MAKING AND BREAKING MATHEMATICAL SENSE
ROI WAGNER

Reviewed by David Corfield

Making and Breaking Mathematical Sense: Histories and Philosophies of Mathematical Practice
Roi Wagner
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Reading the opening pages of this book, an enormous sympathy welled up in me towards its author. The introduction depicts Wagner frustrated by the limited repertoire of questions covered today within the philosophy of mathematics, where by his calculation forty percent of the literature is dedicated to the sole issue of the existence of mathematical objects, and so seeking to explore ‘What else philosophy of mathematics can be’. He recounts giving a job talk and being told, ‘this is not philosophy of mathematics’. I was immediately reminded of a time around fifteen years ago, as I was finishing off my own book (Corfield [2003]), which itself arose from similar frustrations about the state of the field. I gave a talk on the subject of the role of analogy in mathematics, later to become Chapter 4 of my book, to what must have been a politer audience. Rather than bluntly being told it was not philosophy, instead it was gently enquired why such an investigation as mine should count as philosophy.

The similarity of our stances apparently runs deeper. Wagner places at the centre of his book the idea of mathematics moving forward under constraints of various kinds—natural, social, practical, cognitive, and so on. In the introduction to my book, I too outline a way forward for philosophy in treating mathematics in terms of the factors governing change: formal, psychological, technological, sociological, structural, relations with the sciences, and so on. As you may imagine then, with such a common base, I expected to enjoy reading Wagner's thoughts on plenty of fresh topics. Indeed, there are plenty of novel considerations in this book, and I found myself more engaged by the material he includes than I would be by yet another twist on the indispensability argument. And yet I finished the book largely dissatisfied. Let me explain why.

First, an outline of the book: Chapter 1 gathers together a number of historical vignettes of a range of philosophically contested topics—the proper grounding of mathematics, its conceptual freedom, how to deal with what appear as monsters, its sources of authority. Where the philosopher will tend to take sides on these topics, advocating with respect to the first of these topics, say, a formalist, an empiricist, or an intuitionist position, Wagner wants us to say, 'Yes, please! All of the above'. For him, mathematics is not the kind of monolithic entity that warrants claims in favour of any single outcome in these debates. All answers reflect facets of the complex operations of mathematics through its practical and theoretical development. Chapter 2 then looks to illustrate the functioning of these themes in the two cases of early financial arithmetic and Renaissance algebra.

Chapter 3 presents the constraint idea, worked out in relation to a set of questions: How is mathematics used? How do people agree about mathematical statements? How are they interpreted? Chapter 4 plays an illustrative role again, this time picking up on the theme of interpretation by showing the latent power of ambiguity in two case studies: generating functions and matching problems in combinatorics. Chapter 5, the longest chapter, considers cognition acting as a constraint, passing by Dehaene's 'number sense' and Lakoff and Nuñez's mathematical metaphors, and ending with Wagner showing one of his most important philosophical sources of inspiration, with a largely jargon-free treatment of Gilles Deleuze's *The Logic of Sensation*. The question raised in this chapter is whether our conceptual freedom is limited by cognitive schemas wired into our brains. Where Dehaene looks for neural circuitry, Lakoff and Nuñez look for 'inference-preserving, cross-domain mappings' grounded in our embodied minds. From comments on p. 163, we learn of the importance of Deleuze for Wagner, his thesis summarized thus:

> It is the haptic eye's articulation and recomposition of orders of sensation into a serial development or 'movement in all its continuity' that regulates what Deleuze calls the 'logic of sensation' [...] This is the logic that turns noise into creative input, and binds together conflicting constraints and superposing interpretations into a system of layers that are piecemeal formaliz*able*, but not *actually* subject to any global formalism. This logic is what the various case studies and arguments of this book attempt to illustrate.

The brief Chapter 6 ('Mathematical Metaphors Gone Wild') considers two case studies on metaphor, involving the mediaeval and early modern transfers between algebra and geometry, and the metaphors supposed by Lakoff and Nuñez to underpin conceptions of the infinite. Wagner argues that each case shows metaphor to be a 'wilder' phenomenon than might have been supposed, and certainly not to be circumscribed by simple metaphoric schemes that see the transfer of mathematical knowledge between domains as mere correspondences between entities and inferences. Chapter 7 completes the book with the treatment of the subtlest of the several constraints, that which arises from the feedback of our shaping the world through the application of mathematics and so giving rise to material for further mathematization. This is done through the lens of less commonly encountered philosophers: Fichte, Schelling, and Cohen.

