle Platonism', on which the values of gauge fields are Platonic universals instead.

a space of field values, however, the move from a pair of distinct value spaces to a unified value space also indicates a change in ontology: the ontology of classical electromagnetism consisted of a separate electric field space and magnetic field space; the ontology of relativistic electromagnetism consists of a single electromagnetic field space. The lesson from this section thus generalises to these more realistic theories.

Therefore, paying closer attention to the logical structure of physical theories forces a reconfiguration of the ontology/ideology dichotomy. This does not mean that ideology has vanished from physics. On the contrary, these considerations lead to a more fine-grained perspective on physical ideology. The ideology of our toy theory was two-fold: it consisted of (i) the structural relations between spacetime points and mass properties respectively; and (ii) the position and mass functions. I will call the former *structure* and the latter *quantities*. Generally, within the category of ideology there is structure, which consists of relations between collections of a theory's ontological posits; and quantities, which are represented by functions between different such collections. These are universal features of contemporary physical theories. The concepts of quantities and structure are central to the philosophy of physics, so ideology remains of profound importance. The next section concers quantities; I will return to the topic of structure in §4.

3 Quantities: Internal and External

In this section I argue that there is a sense in which the ontology/ideology distinction is superseded by a different one: the distinction between *internal* and *external* degrees of freedom (DOFs). This latter distinction differs from Quine's in that it is not absolute, but relative to the quantities a theory posits. In this section I introduce this distinction, which may not be familiar to most metaphysicians. I then show that this distinction more closely tracks the intuitions about quantities discussed in the introduction.

The internal/external distinction is most familiar from the literature on symmetries. The definition of symmetries is a fraught question which I will not consider in detail here.[8] For our purposes it is only relevant to note that symmetries are often said to 'act' on some part of the theory's ontology (in our expanded sense, so inclusive of value spaces). For instance, spacetime symmetries act on $\mathcal{M}$, the structured manifold of spacetime points. Likewise, mass symmetries, such as uniform scalings, act on the mass value

[8] For some recent discussions on how (not) to define symmetries, see Belot (2013), Dasgupta (2016), Wallace (2019) and Read and Møller-Nielsen (2020).

space $\mathcal{V}$.[9] The distinction between internal and external symmetries is then drawn in terms of this action. Here the literature diverges. On the one hand, external symmetries are often defined as those that act on spacetime—all other symmetries are internal. For instance, Kosso (2000, 84) writes that "[a]n *external* symmetry is one in which the transformation is a change of a spatial or temporal variable. An *internal* symmetry is one in which the transformation is not a change of spatial or temporal parameters". On the other hand, one also comes across definitions of external symmetries as those that act on a theory's *independent* DOFs, while internal symmetries act on the theory's *dependent* DOFs. For instance, Belot (2013, 330) defines internal symmetries as symmetries "that involve transformations of dependent variables of the theory that do not affect the independent variables". It seems that the first definition is slightly more common, especially in the philosophical literature. For reasons that will become apparent, I prefer the second definition instead.[10]

Recall that a theory's quantities are functions between (sub)domains. The mass density field of Newtonian Gravitation, for instance, is typically represented by a function from spacetime $\mathcal{M}$ into mass value space $\mathcal{V}$. For any such function, we call its domain the independent DOF and its codomain the dependent DOF. In the case of mass density, denoted ρ, the spacetime coordinates x are the independent DOFs and the mass density values $\rho(x)$ are the dependent DOFs. This definition accords with common usage in physics. Traditionally, the independent variables are identified with the x-axis of a graph, and the dependent variables with the y-axis: think of how an x-vs-t graph plots location as a function of time.

It may seem that the distinction between dependent and independent DOFs is arbitrary. After all, couldn't one introduce an inverse function ρ^{-1} from mass density values back into spacetime? In that case, spacetime is the dependent DOF and mass density the independent DOF. The answer is 'no': there is an asymmetry between dependent and independent variables. The reason is that quantities are typically not bijections.[11] In particular, they

[9] Whether mass scalings *are* symmetries of Newtonian Gravitation is disputed: for discussion, see Martens (2019a); Baker (2020); Dasgupta (2020); Wolff (2020); Jacobs (2023b).

