Foundations of Physics manuscript No.
(will be inserted by the editor)

Life, the Multiverse, and Fine-Tuning

Fact, Fiction, and Misconceptions

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Received: date / Accepted: date

Abstract Few topics in cosmology are as hotly debated as the Multiverse: for some it is untestable and hence unscientific; for others it is unavoidable and a natural extension of previous science. The idea of fine-tuning has a similar status. Some of this disagreement might be due to misunderstanding, in particular the degree to which probability distributions are necessary to interpret conclusions based on the Multiverse, especially with regard to the Anthropic Principle. I present undisputed facts, discuss some common misunderstandings, and investigate the role played by probability. The Multiverse is perhaps an important component necessary for interpreting cosmological and other physical parameters.

Keywords cosmology · fine-tuning · multiverse · anthropic principle

1 Introduction

This article is not a review of any, much less all, of the topics mentioned in the title. It is not even a review of the intersection of those topics. Rather, it aims to address a specific problem, namely confusion in discussions about those topics due to different definitions of the term fine-tuning and, to a lesser extent, confusion regarding different definitions of the Multiverse and the Anthropic Principle (or, equivalently, which of the many types of each is meant). Sometimes, confusion about absolute and conditional probability and the association of fine-tuning with improbability also play a role. Although the topics are sometimes intertwined, I first discuss fine-tuning, then the Multiverse, then the Anthropic Principle, before assessing the confusion in the last section.

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2 Fine-tuning

2.1 Two definitions

The term "fine-tuning" is used in physics in two distinct senses in connection with the values of parameters. One is concerned with the (near) equality of two numbers; the other is concerned with an unstable value, i.e. one such that a small change in that value would produce a large change in quantities which depend on it.[1] The latter sense is also important with regard to the instability with respect to initial conditions in chaotic phenomena.

2.2 Equality, more or less

At first glance, one might get the impression from the literature that not only is the (near) equality of two numbers an indication of fine-tuning, but also two vastly different numbers. The latter seems counter-intuitive: if one selects two numbers at random, corresponding to the lack of fine-tuning, then, in almost all cases, their ratio would be very small.[2] For example, it is often claimed that the observed value of vacuum energy density, if the cosmological constant is interpreted as such, is extremely small and that that is an example of fine-tuning, since $(\rho_{\mathrm{observed}})/(\rho_{\mathrm{expected}})$ is small. What is actually meant is the following: There is a naïve expectation that the vacuum energy should be about 120 orders of magnitude larger than observed; the low value is explained by some unknown cancellation mechanism; that cancellation mechanism must be very exact, so that the quantity $(\rho_{\mathrm{expected}}-\rho_{\mathrm{cancelling}})/(\rho_{\mathrm{expected}}+\rho_{\mathrm{calcelling}})$ is very small. So the actual fine-tuning is the near equality $\rho_{\mathrm{expected}} \approx \rho_{\mathrm{cancelling}}$. Of course, if the naïve expectation is wrong (which does not seem a priori less likely than an unknown cancellation mechanism), then there is nothing at all puzzling about the observed value of the cosmological constant. [e.g. 2]. Fine-tuning, in the first sense mentioned above, thus always involves the (near) equality of two quantities.

One might think that the concept of naturalness in particle physics, which is a lack of fine-tuning, is the other way around, because "natural" dimension-

[1] According to Adams [1], the second sense is the usual one: 'The usual meaning of "fine-tuning" is that small changes in the value of a parameter can lead to significant changes in the system as a whole.' He also discusses '[a] second type of fine-tuning ... when a parameter has a vastly different value from that expected' and in the context of such hierarchical fine-tuning discusses the near equality of two large numbers necessary for them to almost cancel and the concept of naturalness, all also mentioned in this section.

[2] Of course, the reciprocal of a small number is a large number. I will always assume that the smaller number is the numerator when discussing ratios. One should also concentrate on ratios and not differences. One reason is that differences depend on the dimensions used, which are arbitrary; ratios are automatically dimensionless. Another is that, even when the difference between a and b is important, one should always consider the dimensionless quantity $(|(a-b)|)/(|a|+|b|)$ (if a and b have the same sign, that is equivalent to $|(a-b)/(a+b)|$) since the important quantity is the size of the difference compared to the size of the numbers involved.

less numbers should be of $\mathcal{O}1$. However, that is also a case where fine-tuning is equated to a (near) equality. In an effective theory, $g_{\mathrm{effective}} = g + f(g)$, where g is the “bare” quantity in the Lagrangian, $g_{\mathrm{effective}}$ the observed value, and $f(g)$ represents quantum corrections. Since such corrections usually preserve symmetries, $f(0) = 0$, so $f(g)$ is linear with the addition of subleading terms, implying that $g_{\mathrm{effective}} \approx g$. If $g_{\mathrm{effective}}$ is measured to be small, and if the smallness is natural, then $g \lessapprox g_{\mathrm{effective}}$; if the smallness is not natural, then $g_{\mathrm{effective}}$ must be fine-tuned so that g and $f(g)$ almost exactly cancel. (Strictly speaking, that is an example of technical naturalness; one still has to explain why g is small. The more extreme concept of Dirac naturalness, i.e. that all dimensionless parameters g in a theory should be of $\mathcal{O}1$, is rarely used anymore.) I mention that to avoid confusion; for the rest of this paper, the definitions of fine-tuning in Sect. 2.1 are sufficient. See Grinbaum [3] for more on the concept of naturalness and its history.