As you see then, a diverse variety of ways to say what philosophy of mathematics can be, ranging far and wide over an immense historical terrain. And yet, to my mind, a curious ahistoricism haunts the book despite its temporal scope. Just as mathematics is to be understood through its constraints, the philosophy of mathematical practice—as those who belong to what Aspray and Kitcher ([1988]) termed the 'Maverick approach' like to call the discipline these days—must likewise be constrained, but by what? I would suggest primarily through pegging itself to the historical unfolding of mathematics.

As I am sure he would acknowledge, the philosophical topics and themes of Wagner's Chapters 1 and 3 are time-dependent matters. Levels of agreement in mathematical statements, for example, depend on the era, and often on specific communities of these eras. Changing standards ensued from debates within and between these communities. Again the demands of science, in particular physics, provoked enormous changes in the content of mathematics over the centuries, and in turn major shifts in its self-understanding. Now, if mathematics is to be understood as a rational activity, we need to hear how these changes can be seen as warranted, for example, how difficulties in achieving consensus drove certain innovations in formal techniques. Someone to help us here is the rather overlooked philosopher of science Dudley Shapere, who argued forcefully that any attempt to separate the content of scientific theories from the meta-scientific conceptualization of observation, explanation, and confirmation was doomed to fail. There is a permeable boundary operating here, where, for instance, the content of scientific claims as to what there is in the universe has a direct bearing on what counts as an observation. All the same, it was possible to escape Kuhnian concerns of theory-ladenness and maintain, Shapere ([1989]) argued, that science be seen as a rational process due to the 'chain of reasoning' that accompanies shifts in our scientific conception. As we proceed, our understanding improves of how we have learned how to learn from the world.

But then one would expect that the similar subtle interrelation of theoretical content and meta-theoretical conceptualization in *mathematics* needs careful, historically sensitive treatment. One place where the philosophy of mathematical practice has achieved just this is in the fine work carried out on that exceptional period of the decades surrounding 1900, when the philosophically educated mathematicians of their day, some of them giants according to any age, held sway: Weierstrass, Dedekind, Kronecker, Frege, Cantor, Pasch, Poincaré, Hilbert, Borel, Brouwer, Weyl. Here we are seeing a great deal of careful scholarship unpick a vast array of myths. All the same, one might worry that even if work in this field has undermined the simplistic and indeed false stories that are told about Frege the logicist, Hilbert the formalist, and so on, we ought to push beyond the date of the supposed grounding of mathematics within first-order logic and set theory, lest we give support to the idea that such dynamic changes within mathematics came to an end. So we need to place our philosopher–mathematicians listed above in their settings, after the earlier Gauss, Cauchy, and Riemann, and before Noether, Weil, and Grothendieck. One can expect not to be hopelessly wrong in picking out from more recent times some names from the continuation: Langlands, Wiles, Kontsevich, Voevodsky, and Lurie. For a number of reasons, it is their interest in category theory that has driven several philosophers closer to the present (see Landry [2017]).

This may appear to be an elitist notion of mathematics, and it is certainly resisted by another large wing of the philosophy of mathematical practice that sets its stall in the classroom. This democratization of mathematics is also encountered in the work of some historians, such as in Cuomo's *Ancient Mathematics* ([2001]), where as much or more time is devoted to the arithmetic of the accountants at the Port of Piraeus as to the geometry of those few people engaged in the codification of what became Euclid's *Elements*. Well, of course a philosophy of mathematics should be able to search wherever mathematics is used, both in everyday practices of finance and measurement, and in science. It can call into question these uses and indeed abuses, not least the false quantification so common in everyday life. But at the heart of our discipline there must be reserved sufficient space for what constitutes the cutting-edge of mathematical conceptualization, such as was realized by those contributors to Euclidean geometry.

My concern then with this book is that aside from opening up a range of starting points, Wagner's busy tour of some fragments of mathematical practice does little to add to the scholarly literature. In the section of Chapter 6 entitled 'What Passes between Algebra and Geometry', he admits that he 'shuffled the historical order in favour of the logical buildup of the analysis' (pp. 177–8). If we are to be given a 'logical buildup'—or perhaps better, 'rational narrative'—of the exchange between algebra and geometry, a very fruitful one running right up to the profoundest dualities of the present day (see, for example, 'Isbell duality' (nLab)), then it had surely better be done without such shuffling. In his *Lectures on the Foundations of Mathematics*, Wittgenstein, who looms large in Wagner's third chapter, likens his work there to that of a guide in a strange town. However, he warned his audience, 'I am an extremely bad guide, and am apt to be led astray by little places of interest, and to dash down side streets before I have shown you the main streets' ([1975], p. 44). What we need now, above all, are guides whose tours are carefully constructed and shaped to the contours of the town around which they are directing us.

David Corfield
Department of Philosophy
University of Kent
d.corfield@kent.ac.uk

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