Relativistic Locality from Electromagnetism to Quantum Field Theory

Eugene Y. S. Chua
Nanyang Technological University, Singapore
eugene.chuays@ntu.edu.sg

Charles T. Sebens
California Institute of Technology
csebens@caltech.edu

December 16, 2024 Preprint
Invited submission to *Everett and Locality* (ed. Alyssa Ney)

Abstract

Electromagnetism is the paradigm case of a theory that satisfies relativistic locality. This can be proven by demonstrating that, once the theory's laws are imposed, what is happening within a region fixes what will happen in the contracting light-cone with that region as its base. The Klein-Gordon and Dirac equations meet the same standard. We show that this standard can also be applied to quantum field theory (without collapse), examining two different ways of assigning reduced density matrix states to regions of space. Our preferred method begins from field wave functionals and judges quantum field theory to be local. Another method begins from particle wave functions (states in Fock space) and leads to either non-locality or an inability to assign states to regions, depending on the choice of creation operators. We take this analysis of quantum field theory (without collapse) to show that the many-worlds interpretation of quantum physics is local at the fundamental level. We argue that this fundamental locality is compatible with either local or global accounts of the non-fundamental branching of worlds, countering an objection that has been raised to the Sebens-Carroll derivation of the Born Rule from self-locating uncertainty.

Contents

1 Introduction

There is a well-known tension between special relativity and quantum physics. Special relativity prohibits any instantaneous action-at-a-distance and, more generally, any interaction across space-like separation. This is the requirement of relativistic locality. Quantum physics, by contrast, appears to involve just this sort of action-at-a-distance. The EPR argument and Bell's theorem together show (from certain assumptions) that any theory that hopes to make the right probabilistic predictions (in cases like spin measurements of entangled particles located far apart from one another) must violate relativistic locality. One can respond by embracing this non-locality and adopting a version of quantum physics that explicitly describes the non-local dynamics (as is done in Bohmian approaches). Alternatively, one might hope to resolve the tension by abandoning one of the assumptions that goes into the EPR-plus-Bell argument for non-locality. Maudlin (2014) highlights the assumption that measurements have unique results, noting that the argument does not directly apply to the many-worlds interpretation of quantum mechanics (where every possible result occurs on some branch). He then writes:

> "That does not prove that Many Worlds is local: it just shows that Bell's result does not prove that it isn't local. In order to even address the question of the locality of Many Worlds a tremendous amount of interpretive work has to be done. This is not the place to attempt such a task." (Maudlin 2014, pg. 23)[1]

Here we attempt that task, arguing that the many-worlds interpretation is indeed local by analyzing quantum field theory (QFT) in a "many-worlds" or "Everettian" form, where there is no true collapse of the quantum state and no supplementation of that state with additional ontology (like point particles with definite locations or fields with definite configurations). Determining whether the collapse-free Schrödinger dynamics in QFT is local is important for accurately assessing the merits of the many-worlds interpretation, and also for better understanding QFT independently of that interpretation.

In section 2, we begin with classical electromagnetism. Electromagnetism is a local theory because once you have specified what is happening in a spherical region, the laws of the theory only allow a single possible future in the contracting light-cone that has that region as its base. Put another way: the future of that spherical region is fixed within a zone that shrinks at the speed of light. This is a standard for relativistic locality that can be applied across many domains of physics.

In section 3, we see that the Klein-Gordon and Dirac equations meet the same standard. In section 3.1, we prove this for the Klein-Gordon equation. There have been worries raised about superluminal propagation for both the Klein-Gordon and Dirac equations. In section

[1] See Norsen (2017, sec. 10.4) for more on this challenge.

3.2, we explain how the fundamental dynamics avoids any such propagation. In section 3.3, we offer a proof of locality for the Dirac equation that closely mirrors the proofs of locality for electromagnetism and the Klein-Gordon equation (something that we have not seen done before).

We turn to QFT in section 4. To apply the standard for relativistic locality from section 2, we must first find a way to specify the state of a region of space at a time. Wallace and Timpson (2010) propose a general way of assigning states to spacetime regions and sketch how locality can be proved for such states. We aim to add fine lines to their broad brushstrokes, and consider two strategies for making the proposal more ontologically precise. First, one could represent the universal state by a field wave functional and construct an appropriate field reduced density matrix for a region by tracing over the field degrees of freedom outside the region. Second, one could represent the universal state by a particle wave function for a variable number of particles (a state in Fock space) and attempt to construct an appropriate particle reduced density matrix for a region by tracing over the particle degrees of freedom outside the region. We explore both options in detail and show, at least in outline, how relativistic locality can be proven within a field approach but *not* a particle approach (where the standard creation operators prevent one from assigning states to regions, and the alternative Newton-Wigner creation operators lead to non-locality). We prefer the field wave functional approach for a variety of reasons and see its locality as another point in its favor.

In section 5, which can be read on its own, we move from the fundamental dynamics of Everettian QFT to the non-fundamental carving of the quantum state into distinct worlds. There has been disagreement in the literature as to how this carving should be performed. Wallace (2012) advocates a 'local branching' view, according to which quantum measurements cause a local division of the one world into many, so that: first the measuring device branches, then the experimenter, later her friend down the hall, and so forth. Branching propagates no faster than the speed of light. By contrast, Sebens and Carroll (2018) argue that one can take a global branching view where a quantum measurement anywhere causes the entire world (and everything in it) to branch. This global picture of branching plays a role in the Sebens-Carroll derivation of the Born Rule and has been challenged by Kent (2015) and McQueen and Vaidman (2019) (and defended by Ney 2024; Ney n.d.). We argue that, despite its apparent non-locality, the global branching view does not violate relativistic locality because (i) the fundamental dynamics are local, and (ii) there is precedent, in cases like Newtonian gravity and electrostatics, for unproblematic non-fundamental non-locality emerging in an approximation to deeper, local physics.

2 Electromagnetism and the Standard for Relativistic Locality

Electromagnetism sets the gold standard for relativistic locality. It is widely understood to be a local theory where causal influences cannot propagate faster than the speed of light. Let us now examine how that can be proven (employing Gaussian CGS units).[2]

Let us begin by studying the behavior of the vector and scalar potentials $\vec{A}$ and ϕ, related to the electric and magnetic fields via

$$\begin{aligned} \vec{E} &= -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \vec{\nabla} \times \vec{A}\,, \end{aligned} \tag{1}$$

adopting the Lorenz gauge as a partial gauge fixing condition,

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0\,. \tag{2}$$

Using potentials that satisfy (1) ensures that two of Maxwell's equations are automatically satisfied. In the Lorenz gauge, the other two of Maxwell's equations become wave equations for the potentials, with the charge and current densities acting as source terms,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = -4\pi\rho \tag{3}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}\,. \tag{4}$$

To prove locality for a complete theory where the electromagnetic field interacts with charged matter, we would also need to introduce equations describing the way that the charge and current densities change in response to the electromagnetic field—and use those equations show that ϕ, $\vec{A}$, ρ, and $\vec{J}$ evolve together in a local way. For our purposes in this section, we would like to leave the type of matter and its response to electromagnetic fields unspecified. So, we will follow standard practice and treat ρ and $\vec{J}$ as fixed source functions given at every point in space and time (obeying the equation for local conservation of charge, $\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$). The goal then is to show that, for specified sources, the above wave equations yield a local evolution for the potentials.

For electromagnetism (formulated in terms of potentials in the Lorenz gauge) to be relativistically local, it must be the case that specifying what is happening inside a spherical region R, at some time that we can call $t = 0$, fixes what happens everywhere inside the contracting light-cone with R as its base. Put more precisely, any two solutions to the wave

[2] Wald (1984, sec. 10.2); Wald (2022, sec. 5.4) presents a slightly different way of proving the same result.

equations that agree on the values of ϕ, $\frac{\partial \phi}{\partial t}$, $\vec{A}$, and $\frac{\partial \vec{A}}{\partial t}$ within R at $t = 0$ must agree on the values of ϕ and $\vec{A}$ within the entire contracting light-cone with R as its base (given the fixed source terms specified throughout the light-cone). This standard of relativistic locality asks what R determines in the future. We can alternatively phrase it as a standard about what R can influence: any two solutions to the wave equations that disagree on the values of ϕ, $\frac{\partial \phi}{\partial t}$, $\vec{A}$, and $\frac{\partial \vec{A}}{\partial t}$ only within R at $t = 0$ can later disagree on the values of ϕ and $\vec{A}$ only within the expanding light-cone that has R as its base (given the fixed source terms specified throughout the light-cone). See figure 1.

The next steps largely follow standard proofs from partial differential equations textbooks that three-dimensional wave equations like (3) and (4) satisfy a "principle of causality" (another name for relativistic locality).[3] These steps can be skipped or skimmed on a first reading of this paper, but they provide a useful illustration of what it takes to prove locality, and it is by modifying this proof that we will arrive at proofs of locality for the Klein-Gordon and Dirac equations.

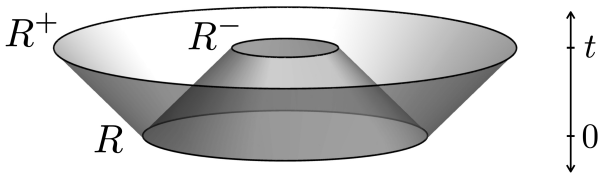

Figure 1: This figure shows expanding and contracting light-cones from R at $t = 0$, picking out the regions R^+ and R^- at a later time t. In a local theory, fixing what is happening within R at $t = 0$ should at that later time fully determine what happens within R^- and at most influence what happens within R^+.

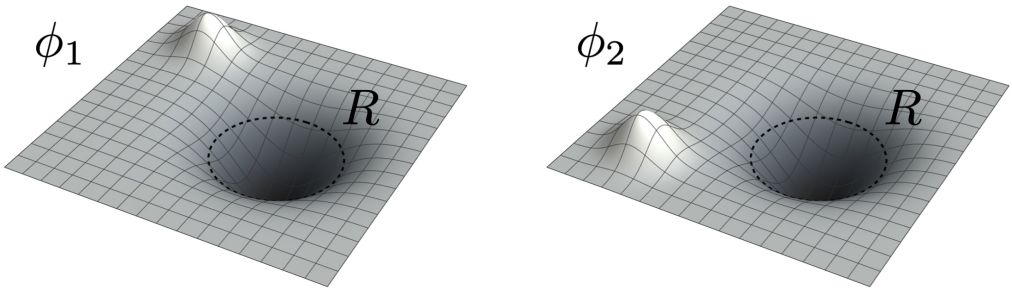

Figure 2: Scalar potentials ϕ_1 and ϕ_2 that initially only differ outside of R will agree within the contracting light-cone of R.