[10] Menon (2021) presents a third option: the external symmetries are those common to the dynamics of all matter fields. When there are distinct types of matter, this automatically entails the non-injectivity discussed in fn. 11. While I am sympathetic to Menon's suggestion, I will continue to work with Belot's definition.

[11] Gauge quantities present an anomaly. In the fibre bundle framework, such quantities are represented as functions from spacetime points to a fibre bundle. But since each point possesses its own value space—the 'typical fibre—these functions *are* injective. Why then

are not injective. It is possible for distinct particles to have the same mass or colour, or for the mass density field to have the same value at distinct spacetime points. But it is not possible for the same particle to have a pair of distinct masses at once, or for the mass density field to possess multiple values at the same spacetime point. Therefore, the distinction between independent and dependent DOFs is justified by the formal features of the functions between them. Carnap (1928, §158) already drew the same conclusion, calling the different classes of DOFs 'objects of the first type' and 'objects of the second type' respectively.[12]

The internal/external distinction is thus drawn relative to a choice of quantity-function. It is possible, in principle, for some value space to be both the domain of one function and codomain of another. For example, in classical particle mechanics the spacetime manifold $\mathcal{M}$ is the codomain of the particle position function $x(i)$, but also the domain of the gravitational potential field $\varphi(x)$. In this case it is not clear whether spacetime is internal or external. It might be said that there is no fact of the matter. This is a crucial difference with the ontology/ideology distinction. Whether something counts as ontology or ideology depends only on what it *is*, namely an object or a predicative concept. Whether a value space is internal or external, on the other hand, depends on whether it is the domain or the codomain of some quantity. Different theories may posit the same value spaces, yet differ over which ones are internal. It is at least conceivable, for example, for a theory to posit a spacetime and a mass density value space, but whose fundamental quantity is represented by a (non-injective) function from the latter into the former. On such a theory, mass is an external quantity: mass density values instantiate locations, rather than vice versa.

For a concrete example, consider Wallace's (2015) reformulation of gravitational theory as a gauge theory. As discussed in more detail below, local field theories typically represent matter fields as functions on a spacetime

call spacetime the independent DOF in this case? Because *for any choice of gauge*, the function from spacetime points to the typical fibre is non-injective.

[12] Notice that it is not an *a priori* fact that quantities are represented by non-injective functions. It is possible to imagine a theory for which certain quantities are represented by bijective functions. The distinction between independent and dependent DOFs would then vanish. We can think of this puzzle—why *are* nature's quantities represented by non-injective functions?—as the complement to Wittgenstein's infamous *exclusion problem* (cf. (Bricker, 2017)): why are nature's quantities represented by *functions* in the first place? Kim (2016) calls this assumption the *Functionality of Quantity*. It is far beyond the scope of this paper to solve either puzzle, let alone both. Suffice it to say that the independent/dependent distinction depends on the fact that nature seems to prefer non-injective functions, for some reason or another.

manifold: hence spacetime is external whereas field values are internal. The spacetime points *have* field values. On Wallace's version of the theory, on the other hand, the matter field is itself represented as a smooth manifold. The field is thus an extended body. The location of a part of the field is represented by a function *from* the field *to* (a local copy of) Minkowski spacetime: the field is external whereas spacetime is internal. The spacetime points are *had* by parts of the field. Although both formulations posit a spacetime manifold, the status of this manifold as dependent or independent differs across them. This illustrates the way in which the internal/external distinction is relative to the theory's quantities. Whether spacetime points exist (i.e. are part of the ontology), on the other hand, seems entirely independent of such issues. This is a crucial difference between the internal/external distinction and the ontology/ideology one.[13]