A famous example of the near coincidence of numbers are the various numbers $\approx 10^{40}$ noted by Dirac [4], who believed that such a coincidence is unlikely. Since one such number, the size of the Universe compared to the size of a subatomic particle (N_2), changes as the Universe expands, then if another, the ratio of the strengths of the electromagnetic and gravitational interactions (N_1), did not change, then the coincidence would be even stronger, as it would hold only at our epoch. Dirac thus suggested that the gravitational constant G decreases with time which, however, has been ruled out observationally. There is, however, an explanation, at least in some cases, as shown by Dicke [5], which is a weak-anthropic explanation.[3] The coincidence does hold only at our epoch, but that shouldn’t be surprising since our existence depends on it. There are, however, other quantities corresponding to simple powers of N, not all of which have such explanations [e.g. 7, in which Harrison notes ‘This ingenious explanation leaves me, at least, with a vague and uneasy feeling that possibly some unknown fundamental relation still lurks between N_1 and N_2.’]

2.3 Sensitivity

Another way of expressing the second sense of fine-tuning is that a quantity must be within a certain range in order that something else occur. For example, the velocity of an object near the Earth must be within a narrow range in order for it to go to into orbit, rather than escaping from or falling onto the Earth. Note that the second sense implies the first (i.e., the velocity must be approximately equal to some fiducial velocity in order for the orbit to occur), but not vice versa (i.e., the (near) equality of two quantities does not necessarily imply that something special must happen in the case of that (near) equality).

[3] Note that Dicke’s argument is an example of the use of the Anthropic Principle which does not involve the Multiverse. There is also the subtle issue as to the difference between invoking the Multiverse as an explanation for fine-tuning and seeing fine-tuning as evidence for the Multiverse [e.g. 6].

2.4 Fine-tuning and probability

Another source of confusion is that "fine-tuning" is often used as a synonym for "improbable". However, it makes sense to reserve "fine-tuning" for the two cases discussed above; whether or not a given (near) equality is improbable is a different question. All four cases are possible (using the "sensitivity" definition of fine-tuning):

fine-tuned and improbable The radius of the Earth's orbit: It is fine-tuned because a slightly smaller or larger value would have disastrous consequences for life on Earth; it is improbable because most planets are not at such a distance from their star. (Of course, that is a classic example where the weak Anthropic Principle (see below) can be used to explain an improbable fine-tuned quantity.)

fine-tuned and not improbable A functioning pocket watch: It is fine-tuned because a slightly different arrangement of the parts would not function as a watch at all. It is, however, not improbable, because almost all watches one observes function. (Historically, and by some even today, it is often argued that a watch implies a watchmaker. While that might be true, it is not necessarily true of other fine-tuned and probable things, e.g. the fact that insects which spend several years as nymphs do so for a prime number of years – a predator with a cycle of a smaller number of years could evolve its cycle to be a factor of the longer cycle and would thus be abundant at every metamorphosis in a non-prime case, but in the prime-number case only in cycles corresponding to the product of the two cycles. Evolution is a prime example of the production of fine-tuning which is not unlikely in any meaningful sense.[4])

not fine-tuned and improbable Winning the lottery: Any given draw is improbable, but, since it doesn't matter (except to the winner) if a given draw corresponds to a specific choice, and thus who wins, it is not fine-tuned.

not fine-tuned and not improbable The masses of stars: Stars have a relatively small range of mass, from about 0.08 to about 150 solar masses. There is no fine-tuning because the presence of stars of, say, 0.005 or 2000 solar masses would not greatly change, say, the structure and evolution of a galaxy. It is not improbable because the mass range follows from the underlying physics.

The statement that the Universe[5] is fine-tuned for life is independent of the question whether the combination of parameters which specifies our Universe is in some sense improbable. **That is one source of confusion in the discussion of fine-tuning in cosmology.** (In daily life, of course, many

[4] Of course, specific outcomes are highly contingent and random [e.g. 8], but the basic concept, e.g. predators have a fine-tuned digestive system so that they can subsist on whatever prey is abundant, is not.

[5] "Universe" refers to *our* Universe, while "universe" refers to a model universe in the sense of a cosmological model or to a different physical universe in the Multiverse.

fine-tuned events are improbable, though most improbable events are not fine-tuned. One must be careful to distinguish the two concepts.)

That sense of fine-tuning is distinct from another common use of the term, namely the lack of technical naturalness in the particle-physics sense. **That is another source of confusion in the discussion of fine-tuning in cosmology.** (See the second paragraph of Sect. 2.2; the lack of technical naturalness is a specific example of the near-equality definition of fine-tuning.) Unless noted otherwise, in this paper I use "fine-tuning" to mean "fine-tuned for life".

2.5 *Special* state or *improbable* state?

What needs to be explained? Of the four examples above, only "not fine-tuned and improbable" doesn't need an explanation. Things which are fine-tuned need an explanation because, by definition, a fine-tuned value is a *special* value. Things which are probable need an explanation as to why they are probable.

Fine-tuned (in the first sense) values are special because they involve the (near) equality of two numbers (which, without further explanation, is improbable), perhaps with the additional implication that small deviations from the observed values would lead to large changes in outcomes which depend on those values (the second sense). However, there are also numbers which are neither per se more improbable than others nor would a deviation from them have disproportionate consequences, but nevertheless play a role in the discussion of fine-tuning and probability. Consider, for example, someone watching (unknown to the tosser) someone else tossing a coin, and observing a sequence of 100 heads. In itself, that sequence is not fine-tuned, since a slight variation of it would have no consequences (other than those based on bets which, however, are due to the perceived improbability and/or specialness of such a sequence). Neither is it more improbable than any other sequence of 100 tosses. Nevertheless, someone who said that it is nothing special because any other sequence is just as probable[6] would be, in the view of most, wrong, and their perception would be correct. Why? The relevant question is not 'Given a fair coin, how probable is a sequence of 100 heads?', but rather 'Given an observation of 100 tosses, all of which result in heads, how probable is it that that is the observation of a fair coin?'