To show that what is happening in the sphere R at $t = 0$ fixes what happens within the entire contracting light-cone, let us select an arbitrary time t (after $t = 0$ but before the

[3] Our proof will most closely parallel Strauss (2008, sec. 9.1), straightforwardly generalized to include source terms. Similar proofs appear in Zachmanoglou and Thoe (1976, ch. 8), Folland (1995, ch. 5), and Evans (1998, sec. 2.4.3).

contracting light-cone vanishes), at which point the contracting light-cone from R has shrunk to the smaller sphere R^-, and show that the state on R^- is determined uniquely. Let us focus first on the scalar potential ϕ and its wave equation (3). Consider two separate solutions to (3), $\phi_1(\vec{x},t)$ and $\phi_2(\vec{x},t)$, that agree on their values and their first time derivatives within R at $t=0$ (figure 2). What we would like to show is that these solutions must agree within R^- at t. To do so, let us begin by defining their difference as

$$\phi_d = \phi_1 - \phi_2 \,. \tag{5}$$

If ϕ_1 and ϕ_2 obey the wave equation (3) for a specified source ρ, then ϕ_d must obey the source-free (homogeneous) wave equation:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_d = 0 \,. \tag{6}$$

Multiplying through by $-c^2\frac{\partial\phi_d}{\partial t}$ and rearranging yields

$$\begin{aligned} 0 &= -c^2\frac{\partial\phi_d}{\partial t}\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_d \\ &= \frac{\partial\phi_d}{\partial t}\frac{\partial^2\phi_d}{\partial t^2} + \frac{c^2}{2}\frac{\partial}{\partial t}|\vec{\nabla}\phi_d|^2 - c^2\vec{\nabla}\cdot\left(\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right) \\ &= \frac{\partial}{\partial t}\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right] - c^2\vec{\nabla}\cdot\left(\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right) . \end{aligned} \tag{7}$$

To show that ϕ_d vanishes on R^-, it will turn out to be useful to integrate the above expression over the solid portion F of the light-cone that has R at its base, R^- as its top, and E as its edge (figure 3). (This shape is called a "frustum," hence the capital F.) The integral over the frustum F can be transformed (using the four-dimensional divergence theorem) from a four-dimensional integral over the entirety of F to an integral over the three-dimensional "surfaces" of F (R, R^-, and E),

$$\begin{aligned} 0 &= \int_F d^3\vec{x}dt\left(\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right] - c^2\vec{\nabla}\cdot\left(\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right) \\ &= \int_{R\cup R^-\cup E} d^3\vec{x}\left(n_t\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right] - \vec{n}\cdot\left(c^2\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right) . \end{aligned} \tag{8}$$

Here, $(n_t,\vec{n})$ is the unit normal four-vector pointing outward from F on the three-dimensional boundary of F. For the bottom of the frustum, R, the unit normal four-vector points temporally downward $(-1,0)$ and for the top, R^-, this points temporally upwards $(1,0)$. The integral can

thus be broken up into the following three contributions:

$$
\begin{aligned}
0 = &- \int_R d^3\vec{x}\left(\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right) \\
&+ \int_{R^-} d^3\vec{x}\left(\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right) \\
&+ \int_E d^3\vec{x}\left(n_t\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right] - \vec{n}\cdot\left(c^2\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right) . \qquad (9)
\end{aligned}
$$

The first integral vanishes because we assumed earlier that the two solutions ϕ_1 and ϕ_2 agreed on what was happening within R at $t = 0$: $\phi_1 = \phi_2$ and $\frac{\partial\phi_1}{\partial t} = \frac{\partial\phi_2}{\partial t}$, thus $\phi_d = \frac{\partial\phi_d}{\partial t} = 0$. We are left with the sum of the integrals over R^- and E being zero,

$$
\begin{aligned}
&\int_{R^-} d^3\vec{x}\left(\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right) \\
&\quad + \int_E d^3\vec{x}\left(n_t\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right] - \vec{n}\cdot\left(c^2\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right) = 0 . \qquad (10)
\end{aligned}
$$

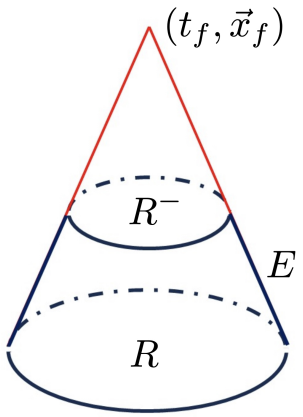

Figure 3: The frustum F is the portion of the light-cone bounded by the base, R, the top, R^-, and the edge, E.

Letting $(t_f, \vec{x}_f)$ denote the tip of the light-cone (floating above the frustum in figure 3), the unit outward normal four-vector on E can be written as[4]

$$
(n_t, \vec{n}) = \frac{c}{\sqrt{c^2+1}}\left(1, \frac{\vec{x}-\vec{x}_f}{c|\vec{x}-\vec{x}_f|}\right) . \qquad (11)
$$

[4]See Strauss (2008, pg. 229, 232).

Inserting this four-vector, the integral over E in (10) becomes

$$\int_E d^3\vec{x}\left(n_t\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2+\frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right]-\vec{n}\cdot\left(c^2\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right)$$
$$=\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left(\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2+\frac{c^2}{2}|\vec{\nabla}\phi_d|^2-c^2\frac{\partial\phi_d}{\partial t}\frac{\vec{x}-\vec{x}_f}{c|\vec{x}-\vec{x}_f|}\cdot\vec{\nabla}\phi_d\right). \tag{12}$$

This can equivalently be written as

$$\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}-c\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\vec{\nabla}\phi_d\right)^2+\frac{c^2}{2}\left(\vec{\nabla}\phi_d-\left(\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\vec{\nabla}\phi_d\right)\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\right)^2\right]. \tag{13}$$

The equivalence of (12) and (13) can be checked by expanding the two squared expressions in (13).

Because (13) is an integral of two squares that each must make a non-negative contribution, it must be greater than or equal to zero. Returning to (10), this means that the integral over R^- must be less than or equal to zero,

$$\int_{R^-} d^3\vec{x}\left(\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2+\frac{c^2}{2}|\vec{\nabla}\phi_d|^2\right)\leq 0\,. \tag{14}$$

In this integral, $\left(\frac{\partial\phi_d}{\partial t}\right)^2$ and $\frac{c^2}{2}|\vec{\nabla}\phi_d|^2$ are squares that are non-negative everywhere. So, the $\leq$ relation can only be satisfied if $\frac{\partial\phi_d}{\partial t}$ and $\vec{\nabla}\phi_d$ both vanish on the entirety of R^-. Because t was chosen arbitrarily in picking out this particular frustum, $\frac{\partial\phi_d}{\partial t}$ and $\vec{\nabla}\phi_d$ must be zero throughout the contracting light-cone. As $\phi_d=0$ at the base of the light-cone R, it thus must be zero throughout the light-cone. That is, the two solutions ϕ_1 and ϕ_2 that agreed on what was happening in R must agree throughout the contracting light-cone. This completes the proof of locality for the scalar potential ϕ.

Because the wave equations for each component of the vector potential $\vec{A}$ take the exact same form as the wave equation for ϕ, the very same techniques as those employed above can be used to prove that the evolution of $\vec{A}$ in the Lorenz gauge is local. If two solutions to (4) agree on $\vec{A}$ and $\frac{\partial\vec{A}}{\partial t}$ within R at $t=0$, they will agree throughout the contracting light-cone. Looking at ϕ and $\vec{A}$ together: a specification of ϕ, $\frac{\partial\phi}{\partial t}$, $\vec{A}$, and $\frac{\partial\vec{A}}{\partial t}$ on R—satisfying the Lorenz gauge condition (2)—determines uniquely the evolution of ϕ and $\vec{A}$ within the contracting light-cone.

If you take the fundamental ontology of electromagnetism to be the scalar and vector potentials in the Lorenz gauge, then our proof of locality could end here. The dynamical laws, (3) and (4), are second-order and thus you need to know both the values of the potentials and their derivatives (along with any source terms) to generate time evolution. Specifying these quantities within a sphere fully determines what will happen in the contracting light-cone

with that sphere as its base. If, on the other hand, you take the fundamental ontology to be the gauge-invariant electric and magnetic fields, then there are a few more steps to prove locality.[5] The dynamical equations for these fields (the two of Maxwell's equations that feature derivatives of the fields) are first-order and thus you need to know only the values of the electric and magnetic fields at a moment (along with any source terms) to generate time evolution. To prove locality, we must show that a specification of $\vec{E}$ and $\vec{B}$ within a sphere R fixes what happens in the contracting light-cone (assuming that the source terms are given).

Let us begin by drawing a relation between two different ways of presenting the initial conditions: It is a fact that specifying the values of $\vec{E}$ and $\vec{B}$ within a sphere R at $t = 0$ (satisfying Maxwell's equations $\vec{\nabla} \cdot \vec{B} = 0$ and $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$) will fix ϕ, $\frac{\partial \phi}{\partial t}$, $\vec{A}$, and $\frac{\partial \vec{A}}{\partial t}$ in the Lorenz gauge on R, up to the gauge transformation that remains because the Lorenz gauge is only a partial gauge fixing condition, leaving the freedom to transform

$$\begin{aligned} \phi &\to \phi - \frac{\partial \Lambda}{\partial t} \\ \vec{A} &\to \vec{A} + \vec{\nabla}\Lambda \end{aligned} \tag{15}$$

for any scalar field Λ such that $\nabla^2\Lambda - \frac{1}{c^2}\frac{\partial \Lambda}{\partial t^2} = 0$.[6] From the given $\vec{E}$ and $\vec{B}$ fields on R, let us arbitrarily pick one set of ϕ, $\frac{\partial \phi}{\partial t}$, $\vec{A}$, and $\frac{\partial \vec{A}}{\partial t}$ in the Lorenz gauge to encode the fields among those sets allowed by the remaining gauge freedom (15). Then, our earlier proof shows that there is only one possible evolution of these potentials within the contracting light-cone. By (1), this means that there is only one possible evolution for $\vec{E}$ and $\vec{B}$ within the contracting light-cone. Other choices for the potentials that differ by the gauge transformations in (15) must yield the same dynamics for $\vec{E}$ and $\vec{B}$ because they will only ever differ by a gauge transformation. We have thus proven locality for the dynamics of the $\vec{E}$ and $\vec{B}$ fields.

Before moving on, let us carefully state the general standard for locality (in deterministic theories) that we have applied in this section.[7]

---

[5] See Wald (2022, pg. 96–97).

[6] To see that this fact holds, note that (a) $\vec{B} = \vec{\nabla} \times \vec{A}$ specifies $\vec{A}$ up to the gauge transformation $\vec{A} \to \vec{A} + \vec{\nabla}\Lambda$; (b) because $\vec{B} = \vec{\nabla} \times \vec{A}$ holds at all times, $\frac{\partial \vec{A}}{\partial t}$ must transform as $\frac{\partial \vec{A}}{\partial t} \to \frac{\partial \vec{A}}{\partial t} + \frac{\partial}{\partial t}\vec{\nabla}\Lambda$; (c) given that $\frac{\partial \vec{A}}{\partial t}$ transforms this way, $\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ allows us to determine ϕ and $\frac{\partial \vec{A}}{\partial t}$ up to the gauge transformations $\phi \to \phi - \frac{\partial \Lambda}{\partial t}$ and $\frac{\partial \vec{A}}{\partial t} \to \frac{\partial \vec{A}}{\partial t} + \frac{\partial}{\partial t}\vec{\nabla}\Lambda$; (d) for Gauss's law $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ to be satisfied in the Lorenz gauge, it must be the case that $\nabla^2\Lambda - \frac{1}{c^2}\frac{\partial \Lambda}{\partial t^2} = 0$ as at the end of (15); (e) using this relation, the Lorenz gauge condition (2) allows one to go from the specification of $\vec{A}$ (up to a gauge transformation) from part (a) to a determination of $\frac{\partial \phi}{\partial t}$ up to the gauge transformation $\frac{\partial \phi}{\partial t} \to \frac{\partial \phi}{\partial t} - \frac{\partial^2 \Lambda}{\partial t^2}$.

[7] Relativistic locality can be contrasted with the related concept of spatiotemporal locality or continuous action (Lange 2002, pg. 13–17; Wharton and Argaman 2020). A theory is spatiotemporally local if interactions occur without any gaps in space or time. This standard allows influences to travel arbitrary quickly, provided they do not jump any gaps when doing so. By contrast, relativistic locality is a stricter standard that enforces a light-speed limit on interactions. (Adlam n.d., this volume, introduces a standard for continuous action that includes a light-speed limit and should be satisfied by theories that meet our standard for relativistic locality. Such theories should also meet her condition prohibiting action-at-a-distance.)

The Standard for Relativistic Locality:

A deterministic theory with **first-order dynamics** is **relativistically local** if and only if, given the theory's laws, the instantaneous physical state within a sphere R at some time uniquely determines, at any future time, the instantaneous physical state within the smaller sphere R^- that is a slice of the contracting light-cone with R as its base (figure 1).

