Quantum Sortal Predicates*

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Abstract

Sortal predicates have been associated with a counting process, which acts as a criterion of identity for the individuals they correctly apply to. We discuss in what sense certain types of predicates suggested by quantum physics deserve the title of 'sortal' as well, although they do not characterize either a process of counting or a criterion of identity for the entities that fall under them. We call such predicates 'quantum-sortal predicates' and, instead of a process of counting, to them is associated a 'criterion of cardinality'. After their general characterization, it is discussed how these predicates can be formally described.

1 Introduction

The difficulties in providing a clear distinction between *sortal predicates*, which originate from terms like 'man', 'tree' and 'book' and other predicates, which come from terms like 'green' and 'thing', have been acknowledged in the philosophical literature. The subject is related to a more general discussion on the nature of general terms and has its roots (at least) in Aristotle's concept of second substance. Frege's reference to those predicates that isolate what fall under them in a definite manner, and Quine's consideration of predicates that divide their reference are also recalled in dealing with the subject (Wallace 1965; Stevenson 1975).

However, little has been done to link aspects of the epistemology of quantum physics with this discussion on sortal predication. In this paper, we shall present some guidelines for providing such a relationship by presenting a kind of predicate, here termed 'quantum-sortal', which also deserve to be included in the pantheon of sortal predicates. The differences between quantum-sortal and standard sortal predicates are emphasized, and a way of dealing with them within a logical system is outlined. Let us begin by recalling in brief some of the main ideas concerning sortal predication.

Two philosophers have been particularly referred to in the discussions on sortal predication, namely, P. F. Strawson and P. T. Geach. According to

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Strawson, general terms are divided up into *characterizing* and *sortal* terms (Strawson 1959, p. 168), while Geach talks in terms of *adjectival* and *substantival* general terms respectively (Geach 1962). Jonathan Lowe, in discussing the subject, has chosen *adjectival* and *sortal* respectively to designate them, and has mentioned M. Dummett's criteria for distinguishing between these two kinds of general terms (Lowe 1995, Chap. 5). Dummett's criteria, according to Lowe, may be summed up as follows:

(i) Adjectival terms have associated with them a *criterion of application*, and by this we should understand a general principle determining which *individuals* the considered term correctly applies to (we have emphasized the word 'individuals' for the purposes to be mentioned below).

(ii) In addition to a criterion of application, a sortal term has also a *criterion of (numerical) identity* associated to it. This is to be a principle determining the conditions under which one individual to which the term applies can be said to be the *same* or *distinct* as another.

As recalled by Wallace (*op. cit.*), the distinction between adjectival and sortal terms is very subtle, and it is difficult to find a clear way of providing an 'objective' description of what should be understood by the mentioned criteria. Notwithstanding, in general the criterion of identity has been associated with a *counting process*, that is, something which should enables us to *count* things that fall under a certain sortal term. Take for instance (Lowe's example), the term 'tree'. Of course we may suppose that we have a criterion of application which enables us to apply the term to some *given* object; furthermore, we can *count* trees, at least in principle. So, 'tree' is a sortal concept, and 'to be a tree' is a sortal predicate. But 'green' (another of the Lowe's examples) is not, for while we have a criterion of application for identifying green things, despite the vagueness associated with this predicate (this point shall be mentioned below), we don't have an associated counting process for green things. As realized by Lowe, in trying to count green things in a wood, probably we would not know what things to count; should we count only the trees in a wood as green things? If the grass is also to be counted, then should the grass be counted as just one object of should we count also every leaf of grass? The same holds for the leaves of the trees. And what about the parts of the leaves? Since a leaf can be divided up, say, in circular or triangular parts, are these parts also to be counted as distinct green things? Similarly, the same holds with terms like 'thing'; the reader may try to count the things there are in the room you are now. How many things are there? Are the parts of the things (say, the lock of a door) also 'things' which should be added to the list? And what about the components of the lock? All of this exemplifies, albeit so roughly, the claim that 'green' and 'thing' are examples of non-sortal terms.

The counting process is acknowledged by the philosophers who have discussed the subject as the distinctive feature of sortal terms, and of sortal predicates in particular. So, Wallace says: "a sortal predicate 'F' provides a criterion for counting things that are 'F' ", and continues: "[i]f 'F' is a sortal predicate, you can find out how many F's there are in such and such a space by counting" (*op. cit.*). Other philosophers talk quite similarly, so that we don't need to quote them here (but see the references). The interesting thing is that all the discussion is developed in connection with, say, 'macroscopic' objects, that is, the efforts are given to describe a *given thing* as falling under either a sortal or as adjectival predicate; really, the examples concern 'trees', 'mountains', 'green things', 'bald men' and so on, that is, *individuals* of a sort (recall what was said above about Dummett's criteria). This is of course not so strange for, as says S. Auyang, "the paradigms of objects are the things we handle everyday" (Auyang 1995, p. 11).