In the case of a coin, the answer should be obvious: It is relatively easy to think of a mechanism which biases the coin so that (almost) always heads results from a toss, so one should conclude that it is more likely that such a mechanism is in operation than that one has observed 100 heads. A "random" sequence will be seen as just that. More difficult is the case of a *special*

[6] I would be willing to bet a rather large sum that the next toss is also heads against someone who takes the view that such a sequence is just as probable as any other and, thus, we shouldn't be surprised to have seen one-hundred heads in a row, but also that the chances of the 101st head is 50 per cent.

sequence, say, any binary sequence[7] in The On-Line Encyclopedia of Integer Sequences (OLEIS) [9]. Those are special in the sense that a sequence of 100 heads is special, yet there is no obvious way to produce them. So what should one conclude if one has observed such a sequence? Of course, the interesting question is the probability of observing *any* sequence from the list at the OLEIS, unless the sequence is specified in advance, and also how many tosses are observed until an interesting sequence occurs. Since, for a length of 100 or more, most sequences are not listed at the OLEIS, such an observation would indeed be surprising, but in any case not a problem until it is actually observed. When that happens, we can return to the question whether the fact that it is 'just as probable as any other sequence' is a sufficient answer.

An objection could be that being special is subjective. That is certainly true to some extent, but a practical solution is to consider any situation to be special which was discussed before it was observed. An alleged problem in classical cosmology is the so-called flatness problem, namely the observation that $\lambda_0 + \Omega_0 \approx 1$; when $\lambda_0 + \Omega_0 = 1$, the universe is spatially flat.[8] Since a flat universe was regarded as a special case before observations indicated that $\lambda_0 + \Omega_0 \approx 1$, '[i]t's an initial value that's constrained by observation and that's really all there is to say about it' [16] is *not* a valid explanation.[9] Considering

[7] That should be extended to obvious derivatives, such as two-number sequences with digits other than 0 and 1, compressed sequences in which the number of successive heads or tails codes for the corresponding number, and so on.

[8] λ_0 and Ω_0 are the current values of λ and Ω, the normalized cosmological constant and density parameter, defined as $\Lambda/(3H^2)$ and $(8\pi G\rho)/(3H^2)$, respectively, where Λ is the cosmological constant (here in units of time^{-2}), G the gravitational constant, H the Hubble constant, and ρ the density. See Helbig [10], Holman [11], Helbig [12, 13] and references therein for further details on the notation and discussion of the flatness problem in general, including discussions of fine-tuning related to the flatness problem, which I don't address here since it is *not* an example of problematic fine-tuning: although the Universe is fine-tuned in the second sense mentioned above, most literature on the flatness problem is concerned with the alleged improbability, but that is due to a misunderstanding. Interestingly, it seems that it was not really perceived to be much of a problem before Guth [14] claimed it to be a problem [15].

[9] That blog post illustrates many of the types of confusion which I address in this paper, but is confusing for other reasons as well. (One which I don't hold against her is that by Ω she means, in my notation above, $\Omega + \lambda$. Both conventions are common.) Her 'curvature density parameter' is apparently $\Omega - 1$ ('its value today is smaller than 0.1 or so'; actually, current observations suggest that it is smaller than 0.01 in absolute value, but with unknown sign) but by 'curvature density' many will assume that she means Ω ('at early times should have been close to 1'), but the only consistent reading is that she means that $|\Omega - 1|$ should be close to 1 in the early Universe. She claims to have 'no idea' why 'cosmologists ... think a likely value for the curvature parameter at early times should have been close to 1'. I have no idea where she got the idea that cosmologists believe that. If we are concerned with the classical flatness problem, then assuming a Friedmann–Robertson–Walker (FRW) model, we can *calculate* what Ω was in the early Universe, and indeed it was very close to 1, hence $|\Omega - 1|$ was close to zero. The debate about the flatness problem hinges around whether such a small value of $|\Omega - 1|$ in the early Universe is somehow unexpected. However, the typical formulation is not that $|\Omega - 1|$ should be ≈ 0, but rather that it could be anything. (She rightly complains that, with respect to the flatness problem, 'so many of them tell a story that is nonsense' and 'keep teaching it to students, print it in textbooks, and repeat it in popular science books', but does so herself, just with a *different* wrong story.) It then

the history of the flatness problem [e.g. 10, 11, 12, 13, and references therein], were that a valid explanation I am sure that someone would have thought of it before, so was surprised to see it mentioned by Hossenfelder [16, 19]. Actually, the flatness problem has been solved [e.g. 10, 11, 12, 13, and references therein], though for some reason a large fraction of the cosmological community seems to be unaware of that fact. Nevertheless, it is a good example of an example of (in this case, apparently improbable) fine-tuning which cannot be solved simply by saying that the observed values are as likely as any other and so there is no problem. (One of the key ideas in the resolution of the problem is the realization that all values are not equally likely. The observed values are not unlikely. Whether fine-tuning exists for nearly flat universes essentially depends on the measure chosen [20]; a change in the values of the cosmological parameters would have disproportionate consequences, but such a change is not small in the appropriate sense. Another issue is that, in general, the cosmological parameters change with time, so it can be more meaningful to