A deterministic theory with **second-order dynamics** is **relativistically local** if and only if, given the theory's laws, the instantaneous physical state *and rates of change* within a sphere R at some time uniquely determines, at any future time, the instantaneous physical state within the smaller sphere R^- that is a slice of the contracting light-cone with R as its base (figure 1).

Here the slogan that "what is happening in R" should fix what happens in its contracting light-cone gets filled in differently depending on whether we are dealing with first-order or second-order dynamics. For first-order dynamics (like Maxwell's equations for the evolution of the electric and magnetic fields), all that needs to be specified is the instantaneous physical state at $t = 0$ (such as $\vec{E}$ and $\vec{B}$). For second-order dynamics (like the wave equations for the potentials in the Lorenz gauge), we must specify the instantaneous physical state and rates of change at $t = 0$ (such as ϕ, $\frac{\partial \phi}{\partial t}$, $\vec{A}$, and $\frac{\partial \vec{A}}{\partial t}$).

The above standard is applied within a particular reference frame, but it is not frame-dependent. In a relativistic theory the same laws hold in all inertial frames, and thus if the condition is met in one inertial frame it is met in all inertial frames.

The above standard is appropriate for deterministic theories and might be called a standard of "local determinism." For theories where there is randomness in the dynamics, one would not expect specifying what is initially happening within R to uniquely determine the state of R^-. Such a theory would be relativistically local if specifying what is initially happening within R gives a probability distribution over different states of R^- that is independent of what is initially happening outside of R.

The standard for relativistic locality that we have put forward focuses on the contracting light-cone of a region. We could alternatively (equivalently) look at the expanding light-cone of a region and ask whether two initial specifications of what is happening that only disagree within R later only disagree within R^+ (figure 1). Vaidman (n.d., this volume) formulates his similar standard (which he describes as a prohibition on action-at-a-distance) this way.

3 A Bridge Between Electromagnetism and Quantum Field Theory

As a bridge between the proof of locality that we have just seen for electromagnetism and the question of locality in QFT, this section discusses the Klein-Gordon and Dirac equations.

These equations are often presented as part of "relativistic quantum mechanics," where they are seen as giving the dynamics for quantum wave functions (of particles with spin-0 or spin-$1/2$, respectively). QFT can then potentially be arrived at by extending such single-particle relativistic equations to scenarios involving particle creation and annihilation. Alternatively, the Klein-Gordon and Dirac equations are sometimes interpreted as field equations giving the dynamics of classical relativistic fields. The path to QFT is then to quantize such classical fields. The merits of these two routes to QFT are discussed in Dürr and Lazarovici (2020, ch. 11); Sebens (2022); Tumulka (2022, sec. 6.5). For our purposes here, we can analyze locality for the Klein-Gordon and Dirac equations while remembering that they might be interpreted either as equations of relativistic quantum mechanics or relativistic classical field theory. Studying these equations serves as a useful warm-up to QFT.

3.1 The Klein-Gordon Equation

It is straightforward to prove relativistic locality for the second-order Klein-Gordon equation in a way that parallels our earlier proof of relativistic locality for the wave equation governing the evolution of the electromagnetic scalar potential in the Lorenz gauge.[8] Let us begin with the free Klein-Gordon equation,

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi = 0\,, \tag{16}$$

which differs from the wave equation for the scalar potential (3) only by the addition of a mass term (and puts the symbol ϕ to a new use). To prove locality, we must show that a specification of ϕ and $\frac{\partial\phi}{\partial t}$ within some sphere R fixes what will happen within the contracting light-cone that has R as its base (figure 3). As in the earlier proof, we can consider two solutions ϕ_1 and ϕ_2 that agree on their values and their first time derivatives within R at $t = 0$. Their difference $\phi_d = \phi_1 - \phi_2$ will also be a solution to the Klein-Gordon equation:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi_d = 0\,, \tag{17}$$

As before, we can multiply through by $c^2\frac{\partial\phi_d}{\partial t}$, rearrange, and integrate over the frustum F to get

$$0 = \int_F d^3\vec{x}dt\left(\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\frac{\partial\phi_d}{\partial t}\right)^2 + \frac{c^2}{2}|\vec{\nabla}\phi_d|^2 + \frac{mc^4}{2\hbar^2}\phi_d^2\right] - c^2\vec{\nabla}\cdot\left(\frac{\partial\phi_d}{\partial t}\vec{\nabla}\phi_d\right)\right)\,, \tag{18}$$

[8] Strauss (2008, pg. 234) and Wald (1984, pg. 95) both mention that their proofs of "causality" (locality) can be applied to the Klein-Gordon equation as well.

in parallel to (8). Using the four-dimensional divergence theorem, this can again be broken up into integrals over R, R^- and E. The integral over R will again vanish because ϕ_d and $\frac{\partial \phi_d}{\partial t}$ are (by supposition) 0 on R at $t = 0$. The integral over E is as before (13), with an additional term that must make a positive contribution,

$$\frac{c}{\sqrt{c^2+1}} \int_E d^3\vec{x} \left[\frac{1}{2} \left(\frac{\partial \phi_d}{\partial t} - c \frac{\vec{x} - \vec{x}_f}{|\vec{x} - \vec{x}_f|} \cdot \vec{\nabla} \phi_d \right)^2 + \frac{c^2}{2} \left(\vec{\nabla} \phi_d - \left(\frac{\vec{x} - \vec{x}_f}{|\vec{x} - \vec{x}_f|} \cdot \vec{\nabla} \phi_d \right) \frac{\vec{x} - \vec{x}_f}{|\vec{x} - \vec{x}_f|} \right)^2 + \frac{mc^4}{2\hbar^2} \phi_d^2 \right] . \tag{19}$$

So, again the integral over E is ≥ 0. The integral over R^- must then satisfy the inequality,

$$\int_{R^-} d^3\vec{x} \left(\frac{1}{2} \left(\frac{\partial \phi_d}{\partial t} \right)^2 + \frac{c^2}{2} |\vec{\nabla} \phi_d|^2 \right) \leq 0 \, , \tag{20}$$

as in (14). This can only be achieved if $\frac{\partial \phi_d}{\partial t}$, $\vec{\nabla} \phi_d$, and ϕ_d all vanish on the entirety of R^-. Because this must hold for any choice of t and corresponding slice of the contracting light-cone R^-, ϕ_1, and ϕ_2 must agree throughout the contracting light-cone. This establishes locality for the free Klein-Gordon equation.

3.2 Interlude: Superluminal Propagation

If you interpret the ϕ that appears in the Klein-Gordon equation (16) to be a single-particle quantum wave function (as opposed to a classical field), then it is natural to ask what the probability density is for finding the particle at a given location. The 0-component of the usual Klein-Gordon four-current fails to yield a probability density that is always positive. A proposal from Newton and Wigner is generally considered to give a more satisfactory probability density.[9] This Newton-Wigner probability density can be applied to the subset of solutions to the Klein-Gordon equation that are formed by superposing positive energy modes (leaving the negative energy modes to be interpreted as antiparticle modes). We do not need to settle the question of the correct position probability density here because our concern is only with the fundamental collapse-free dynamics of ϕ. Taking an Everettian (many-worlds) approach, one would eventually need to assign probabilities to branches of the universal wave function that correspond to different outcomes for measurements of the positions of particles. But, that is a project that can be embarked upon after positing particular fundamental dynamics that may or may not be local.

In the literature on probability densities and position operators for the Klein-Gordon wave function, you sometimes see claims that certain states will propagate superluminally—in apparent contradiction with the proof of locality that we have just presented. Let us now address that apparent contradiction to assuage concerns about superluminal propagation.

[9] See Schweber (1961, sec. 3c).

Fleming and Butterfield (1999) provide a way of dissolving the tension:

> "… the restriction to subluminal propagation refers to wavefunctions for which *both* ϕ and its time derivative $\partial\phi/\partial x^0$ have momentarily compact support. … there are no such wave functions in the positive (or indeed, the negative) energy subspace." (Fleming and Butterfield 1999, pg. 124)

One can construct states where, at a moment, ϕ is only non-zero within a given region and then, very soon after, ϕ is non-zero far away (at spacetime points space-like separated from the original region). At first glance, that might look like superluminal propagation. But, by our earlier second-order standard for locality it is not. If $\partial\phi/\partial t$ is non-zero outside the region then there are important things happening out there, even if ϕ itself happens to be momentarily zero. When you use the Klein-Gordon equation (which is second-order) to determine the evolution of ϕ somewhere far from the specified region, that evolution will take as input both ϕ (which is zero) and $\partial\phi/\partial t$ (which is not zero).

This dissolution of the superluminal propagation problem relies on the common understanding of the Klein-Gordon dynamics as second-order, taking as input both ϕ and its time derivative, $\partial\phi/\partial t$. By contrast, if the dynamics were first-order then perhaps we should be troubled by cases where ϕ is initially only non-zero in some region and soon after is non-zero at far space-like separated points. Fleming and Butterfield (1999) and Tumulka (2022, pg. 359) explicitly argue that the Klein-Gordon dynamics should be viewed as first-order. Fleming and Butterfield (1999, pg. 123–124) write that the first-order equation

$$i\hbar\frac{\partial}{\partial t}\phi = \sqrt{m^2c^4 - \hbar^2c^2\nabla^2}\phi \tag{21}$$

> "supercedes the KG [Klein-Gordon] equation, because there is a one-to-one correspondence between positive energy solutions of the Klein-Gordon equation and specifications of (smooth) ϕ on a spacelike hyperplane and [(21)] implies the KG equation by iteration."

In favor of (21), Tumulka (2022, pg. 359) writes that

> "the second-order Klein-Gordon equation includes contributions of negative energy and is therefore widely avoided."

The operator under the square root in (21) looks a bit odd, but can be rendered precise by Fourier transforming ϕ, writing the operator in terms of the wave vector $\vec{k}$, and Fourier transforming back (as in Schweber 1961, pg. 56):

$$i\hbar\frac{\partial}{\partial t}\phi(\vec{x}\,) = \frac{1}{(2\pi)^3}\int d^3k\; d^3\vec{y}\; e^{i\vec{k}\cdot(\vec{x}-\vec{y}\,)}\sqrt{m^2c^4 + \hbar^2c^2k^2}\phi(\vec{y}\,)\;. \tag{22}$$

This first-order equation for the evolution of ϕ is clearly non-local, but Fleming and Butterfield are correct that we can use the equation to pick out the allowed free evolutions for positive

energy states. We do not see a compelling argument in the above quotations from Fleming, Butterfield, and Tumulka as to why the first-order equation should be viewed as more fundamental. In response to Fleming and Butterfield: It is true that one can derive the second-order equation from the first-order, but you can also go the other way and use the second-order equation to show that states formed from positive energy modes will satisfy the first-order equation. In response to Tumulka: Using the second-order equation only fixes the dynamics and we still have the freedom to choose whether to allow physical states that include negative energy modes (depending on their ultimate utility for the representation of either particles or antiparticles).

Before proceeding, let us also mention that there are other first-order reformulations of the Klein-Gordon dynamics: Greiner (2000, sec. 1.6 and 1.8) gives a first-order Schrödinger form of the Klein-Gordon equation with a two-component wave function and modifies that to arrive at the first-order Feshbach-Villars representation. If it turns out that some first-order formulation gives the fundamental law governing the evolution of ϕ, then there may indeed be superluminal propagation and non-locality here. But if it is the Klein-Gordon equation, we are safe.