Nevertheless, the internal/external distinction captures many of the intuitions attributed to the ontology/ideology split. For example, mass and charge are internal quantities (this is true whether one considers particle or field theories), whereas spacetime is an external quantity when one considers local field theories. For most particle theories, spacetime is internal only relative to location but external otherwise. Although it is correct to say that there is a sense in which particles *have* masses, then, it is a mistake to attribute this to a perceived difference between ontology and ideology. The difference is rather that mass is an internal quantity, whereas particle label is an external one. More generally, recall that Duerr and Read (2019) claimed that a theory's ideology consists of certain DOFs. If the DOFs intended here are the internal ones—as seems plausible from the quote—then the category of 'ideology' plays no further role. Duerr and Read identify ideology with internal DOFs, whereas it is more correct to say that the latter category captures the sense in which quantities like mass or charge are had by particles or points. The standard gloss on ideology is that it concerns what one can say *of* the ontological posits of a theory. In first-order logic this is typically cashed out in terms of predication. But there is no reason to think of predication as the sole formal method of property-ascription.[14] In particular, one can also use non-injective functions to represent the ascription of some 'property' to some 'object', since the function's non-injectivity ensures the requisite asymmetry. Within the framework of modern physics, then, there is a sense in which the internal/external distinction supersedes

[13] For further illustration of the metaphysical consequences of redrawing the internal/external distinction, see Menon's (2021) discussion of supersymmetry.

[14] In this, my view echoes Ramsey's (1925) rejection of the universal/particular distinction.

the ontology/ideology distinction insofar as it is intended to capture the intuition that quantities are predicative.

4 Symmetries as a Guide to the Structure of Physical Quantities

It may seem as if ideology has become all but irrelevant. The values of quantities are ontological posits, on a par with particles and points. Meanwhile, the notion of predication is taken over by the internal/external distinction, which is relative to a choice of quantity function. In this section, I want to show that ideology *is* crucial to modern physics, but in a different place from where one would expect it.

Recall from §2 that a theory's value spaces are *structured*: the elements of such a space stand in certain relations to each other, for instance an order relation. These relations are part of the theory's ideology. The most well-known example of this is spacetime structure, that is, the relations that obtain between spacetime points (Barrett, 2015). To name just one example, it is a contested issue whether the spacetime that is most appropriate to the dynamics of Newtonian Gravitation has an absolute acceleration structure or not (Saunders, 2013; Dewar, 2018). Similarly, philosophers have recently shown interest in the structure of internal value spaces, such as mass value space (Wolff, 2020; Dewar, 2021; Jacobs, 2023b).

Quine famously believed that one could read off a theory's ontological commitments from its formalism: the theory is committed to whatever one needs to quantify over in order for the theory's claims to come out as true. This is still the case for modern physical theories, since one can read off the theory's ontology from the value spaces it posits. Since quantities themselves are part of a theory's ideology, one can also read off aspects of the latter from the formalism. But can one also read off a theory's *structure*—the second half of ideology—in the same way? I believe that the answer is 'no'. For history is familiar with instances of empirically successful theories that exhibit superfluous structure. For example, Newton posited an absolute space, but absolute positions and velocities play no role in the novel predictions or scientific explanations of classical mechanics. Therefore, a 'literal' interpretation of a theory does not usually reveal the appropriate structure.

Fortunately, history is also familiar with an indirect method for the discovery of structure: symmetries. Again, the most well-known examples here involve spacetime. To return to Newton's absolute space, it is the theory's invariance under the Galilean symmetry transformations—which include uniform translations and boosts of all matter content—that provides a reason to believe that classical spacetime must have a weaker structure, appropriately called Galilean spacetime. Although it has received comparatively less attention, a similar method exists for the structure of internal value spaces (Jacobs, 2021b; Dewar, 2021). It is thus possible to probe a theory's ideology indirectly. The remainder of this section is devoted to elaborate on this method. This reveals a hitherto undiscovered connection between symmetries and ideology.