becomes clear that she thinks that the assumption (which it is not) that Ω was very close to 1 in the early Universe is something akin to particle-physics naturalness: 'Numbers close to 1 are good. Small or large numbers are bad'. That is a classic example of confusing two different types of fine-tuning. She even makes the confusion explicit: 'Therefore, cosmologists and high-energy physicists believe that numbers close to 1 are more likely initial conditions. It's like a bizarre cult that you're not allowed to question.' That is certainly not the case in cosmology, even among those who get the flatness problem wrong. To be sure, the flatness problem is the idea that $\Omega - 1 = 0$ to very high accuracy in the early Universe is *un*likely, but not because of the perceived likelihood that $|\Omega - 1| \approx 1$ in the early Universe. While the classical flatness problem is bogus [e.g. 10, 11, 12, 13, and references therein], and hence inflation is not needed to solve the flatness problem as the latter is usually understood (which does not mean that inflation cannot have happened), it is still legitimate to question, as she does, whether the cure of inflation is worse than the disease in that, in order to work, it requires more improbable initial conditions than the improbable conditions it is trying to explain [e.g. 17]. However, for some reason Hossenfelder thinks that Penrose's initial condition is $|\Omega - 1| \approx 1$. Furthermore, she presents a completely wrong-headed characterization of inflation, namely that it is just 'pulling exponential factors out of thin air' while it would make more sense to 'put them into the probability distribution instead'. The whole point of inflation in connection with the flatness problem is that it makes the Universe flat today regardless of the initial conditions. She seems to think that the assumption that $|\Omega - 1| \approx 1$ is *necessary* because inflation reduces it by 'I dunno, 100 or so orders of magnitude' and hence would not work if the initial value of $|\Omega - 1|$ were 'some very large value, say 10^{60}'. Actually, however, no-one claims to know the number of e-foldings produced by inflation, and that is essentially a free parameter of the theory. Whether that is good or bad is beside the point, but the assumption that $|\Omega - 1| \approx 1$ in the early Universe is needed in order for inflation to work is wrong. Indeed, most who believe that inflation solves the flatness problem probably *do* think that the initial value was 10^{60} or something. Ironically, later in the post she actually mentions the correct explanation: 'you should look for a mechanism that explains the initial probability distribution and not a dynamical mechanism to change the uniform distribution later', which is an important part of the solution of the flatness problem – or, rather, the realization that it does not exist [e.g. 10, 11, 12, 13, and references therein], and, of course, completely different from her 'initial value that's constrained by observation and that's really all there is to say about it', i.e. things are as the are because they were as they were, which explains nothing. For values which are *not* special, that is usually sufficient, but not for *special* values. She makes a similar mistake [18] by claiming that matter-antimatter asymmetry needs no explanation since allegedly the problem exists only because 'physicists think that …the number 1.0000000000 is prettier than 1.0000000001'.

characterize different cosmological models via quantities which are conserved during the evolution of the universe.)

To take a rather different example from cosmology, one could explain the m-z relation for standard candles [e.g. 21, 22], say, not via fitting the parameters of an FRW model (a cosmological model of a homogeneous and isotropic universe with dynamics according to general relativity) to the data, but rather by a Lemaître–Tolman–Bondi model [23, 24, 25] model [e.g. 26] that requires us to be at the centre of concentric shells of varying density. Why do most discount that explanation, *when other locations within that model are just as unlikely*? The reason is clearly that the centre is not just an *unlikely* place, but is also a *special* place. *Special locations (in real or parameter space) need explanations while other locations of the same probability do not.* **That is another source of confusion in the discussion of fine-tuning in cosmology.** Thus, dismissing certain parameter combinations as being just as likely as any other, just 'choosing a value that's compatible with observation' [e.g. 19], or claiming that no statement about the likelihood can be made since the underlying probability distribution is unknown [e.g. 27, 19] is not a sufficient explanation if the observed parameters are *special* in some way. As mentioned above, if $\Omega_0 + \lambda_0 \approx 1$ (meaning that the Universe is nearly spatially flat), then that *does* require an explanation, even if other sums are just as likely, because the spatially flat case is, in some sense, not only a set of measure 0, but is also *special*, since all other values of $\Omega_0 + \lambda_0$ imply a finite radius of curvature; an infinite radius of curvature is obviously special if one regards λ and Ω as free parameters (which is wrong but is what Hossenfelder and many others do). The probability distribution is a red herring. While it is true that without knowing it we cannot explain the value of the likelihood, i.e. we cannot explain why we observe a value which a priori looks to be unlikely, that is not the point; the point is to explain a *special* value. In other words, the Copernican Principle [e.g. 28] says that we should not explain our observations by our being in a *special* place (in real or parameter space), not that we should demand that we are not at an improbable place: if many others are just as improbable, then nevertheless our position can still be typical. (Also, as mentioned below, our existence might be possible only in a relatively small subset of all places.) The correct response after observing one-hundred coin flips come up "heads" is that the coin is very probably not fair, even though that particular sequence is not more unlikely than *any* other sequence.

2.6 The flatness problem and fine-tuning?

As the flatness problem indicates, not everything which appears to be fine-tuned is improbable. (Note that most discussions of the flatness problem *assume* that fine-tuning implies improbability.) Also, as mentioned above, there are other things might be fine-tuned but not unlikely, such as the adaptation of living organisms to their environment.

To be precise, the Universe *is* fine-tuned in the sense used in this paper: a small (relative to the possible ranges, which are infinite) change to λ or Ω in the very early universe *would* drastically change the late-time universe. But it is not fine-tuned in the sense in which that term is often used, i.e. the values of λ or Ω in the early Universe are not unlikely in the sense that the likelihood, though extending over an infinite range, is strongly peaked near $\lambda = 0$ and $\Omega = 1$ (which of course is just a consequence of the Universe being well described by an FRW model). In other words, flat distributions of other parameters, such as λ_0 and Ω_0 at late times or, better, invariant parameters such as α [29], correspond to distributions of λ or Ω in the early universe which are strongly peaked. In a nutshell, that is why there is no flatness problem in classical cosmology, in the sense of what is known as the fine-tuning problem [11] or the qualitative flatness problem [10].

2.7 Fine-tuning for life

The Universe is fine-tuned for life in the sense that small changes in one or more physical constants would make life impossible [e.g. 30, 31, 32, 33, 1, and references therein]. There should be no debate about that. (There is, however, not a consensus regarding the *strength* of the fine-tuning in our Universe.) The common objection that that no longer holds if one varies combinations of constants [e.g. 27, p. 114] is wrong in the sense that most of the parameter space remains hostile to life. It is also irrelevant whether some other far-removed region of parameter space could allow life as well as our region [e.g. 34]; the fact remains that most of the parameters space spanned by the physical constants is hostile to life. Such fine-tuning has been well documented and those and other common objections rebutted [33, 1].