For the first-order Dirac equation, you can prove that the dynamics of the four-component wave function (or classical field) ψ are local. As Thaller (1992, pg. 28) puts it: "any solution of the Dirac equation … whether it has positive energy or not, cannot propagate faster than with the velocity of light." We will see in the next section that simply fixing ψ within a sphere R is enough to fix what happens in the contracting light-cone with R as its base (there is no need to fix $\partial\psi/\partial t$, as should be the case for a first-order theory). As with the Klein-Gordon equation, if we interpret the Dirac equation as a quantum equation (as opposed to a classical field equation) there is room to debate the correct position probability density and the appropriate position operator. The standard probability density $\psi^\dagger\psi$ is a viable option, as it is always positive and part of a well-behaved probability four-current. Alternatively, one can take an approach that more closely parallels the Newton-Wigner probability density for the Klein-Gordon equation and introduce a Newton-Wigner position operator for positive energy Dirac wave functions (Schweber 1961, pg. 94; Thaller 1992, sec. 1.7.2). This operator takes a simple form in the Foldy-Wouthuysen representation (which we will not present here) and, if that representation is taken as fundamental, then we have a non-local theory (Thaller 1992, pg. 28). However, if we take the ordinary ψ to be our basic entity and the standard Dirac equation to always give its evolution, the dynamics will be local.

3.3 The Dirac Equation

In this section, we prove the locality of the Dirac equation. While this property of the Dirac equation is known, we have not seen a proof that neatly parallels the proofs of locality for the

Maxwell and Klein-Gordon equations presented earlier.[10] Here we present such a proof.

Let us begin with the free Dirac equation:

$$\left(i\hbar\frac{\partial}{\partial t} + i\hbar c\,\gamma^0\vec{\gamma}\cdot\vec{\nabla} - \gamma^0 mc^2\right)\psi = 0\,. \tag{23}$$

Because this is a first-order equation, the standard for locality (from the end of section 2) is whether two solutions ψ_1 and ψ_2 that agree within some sphere R at $t = 0$ will agree within the contracting light-cone with R as its base (figure 3)—with no requirement that the first time-derivatives of ψ_1 and ψ_2 agree on R. Let us define ψ_d to be the difference between ψ_1 and ψ_2: $\psi_d = \psi_1 - \psi_2$. Because the free Dirac equation is linear, ψ_d will obey the free Dirac equation if ψ_1 and ψ_2 do.

From the Dirac equation, one can prove a conservation law that might be regarded as describing the local conservation of probability or charge (depending on whether the Dirac equation is being viewed as part of relativistic quantum mechanics or classical field theory). The difference ψ_d must thus obey:

$$\frac{\partial}{\partial t}|\psi|^2 + \vec{\nabla}\cdot\left(c\psi^\dagger\gamma^0\vec{\gamma}\psi\right) = 0\,. \tag{24}$$

This continuity equation is of the right form to apply the four-dimensional divergence theorem, as in (8). Let us integrate (24) over the interior of the frustum F and use the four-dimensional divergence theorem to rewrite this four-dimensional volume integral as an integral over the three-dimensional "surface" comprised of the base R, the top R^-, and the edge E,

$$\begin{aligned} 0 &= \int_{R\cup R^-\cup E} d^3\vec{x}\left(n_t|\psi_d|^2 - \vec{n}\cdot\left(c\psi_d^\dagger\gamma^0\vec{\gamma}\psi_d\right)\right) \\ &= -\int_R d^3\vec{x}|\psi_d|^2 + \int_{R^-} d^3\vec{x}|\psi_d|^2 \\ &\qquad + \int_E d^3\vec{x}\left[n_t|\psi_d|^2 - \vec{n}\cdot\left(c\psi_d^\dagger\gamma^0\vec{\gamma}\psi_d\right)\right]\,. \end{aligned} \tag{25}$$

The integral over R vanishes because, by supposition, $\psi_1 = \psi_2$ on R and thus ψ_d is zero on R. We can insert our earlier expression for the four-dimensional unit vector (11) normal to the edge E of the frustum to express the integral over E as

$$\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left[|\psi_d|^2 - \frac{\vec{x}-\vec{x}_f}{c|\vec{x}-\vec{x}_f|}\cdot\left(c\psi_d^\dagger\gamma^0\vec{\gamma}\psi_d\right)\right] \tag{26}$$

[10] Thaller (1992, sec. 1.5) gives a very different proof of the same result.

This can be rewritten as a sum of squares

$$\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left[\left|\psi_d - \frac{1}{2}\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\left(\gamma^0\vec{\gamma}\psi_d\right)\right|^2 - \frac{1}{4}\left|\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\left(\gamma^0\vec{\gamma}\psi_d\right)\right|^2\right], \tag{27}$$

using the properties of the Dirac gamma matrices that (a) the zero matrix is Hermitian: $\gamma^{0\dagger} = \gamma^0$, (b) the x, y, and z matrices are anti-Hermitian: $\gamma^{i\dagger} = -\gamma^i$, and (c) the matrices obey the anticommutation relations $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ with metric signature (+ - - -). Employing the same tools again, the final term becomes

$$\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left[\left|\psi_d - \frac{1}{2}\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\left(\gamma^0\vec{\gamma}\psi_d\right)\right|^2 - \frac{1}{4}\psi_d^\dagger\left(\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\vec{\gamma}\right)\left(\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\vec{\gamma}\right)\psi_d\right]. \tag{28}$$

The unit three-vector $\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}$ picks out a direction and taking the dot product with $\vec{\gamma}$ gives us a gamma matrix associated with that direction. Like γ^1, γ^2, or γ^3, $\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\vec{\gamma}$ will square to -1. Thus, our integral over E becomes

$$\frac{c}{\sqrt{c^2+1}}\int_E d^3\vec{x}\left[\left|\psi_d - \frac{1}{2}\frac{\vec{x}-\vec{x}_f}{|\vec{x}-\vec{x}_f|}\cdot\left(\gamma^0\vec{\gamma}\psi_d\right)\right|^2 + \frac{1}{4}|\psi_d|^2\right]. \tag{29}$$

This is a sum of squares and thus must be ≥ 0. Returning to (25), this means that the integral over R^- must satisfy

$$\int_{R^-} d^3\vec{x}|\psi_d|^2 \leq 0 \tag{30}$$

The only way that this inequality can hold is if $|\psi_d|^2 = 0$ (and thus $\psi_d = 0$) throughout R^-. The two solutions ψ_1 and ψ_2 that agreed on R must agree on R^-. This is true regardless of where in time we cut the slice R^- along the contracting light-cone, and thus ψ_1 and ψ_2 must agree throughout the contracting light-cone. This completes our proof that the Dirac equation satisfies the same standard of relativistic locality as electromagnetism and the Klein-Gordon equation.

4 Quantum Field Theory

Having seen that a single standard for locality holds for classical electromagnetism, the Klein-Gordon, and the Dirac equation, we next embark on the task of showing that the same standard holds for QFT. Because our most fundamental quantum theories are cast in the framework of QFT, we believe that establishing the locality of QFT (without collapse) suffices to show that the many-worlds interpretation is local. In this section, we will follow the standard convention for QFT and set $\hbar = c = 1$.

To apply the standard for relativistic locality from the end of section 2, we first need a way of assigning states to regions at a time in QFT. Wallace and Timpson (2010) and Wallace (2012, ch. 8) have considered this question in the context of the many-worlds interpretation and proposed a way of assigning states to regions of spacetime and to regions of space at a time.[11] Wallace and Timpson also claim that their spacetime states evolve locally. Although we broadly agree with their approach and see ours as compatible with theirs, we think that there are many details left to be filled in by their quick treatment.

Wallace and Timpson propose that we should assign a "density operator" or "reduced density matrix" to a region of space at a time to give the state of that region. That much we agree on. However, when presenting the nature of this density operator in relativistic QFT, they seek to avoid adopting a preferred basis[12] (such as particle or field configurations) and end up taking an algebraic approach that we see as overly abstract and general (see Wallace and Timpson 2010; Wallace 2012, pg. 301; Swanson 2020). In this section, we explore two competing proposals as to how one might assign density matrix states to regions that we take to be more ontologically precise. The first option, which we prefer, begins from the field wave functional approach to QFT and leads to locality. The second option begins from the competing particle Fock space approach and either fails in assigning states to regions or leads to non-locality.

Wallace (2012, pg. 302–303) appeals to a standard result from QFT to argue that the states that he and Timpson identify will evolve locally:

> "In a quantum field theory, the quantum state of any region depends only on the quantum state of some cross-section of the past lightcone of that region. Disturbances cannot propagate into that lightcone. This follows from the well-known fact that spacelike separated field operators commute: any disturbance outside the past lightcone of a region R can be represented on the global quantum state as the action on that state of a unitary operator built out of some operators localized outside the past light cone of R, and since those operators commute with all operators localized in R, they have no effect on the expectation values of operators in R, and so no effect on the physical state of R."

[11] Wallace and Timpson (2010) make their proposal about how to assign states to regions in QFT as part of a response to "wave function realism." For the non-relativistic quantum mechanics of a fixed number of particles, N, wave function realism is the view that: the fundamental space is $3N$-dimensional, not 3-dimensional, and the universal quantum state should be understood as a kind of field on this space. There are ways that one might attempt to extend wave function realism to QFT (Wallace and Timpson 2010, pg. 707–708), but Wallace and Timpson reject such strategies and put forward an alternative view, "spacetime state realism," that preserves ordinary 3-dimensional space. We also take space to be 3-dimensional, and here consider how states might be assigned to regions of that space or the entirety of it.

[12] The approach that we prefer for assigning states to regions (section 4.1) makes use of a particular basis (in our example, configurations of the field ϕ). Still, one need not take this to be a metaphysically preferred basis. One could maintain that the states of regions can be expressed equally well in other bases. This would give a way of responding to the following concern that Wallace and Timpson (2010, pg. 707–708) raise: "there is no single preferred choice of fields by which a QFT can be specified. A number of results from QFT ... suggest that a single QFT can be equivalently described in terms of several different choices of field observable, with nothing in particular to choose between them."

While we agree with the verdict in the first sentence and will make use of the same result about the commutation of space-like separated operators to prove it, we do not see this quick argument as sufficient. As will become clear in what follows, it takes a significant amount of work to settle the question of fundamental locality for a particular concrete choice of ontology (applying the standard for locality from section 2).

In what follows, we do not adopt a particular strategy for handling the UV divergences that arise in QFT, but assume that they can be handled somehow. Our proof of locality for a field approach to QFT is not watertight and certainly not up to the standards of mathematical rigor found in algebraic approaches to quantum field theory.[13] Still, we think it suffices to illustrate the locality of QFT.

4.1 Wave Functionals and Field States for Regions

On what can be called a "field approach" to QFT, the universal quantum state is given by a wave functional Ψ that assigns amplitudes to different possible classical field configurations (different specifications of the field values at every point in space).[14] This is analogous to the way that a particle wave function assigns amplitudes to different possible classical particle configurations (different specifications of the locations of point particles). Adopting the Schrödinger picture, the wave functional evolves by a Schrödinger equation that takes the usual form,

$$i\frac{d}{dt}\Psi = \hat{H}\Psi \; . \tag{31}$$

To see how a state can be assigned to a region within the field approach, let us warm up by reviewing an example from non-relativistic quantum mechanics. Let us suppose that we have an isolated system of $n+m$ non-identical particles (to set aside issues of symmetrization) without spin, whose state is given by the wave function

$$\Psi(\vec{x}_1, \ldots, \vec{x}_n, \vec{x}_{n+1}, \ldots, \vec{x}_{n+m}) \; , \tag{32}$$

where indices 1 through n pick out the particles in one subsystem and $n+1$ through $n+m$ pick out the particles in another subsystem. Because Ψ takes particle positions as input, it is straightforward to ignore certain particle positions by tracing over them to arrive at a reduced

[13] We seek to operate at the level of rigor of a typical QFT textbook. Problems may arise at higher levels of rigor. For instance, the way that we assign density operators to regions by taking partial traces (35) might run into technical difficulties having to do with physics at the UV scale and the question of whether the Hilbert space is separable (see Swanson 2020). Setting these issues aside and working at a lower level of rigor, it is already non-trivial to prove the relativistic locality of unitary quantum field theory in terms of state evolution.