4.1 Defining Symmetries

I first present a more precise definition of symmetries. Dasgupta (2016) develops a useful tripartite classification of symmetry definitions. The first type, *formal* definitions, define symmetries in purely formal terms, absent any physical interpretation of this formalism. The second, *ontic* type define symmetries as those transformations that preserve certain *physical* features, such as a distinguished set of quantities. This requires a physical interpretation of the theory's formalism. The third type of definition is *epistemic*: symmetries are defined as transformations that preserve empirical content (this is a necessary but not a sufficient condition).

The definition of symmetries I offer is a formal one. Formal definitions have recently come under attack from Belot (2013) and Dasgupta (2016), who argue that neither formal nor ontic accounts can ensure that symmetry-related states are empirically equivalent. Since symmetry transformations are philosophically relevant because they seem to relate empirically equivalent states of affairs, this is a problem for formal definitions of symmetries. It is not my aim here to defend the claim that symmetries under some formal definition invariably relate empirically equivalent states of affairs.[15] Rather, my more limited aim is to offer a definition that encompasses a broad class of symmetries of philosophical interest—including both external and internal symmetries—but excludes more trivial transformations, such as arbitrary permutations of possibilities or symmetries of particular subsystems. This will suffice to establish the required connection between symmetries, structure and ideology. With this aim in mind, I can find no fault with a formal definition.

Let a *local spacetime theory* consist of a spacetime $\mathcal{M}$ and a set of functions ϕ_i from spacetime into associated value spaces $\mathcal{V}_i$. Notice that for a

[15] For a to my mind convincing response to Dasgupta's and Belot's criticisms, see Wallace (2019).

local spacetime theory, spacetime is the only external DOF; all other quantities are internal. Denote a model of such a theory $\langle\mathcal{M}, \mathcal{V}, \phi\rangle$. The *space of solutions* of a theory consists of those models that satisfy the theory's dynamical equations.

I then define *external symmetries* as follows:[16]

> *External Symmetry*: Let d be a diffeomorphism that acts on $\mathcal{M}$; then d is an external symmetry iff the map $g_d : \langle\mathcal{M}, \mathcal{V}, \phi\rangle \rightarrow \langle\mathcal{M}, \mathcal{V}, d_*\phi\rangle$ maps the space of solutions onto itself (where d_* is the pushforward map of d).

This definition of external symmetries is standard; cf. Earman (1989, 45). Let me briefly gloss the technical vocabulary. Firstly, a diffeomorphism is a smooth bijection between manifolds, i.e. it preserves the topological structure of $\mathcal{M}$. Secondly, the pushforward of ϕ by d, denoted $d_*\phi$, is such that for any point p, $d_*\phi$ has the 'same' value at $d(p)$ as ϕ has at p. So, if d maps the location of the centre of the sun to a location somewhere in empty space, then the pushforward of the mass density field assigns a high mass density to the latter point. This transformation is an external symmetry iff it preserves the theory's dynamics when applied to *all* of the theory's quantities. The above definition encompasses all of the usual spacetime symmetries. For example, a static shift is induced by a diffeomorphism that maps each point to another point some fixed distance away from the first in some particular direction. On the other hand, the definition excludes arbitrary permutations of a theory's DPMs, since not all maps on a theory's space of solutions are expressible as the 'lift' of a diffeomorphism on the theory's base manifold (cf. Dasgupta's (2016) notion of a 'generated' versus a 'bare' transformation).

It is possible to define internal symmetries analogously. I am not aware of any previous definition of internal symmetries along these exact lines, but the parallels between it and the definition of external symmetries are clear:[17]

> *Internal Symmetry*: Let f be a bijection of $\mathcal{V}$; then f is an internal symmetry iff the map $g_f : \langle\mathcal{M}, \mathcal{V}, \phi\rangle \rightarrow \langle\mathcal{M}, \mathcal{V}, f \circ \phi\rangle$ maps the space of solutions onto itself.

[16] This definition fails in the context of theories set on fibre bundles, since for such theories the action of the push-forward map on a section of the bundle is ill-defined. Dewar (2020) argues that in that context it is more natural to interpret external symmetries as a subset of internal symmetries.