Furthermore, even among sortal terms there are difficult distinctions to be taken into account. For example, as recalled by Lowe (*op. cit.*), sortal terms like 'green' and 'mountain' have different criteria of identity. Let us follow Lowe for a moment; he says:

> "The criterion of identity for *trees*, for instance, is very different from the criterion of identity for *mountains* (...) *Trees* (...) can undergo very considerable changes of shape and position [he is referring to the possibility of a tree we know very well to be transplanted to another place in our garden during our absence] while remaining numerically the same, that is, while persisting identically through time. By contrast, it does not make much sense to talk of *mountains* undergoing radical changes of shape and position (...) If the land falls in one place and rises in another, we do not say that a mountain has *moved*, but rather that the mountain has ceased to exist and another has been created. (To be sure, we do allow 'small' changes in the shapes and positions of mountains, and this does potentially lay us open to paradox, since a long series of small changes can add up to a large change –as in the notorious paradox of the bald man. This, however, just shows that 'mountain', like many other general terms in ordinary language –such as 'red' and, indeed, 'bald'– is a *vague* term.)" (Lowe 1995, pp. 95-6)

This criterion of remaining the same in time, as is well known, lies within the old discussion concerning identity through time (Magee 1973, pp. 58ff), but will be not discussed here. Although Lowe purposely does not discuss vague terms, his above quotation suggests that among sortal predicates (like 'to be bald') there are also those which are *vague*, and of course these should also deserve some attention.

The discussion in the pertinent literature pushes the topic on several interesting and important points, but we would like to contribute to the subject by presenting a different kind of problem. So, instead of trying to discuss the given examples and cases, let us complicate a little bit the already confused discussion on general terms by presenting other kind of terms which, as far as we know, have not yet been taken into account in such contexts. These are general terms like 'electron', 'proton' and other terms provided by quantum physics (we could include here 'strings', 'membranes' and so on). Are these terms mere adjectival or are they sortal terms? In the second case, do they have also a criterion of counting despite the indiscernibility of the objects they apply to? This is what we shall discuss in the next section. In doing that, we shall also mention the case of vague sortal terms.

2 Quantum-Sortal Predicates

Let us consider a predicate like 'to be a proton of a Lithium atom', which we shall term P (the reader may think of the ^{7}Li atom, which has 3 protons and 4 neutrons in the nucleus). Alternatively, we should consider the following predicate (which we could use to communicate a certain experience to our audience): '(to be) the atom which was ionized negatively by capturing an electron and, after a short time later, has reverted to a neutral state by releasing an electron' (taken from Lowe 1994).

These cases indicate that, first of all, it should be realized that general terms like 'proton', 'electron', etc. are not mere adjectival general terms (in the above sense), since they do not merely *characterize* (to use Strawson's words) an object as a such-an-such (cf. Strawson *op. cit.*, p. 167). Perhaps we could say, in Quine's sense, that they also do not divide their reference: a proton cannot (so far as we know) be divided up into smaller parts, and the same holds for other kinds of particles.[1] Terms like these apparently refer to certain kinds (sorts) of entities which are closer to *trees* in a wood or of *dogs* in a city than to *green* things in a wood or *things* in a room. Really, we may talk of the three protons in the nucleus of the Lithium atom, or of the three electrons it has without confusion of concepts. In other words, terms like 'electron', 'proton' and so on should be treated as sortal terms of a kind. But, of what kind? Do they have the above mentioned characteristics attributed to sortal terms? The very surprising thing is that they do not. Let us see why.

Quantum objects are very interesting and strange entities, of course. Even the word 'object' is to be used with some care in this context; some like J. M. Lévy-Leblond and F. Balibar have used the word *quanton* to designate this "different kind of entity" (Lévy-Leblond and Balibar 1990, p. 69).[2] Even so, we continue to *talk* about them as we talk about the usual objects of our surroundings. Perhaps this is due to our ways of reasoning and to the way we make use of languages. G. Toraldo di Francia reminds us that we usually divide the world into objects to talk about them so as in order to enunciate the laws of physics (Toraldo di Francia 1981, Chap. 4), and this happens even with respect to *quantons* (including strings and other entities introduced more recently). These, we should remark, sometimes *can* be treated as individuals, but at the expense of introducing restrictions on the states they may be in, as shown by French and Redhead (1988). But another common talk is in the

[1] Even if we consider the annihilation/creation processes, we may accept the fact that, after interaction, a proton remains being a proton.

[2] Apparently, this terminology came from Mario Bunge.

sense that they are *non-individuals* of a kind, as pointed out by E. Schrödinger, M. Hesse, M. Born and H. Weyl for instance, having no criterion of individuation, no identity (see French 2000 for further discussion on this point). This hypothesis lies at the core of quantum statistics, as is well known. Of course we should begin by posing that the very concept of *individual* is problematic here, and if we are to accept that the above mentioned 'quantum terms' define some predicates, then we should also accept that a possible revision on Dummett's criteria above would be indicated, since, as we have emphasized, they speak in terms of individuals.