[14] The field approach is presented in Jackiw (1987); Hatfield (1992); Bohm and Hiley (1993, ch. 11); Struyve (2010); Sebens (2022).

density matrix for the first subsystem,[15]

$$\rho^n(\vec{x}_1,\ldots,\vec{x}_n;\vec{x}'_1,\ldots,\vec{x}'_n)$$
$$= \int d^3\vec{x}_{n+1}\ldots d^3\vec{x}_{n+m}\ \Psi(\vec{x}_1,\ldots,\vec{x}_n,\vec{x}_{n+1},\ldots,\vec{x}_{n+m})\Psi^*(\vec{x}'_1,\ldots,\vec{x}'_n,\vec{x}_{n+1},\ldots,\vec{x}_{n+m})\ . \tag{33}$$

This ability to write a state for a subset of particles in non-relativistic quantum mechanics does not immediately yield a way to assign states to regions of space, though you can get something close enough for certain purposes when the particles have fairly well-localized positions.

In a field approach to QFT, the wave functional takes values of each field at every point in space as input instead of particle positions. This makes it straightforward to identify systems with regions of space. Whereas before we could trace out certain particles to arrive at a state for a subset of particles, here we can trace out the field values at certain locations to arrive at a state for the fields at all other locations.

Schematically, one could index the points of space in the region of interest R to be x_1 through x_n and the points outside of R (in the complement of R, $\bar{R}$) to be x_{n+1} through x_{n+m} (pretending for the moment that there are only a finite number of points in each region) and write the wave functional of a single scalar field ϕ as

$$\Psi\left[\phi(x_1),\ldots,\phi(x_n),\phi(x_{n+1}),\ldots,\phi(x_{n+m})\right]\ , \tag{34}$$

in analogy with (32). Then, one can simply trace over the field values at the points outside the region to get a reduced density matrix for the region of interest

$$\rho^R\left[\phi(x_1),\ldots,\phi(x_n);\phi'(x_1),\ldots,\phi'(x_n)\right]$$
$$= \int d\phi(x_{n+1})\ldots d\phi(x_{n+m})$$
$$\Psi\left[\phi(x_1),\ldots,\phi(x_n),\phi(x_{n+1}),\ldots,\phi(x_{n+m})\right]\Psi^*\left[\phi'(x_1),\ldots,\phi'(x_n),\phi(x_{n+1}),\ldots,\phi(x_{n+m})\right]\ , \tag{35}$$

as in (33).

Dropping the fiction of a finite number of points of space, the universal wave functional for a single field ϕ can be written as

$$\Psi\left[\phi\right] = \Psi\left[\phi^R;\phi^{\bar{R}}\right]\ , \tag{36}$$

where ϕ is understood to be a function that ranges over the entirety of space, ϕ^R only assigns field values to points within R, and $\phi^{\bar{R}}$ only assigns field values to points within $\bar{R}$. The

[15] This kind of representation of the reduced density matrix is given in Dürr, Goldstein, et al. (2005, sec. 3).

universal density matrix (which we have assumed to be pure) can be written in terms of the universal wave functional as

$$\rho[\phi;\phi'] = \Psi\left[\phi\right]\Psi^*\left[\phi'\right] \ , \tag{37}$$

We can write the reduced density matrix for a region R as

$$\rho^R[\phi^R;\phi'^R] = \int \mathcal{D}\phi^{\bar{R}}\ \Psi\left[\phi^R;\phi^{\bar{R}}\right]\Psi^*\left[\phi'^R;\phi^{\bar{R}}\right] \ , \tag{38}$$

where $\int \mathcal{D}\phi^{\bar{R}}$ denotes integrals over $\phi^{\bar{R}}(\vec{x})$ at each point $\vec{x}$ within $\bar{R}$.[16] Thus, we have arrived at precise states for regions in relativistic QFT. (Such states are described in, e.g., Holzhey, Larsen, and Wilczek 1994, pg. 445; Susskind and Lindesay 2005, eq. 3.4.28.)

We can move from the above expressions for the universal and reduced density matrices as functions of two field configurations to alternative operator expressions[17] (which will be of use later) by inserting integrals over possible field configurations and field eigenstates,

$$\begin{aligned}\hat{\rho} &= \int \mathcal{D}\phi\mathcal{D}\phi'\ \rho[\phi;\phi']|\phi\rangle\langle\phi'| \\ \hat{\rho}^R &= \int \mathcal{D}\phi^R\mathcal{D}\phi'^R\ \rho^R[\phi^R;\phi'^R]|\phi^R\rangle\langle\phi'^R| \ .\end{aligned} \tag{39}$$

The field eigenstates $|\phi\rangle$ are such that a field operator $\hat{\phi}(\vec{x})$ acting on $|\phi\rangle$ returns the value of ϕ at $\vec{x}$.[18] The field eigenstates $|\phi^R\rangle$ within R return the values of ϕ^R when acted upon by a field operator within R. Note that the eigenstates $|\phi^R\rangle$ are partial states, only describing the field degrees of freedom within R. To get a full eigenstate for the field everywhere, you would need to combine this with a field configuration $\phi^{\bar{R}}$ in the complement of R and write the state as $|\phi^{\bar{R}},\phi^R\rangle$ or $|\phi^{\bar{R}}\rangle|\phi^R\rangle$. (This is similar to how in non-relativistic quantum mechanics the state of two distinct particles in position eigenstates might be written as $|\vec{x}_1\rangle|\vec{x}_2\rangle$.)

Here we have focused on finding reduced density matrix states for regions starting from a wave functional for a single bosonic scalar field. The extension to other bosonic fields (or multiple bosonic fields) is straightforward. Formally, one could use the same method to find reduced density matrix states for regions starting from a wave functional for a single fermionic field. However, puzzles arise relating to the use of Grassmann numbers in such wave functionals. (The field approach to QFT faces serious challenges regarding Grassmann numbers that must be addressed for the approach to be viable—see Sebens 2022, sec. 5.1.)

[16] See Hatfield (1992, sec. 9.3) for the $\mathcal{D}\phi$ integral notation.

[17] Dürr, Goldstein, et al. (2005, sec. 3) present similar operator and function versions of the reduced density matrices for collections of particles in non-relativistic quantum mechanics.

[18] See Hatfield (1992, eq. 10.5).

4.2 Wave Functional Dynamics and Relativistic Locality

To show that QFT meets the standard for relativistic locality from the end of section 2, we must show that specifying the state of some sphere R at $t = 0$ fixes the future states within the contracting light-cone of R. Because the dynamics of QFT (31) are first-order, the state at R itself should be sufficient to determine that evolution and we should not need to specify any time derivatives.

Let us focus first on the simplest case of a bosonic QFT for a single free real scalar field (the Klein-Gordon field). Adopting the Schrödinger picture (as is rarely done but perfectly permissible in QFT), the dynamics of the universal density matrix (39) are given by the von Neumann equation,

$$i\frac{d}{dt}\hat{\rho} = [\hat{H}, \hat{\rho}] \,, \tag{40}$$

which (for a time-independent Hamiltonian) has the solutions

$$\hat{\rho}(t) = e^{-i\hat{H}t}\hat{\rho}(0)e^{i\hat{H}t} \,. \tag{41}$$

For a free real scalar quantum field, we can take the Hamiltonian $\hat{H}$ to be the integral over all space of the Hamiltonian density operator

$$\hat{\mathscr{H}}(\vec{x}) = \frac{1}{2}\left(\hat{\pi}^2(\vec{x}) + |\vec{\nabla}\hat{\phi}(\vec{x})|^2 + m^2\hat{\phi}(\vec{x})^2\right) \,, \tag{42}$$

where the $\hat{\pi}$ operator (called the "conjugate field momentum") that appears here can be understood as a functional derivative: $\hat{\pi}(\vec{x}) = -i\frac{\delta}{\delta\phi(\vec{x})}$ (Hatfield 1992, eq. 10.9). (The common expression for the Hamiltonian above should arguably be normal-ordered, but let us set that complication aside here.[19]) Although at this point it makes things more concrete to have a particular Hamiltonian in mind, the proof will end up applying more generally across different Hamiltonians.

The key property of the above free Hamiltonian, for the purposes of proving locality, is that when it is used to construct Heisenberg picture operators, local operators will commute at space-like separation,

$$[\hat{\mathcal{O}}_1(\vec{x}, t), \hat{\mathcal{O}}_2(\vec{x}', t')] = 0 \quad \text{if } (\vec{x}, t) \text{ and } (\vec{x}', t') \text{ are space-like separated.} \tag{43}$$

This commutation property is standardly taken to be central in textbook proofs of "causality" (or "local causality" or "microcausality") and it is among the axioms in axiomatic approaches to relativistic QFT.[20] We will not use a particular Hamiltonian, like (42), to prove that (43) holds for certain local operators here, but instead simply show how this commutation property (once

[19] See Hatfield (1992, pg. 45–46); Greiner and Reinhardt (1996, pg. 81).
[20] See Earman and Valente (2014); Calderón (2024).

established) can be used to prove locality. Although (43) may sometimes be presented as a locality condition, we will see that it takes some work to go from this commutation property to a demonstration that QFT satisfied the standard for relativistic locality from section 2. From (43), with $t' = 0$, it follows that

$$[e^{-i\hat{H}t}\hat{\mathcal{O}}_2(\vec{x}')e^{i\hat{H}t}, \hat{\mathcal{O}}_1(\vec{x})] = 0 \quad \text{if } (\vec{x}, t) \text{ and } (\vec{x}', 0) \text{ are space-like separated,} \tag{44}$$

which is a form that will be useful later.

As a special case of (43), the Heisenberg picture field operators commute at space-like separation,[21]

$$[\hat{\phi}(\vec{x}, t), \hat{\phi}(\vec{x}', t')] = 0 \quad \text{if } (\vec{x}, t) \text{ and } (\vec{x}', t') \text{ are space-like separated.} \tag{45}$$

In their discussion of (45), Peskin and Schroeder first show that $\langle\Omega|\hat{\phi}(\vec{x}, t)\hat{\phi}(\vec{x}', t')|\Omega\rangle \neq 0$ (where $|\Omega\rangle$ is the vacuum) when $(\vec{x}, t)$ and $(\vec{x}', t')$ are space-like separated, which they interpret as showing that a particle *can* propagate from $(\vec{x}, t)$ to $(\vec{x}', t')$—a faster-than-light motion. That seems troubling, but Peskin and Schroeder reassure the reader:

> "To really discuss causality, however, we should ask not whether particles can propagate over spacelike intervals, but whether a *measurement* performed at one point can affect a measurement at another point whose separation from the first is spacelike."

The commutation of field operators at space-like separation (45) is then taken to show that such measurements will be unable to affect one another.

Peskin and Schroeder's focus on measurements is standard in QFT textbooks,[22] but we do not need to be deterred by Peskin and Schroeder's warning. To "really discuss causality," we should ask what is happening in nature, whether or not measurements are being conducted. In an Everettian (many-worlds) approach to QFT, we can ask such questions and we will see that there is no superluminal propagation. The commutation of operators at space-like separation will be central to that proof, but there are further steps that must be traversed to go from (43) to a proof of locality that resembles our earlier proofs for simpler theories.

As with our earlier proofs of locality for the Maxwell, Klein-Gordon, and Dirac equations, let us begin with two distinct evolving density matrices that are solutions to the von Neumann equation (40), $\hat{\rho}_1(t)$ and $\hat{\rho}_2(t)$, and agree on the state within R at $t = 0$. One can introduce a matrix $\hat{\rho}_d(t) = \hat{\rho}_1(t) - \hat{\rho}_2(t)$ that is the difference between these solutions. This matrix is a

[21] See, e.g., Peskin and Schroeder (1995, sec. 2.4); Greiner and Reinhardt (1996, sec. 4.4); Hatfield (1992, pg. 46); Tong (2007, sec. 2.6.1).