The fine-tuning of the Universe has nothing to do with the probability of our Universe; it merely means that most of the parameter space is inhospitable without any statement about which parts of parameter space are likely, or even whether other values of the parameters are possible even in principle.

Many objections to the idea that the Universe is fine-tuned for life are due to confusing fine-tuning with low probability.

It is possible to compute things such as whether the deuteron would be stable if various input parameters to the calculation were different. If stability is possible for only a small range of those parameters (compared to the total possible range), then the stability of the deuteron is a fine-tuned quantity. That statement is independent of how probable the observed parameters are, assumed to be drawn from some real ensemble. It could be that there is a real ensemble, but strongly peaked around the observed values, so that they would not be improbable, but the stability would still be fine-tuned. Even if there is no ensemble (or, equivalently, delta-function probabilities for the input parameters), something can still be fine-tuned in the sense defined above.

A common objection to the ensemble hypothesis is that we don't know whether the parameters can differ from the observed ones. That is true; we don't know for certain that such an ensemble exists. However, it is also a hypothesis that there is *no* ensemble. In any other context, a flat prior would be considered less prejudiced than a delta-function prior in the case of complete ignorance on our part. Given the fact that the Universe is fine-tuned for life, then, if there is no ensemble, an explanation is still needed for the fine-tuning, even if the values are not improbable, at least if we want an explanation at all. In other words, it could very well be that the Universe could not be other than it is, but I see no objection to not believing that as long as there is no evidence for a theory which claims to prove it.

2.8 Possible explanations

There are several possible explanations for such fine-tuning. It could be coincidence; there are other unlikely events which, as far as I know, have no explanation, such as the equal angular sizes of the Sun and Moon (which moreover are equal only near the present time). Coincidences involving basic physics, though, are usually perceived as more puzzling, and the degree of fine-tuning in the case of life in the Universe is much greater. Or could the Universe be no other way? Perhaps, but that remains to be shown. As long as it is not possible to calculate basic quantities such as the value of the gravitational constant, mass of the electron, etc., it makes sense to assume that they could have been different. Was it designed? (A variant of the design hypothesis is that our Universe is a simulation within another universe.) Did it evolve? Or are there many universes in a Multiverse, and we shouldn't be surprised that we live in one which allows life?[10]

While "just coincidence" might be true, that is not a scientific explanation. Perhaps the most interesting possibility is that the Universe must be as it is, for reasons which we don't know. An explanation such as this has been seen as a goal of "theories of everything", though such a theory can probably not explain *everything* in a *practical* sense [36]. The burden of proof is on those who favour such an explanation; probably the only way to prove that such a theory exists is for the theory to be known. Perhaps the Universe somehow evolved [37] to be fine-tuned for life, though that proposal probably creates more problems than it solves. (Evolution, of course, does explain why organisms are fine-tuned to their environment, but that is still another type of fine-tuning.) That leaves the Multiverse and a designer as possible explanations.[11] A designer does not necessarily have to be some sort of supernatural being; that would also be the case if our Universe, or at least what we think is our perception of it, were some sort of simulation [e.g. 38, 39, 40]. Leaving aside the theological case as unscientific, a simulation would move the question of fine-tuning to the universe in which the simulation is running, about which we know nothing (not

[10] See Smeenk and Ellis [35, sect. 4.2] for a similar list.

[11] One could claim that the Universe does not have to be fine-tuned for life because *some* sort of life would arise whatever the conditions. However, since most of the parameter space is "boring" [e.g. 33], that seems unlikely.

even whether there is any sort of fine-tuning problem there – the simulated physics could also be completely different from the real physics).

So, according to the present stage of our knowledge, the best explanation for fine-tuning is the Multiverse – more specifically, the Anthropic Principle applied to the Multiverse. Of course, that does not rule out that some other explanation is possible, and such an explanation would be, at least to many, preferable to an explanation involving the Anthropic Principle applied to the Multiverse. For example, there have been attempts to relate the value of the cosmological constant Λ to the values of the gravitational constant G and the fine-structure constant α [e.g. 41, 42], although in my view it is fair to say that such ideas are still too speculative to be a better explanation than the Anthropic Principle applied to the Multiverse.

3 The Multiverse

3.1 Types of Multiverses

A typical argument used against something one does not like is to claim that it is a recent ad-hoc invention. Despite the evidence to the contrary, such claims have been made even regarding dark matter and dark energy [43], even though they are easy to disprove [44]. So it is not surprising that such claims have been made against the idea of the Multiverse, even though it is a concept with a long history [e.g. 45]. Almost by definition, another universe in the Multiverse cannot be observed. That is probably why Tegmark [46, 47] includes stuff outside of our particle horizon in his Level I Multiverse, even though, at least in some cosmological models, that horizon grows with time, i.e. more and more of the region now hidden comes into view. Most people wouldn't think of the stuff outside of our horizon as in another universe or as being part of the Multiverse, but at least Tegmark is consistent in his terminology. (Tegmark's Level I Multiverse is what many call the Universe, with other universes being in his Level II Multiverse.) What most people refer to as the Multiverse is Tegmark's Level II Multiverse, i.e. a (perhaps infinite) collection of physical universes, of which our Universe is one example; that is the sense used in this paper. (His Level III Multiverse are the many worlds in the many-worlds interpretation of quantum mechanics [48] and his Level IV is his Mathematical Universe. Here, we are concerned only with his Level II Multiverse. Note that Tegmark [47] discusses fine-tuning within the context of the Multiverse, while Lewis and Barnes [33] discuss the Multiverse within the context of fine-tuning.) The volume edited by Carr [49] demonstrates that the Multiverse is now part of mainstream science and not some sort of fringe idea, despite claims that it is unscientific [e.g. 50, 51].[12]