All of this suggests the above claim that terms like 'proton', 'electron' and so on have associated to them a criterion of application, although we don't have the 'individuals' properly to apply this criterion. In Toraldo di Francia's sense, these entities *came with the theory*:[3] "[i]n some way, physical objects are today knots of properties, prescribed by physical law" (Toraldo di Francia 1978). But we realize that these terms are not merely adjectival. So, let us call *quantum-sortal predicates* (q-s-predicates) those predicates we are discussing. Then we could say the following regarding them:

(i) Quantum-sortal predicates have a criterion of applicability which tells us to what kind of entity they apply to. For instance, the predicate P above applies to protons, and not to electrons, and the distinction between these two categories of quantum entities may be assumed to be no less clear to the physicist than the terms 'tree' or 'mountain' are to the average person, for physicists have the possibility of recognizing (either by theoretical or by experimental means), whether a given physical system is, say, an electron system or not.

(ii) Although we generally cannot say that there is a well defined criterion of identity which enables us to distinguish between 'two' objects that fall under a certain 'q-s-concept', as for instance to distinguish among the three protons of the Lithium atom, even so we usually refer to a *quantity* of them: "the objects of physics are associated with natural numbers" (Toraldo di Francia 1981, p. 306).[4]

This associated *number*, although obtained in different ways, as for instance by means of Feynman diagrams (*ibid.*, pp. 302-4), are of course not given by *counting*, if by this we understand the usual attribution of an ordinal to their collection. So, we have an interesting situation where a certain collection of 'objects' (*quantons*, to use the terminology from the above), may have a cardinal, but not an ordinal. These collections shall be identified with *quasi-sets* below.

To provide a further characterization of the predicates we are introducing, let us make a comparision with other 'more usual' ones which could come to the mind. So, following Terricabras and Trillas (1989), let us call *Fregean* a predicate F which induces a bipartition in the domain of discourse into two

[3] This is particularly clear today if we think of the Higg's bosons.

[4] Toraldo di Francia also considers the case of virtual particles, when even the cardinal of a collection of them may be not well defined, but we shall not discuss this case here.

disjointed subsets whose union gives again the whole domain. In other words, a Fregean predicate F enables us to define a mapping $f : D \longrightarrow \{0, 1\}$, where D is the domain, such that:

$$D = f^{-1}(0) \cup f^{-1}(1),$$

and these two sets are disjoint. In the standard semantics, the set $f^{-1}(1)$ is the *extension* of the predicate F, while $f^{-1}(0)$ is its complement relative to D. So, given any $x \in D$, one of the two possibilities holds: either $F(x)$ is true (when $x \in f^{-1}(1)$) or $\neg F(x)$ is true (when $x \in f^{-1}(0)$). These are the predicates we deal with in classical standard logic.

Vague predicates may be characterized as follows (Terricabras and Trillas *op. cit.*): a vague predicate V induces a mapping $v : D \longrightarrow [0, 1]$ (the closed interval of real numbers) so that

$$D = v^{-1}(0) \cup v^{-1}(1) \cup \bigcup_{r \in (0,1)} v^{-1}(r),$$

where $\bigcup_{r\in(0,1)} v^{-1}(r)$ is a non empty set and these three sets are pair-wise disjointed. Of course the case of Fregean predicates can be incorporated within this framework by supposing that in this case this last set is empty. So, if $\bigcup_{r\in(0,1)} v^{-1}(r)$ is not empty, we can say that there are objects in D for which we cannot assert neither that they have the property V nor that they have not. For instance, if $V(x)$ means 'x is bald', then if $x \in v^{-1}(0)$, we say that x is not bald; if $x \in v^{-1}(1)$, we say that x is bald, but if $x \in \bigcup_{r\in(0,1)} v^{-1}(r)$, we should say that x is 'more or less' bald, depending on the place of the r in the interval $[0, 1]$; the more r is closer to 1, the more x is bald. As remarked by Terricabras and Trillas, the semantic analysis of vague predicates could be done by using fuzzy sets, as seems clear from the above discussion.

But in both the considered cases (Fregean and vague predicates) we are dealing with *individuals*; the worst case (involving vague predicates) is the situation where we have a certain 'well defined' individual, say the well known Mr. X, and we are only in doubt whether he is or not bald. The uncertainty is *epistemological* only. A different situation is posed by quantum objects, for in this case we don't have the 'individual' to look at and to classify according to our standards. The objects of quantum physics come to us already pre-packaged as entities of a sort, given by theory as such. Toraldo di Francia says that they are *nomological objects*, given by physical law (*op. cit.*, p. 222), having fixed and prescribed characteristics: "the *naked* particle is not observable" (*ibid.*, p. 305). Sunny Auyanng helps in fixing this idea; as she says, "[p]eople had a fairly clear notion of planets before the advent of Newtonian mechanics. The same cannot be said of quarks: no one had dreamed of them in the absence of quantum mechanics" (*op. cit.*, p. 7); with the due qualifications, the same of course could be said of the other quantum particles (understood as 'quantons').