[22] See Schweber (1961, pg. 222–223); Hatfield (1992, pg. 38); Weinberg (2005, pg. 198); Tong (2007, sec. 2.6.1); Schwartz (2014, pg. 219); Greiner and Reinhardt (1996, pg. 102–103); Tong (2007, sec. 2.6–2.7).

solution to the von Neumann equation but is not a *density matrix* because it has trace zero:

$$\mathrm{tr}\,\hat{\rho}_d(t) = \mathrm{tr}\,\hat{\rho}_1(t) - \mathrm{tr}\,\hat{\rho}_2(t) = 1 - 1 = 0\,, \tag{46}$$

where $\mathrm{tr}\,\hat{\rho}_1(t) = \mathrm{tr}\,\hat{\rho}_2(t) = 1$ as these are properly normalized states. Noting that it is not a density matrix, let us call $\hat{\rho}_d(t)$ a "difference matrix".

By the assumption that $\hat{\rho}_1(t)$ and $\hat{\rho}_2(t)$ agree on the state within R at $t = 0$, the difference matrix $\hat{\rho}_d(t)$ will yield zero as the reduced difference matrix for R at $t = 0$ (integrating over the possible field configurations for $\bar{R}$):

$$\begin{aligned}\hat{\rho}_d^R(0) &= \mathrm{tr}_{\bar{R}}\hat{\rho}_d(0)\\ &= \int \mathcal{D}\phi^{\bar{R}}\ \langle\phi^{\bar{R}}|\hat{\rho}_d(0)|\phi^{\bar{R}}\rangle\\ &= 0\,.\end{aligned} \tag{47}$$

The task at hand is then to show that at an arbitrary later time t, $\hat{\rho}_d^{R^-}(t) = 0$, or in other words, that $\hat{\rho}_1(t)$ and $\hat{\rho}_2(t)$ agree on the state within the slice R^- of the contracting light-cone that has R as its base (recall figure 1).

Because $\hat{\rho}_1(0)$ and $\hat{\rho}_2(0)$ agree within R, the universal difference matrix $\hat{\rho}_d(0)$ initially assigns no reduced difference matrix to R and $\hat{\rho}_d(0)$ can be written as a local operator restricted to $\bar{R}$ (as in (39)),

$$\hat{\rho}_d(0) = \hat{\rho}_d^{\bar{R}}(0) = \int \mathcal{D}\phi^{\bar{R}}\mathcal{D}\phi'^{\bar{R}}\ \rho_d^{\bar{R}}[\phi^{\bar{R}};\phi'^{\bar{R}};0]|\phi^{\bar{R}}\rangle\langle\phi'^{\bar{R}}|\,, \tag{48}$$

where $\rho_d^{\bar{R}}[\phi^{\bar{R}};\phi'^{\bar{R}};0]$ are just complex numbers associated with each pair of possible field configurations $\phi^{\bar{R}}$ and $\phi'^{\bar{R}}$ on the complement of R at $t = 0$. The next step is to show that $\hat{\rho}_d(t) = e^{-i\hat{H}t}\hat{\rho}_d(0)e^{i\hat{H}t}$ is a local operator restricted to $\bar{R}^-$. To do so, it should be sufficient to show that it commutes with any operator in R^-:

$$[e^{-i\hat{H}t}\hat{\rho}_d(0)e^{i\hat{H}t}, \hat{\mathcal{O}}(\vec{x})] = 0 \quad \text{for any } \hat{\mathcal{O}}(\vec{x}) \text{ with } \vec{x} \in R^-. \tag{49}$$

Here we can appeal to the commutation at space-like separation that is standardly taken to establish "causality" in QFT, (43) in the form of (44). Assuming that $\hat{\rho}_d(0)$ can be expanded in terms of local operators at points within $\bar{R}$ (which should be possible given (48)), (44) ensures that each such operator will commute with $\hat{\mathcal{O}}(\vec{x})$ and thus that (49) holds. Having established that $\hat{\rho}_d(t)$ is a local operator restricted to $\bar{R}^-$, tracing over $\bar{R}^-$ will yield zero:

$$\begin{aligned}\hat{\rho}_d^{R^-}(t) = \mathrm{tr}_{\bar{R}^-}\hat{\rho}_d(t) &= \int \mathcal{D}\phi^{\bar{R}^-}\ \langle\phi^{\bar{R}^-}|e^{-i\hat{H}t}\hat{\rho}_d(0)e^{i\hat{H}t}|\phi^{\bar{R}^-}\rangle \\ &= \int \mathcal{D}\phi^{\bar{R}^-}\mathcal{D}\phi^{R^-}\ \langle\phi^{\bar{R}^-}|\langle\phi^{R^-}|e^{-i\hat{H}t}\hat{\rho}_d(0)e^{i\hat{H}t}|\phi^{R^-}\rangle|\phi^{\bar{R}^-}\rangle = \mathrm{tr}\hat{\rho}_d(t) = 0\,,\end{aligned} \tag{50}$$

where the second line inserts $\int \mathcal{D}\phi^{R^-} \langle \phi^{R^-} | \phi^{R^-} \rangle = 1$ and uses (46).[23] The fact that $\hat{\rho}_d^{R^-}(t) = 0$ means that $\hat{\rho}_1^{R^-}(t)$ and $\hat{\rho}_2^{R^-}(t)$ must agree and our proof is finished. (The two recent uses of "should be," before and after (49), leave gaps that would need to be filled in to arrive at a more thorough proof.)

This proof can be extended immediately to interacting theories because the commutation of local operators at space-like separation (43) is a result that holds across bosonic relativistic quantum field theories, not just for the free Hamiltonian (42). Indeed, that commutation condition is a standard that must be met for a bosonic quantum field theory to be classified as a "local quantum field theory" (see Tong 2007, pg. 37).[24]

For fermionic fields, a common textbook line is that even though the Heisenberg picture field operators satisfy *anticommutation* relations instead of commutation relations, there is still no violation of causality (locality) because pairs of field operators ("bilinears") commute at space-like separation and *observables* (the kinds of operators that you can actually measure) will be pairs of field operators (see Tong 2007, sec. 5.4; Schwartz 2014, pg. 219; Greiner and Reinhardt 1996, exercise 5.5). That discussion of measurement is insufficient for our purposes, as we would like to prove that relativistic locality holds for the time evolution of quantum fields regardless of whether any measurements are being conducted. Still, we can potentially use the standard results about anticommutation at space-like separation to prove locality for fermionic fields. If it can be proven that, for a fermionic field, the initial difference matrix $\hat{\rho}_d(0) = \hat{\rho}_d^{\bar{R}}(0)$ is a sum of pairs of anticommuting local operators, then that will commute with operators at space-like separation as in (49) and locality can be proven as above.

4.3 Interlude: Local Dynamics, Non-Separable States

We have now seen that QFT, as a theory of fields, can be shown to meet the same standard for relativistic locality as electromagnetism, the Klein-Gordon equation, the Dirac equation. In this sense, QFT is relativistically local and there is no influence from outside the past light-cone on any spatial region.

However, one might be concerned that our construction of the reduced density matrix giving the state of a region R requires starting with the full universal state and tracing over the region *outside* of R. One could argue that the reduced density matrix for R is not local to R because it depends by its definition on what is happening outside of R.

To address this concern, we want to emphasize the distinction between two species of non-locality, *non-separability* and *relativistic non-locality*. Following Wallace (2012), we can define non-separability as such:[25]

[23] The equation $\int \mathcal{D}\phi^{R^-} \langle \phi^{R^-} | \phi^{R^-} \rangle = 1$ is similar to how in non-relativistic quantum mechanics the integral of the inner product of a position eigenstate with itself is 1: $\int d^3\vec{x} \langle \vec{x} | \vec{x} \rangle = 1$.

[24] Williams, Dougherty, and Miller (2024, sec. 6) formulate Weinberg's principle of "cluster decomposition" as a constraint on acceptable Hamiltonians that might suffice for guaranteeing the necessary commutation relations hold.

[25] See also Healey (1991), Wallace and Timpson (2010), Myrvold (2015), and Williams, Dougherty, and Miller (2024,

> A theory is non-separable if, given two regions A and B, a complete specification of the states of A and B separately fails to fix the state of the combined system $A + B$. That is, there are additional facts – nonlocal facts, if we take A and B to be spatially separated – about the combined system, in addition to the facts about the two individual systems. (Wallace 2012, p. 293)

As an example, consider an idealized setup where there is a single spin-$1/2$ particle in region A, another in region B, and we are only concerned with representing facts about the spins of each particle. The total wave function might be in the spin singlet state where the spins are opposite,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_z\rangle_A|\downarrow_z\rangle_B - |\downarrow_z\rangle_A|\uparrow_z\rangle_B\Big) \tag{51}$$

or the triplet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow_z\rangle_A|\downarrow_z\rangle_B + |\downarrow_z\rangle_A|\uparrow_z\rangle_B\Big)\,. \tag{52}$$

These are distinct total states (that make different predictions as to whether the x spins of the two particles could be aligned when measured) that yield the same reduced density matrices for A and B. Similarly, in QFT the reduced density matrix states of two regions A and B will not fix the state of $A \cup B$ because we will be missing facts about the entanglement between these regions.

Non-separability is not the sort of non-locality that is ruled out by special relativity. Special relativity only forbids violations of relativistic locality (as defined in section 2). Relativistic locality is a property of the dynamics, ruling out instantaneous action-at-a-distance and any other faster-than-light interactions. Newtonian gravity violates this standard, though it has a separable ontology. QFT, by contrast, has a non-separable ontology but satisfies the standard of relativistic locality.

One might worry that non-separability could lead to violations of relativistic causality because the non-separable states of widely spread-out composite systems can immediately change when acted upon at one location.[26] As an example, consider taking the spin singlet state in (51) and applying a unitary transformation on the particle in A alone (e.g., via the application of an appropriate magnetic field) which flips its z-spin, yielding:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\downarrow_z\rangle_A|\downarrow_z\rangle_B - |\uparrow_z\rangle_A|\uparrow_z\rangle_B\Big)\,. \tag{53}$$

That is, via a unitary transformation acting locally on the particle in A, the entanglement relation of perfect anti-correlation between the z-spins of the two particles is transformed into an entanglement relation of perfect correlation. A local action within A has changed the global

sec. 6) for similar discussions of non-separability.

[26] We would like to thank Jacob Barandes for discussion of this point.

state of the two particles, a state that extends into a region, B, that is space-like separated from A. Furthermore, note that the reduced density matrices of the two particles are the same in both (51) and (53). This local action leaves the local states of the A-particle and B-particle unchanged while causing global changes in the state of the two particles together.

Although this might appear to be a violation of relativistic locality, it is not. The global state of the two particles, located in region $A \cup B$, has changed due to a cause within region A, a part of $A \cup B$. The cause is local to the effect. The cause happens at a place where the global state is, rather than at any spatial separation from the global state. An analogy: a country's global state of having a certain population changes when a birth occurs on one coast, but this does not involve any action-at-a-distance or superluminal interaction with the opposite coast. Granted, the situation with quantum entanglement is not entirely analogous. The birth changes the local state of the town on the coast, whereas the transformation of the particle in A leaves its local state unchanged (while still changing the global state of the two particles). However, because in both cases the cause is local to the effect, neither presents a problem for relativistic locality.

4.4 Fock Space and Particle States for Regions

While we think that QFT and its local states are best discussed in terms of wave functionals (interpreting QFT as a theory of fields),[27] QFT is sometimes presented as a theory of particles using a Fock space representation (interpreting QFT as a theory of particles).[28] Wallace and Timpson (2010) and Wallace (2012, sec. 8) use a Fock space representation to assign states to regions within non-relativistic quantum physics. They say little as to why that representation cannot (or should not) be used for relativistic quantum field theory. Here we consider two ways one might attempt to use a Fock representation to obtain states for regions of space within QFT and survey the conceptual and technical problems that each approach runs into, showing that one strategy presents us from assigning states to regions of space and the other leads to violations of relativistic locality. We take these problems to give further reasons (beyond those identified elsewhere[29]) to prefer the wave functional approach. Put another way, these

[27] See Sebens (2022) for a defense of the field approach and an acknowledgment of the challenges facing the approach (including the use of Grassmann numbers when dealing with fermionic fields).