[12] While both Ellis and Silk are very well known and influential cosmologists, note that Martin Rees, probably the most famous living astronomer, is a supporter of the Multiverse [e.g. 52, 53, 54, 55]. Also note that while Ellis is sceptical, he by no means dismisses the concept entirely. An overview is provided by Carr and Ellis [56], arguments against some

3.2 Why believe in the Multiverse?

One argument for the Multiverse is that it is a consequence of theories which we otherwise accept. Assuming that we accept those other theories, that is not a problem. (At least classically, we can never observe what happens in a black hole, but nevertheless we tend to believe what GR tells us about that region.) Often, the Multiverse is discussed in the context of eternal inflation [e.g. 60, 61] or the string-theory “landscape” [e.g. 62]. But what if we don’t believe any theory which has the Multiverse as a consequence? There are at least two other arguments. (In such a case, the Multiverse is indeed “just” a hypothesis, though of course there is nothing wrong with that, and it still might be shown later that it is a consequence of some theory which we accept for other reasons.)

One is that there is no other good explanation for fine-tuning. That is similar to the answer to the question why the Earth is just at the right distance from the Sun for life to exist. Just as the “plurality of worlds” (which meant not just unobserved planets but whole “universes” in the sense the word was use in the Renaissance, i.e. a shell of fixed stars surrounding a solar system (or even a system with the equivalent of Earth at the centre)) was put forward as an idea before there was any evidence of other solar systems [63], one can put forward the Multiverse as a hypothesis. It is more or less an accident of history whether the observation or the theory comes first: some times theories predict things, other times they explain what is already known. Of course, a postdiction is, at least intuitively, less convincing than a prediction, and the corresponding theory probably needs to make a prediction which is later confirmed in order to be accepted. For example, in the case of the planets in the Solar System, Kepler’s mathematical “theory of everything” explanation involving Platonic solids turned out to have no basis in reality, the correct explanation being that, although they are not completely random, the distances of the planets are essentially contingent, with the weak Anthropic Principle explaining the fine-tuning regarding the distance of the Earth from the Sun (see Sect. 4). On the other hand, Planck’s ad-hoc hypothesis that radiation could be emitted and absorbed only in discrete packets turned out to be true. The credibility of such an explanation should not depend on whether we have independent evidence that it is true [e.g. 64, 65, 6, 66, 67].

Another reason is “why not?” Although not absolutely necessary, one can think of our 3+1 dimensional space as being embedded in a higher-dimensional space. Why should our Universe be the only one there? Whatever caused the origin of our Universe, why should it have happened only once, or indeed any finite or even infinite number of times?

We must assume that the constants of nature can vary from universe to universe in order to explain fine-tuning via the Anthropic Principle as long as we have no theory which has the Multiverse as a consequence and in which

aspects of the Multiverse by Ellis [57], and a rebuttal by Carr [58]. An excellent impartial discussion is given by Friederich [59].

that is the case. That seems a valid assumption, though, as long as we have no reason to believe that they can't; that is supported by the fact that many constants of nature are consistent with being random [e.g. 68]. (Of course, if they don't vary, then we have the same problems as if there were only one universe, namely our Universe.)

3.3 The Multiverse and fine-tuning

Either the Multiverse exists or there is only one Universe.

If the Multiverse exists, in the sense that the fundamental constants can take on different values in different universes, then we are done. Some universes will be hospitable, in the same sense that some planets are hospitable. It doesn't need a fundamental explanation. *It also doesn't matter how unlikely our Universe is, as long as it has a finite probability.* The fact that most other universes are uninhabitable is no more puzzling than the fact that most places in our Universe, in our Galaxy, in our Solar System, even in our Earth (if we include the entire Earth and not just the surface) are uninhabitable. If we have a theory which predicts the Multiverse (consequences of theories do not themselves have to be testable; we believe what GR says about the interior of black holes), even better one which has some observational support, then that is nice. It is not necessary, however. Someone could have told Kepler that his effort to explain the relative distances of the planets from the Sun was a waste of time: just imagine very many stars, many with planets; what we observe is just a more or less random outcome. Also, by chance some planets will have the right distance from the Sun for life, so no deeper explanation is needed. That would be a good theory even without observational support, just as natural selection was a good theory even though Darwin didn't have a mechanism to explain heredity.

If the Multiverse does not exist, then either the Universe could not have been any other way or it contains some random features (without the implication of a real Multiverse).

In the former case, the burden of proof is on those who make that claim. Otherwise, we give up science: anything is explained away by claiming that that's just how it is, it couldn't be any other way, with no underlying theory. That is clearly not scientific.

In the latter case, either the constants in our Universe are just "random" values, or in some sense they are likely.

In the case of random values for the constants of nature, we are essentially back to the just-so story: that's just the way it is, and we have no idea why. Again, not scientific. Science seeks to explain why we observe what we observe. Of course, the Anthropic Principle applies even if there is no Multiverse, thus our Universe must support life. If the Universe were not fine-tuned, then there would be no puzzle as to why it supports life, though we still wouldn't know why those parameters and not others.

If the values are in some sense more likely, that is, if there is some distribution sharply peaked at the observed values (in the limit of a delta function, we are back at the case that the Universe could not have been any other way), then we need an explanation for that, and the burden of proof is on those making the claim. (It is of course difficult if not impossible to apply the concept of probability to just one universe, though some, such as McCoy [69], discuss probability in cosmology without invoking a physical multiverse. His paper is a good introduction to the concept of probability in cosmology.)

Of course, it is entirely possible that some explanation of the fine-tuning of the Universe for life might be found which doesn't involve the Multiverse. Just because the Multiverse *can* explain everything doesn't mean that it *does* explain everything. On the other hand, if, as in the case of the flatness problem, another, and presumably better, explanation is found, it does not follow that *everything* explained by the Anthropic Principle *must* have another, and presumably better, explanation.