Despite the difficulties also in characterizing precisely these entities as nomological in Toraldo di Francia's sense, for it is not clear how these alleged properties are 'fixed and prescribed', or in what sense are these objects 'prescribed' by physical law, the idea is useful in helping us to point out that we are faced with a different kind of uncertainty: an *ontological* one (and this reminds us again of Lévy-Leblond and Balibar's use of the word 'quantons', as mentioned above). But the problem now is not with the predicates properly: they are not vague at all, for the physicist knows very well what a certain object must satisfy, say, to be classified as a proton. He *knows* protons via theory. The uncertainty is concerning the entity itself. In other words, since indistinguishable quantum objects (those sharing all their state-independent, or intrinsic, properties) cannot be distinguished from one another, the problem is to identify the extension of a predicate like 'to be a proton of a ^{7}Li atom', for whatever collection with three protons will do just as well, and this is quite different from, say, 'to be a U. K. Prime Minister' (put another way, the extension of this last predicate obeys the Axiom of Extensionality of set theory, while the extension of the former does not). This last point, let us recall, is linked to one of the most basic assumptions of quantum theory, namely, the Indistinguishability Postulate (Redhead and Teller 1991), which roughly says that permutations of indistinguishable quanta are not regarded as observable. So, in the semantic analysis of such predicates, even fuzzy sets are not of much help, for while they enable us to deal with epistemological uncertainty in the above sense, they do not help us in dealing with ontological uncertainty.

So, we may say that the *quantum-sortal predicates*, that is, those predicates such as the above suggested by quantum physics, have the following main characteristics:

(i) They have a criterion of applicability in Dummett's sense mentioned above.

(ii) Instead of a criterion of identity, there is a *criterion of cardinality*, a principle which enables us to say that in certain situations the predicate truly applies to a certain number (generally finite) of entities, yet sometimes there is no counting process associated with them. This number is sometimes called the 'occupation number' (see Auyang *op. cit.*, pp. 159-60), and may vary from one application to another.

(iii) In certain situations, such as those involving indistinguishable quantum entities, the extension of the predicate is not well defined, in the sense that another collection of similar objects with the same cardinality may act as its extension as well. So, we may say that there is a kind of *opacity* involving at least some objects of the domain, for the issue becomes not that of involving predicates lacking 'sharp boundaries' (as in the situations involving vague predicates), but rather of the objects to which the predicates apply lacking individuality (Krause and French 1999).

Terms (and the corresponding predicates) like these should be included among the pantheon of general terms and they should be considered in the semantic analysis involving predication and reference in general. So, we should ask for a way of characterizing them formally. This is what we shall do in the next sections.

3 Sortal Logics

Sortal predication sometimes has been treated formally by means of the related concept of relative identity. Peter Geach, in the 60's, suggested that there is no 'absolute' identity, and that all identity statements are relative. So, according to him, when we say that 'x is identical with y', we aim to say that 'x is the same S as y', where S is understood as a sortal predicate ("a count noun", according to Geach) (Geach 1967). The aim of sortal logics is to treat these predicates differently from standard one-placed predicates, but this is not so easy; as recalled by Stevenson (*op. cit.*, who does not follows Geach's ideas) in usual first-order logic we can write $x = y \wedge S(x)$ to mean 'x is the same S as y'. But, in this case, what should distinguish S as a sortal predicate in the sense already explained? The way this distinction is achieved is a source of controversies.

Really, there has not been a 'proliferation' of sortal (formal) systems in the literature. Pelletier's review (1992) provides a general overview of the subject, and he mentions the works of Smiley, Wallace, Stevenson (*op. cit.*) and Tennant on sortal logic (Pelletier 1992, where the references are given). Since we do not aim to revise these systems here, we shall stay with Pelletier's analysis taken for granted. Anyway, our arguments here do not depend on a revision of the proposed systems.

Pelletier says that in trying to characterize a sortal logic, these systems have provided only 'syntactic sugar', for according to him none of them provides a clear distinction between sortal and standard one-placed predicates: as he says,

> "... the accounts produced are merely notational variants of classical restricted quantification theory –which of course is a mere notational variant of classical quantification theory (...) In restricted quantification theory we 'abbreviate' formulas of the form $\forall x(Fx \to Gx)$ and $\exists x(Fx \& Gx)$ respectively as $(\forall x : Fx)Gx$ and $(\exists x : Fx)Gx$. The latter formulas appear to have the syntactic unit 'quantifier phrase' (if F stood for 'dog', then the quantifier phrases would be 'every dog' and 'some dog'). But in restricted quantification theory this is *mere* appearance, for these formulas have precisely the same truth conditions as the original unrestricted formulas. Exactly the same formulas are theorems; exactly the same arguments are valid, after translation from one idiom to another (...) True sortal logic resides in restricted quantification theory exactly to the same extent that it resides in unrestricted quantification theory (...) and then one concludes that *none* of these alleged 'sortal logics' adequately represents the desired doctrine." (Pelletier *op. cit.*)