[28] See Schweber (1961), Dürr, Goldstein, et al. (2003), Oldofredi (2018), Deckert, Esfeld, and Oldofredi (2019), Dürr and Lazarovici (2020), and Tumulka (2022).

[29] Although we have not seen these problems for a particle approach raised in this way elsewhere, these issues are arguably manifestations of well-known limitations facing particle approaches. Malament (1996), for example, rejects a fundamental ontology of particles in quantum field theory because one cannot simultaneously satisfy four desirable conditions—including a localizability condition (that appears to be violated by the strategy in section 4.4.1) and a locality condition (that appears to be violated by the strategies in sections 4.4.1 and 4.4.2). Malament's localizability condition is distinct from our requirement that one be able to assign reduced density matrix states to regions and his locality condition is distinct from the standard of relativistic locality that we introduced in section 2 and have been applying across theories. Our discussion can be seen as complementing existing literature on localizability and locality by showing explicitly how these problems arise when one attempts to prove that quantum field theory meets the standard for relativistic locality that is applied in electromagnetism, by using either the standard or the Newton-Wigner creation operators to construct reduced density matrix states for regions.

are reasons to prefer a fundamental ontology of fields over particles. Readers may skip this section if they are already convinced that one should not treat Fock space representations as fundamental within QFT. To keep things simple and avoid the issues that arise for theories with interactions,[30] we focus our attention in this section on a free scalar Klein-Gordon field, as in section 4.2.

In a Fock space representation, the universal quantum state is a wave function living in Fock space $\mathcal{F}$, the direct sum of n-particle Hilbert spaces $\mathcal{H}$ defined by:

$$\mathcal{F} = \bigoplus_{N=0}^{\infty} \mathcal{H}^{(n)} \ . \tag{54}$$

These states generalize ordinary quantum mechanical states for a fixed number of particles by allowing for variable particle number. In particular, these states allow for the creation and annihilation of particles as well as superpositions of different numbers of particles.

We can decompose the quantum state at some time t (adopting the Schrödinger picture) in terms of its n-particle components, by:

$$|\Psi(t)\rangle = |\Psi^{(0)}(t)\rangle + |\Psi^{(1)}(t)\rangle + |\Psi^{(2)}(t)\rangle + ...|\Psi^{(n)}(t)\rangle \ . \tag{55}$$

Each component can be written in terms of its position space n-particle wave function $\psi^{(n)}(\vec{x}_1, ..., \vec{x}_n, t)$ as

$$|\Psi^{(n)}(t)\rangle = \frac{1}{\sqrt{n!}} \int d^3\vec{x}_1, ..., d^3\vec{x}_n \ \psi^{(n)}(\vec{x}_1, ..., \vec{x}_n, t)\hat{a}^\dagger(\vec{x}_1)...\hat{a}^\dagger(\vec{x}_n)|\Omega\rangle \ , \tag{56}$$

where $|\Omega\rangle$ is the vacuum state, $\hat{a}^\dagger(\vec{x})$ creates a particle at $\vec{x}$, $\frac{1}{\sqrt{2}}\hat{a}^\dagger(\vec{x})\hat{a}^\dagger(\vec{y})$ creates a symmetric superposition of a particle at $\vec{x}$ and another at $\vec{y}$, and so forth. Because we are dealing with a single type of bosonic particle, the wave function $\psi^{(n)}(\vec{x}_1, ..., \vec{x}_n, t)$ in (56) must be symmetric under particle permutation.

We now face a choice point as to how the creation operators in (56) should be defined. The first strategy for finding Fock space states for regions follows the standard textbook definitions and the second adopts the Newton-Wigner definitions.

[30] The Fock space approach runs into problems when one moves beyond the free case discussed here and considers interactions (Sebens 2022, sec. 4.3), problems that we take to point towards the wave functional approach. Fraser (2008, pg. 847) explains that the original Fock space for the non-interacting theory cannot be used for the interacting theory because "there is no state in the Fock representation for a free field that can be interpreted as containing zero quanta [in the interacting theory]" (a consequence of Haag's theorem; see also Fraser 2022). Put another way, the original Fock representation does not contain the minimum-energy ground state for the Hamiltonian with interactions. For small interaction terms, the Fock representation may remain useful as an approximation (Wallace 2022, pg. 280). But, it arguably cannot give a fundamental description of the quantum state and thus cannot serve our purpose of proving relativistic locality at the fundamental level within QFT. Defenders of a fundamental particle approach to QFT have potential ways to save a Fock representation. Dürr and Lazarovici (2020, pg. 211) appeal to the Dirac Sea. Tumulka (2022, ch. 6) explores using interior-boundary conditions to define Hamiltonians without ultra-violet divergences.

4.4.1 Option 1: Standard Creation Operators

The field operator can be written as an integral over operators that create and annihilate particles with different momenta $\vec{p}$, [31]

$$\hat{\phi}(\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\mathcal{E}_{\vec{p}}}} \left(e^{-i\vec{p}\cdot\vec{x}}\ \hat{a}(\vec{p}) + e^{i\vec{p}\cdot\vec{x}}\ \hat{a}^\dagger(\vec{p})\right)\ . \tag{57}$$

where $\mathcal{E}_{\vec{p}} = \sqrt{|\vec{p}|^2 + m^2}$ is the relativistic energy of a particle with momentum $\vec{p}$. One can interpret the second part of this expression (57) as creating a particle at $\vec{x}$ and the first part as annihilating a particle at $\vec{x}$,[32] yielding the following creation and annihilation operators:

$$\begin{aligned} \hat{a}^\dagger(\vec{x}) &= \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\mathcal{E}_{\vec{p}}}}\ e^{i\vec{p}\cdot\vec{x}}\ \hat{a}^\dagger(\vec{p}) \\ \hat{a}(\vec{x}) &= \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\mathcal{E}_{\vec{p}}}}\ e^{-i\vec{p}\cdot\vec{x}}\ \hat{a}(\vec{p})\ . \end{aligned} \tag{58}$$

From (57) and (58), it straightforwardly follows that:

$$\hat{\phi}(\vec{x}) = \hat{a}(\vec{x}) + \hat{a}^\dagger(\vec{x})\ . \tag{59}$$

A naive first step towards obtaining reduced density matrices for regions, would be to decompose the vacuum $|\Omega\rangle$ into a tensor product of vacua for a region R and its complement $\bar{R}$, as $|\Omega\rangle_R \otimes |\Omega\rangle_{\bar{R}}$—thinking that the union of two regions is in the vacuum state just when each region is in its vacuum state. As Redhead (1994, p. 78) phrases the intuition, "the global vacuum implies a local vacuum. If there are no particles anywhere in space, then there are no particles present in any local region of space." A naive second step would be to assume that each creation operator $\hat{a}^\dagger(\vec{x})$ in (56) either acts in R *or* $\bar{R}$ (leaving the other region in its vacuum state), so that we can rewrite each component of the total state like so:

$$\begin{aligned} |\Psi^{(n)}(t)\rangle = \frac{1}{\sqrt{n!}} \sum_{k=0}^{n} \int_R d^3\vec{x}_1, ..., d^3\vec{x}_k \int_{\bar{R}} d^3\vec{x}_{k+1}, ..., d^3\vec{x}_n \\ \psi_n(\vec{x}_1, ..., \vec{x}_n, t)\hat{a}^\dagger(\vec{x}_1)\ldots\hat{a}^\dagger(\vec{x}_k)|\Omega\rangle_R \otimes \hat{a}^\dagger(\vec{x}_{k+1})\ldots\hat{a}^\dagger(\vec{x}_n)|\Omega\rangle_{\bar{R}}\ , \end{aligned} \tag{60}$$

summing over all the possible ways in which the k of the n particles might be distributed

[31] See Schweber (1961, pg. 177); Peskin and Schroeder (1995, pg. 21).

[32] There are quick arguments for this interpretation presented in quantum field theory textbooks. Peskin and Schroeder (1995, pg. 24) and Schwartz (2014, p. 22) put forth one such argument by showing (i) that the action of the field operator (and hence the creation operator, because the other half of (59) vanishes when acting on the vacuum) on the vacuum looks very similar to the non-relativistic expression for the eigenstate of some definite position state, and (ii) that $\langle 0|\phi(\vec{x})|\vec{p}\rangle$ gives us $e^{i\vec{p}\cdot\vec{x}}$, just like the non-relativistic $\langle\vec{x}|\vec{p}\rangle$. Schweber (1961) makes a similar argument that $\langle 0|\phi(\vec{x})|\vec{p}\rangle$ gives us the probability amplitude for finding a particle at some particular position. Hatfield (1992, pp. 26–27) argues instead from the relationship between the momentum creation/annihilation operators and the field operators, which mimic the position/momentum uncertainty relations.

across R and $n-k$ across $\bar{R}$ (assuming that $\hat{a}^\dagger(\vec{x}_1)\ldots\hat{a}^\dagger(\vec{x}_k)$ act only on $|\Omega\rangle_R$ and $\hat{a}^\dagger(\vec{x}_{k+1})\ldots\hat{a}^\dagger(\vec{x}_n)$ act only on $|\Omega\rangle_{\bar{R}}$). From (55) and (60), one could form a global density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ and find a reduced density matrix for the region R by tracing over possible possible particle arrangements in $\bar{R}$.

While somewhat elegant, this approach is not viable. Both the first and second steps above were flagged as naive and are in fact incorrect. The first step assumed that the global vacuum (or zero-particle state) $|\Omega\rangle$ could be written as $|\Omega\rangle_R \otimes |\Omega\rangle_{\bar{R}}$, an unentangled product state of vacua for R and $\bar{R}$. But, this runs contrary to the well-known fact that in QFTs (even free QFTs) the vacuum is an entangled state.[33] The second step assumed that $\hat{a}^\dagger(\vec{x})$ is a local operator, acting only at $\vec{x}$ and not elsewhere. This is not the case.[34] As Fleming (2000, sec. 4) notes, "unlike the local field $\hat{\phi}$ itself, the positive and negative frequency parts do not commute (with their adjoints) at space-like separation." Since these are precisely the creation and annihilation operators that we adopted in (58) and (in the Heisenberg picture) local operators must commute at spacelike separation (43), we see that the standard creation and annihilation operators are not local operators.[35] Schweber (1961) explicitly writes down the commutator for $\hat{a}(x)$ and $\hat{a}^\dagger(y)$ in terms of a function that falls off exponentially with $x-y$ but does not vanish for space-like separated x, y, showing that they in general do not commute.[36]

Noting the failure of the first step and the entanglement of the vacuum, one might back up and attempt to write the global vacuum state as an entangled state of R and $\bar{R}$. Let us suppose that the global Hilbert space (54) (Fock space) can be factorized into separate Hilbert spaces for R and $\bar{R}$, $\mathcal{F} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$.[37] Then, Schmidt decomposition lets us write the global vacuum, $|\Omega\rangle$, as a sum over tensor product states of $\mathcal{H}_R$ and $\mathcal{H}_{\bar{R}}$:

$$|\Omega\rangle = \sum_{i,j} \lambda_{ij} |\Omega_i\rangle_R \otimes |\Omega_j\rangle_{\bar{R}} \,, \tag{61}$$

where $|\Omega_i\rangle_R$ are orthogonal states of $\mathcal{H}_R$, $|\Omega_j\rangle_{\bar{R}}$ orthogonal states of $\mathcal{H}_{\bar{R}}$, and λ_{ij} are real-valued Schmidt coefficients for each product state.

The naive approach to factorizing the vacuum as an unentangled product state of vacua for R and $\bar{R}$ used earlier, $|\Omega\rangle = |\Omega\rangle_R \otimes |\Omega\rangle_{\bar{R}}$, came with a clear physical meaning: the world as a whole has no particles and each region is also in a vacuum state. The Schmidt decomposition

[33] The fact that the vacuum is entangled can be seen either by examining vacuum wave functionals (Hatfield 1992, sec. 10.1; Huang 1998, p. 34) or via the Reeh-Schlieder theorem (Clifton et al. 1998; Fleming 2000; Halvorson 2001).