3.4 The Multiverse is scientific

A common objection to the Multiverse, at least one containing an infinite number of universes, is that it explains nothing, since anything which *can* happen *will* happen.[13] While some quantities might have fundamental explanations, there is no reason that *all* quantities, such as the temperature where at this moment where you are reading this, *must* have fundamental explanations. The burden of proof is on those who prefer a fundamental explanation to find that explanation. (On the other hand, just because the Multiverse can explain everything, in the sense of the ultimate ensemble theory of Tegmark [46], there is no reason to exclude other explanations. Just like in day-to-day life, some things will have "deep" explanations and some things will just be the way they are.)

Neither is it a valid objection that we cannot directly observe other universes in the Multiverse. Apart from the fact that that is not necessarily true [e.g. 70, 71, 72, 73], it is not uncommon to believe all consequences of a theory for which there is otherwise evidence. For example, entire books have been written about the interior of black holes, i.e. what is contained within the event horizon [e.g. 74], even though that is a region which we cannot observe. Of course, the fact that we cannot directly perceive (whatever that means) other universes in the Multiverse is no more an objection to the Multiverse than the fact that we cannot directly perceive atoms is a valid objection to atomic theory, though it has been raised, famously by Mach [75]. Finally, one should not play word games. Obviously, if one *defines* the Universe to be all

[13] The same idea is sometimes expressed by stating that it explains anything, or that it explains everything. While explaining nothing is clearly not satisfactory, perhaps one shouldn't be so sceptical of something which explains everything, for some definition of 'explain'.

that there is, then there can be no other universes. What matters are not the words but the concepts.[14]

4 The Anthropic Principle

Entire books [e.g. 76] have been written about the Anthropic Principle since its introduction in the modern form by Carter [77]. All that is needed here is the very simple, almost tautological, idea that observers must find themselves in a universe compatible with their existence; in particular, we must find that our Universe is compatible with our own existence. That is usually known as the weak Anthropic Principle.[15] That is true even without the Multiverse, though in that case it provides no real explanation. *In a Multiverse where the constants of nature vary in an essentially random way from universe to universe, and where there are an infinite, or at least a very large, number of universes,* **then we need no further explanation for the values of the constants of nature in *our* Universe**; in such cases, *it does not matter whether our Universe is probable or not.* **That is another source of confusion in the discussion of fine-tuning in cosmology.** Such a Multiverse is a sufficient explanation for parameters which are fine-tuned for life.[16] For other parameters, it can be an explanation, but that does not rule out other explanations, neither explanations involving probabilities and the Multiverse nor other types of explanation. (The fact that many more humans live in China and India does not mean that some sort of special explanation is needed if one finds oneself living in Croatia; indeed, one would expect it to be more likely that one is in Croatia given the additional fact that one speaks Croatian.) To be sure, in cases where a large range of values for the constants of nature are compatible with life, then of course one can use the Multiverse as an explanation for the values in our Universe only if one can show that the values we observe are

[14] Indeed, Tegmark's Level I Multiverse is what many call a universe, e.g. in an FRW model that which is described by that model, which for example in a spatially closed case can have a definite mass, even though part of it might lie beyond our particle horizon, and might even always lie beyond our particle horizon.

[15] Bostrom [78] counted 30 different versions of the Anthropic Principle. Also note that sometimes various authors use the same name for different versions and/or call the same version by different names. The concept has evolved over time; a nice introduction is given by Ellis [50]. Also important is the fact that the Anthropic Principle is not in conflict with the Copernican Principle, i.e., while the latter claims that we are not in a privileged position in time or space, that does not imply that our position is not special in any way; it is perhaps best described as unrepresentative or biased [e.g. 79, 80]. See Williams [81] for a historical overview of the Anthropic Principle.

[16] 'Does the idea that "all that can exist, exists" in the ensemble context provide an explanation for the anthropic puzzles? Yes it does do so. The issue of fine-tuning is the statement that the biophilic set of universes is a very small subset of the set of possible universes; but if all that can exist exists then there are universe models occupying this biophilic subspace.' That is one of the conclusions of Ellis et al. [82], who provide an interesting survey of the topic of Multiverses in physical cosmology. To be sure, Ellis has also been critical of the Multiverse [e.g. 57], but usually with respect to the question as to whether it is a testable hypothesis, rather than a good explanation for observations.

those which a "typical observer" would observe. However, that has no relation to the Anthropic Principle (except in the trivial sense that observers are alive, though if the observed value is at or near the peak of a sharply peaked distribution over *all* universes and not just those which contain life, then the Anthropic Principle is not necessary at all), so the common objection that the Anthropic Principle as applied to the Multiverse is useless unless we know the underlying probability distribution [e.g. 27, p. 115 and elsewhere] is wrong. The fact that other universes with other types of life might be much more probable than our own Universe is no more of a problem than the fact that other universes without life are much more probable than those with life, or that there are more planets without life than with life; the essential idea of the Anthropic Principle as applied to the Multiverse is that one is concerned with a conditional probability (namely, that of our existence).

Even if there are not an infinite number of universes in the Multiverse, there is nothing wrong with low-probability universes being favoured by the Anthropic Principle, nor indeed with our living in such a Universe, as long as there is a reasonable probability that at least one *such universe exists.* **That is another source of confusion in the discussion of fine-tuning in cosmology.** (In a typical lottery, one shouldn't be surprised if someone wins every week as long as there are not significantly fewer players than the number of possible combinations. However, one *should* be surprised – and seek a better explanation than "just coincidence" – if someone wins every week in a lottery in which the number of possible combinations is much greater than the number of players and/or if the same person wins every week.)