In order to make things clear, let us recall in brief what Pelletier says about this 'desired doctrine' of sortal predication. After recalling the points already mentioned above about sortal predicates and sortal general terms, Pelletier makes an interesting remark: according to him, " [a] sortal concept is a (mental? objective?) concept of a *kind* or *sort* of individual. A sortal predicate is a linguistic item which is correlated with a sortal concept. In this view there is no such a thing as an individual *tout court*; instead, individuals come already pre-packaged as individuals-of-the-F-type (where F is a sortal concept)" (*op. cit.*). This is an important and distinguishing point: according to this doctrine, we don't start with certain 'bare' (we could say 'naked') objects to which we progressively ascribe properties, but objects of the domain should come already classified as objects of a sort. The apparent closer relationship of such 'pre-packaged' entities and quantum objects seems evident. This is of course interesting, for in thinking of formal logical systems for expressing that, we should realize that the languages of standard logic and mathematics (set theory) are languages of objects; a set is a collection of *distinct* objects, and the standard interpretations of quantifiers (either objectual or substitutional) make them range over *sets*, hence over collection of 'bare' individuals. In other words, the standard languages operate as if we had *naked* individuals at the beginning (the philosophical literature sometimes refer to *bare particulars* –cf. Teller 1995, Chap. 2), and only a posteriori, little by little, we attribute to them properties (or, alternatively, recognize them as elements of certain sets). On the contrary, according to the view of quantum particles as non-individuals, we do not have this any kind of basic stuff where the properties are anchored; there is nothing which transcends the properties of the particle, no *haecceities*, no 'primitive thisness' (Teller *op. cit.*; Teller 1998; French 1998 and 2000).

This assumption, which has gained the preference of some philosophers, meshes quite well with the above discussed idea that there should be no individuals *tout court* (*bare particulars*), but that the entities would come pre-packaged by theory right from the start as individuals *of a sort*, prescribed by physical law. In considering this, perhaps we can understand why there are no truly sortal logics, as remarked by Pelletier: all of these systems are compromised with standard languages of mathematics (standard set theories) in their semantic aspects. It seems that while physics has moved its paradigm from classical physics to quantum (and relativistic) physics, logic and mathematics still remain using languages which refer to individuals and collections of distinguishable objects (sets). Apparently, these two things do not fit one another with regard to certain assumptions, like the consideration of indiscernible (indistinguishable) objects.

But it seems that a solution can be envisaged, at least with respect to the quantum-sortal predicates; if we regard logic as involving also its semantic aspects, a characterization of such predicates perhaps can be achieved if we would be able to find an adequate mathematical language where we could talk on collections of 'objects' which may have a cardinal, but not an ordinal, that is, aggregates which do not originate a counting process. Then, if these collections are taken to be the extensions of certain predicates, these predicates could legitimately be termed 'quantum sortals'. Furthermore, due to the indistinguishability of the elements of these intended collections, any two of these collections with the same cardinal would be taken as the extension of the predicate, so vindicating the above requirement about quantum objects involving the Indistinguishability Postulate. Of course such collections should not be treated as standard *sets* in the sense of usual set theories. All of these requirements can be achieved within the framework provided by *quasi-set theory*, as we shall discuss in brief below.

We should remark that a similar solution was proposed by Stevenson for characterizing sortal predicates in his mentioned paper; there, he defined a concept of S-sets meaning "those sets which consist of all the individuals to which a given sortal predicate applies –as opposed to arbitrary sets which correspond to one-place predicates in the Tarskian semantics for orthodox quantificational theory" (*op. cit.*).

4 The meta-mathematical framework

Quasi-set theory provides a mathematical way of dealing with collections of indistinguishable but not identical objects (Krause 1992; 1996; Dalla Chiara *et al.* 1998). The axioms are based on ZFU-like axioms (Zermelo-Fraenkel with *Urelemente* (but of course we could develop alternative theories based on the von Neumann-Bernays-Gödel system, or using a higher order logic as its underlying logic and so on). The theory allows the existence of two sorts of atoms, termed m-atoms and M-atoms. The latter are postulated to have the properties of standard *Urelemente* of ZFU, while the former are thought of as representing (*quantons*). Following Schrödinger's ideas, for this kind of object the concept of identity is supposed to lack sense.[5] In quasi set theory, this is achieved by restricting the concept of formula: expressions like $x = y$ are not well formed whether x or y denote m-atoms.