[17] I believe that my definition here is a special case of Dewar's (2020).

Just like an external symmetry is induced by a smooth permutation of spacetime points, an internal symmetry is induced by a permutation of the values of some quantity (since it is left open whether internal value spaces have a manifold structure, the smoothness requirement is dropped). For example, a uniform mass scaling maps each mass value to another one that is a multiple of the first. This permutation induces a transformation of the fields defined over $\mathcal{M}$ as follows: if the value of ϕ at p is $\phi(p)$, then the value of $f \circ \phi$ at p is $f(\phi(p))$. Again, such transformations are symmetries iff they preserve the dynamics when applied to *all* quantities that take value in $\mathcal{V}$. Therefore, this definition too excludes arbitrary permutations of the theory's solutions, since not all maps on the space of KPMs are expressible as the 'lift' of a permutation of some value space.

The similarities between these definitions further support the thesis that the traditional view of points and particles as part of a theory's ontology and their quantities as part of a theory's ideology requires revision. Rather, it seems that the internal and external variables of a theory are to some extent on a par: *both* range over the theory's ontology. The theory's ideology consists of functions between the theory's sub-domains on the one hand, and the structure of these sub-domains on the other. I will say more about the latter in the next sub-section.

4.2 Symmetry Principles

Recall the question at hand: is it possible to indirectly discover the ideology of a theory, in particular the structure of the theory's value spaces? I suggest that this is possible with certain symmetry principles. This leads to the idea of *symmetries as a guide to the structure of physical quantities*.

Earman (1989) famously articulated a pair of symmetry principles that link a theory's (external) symmetries to its spacetime symmetries. Earman's spacetime symmetries are quite unlike the internal and external symmetries defined above—which we will collectively call *dynamical symmetries*—as they are independent from the theory's dynamics; their definition does not involve the notion of solutionhood.[18] Rather, a theory's spacetime symmetries are diffeomorphisms of $\mathcal{M}$ that preserve spacetime structure, for instance the metrical distances between points. In technical terms, space-

[18] Many physicists also refer to so-called 'hidden' symmetries, such as the Lenz-Runge symmetry of a pair of gravitationally interacting bodies, as dynamical. These are excluded by my definition, and in light of the symmetry principles stated below this is the correct verdict: the presence of such accidental symmetries should *not* lead us to postulate a 'weaker' spacetime structure (Bielińska and Jacobs, 2024).

time symmetries are *automorphisms* of $\mathcal{M}$, where $\mathcal{M}$ includes all (absolute) spacetime structure defined over the base manifold. These are important because there is a duality between symmetries and structure. On the one hand: the more structure a spacetime has, the fewer symmetries. For each additional piece of structure, a symmetry transformation must preserve that structure—so the more properties and relations one builds into a spacetime, the more difficult it becomes to preserve spacetime structure. On the other hand: the more symmetries a spacetime has, the less structure. In fact, on some views a spacetime structure is *identified* with its symmetries. The idea is that even if some property or relation is not explicitly included in the definition of a particular spacetime structure, it is implicitly definable from that structure whenever it is invariant under the symmetries of the spacetime in question (Barrett, 2017; Jacobs, 2022). Therefore, *to know a space's symmetries is to know its structure.*

According to Earman, it is possible to determine spacetime symmetries (and therefore spacetime structure) from external symmetries. In particular, Earman postulates the following pair of principles:

$\mathrm{SP1}_{\mathrm{ext}}$ Any external symmetry of T is a spacetime symmetry of T;

$\mathrm{SP2}_{\mathrm{ext}}$ Any spacetime symmetry of T is an external symmetry of T.