Some, but not all, cases of constants of nature being fine-tuned for life also involve fine-tuning in the particle-physics sense of lack of technical naturalness. If only a small range of values is anthropically allowed, then the Anthropic Principle can explain such cases. However, there are other cases, such as the strong-CP-violating angle θ, where life does not seem to be sensitive to the value, even across the entire range from 0 to 2π [e.g. 68]. The fact that the strong-CP-violating angle, at 10^{-10}, is very close to 0 cannot be explained by the Anthropic Principle, though attempts have been made to explain it using the Anthropic Principle together with additional assumptions [83, 84]. (That is an example of a quantity which is neither fine-tuned nor more improbable than any other value, but equal or very close to a special value, namely 0. Note also that that is something which not only the Anthropic Principle cannot explain but neither can the Multiverse – more precisely, the Multiverse could explain the existence of the particular value ≈ 0, but not the fact that we observe it to be equal or very close to a special value.) Of course, it is possible that some examples of lack-of-technical-naturalness fine-tuning can be explained by the Anthropic Principle while others have another, yet unknown, explanation. (The fact that some examples (must) have some other explanation does not rule out the fact that that for others there is no explanation other than the Anthropic Principle.)

Many expected the LHC to find new physics at the TeV scale [e.g. 61], since that would allow a technically natural explanation for the small mass of the

Higgs boson. Since no new physics has been found, it is more plausible that the small Higgs mass can be explained by the Anthropic Principle, the Higgs mass being a parameter to which life is sensitive [e.g. 85, 61]. Weinberg [86] suggested that the observed value of the cosmological constant can be explained by the Anthropic Principle.[17] That explanation is valid whether the one believes that the small (relative to the Planck scale) value of the cosmological constant is fine-tuned (i.e. the "cosmological-constant problem") or that the value itself (apart from the fact that much larger values would be incompatible with life) is unproblematic [2, 87].

Objections to the Anthropic Principle are similar to those to the Multiverse. While the two are often discussed together, one can have one without the other. For example, a theory which is able to explain the observed value of some parameter to which life is not sensitive by calculating that it occurs in a large fraction of universes makes little if any use of the Anthropic Principle. Also, the Anthropic Principle can be invoked without invoking the Multiverse, although the interpretation is not as straightforward as when invoked with the Multiverse. Again, there is no reason to doubt that other explanations *could* exist, but the burden of proof is on those who claim that those other explanations exist. In other contexts, such as the explanation for the distance of the Earth from the Sun being just right for life, it is clear that the Anthropic Principle is a better explanation than an explanation from first principles (which doesn't exist anyway); there is no reason to believe that explanations involving the Anthropic Principle must cease to work at some scale. Certainly in the sense of the weak Anthropic Principle, there is no reason to regard it differently than any other sort of reasoning, though proponents of it, or even those who discuss it without necessarily advocating it, are often greeted with great scepticism. According to Vilenkin [88], 'Steven Weinberg once said that a physicist talking about the anthropic principle "runs the same kind of risk as a cleric talking about pornography. No matter how much you say you are against it, some people will think you are a little too interested."'

5 Summary and Conclusions

Perhaps no topic in modern cosmology is debated as hotly as that of the Multiverse and the distinct but related topics of the Anthropic Principle, fine-tuning (in more than one sense of the term), and necessary conditions for the existence of life – in particular, for our existence. To some extent the debate might be due to confusion of various terms and misunderstandings about the role of probability. One should not be less strict in connection with those topics than with other topics in science, but at the same time one should not be more

[17] The explanation is an interesting one: Weinberg assumed that the naïve particle-physics prediction of an extremely large cosmological constant is correct, and that there is *in addition* a "bare" cosmological constant with a *negative* value which almost but not quite cancels the particle-physics vacuum-energy contribution, the value of the resulting effective cosmological constant being selected according to the Anthropic Principle.

strict just because of personal or philosophical objections. In other contexts, it is clear that not all ramifications of a theory have to be testable for the theory to gain confidence, that "just so" is not a satisfactory explanation for *interesting* coincidences, that typical observers are not necessarily located in a typical position but rather in a typical position compatible with their existence, and that there is a difference between absolute probability and conditional probability. Perhaps less clear is whether the Universe is fine-tuned for life, especially if one takes into account that there might be more bizarre types of life than we can imagine. That has no bearing, however, on the fact that our Universe is fine-tuned for *our* existence. If one desires an explanation for that, then it seems that the Anthropic Principle applied to the Multiverse is the best answer.

In summary, the Universe is fine-tuned for life, but that statement says nothing about probability, only that a small part of parameter space is hospitable to life (i.e. it is not necessary to know the relative probability of different regions of parameter space). The Multiverse explains why the Universe is fine-tuned for life in the same way that the plurality of worlds explains why the Earth is at the right distance from the Sun. In the case of life, whether it is probable is irrelevant. (Since there appear to be parameters which have special values but are not relevant for life, it is clear that at least some cases of such fine-tuning might need explanations other than the Multiverse, if indeed an explanation is needed at all.) Although sometimes probabilities are discussed without invoking the Multiverse, it seems sensible to discuss probabilities only when parameters can be drawn from an actual ensemble. While it is often claimed that the idea of the Multiverse is unscientific, the opposite is true. Rejecting the Multiverse leads to a just-so story – either the Universe could not be any other way or the parameters just happen to be what they are – , which is not scientific.

My impression is that there is actually more agreement than the debate suggests, because much confusion is due to confusion of terminology and not of ideas. We should be optimistic. There are many topics in cosmology where there were similar problems, or even genuine confusion, which were eventually resolved (although one occasionally meets someone who has not noticed that): Olbers's paradox [e.g. 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 28, 97], cosmological horizons [100], the relations between redshift, velocity, and distance [101, 28], and the flatness problem [e.g. 13, and references therein].

Acknowledgements An earlier version of this work was presented at the conference Cosmology 2018 in Dubrovnik. This research has made use of NASA's Astrophysics Data System Bibliographic Services.

Conflict of interest

The author declares that he has no conflict of interest.

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