The consequence is that the axioms permit us to distinguish between the concepts of *identity* (being the same object) and *indistinguishability* (agreement with respect to all attributes), which cannot be done in classical logic and set theory, where there are no indistinguishable but not identical objects. A quasi-set may have a cardinal (termed its *quasi cardinal*) but, in general, not an ordinal; so, the theory admits quasi sets (whose elements are indistinguishable m-atoms) which cannot be ordered. The concept of quasi cardinal is taken as primitive, since it cannot be defined by the usual means (as particular ordinals). This fits the idea that quantum particles cannot be either ordered or counted, but only aggregated in certain amounts. Notwithstanding, due to the concept of quasi cardinal, there is a sense (as in quantum physics) in saying that there may exist a certain quantity of m-atoms obeying certain conditions, although they cannot be labelled.

The language has a primitive binary predicate of indistinguishability ($\equiv$) which is postulated to be an equivalence relation. The standard axiom of ex-

[5]Schrödinger says that "the sameness of a particle is not an absolute concept. It has only a restricted significance and breaks down completely in some cases" (Schrödinger 1998); in another text, in talking about quantum objects, he said that there are cases where: "(...) the 'sameness' becomes entirely meaningless (...) And I beg to emphasize this and I beg you to believe it: It is not a question of our being able to ascertain the identity in some instances and not being able to to so in others. It is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning" (Schrödinger 1952, pp. 17-18). Further details may be found in French and Krause forthcoming.

tensionality does not hold, but a weaker one is used instead, which entails that quasi-sets with the same quasi-cardinality, and whose elements are indistinguishable, are in their turn indistinguishable quasi-sets. These are termed *quasi-similar* quasi-sets. A basic result tells us that if we release a sub-collection of a certain quasi-set and 'substitute' it by another quasi-similar quasi-set, then the resulting collection is indistinguishable from the original one, in the sense of the theory's weak axiom of extensionality (this can be done by *quasi*-set theoretical operations similar to those of standard set theory, that is, if we write $Qsim(y, z)$ to mean that y and z are quasi-similar, then the operations entail something like $y \subseteq x \wedge Qsim(y, z) \rightarrow ((x - y) \cup z) \equiv x)$. Furthermore, a defined concept of 'extensional equality' (here we shall use $=$ for representing this concept) is introduced having all the properties of classical equality, but it does not hold for m-atoms. We shall not present the theory here (see the mentioned references), but what was said enables us to have an idea of the semantical characterization of sortal predicates, which we shall do in the next section.

5 Quantum-Sortal Predicates: a Semantical Analysis

Quantum-sortal predicates can be semantically characterized by using quasi-sets. Predicates like P, considered above (let us recall that $P(x)$ stands for 'x is a proton of a ^{7}Li atom'), have the peculiar characteristic of not having a well defined extension in the sense that whatever collection with three protons acts just as well as its extension (a similar point was firstly emphasized also by Dalla Chiara and Toraldo di Francia 1993; see Toraldo di Francia 1981, p. 306). Then, if the extensions of such predicates are taken to be quasi-sets of indistinguishable objects with a fixed quasi-cardinal, we would have a semantic characterization of these predicates which follows the intuitive accounts mentioned above. Really, as we have seen, whatever quasi-set belonging to a collection C of quasi-similar quasi-sets (recall that these are those quasi-sets which have the same quasi-cardinality and whose elements are related by the indistinguishability relation $\equiv$) may act as the extension of the considered predicate. So, it makes sense to say that they do not have a well defined extension, since all of these quasi-sets are indistinguishable by the weak extensionality axiom. Furthermore, due to the non-individualistic characteristics of the m-atoms, the elements of their (ambiguous) extension cannot be regarded as 'individuals *tout court*', but should be regarded as 'individuals of a sort' instead, namely, of that 'sort' characterized by the properties which (despite ambiguously) defines the considered collection C (Dalla Chiara and Toraldo di Francia *op. cit.* identify the conjunction of these properties with the *intension* of the quasi-set).[6] This is what the quasi-set semantics described below describes.

[6]It should be remarked that the semantic analysis developed by these authors is based on a similar concept of *quaset*, which differ from our quasi-sets, but have similar motivations; for a comparision between these two concepts, see Dalla Chiara *et. al* 1998.

A logical system that can be useful for characterizing quantum-sortal predicates is the 'Intensional Schrödinger logic' developed by da Costa and Krause (1997); here we shall sketch a minimal nucleus of this logic, and provide a slight modification of its quasi-set semantics in order to see how it can be applied to the above discussion.