These principles are meant to ensure that a theory's spacetime has neither too much nor too little structure. If $\mathrm{SP1}_{\mathrm{ext}}$ fails, then the theory contains spatiotemporal structure that is dynamically irrelevant. For example, Newtonian spacetime contains a standard of absolute rest, which means that boosts are not spacetime symmetries. But boosts *are* dynamical symmetries of Newtonian Gravitation, which means that the standard of absolute rest is irrelevant to the theory's dynamics. Conversely, if $\mathrm{SP2}_{\mathrm{ext}}$ fails then the theory has too little spacetime structure to sustain the dynamics. Consequently, the theory has to draw distinctions between regions of spacetime that are qualitatively identical, yet which behave differently dynamically. Jointly, $\mathrm{SP1}_{\mathrm{ext}}$ and $\mathrm{SP2}_{\mathrm{ext}}$ are useful tools to determine the appropriate amount of spacetime structure for local spacetime theories.

But Earman does not consider internal symmetries, nor the structure of internal value spaces such as mass (density) value space I propose to extend Earman's principles to both types of symmetries. This will allow us to use symmetries to determine the structure of both external *and* internal value spaces.

Call the automorphisms of a value space—whether internal or external—their *kinematical symmetries*. The kinematical symmetries of spacetime $\mathcal{M}$

are just the spacetime symmetries; the kinematical symmetries of a value space $\mathcal{V}$, meanwhile, are those bijections on the base set of $\mathcal{V}$ that preserve the relations defined over it. For an example of a non-internal kinematical symmetry, consider once more mass value space. It is usually thought that the automorphisms of mass value space are scale transformations by a constant α, since such transformations preserve both the order of mass values as well as mass addition (if $m_1 + m_2 = m_3$, then $\alpha m_1 + \alpha m_2 = \alpha m_3$). Importantly, both dynamical symmetries and kinematical symmetries are identified with bijections. The upshot of Earman's symmetry principles is that the set of bijections on spacetime that generate external symmetries is identical to the set of bijections that are automorphisms of spacetime. The obvious extension of these principles to internal quantities then says that that the set of bijections of a quantity's value space that generate internal symmetries is identical to the set of bijections that are automorphisms of that quantity's value space.

Putting all this together, I propose that the following novel symmetry principles must hold for any well-formulated theory (cf. Hetzroni (2019)):

SP1 Any dynamical symmetry of T is a kinematical symmetry of T;

SP2 Any kinematical symmetry of T is a dynamical symmetry of T.

The motivation behind these principles is analogous to Earman's original motivation for SP1_{ext} and SP2_{ext}. For example, one way in which SP1 can fail for internal symmetries is when the structure of the value space of some dimensionful quantity is identified with that of the real numbers. This in effect endows that quantity with a preferred system of units. But the dynamics are not sensitive to this structure, since the laws of physics remain true when expressed in a different system of units. Conversely, SP2 ensures that the theory has *enough* internal structure to sustain the dynamics. If SP2 fails, then quantity values that are qualitatively identical play distinct dynamical roles. Martens (2019b), for instance, conjectures that the elements of mass value space are qualitatively indiscernible. But this seems incompatible with the fact that strength of the gravitational force on an object depends on the object's mass. The dynamics treat mass values differently even though they are kinematically alike, contrary to SP2.[19]

Jointly, SP1 and SP2 act as guides towards the appropriate structure of a theory's value spaces—whether internal or external. Since such structure is part of a theory's ideology, this means that symmetry principles enable

[19] Jacobs (2023b) defends a proposal on which mass value space is linked to other value spaces by the gravitational constant, arguing that this would satisfy SP2.

one to indirectly infer ideology from a theory's formalism. While one cannot just 'read off' ideology from a theory's formalism, then, that formalism still contains important clues towards the theory's ideology. The above analysis has thus revealed a novel connection between symmetries, structure and ideology. In a slogan: *symmetries are a guide to physical ideology.*

5 Conclusion

The conclusions of this paper are that —

1. The common belief that (the values of) physical quantities are part of a theory's ideology is incompatible with the mathematical structure of modern physical theories;

2. The internal/external distinction supersedes the ontology/ideology distinction, but it is relative to the quantities posited by a theory;

3. Ideology nevertheless plays an important role in physics as the structure of a theory's value spaces; symmetries are a guide to this structure.

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