To begin with, let us introduce the concept of type. The set of *types* is defined as the smallest collection Π such that: (a) $e_1, e_2 \in \Pi$, and (b) if $\tau_1, \ldots, \tau_n \in \Pi$, then $\langle \tau_1, \ldots, \tau_n \rangle \in \Pi$. e_1 and e_2 are the types of the *individuals*; the objects of type e_1 are called m-atoms and are intuitively thought of as denoting *quantons*. Following the above discussion, we suppose that the concept of identity cannot be applied to m-atoms. The language of our logic may be described as follows: it contains the usual connectives, the symbol of equality, auxiliary symbols, quantifiers, and the necessity operator $\Box$. With respect to variables and constants, for each type $\tau \in \Pi$ the language has a denumerably infinite collection of variables $X_1^\tau, X_2^\tau, \ldots$ of type τ and a (possibly empty) set of constants $(A_1^\tau, A_2^\tau, \ldots)$ of that type; we use X^τ, Y^τ, ..., and C^τ and D^τ, ..., perhaps with subscripts as syntactic variables for variables and for constants of type τ respectively.

The *terms* of type τ are the variables and the constants of that type; so, we have individual terms of types e_1 and e_2. We use U^τ, V^τ, perhaps with subscripts, as syntactical variables for terms of type τ. The atomic formulas are defined in the usual way: if U^τ is a term of type $\tau = \langle \tau_1, \ldots, \tau_n \rangle$ and $U^{\tau_1}, \ldots, U^{\tau_n}$ are terms of types $\tau_1, \ldots, \tau_n$ respectively, then $U^\tau(U^{\tau_1}, \ldots, U^{\tau_n})$ is an atomic formula; so is $U^\tau = V^\tau$ if τ is not e_1. So, the language does not enable us to talk either about the identity or about the diversity of the individuals of type e_1. The other formulas are defined as usual. A formula containing at least $U^{\tau_1}, \ldots, U^{\tau_n}$ as free variables sometimes will be written $F(U^{\tau_1}, \ldots, U^{\tau_n})$.

A semantics for such a language can be described as follows (from now on we will be working within quasi-set theory); the equality symbol '=' stands here for the quasi-set theoretical extensional identity (see Krause 1996; Dalla Chiara *et al.* 1998). Let $D = m \cup M$, where $m \neq \emptyset$ is a finite 'pure' qset (that is, a finite qset which has only m-atoms as elements) and $M \neq \emptyset$ is a 'set' (these are the 'copies' of the ZFU-sets). Furthermore, let I be a non-empty set (whose elements are called *index* or *state of affairs*).[7]

By a *frame* for the described language based on D and I we mean an indexed family of quasi-sets $(\mathcal{F}_\tau)_{\tau\in\Pi}$, where:

(i) $\mathcal{F}_{e_1} = m$

(ii) $\mathcal{F}_{e_2} = M$

(iii) $\mathcal{F}_{\langle e_1 \rangle} = [C]^I$, where $C \subseteq m/_{\equiv}$ (the quotient quasi-set of m by the indistinguishability relation), such that the quasi-sets of C are quasi-similar. This

[7] As in Montague's approach to intensional logic, we may suppose that I is the Cartesian product $W \times T$ where W is a (quasi-)set of possible worlds and T is a totally ordered set of instants of time; see da Costa and Krause 1997.

condition says that a predicate of type $\langle e_1 \rangle$ is associated with a relation-in-intension of a finite collection of indistinguishable m-atoms.[8]

(iv) For each $\tau = \langle \tau_1, \dots, \tau_n \rangle \in \Pi$, other than those mentioned in the previous itens, $\mathcal{F}_\tau$ is a non-empty subquasi-set of

$$[\mathcal{P}(\mathcal{F}_{\tau_1} \times \cdots \times \mathcal{F}_{\tau_n})]^I$$

If the equality holds in (iii) and (iv), the frame is *standard*. By a *general model* (*g-model* for short) for our language, based on D and I, we understand an ordered pair

$$\mathcal{M} = \langle \mathcal{F}_\tau, \rho \rangle_{\tau \in \Pi},$$

such that :

(i) $(\mathcal{F}_\tau)_{\tau \in \Pi}$ is a frame for $S_\omega \mathcal{I}$ based on D and I

(ii) ρ is a *quasi-function* which assigns to each constant C^τ an element of $\mathcal{F}_\tau$. Then, in particular $\rho(C^{e_1}) \in m$ and $\rho(C^{e_2}) \in M$.[9]

The axioms of such a logic can be presented without difficulty, as shown in da Costa and Krause 1997, and a generalized completeness theorem can be proven as well, but we shall not present these details here. But, for exploring the ideas delineated above, let us consider some examples which illustrate the 'intensional' counterpart of such a semantics, which links the subject with sortal predication. We shall present four examples which are the 'most paradigmatic' ones. The first two show that the classical intensional case (cf. Gallin 1975) remains valid when the entities are not of the type e_1. The last one exemplifies the specific case of quantum-sortal predicates.

Example 1: Let us consider the constant C^{e_2}. Since $\mathcal{F}_{e_2} = M$, then $\rho(C^{e_2}) \in M$, that is to say, C^{e_2} names an element of a standard 'set' (a copy of a ZFU set). This is in accordance with the standard semantics, since the given constant behave as a 'classical constant'.

Example 2: Now let us take a constant $C^{\langle e_2 \rangle}$. In this case, $\mathcal{F}_{\langle e_2 \rangle} \subseteq [\mathcal{P}(\mathcal{F}_{e_2})]^I = [\mathcal{P}(M)]^I$. Then, $\mathcal{F}_{\langle e_2 \rangle}$ is a class of functions from I in $\mathcal{P}(M)$, also as in the classical case.[10] Intuitively, $C^{\langle e_2 \rangle}$ is a unary predicate (an individual property) whose arguments are individuals of type e_2 (that is, 'classical' individuals). Also in this case, all has happened as in standard semantics.

[8] The terminology is adapted from Gallin 1975, pp. 72ff.

[9] Quasi-functions generalize the standard functions; generally speaking, they map collections of indistinguishable objects into collections of indistinguishable objects, and coincide with usual functions when there are no m-atoms involved (in this case, indistinguishability becomes identity). The details can be found in the mentioned papers.

[10] Since there are no m-atoms involved, these quasi-functions are functions in the standard sense.

Example 3: Let us now take a constant C^{e_1}. In this case, $\mathcal{F}_{e_1} = m$ and then $\rho(C^{e_1}) \in m$, that is, the constant (intuitively) 'names' an m-atom. Since the m-atoms cannot be individualized, counted etc., the denotation of C^{e_1} is ambiguous. We can say that a constant of type e_1 plays the role of a *generalized noun* (*g-noun*). It is by using this kind of constant that we can make reference to 'the electron' which was released from a certain atom by ionization, as mentioned in the previous sections. It is important to mention that this use of the language is distinct from the use of variables; the case resembles the 'parameters' used in mathematics, for instance when we write something like $ax^2 + bx + c = 0$ and regard x as a variable and a, b and c as 'parameters' denoting arbitrary real numbers. But the fundamental difference is that in this case these parameters stand for *individuals*, for real numbers can be named and distinguished from the others, contrary to the constants type e_1, which stand for *non-individual* quantons. The semantical aspects are of course distinct.

Example 4: Now we shall consider a constant $C^{\langle e_1 \rangle}$, which could denote our predicate P described above. In this case, $\mathcal{F}_{\langle e_1 \rangle} = [C]^I \subseteq [\mathcal{P}(\mathcal{F}_{e_1})]^I$. Then, $\rho(C^{\langle e_1 \rangle}) \in \mathcal{F}_{\langle e_1 \rangle}$, that is to say, it is a (quasi-) function from I to C (which, let us recall, is a collection of quasi-similar quasi-sets). In other words, $\rho(C^{\langle e_1 \rangle})$ is a quasi-function from I to C. If m is a pure quasi-set whose elements are all indistinguishable one each other (that is, they stand in the relation $\equiv$), then the denotation function does not distinguish between quasi-sets in C. In this case the only difference among the sub-quasi-sets of m is about their cardinality; that is to say, if $\rho(C^{\langle e_2 \rangle})$ is x, this x has no a precise definition, for whatever quasi-set y such that x and y are similar could act as the denotation of $C^{\langle e_2 \rangle}$ as well. This interpretation accommodates the intuitive idea that a predicate like P does not have a well defined extension.

In other words, since the elements of quasi-sets of indistinguishable m-atoms cannot be named, the terms of type e_1 have no precise denotation; they refer ambiguously to arbitrary elements of these quasi-sets, and so the indistinguishable elements of a pure quasi-set, as non-individuals, can only be aggregated in certain amounts (Teller 1995 explains the similar case involving quantum objects). So, they accurately exemplify collections of *quantons*. In this sense, we may properly say that such constants do not represent anything in particular, for they lack a (precise, well defined) referent. Furthermore, the last example above exemplifies the case where a predicate does not have a well defined extension (in the sense that every quasi-set of a certain class of similar quasi-sets may be considered as their extension); these predicates may be viewed as relations-in-intension of sort $U^{\langle e_1 \rangle}$, and may act as our quantum-sortal predicates, covering the situation where, for instance, a physicist is measuring a certain property of a quantum system, say the spin of a collection of electrons.[11] Suppose that he has chosen the x direction and has stated how many electrons have spin up and how many have spin down. However, he could choose another direction, say the z axis, and then perform again the measurement of the spin of the electrons in

[11]See Dalla Chiara *et al.* 1998.

the x direction (as in the Stern-Gerlach experiment). If he obtains collections of quantum states with the same cardinality, it simply has no meaning to say that these collections are the same or that they are distinct from the first ones. The predicate 'to have spin up in the x-direction' has not a precise denotation, for whatever collection with the right number of electrons act as its extension, and no counting process (in the mathematical sense of attributing it an ordinal) can be achieved. So, we should agree with Torado di Francia in that "the intepretation of the logical concept of extension may definitely need a profound revision in modern physics" (Toraldo di Francia 1981, p. 306), and perhaps the consideration of predicates like the quantum-sortal ones help in pushing this revision a little bit further.

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