[34] See Piazza and Costa (2008) and Halvorson (2001, sec. 3.3).

[35] A technical way to put it is that these operators cannot be part of the local algebra of observables associated with a spatial region (Halvorson 2001, sec. 3.1, point 3).

[36] A similar derivation can be found in Henley and Thirring (1962, pg. 44). See also Duncan (2012, sec. 6.5).

[37] In full QFT, analyzed at a high standard of mathematical rigor, this decomposition may not exist (Swanson 2020, pg. 942–943; Falcone and Conti 2024, footnote 3). However, in section 4.1 we seemed to be able to make such a decomposition. Also, Wallace and Timpson (2010) and Wallace (2012) appeal to such a decomposition when they adopt an algebraic perspective for assigning states to spacetime regions. For our purposes here, we will help ourselves to this decomposition.

in (61) jettisons this physical picture and it is unclear how the superposed states $|\Omega_i\rangle_R$ and $|\Omega_j\rangle_{\bar{R}}$ are to be interpreted. Two ways forward both lead to interpretive puzzles.

First, a natural interpretation of the states of these regional Hilbert spaces would be in terms of particle numbers associated with each region. This interpretation forces us to say (absurdly) that the global vacuum state (with no particles) is a superposition of states where there are various different numbers of particles in R and $\bar{R}$. As Falcone and Conti (2024, pg. 11) put it, "the vacuum is not locally devoid of quanta, but only globally." Second, we might view $|\Omega_i\rangle$ and $|\Omega_j\rangle$ as distinct vacuum states for R and $\bar{R}$ that describe a variety of different zero-particle states for those regions. What then is the distinction between these states and how can it be captured in the particle language that accompanies the Fock space approach? For our purposes here, let us press on, having flagged these interpretive issues.

A general state could be constructed from (61) by acting with creation operators on this vacuum, as in (55) and (56). However, because of the failure of the second step taken earlier, one cannot assume that each creation operator $\hat{a}^\dagger(\vec{x})$ acts only on either the R or $\bar{R}$ states in the Schmidt decomposition (61) (depending on where $\vec{x}$ is in R or $\bar{R}$), as was assumed to get (60). This prevents us from defining the reduced density matrix for R by tracing over possible particle arrangements in $\bar{R}$, as was suggested under (60).

We have thus seen that the strategy for assigning reduced density matrix states to regions outlined earlier—in the paragraph that includes (60)—fails at multiple points and cannot be easily patched up. One could try a radically different approach.[38] At least in the context of non-interacting QFTs, there are recipes for writing particle wave functions (Fock space states) as field wave functionals.[39] As we have seen that field wave functionals do yield reduced density matrix states for regions, one might use the following two-step procedure to find states for a region R: first (i) rewrite the global Fock space state (55) as a global wave functional, and then (ii) use the method from section 4.1 to find a reduced density matrix state for R. The problem with this strategy is that it is parasitic on the field wave functional approach and the reduced density matrices that you end up with are not immediately interpretable in terms of particles. These are not viable *particle* states for regions. If one is going to use the field reduced density matrices from section 4.1 as states for regions, then it seems like one should use wave functionals to represent global states and take a thoroughgoing field approach to quantum field theory (as in sections 4.1 and 4.2).

To attempt a proof of relativistic locality (or its violation), we need to start with states for regions. Unfortunately, we have not been able to write down a general expression for such states within a particle approach to QFT (using the standard creation and annihilation operators). Thus, we reach the inconclusive conclusion that we cannot determine whether this Fock space version of QFT satisfies the standard of relativistic locality from section 2 because

[38] We thank David Baker for this suggestion.

[39] See Hatfield (1992, sec. 10.1); Bohm and Hiley (1993, sec. 11.5); Baker (2009); Sebens (2022, sec. 4.3).

the non-locality of the particle creation operators has prevented us from finding a way to assign states to regions.

4.4.2 Option 2: Newton-Wigner Creation Operators

Let us now back up and consider an alternative proposal as to how we ought to define the creation operators that appear in (56), a proposal that has been advocated by Fleming and Butterfield (1999) and Fleming (2000) and is an extension of the Newton-Wigner maneuvers briefly mentioned in section 3.2. Removing the factors of $\frac{1}{\sqrt{2\mathcal{E}_{\vec{p}}}}$ that appear in (58),[40] we can put forward new creation operators for particles at specific positions (written in terms of the same old creation operators for particles with specific momenta):

$$\begin{aligned}\hat{a}^{\dagger}_{NW}(\vec{x}) &= \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}}\, \hat{a}^{\dagger}(\vec{p}) \\ \hat{a}_{NW}(\vec{x}) &= \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{x}}\, \hat{a}(\vec{p})\ .\end{aligned} \tag{62}$$

These are the Newton-Wigner creation and annihilation operators.[41]

One can introduce a different way of carving the global Hilbert space (Fock space) into Hilbert spaces for regions such that the Newton-Wigner creation and annihilation operators for points in a region act only on that region's Hilbert space. Relative to this revised carving, it is possible to regard the vacuum as an unentangled state[42] such that

$$|\Omega\rangle = |\Omega_{NW}\rangle_R \otimes |\Omega_{NW}\rangle_{\bar{R}}\ . \tag{63}$$

The Newton-Wigner approach thus seems to capture the intuitive physical meaning we sought earlier, whereby "the vacuum ... is devoid of quanta locally as well as globally" (Fleming 2000, pg. 11).

We can proceed to build the n-particle piece (56) of a general state (55) by acting on the vacuum with Newton-Wigner creation operators, operators that will act either on $|\Omega_{NW}\rangle_R$ (if $\vec{x}$ is in R) or $|\Omega_{NW}\rangle_{\bar{R}}$ (if $\vec{x}$ is in $\bar{R}$),

$$\begin{aligned}|\Psi^{(n)}(t)\rangle = \frac{1}{\sqrt{n!}}\sum_{k=0}^{n}\int_R d^3\vec{x}_1,...,d^3\vec{x}_k \int_{\bar{R}} d^3\vec{x}_{k+1},...,d^3\vec{x}_n \\ \psi_n(\vec{x}_1,...,\vec{x}_n,t)\hat{a}^{\dagger}_{NW}(\vec{x}_1)\ldots\hat{a}^{\dagger}_{NW}(\vec{x}_k)|\Omega_{NW}\rangle_R \\ \otimes\, \hat{a}^{\dagger}_{NW}(\vec{x}_{k+1})\ldots\hat{a}^{\dagger}_{NW}(\vec{x}_n)|\Omega_{NW}\rangle_{\bar{R}}\ ,\end{aligned} \tag{64}$$

[40] This seemingly inconsequential move breaks the Lorentz covariance of the measure, which leads to the drastic physical differences for the Newton-Wigner proposal we see below.

[41] See Fleming (2000, pg. 9–10); Piazza and Costa (2008, sec. 2); Falcone and Conti (2024, sec. 2.2).

[42] For technical details, Fleming (2000, sec. 4–5); Halvorson (2001, sec. 4); Piazza and Costa (2008); Falcone and Conti (2024, sec. 5.1.2).

This takes the form that we hoped for in (60), which was blocked by the entanglement of the vacuum (now solved) and the non-locality of the standard creation operators.

The Newton-Wigner creation operators are spatially local, in the sense that creating a particle at a moment has no effect elsewhere, allowing us to write (64). The Schrödinger-picture creation and annihilation operators commute at spatial separation (Fleming 2000, sec. 4; Falcone and Conti 2024, sec. 5.1.2). The Heisenberg-picture creation and annihilation operators obey equal-time commutation relations, but do not commute at space-like separation (Halvorson 2001, sec. 5; Piazza and Costa 2008, pg. 5). That failure to commute at space-like separation means that the Heisenberg-picture operators are not local operators. This leads to violations of relativistic locality, as we will show.

Within the Newton-Wigner approach, one can construct reduced density matrix states for regions. To see how this is done, let us begin by introducing a more compact notation for (64),

$$|\Psi^{(n)}(t)\rangle = \frac{1}{\sqrt{n!}} \sum_{k=0}^{n} \int_R d^3\vec{x}_1, ..., d^3\vec{x}_k \int_{\bar{R}} d^3\vec{x}_{k+1}, ..., d^3\vec{x}_n \\ \psi^{(n)}(\vec{x}_1, ..., \vec{x}_n, t)|\vec{x}_1, ..., \vec{x}_k\rangle_R \otimes |\vec{x}_{k+1}, ..., \vec{x}_n\rangle_{\bar{R}} \,, \tag{65}$$

where the creation operators have been absorbed into the states of R and $\bar{R}$. From (55), it is clear that the universal density matrix at time t, $\hat{\rho}(t)$, is simply:

$$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)| = \sum_{n,m} |\Psi^{(n)}(t)\rangle\langle\Psi^{(m)}(t)| = \sum_{n,m} \rho_{nm}(t) \tag{66}$$

To find the n, m-components of the universal density matrix, $\rho_{nm}(t)$, we can use (65):

$$\begin{aligned} \hat{\rho}_{nm}(t) &= |\Psi^{(n)}(t)\rangle\langle\Psi^{(m)}(t)| \\ &= \frac{1}{\sqrt{n!}\sqrt{m!}} \sum_{k=0}^{n}\sum_{j=0}^{m} \int_R d^3\vec{x}_1 ... d^3\vec{x}_k \int_{\bar{R}} d^3\vec{x}_{k+1} ... d^3\vec{x}_n \int_R d^3\vec{x}'_1 ... d^3\vec{x}'_j \int_{\bar{R}} d^3\vec{x}'_{j+1} ... d^3\vec{x}'_m \\ &\psi^{(n)}(\vec{x}_1, ..., \vec{x}_n, t)\psi^{*(m)}(\vec{x}'_1, ..., \vec{x}'_m, t) \\ &|\vec{x}_1, ..., \vec{x}_k\rangle_R \otimes |\vec{x}_{k+1}, ..., \vec{x}_n\rangle_{\bar{R}} \langle\vec{x}'_1, ..., \vec{x}'_j|_R \otimes \langle\vec{x}'_{j+1}, ..., \vec{x}'_n|_{\bar{R}} \,. \end{aligned} \tag{67}$$

To find the reduced density matrix for R, $\hat{\rho}_R$, we perform the partial trace over $\bar{R}$,

$$\hat{\rho}_R(t) = \mathrm{tr}_{\bar{R}}\left(\hat{\rho}(t)\right) = \sum_n \sum_m \mathrm{tr}_{\bar{R}}\left(\hat{\rho}_{nm}(t)\right) \,. \tag{68}$$

We can enact the trace over $\bar{R}$ by summing over the set of orthonormal states $\{|0\rangle_{\bar{R}}, |\vec{x}_1\rangle_{\bar{R}}, |\vec{x}_1, \vec{x}_2\rangle_{\bar{R}}, ..., |\vec{x}_1, ..., \vec{x}_n\rangle_{\bar{R}}\}$. Thus, for each component of the density matrix

$\rho_{nm}(t)$, we can find its contribution to the reduced density matrix for region R as follows:

$$\begin{aligned}\mathrm{tr}_{\bar{R}}\left(\hat{\rho}_{nm}(t)\right) = \frac{1}{\sqrt{n!}\sqrt{m!}}\sum_{k=0}^{n}\sum_{j=0}^{m}\int_R d^3\vec{x}_1...d^3\vec{x}_k\int_{\bar{R}} d^3\vec{x}_{k+1}...d^3\vec{x}_n\int_R d^3\vec{x}'_1...d^3\vec{x}'_j\int_{\bar{R}} d^3\vec{x}'_{j+1}...d^3\vec{x}'_m \\ \psi^{(n)}(\vec{x}_1,...,\vec{x}_n,t)\psi^{*(m)}(\vec{x}'_1,...,\vec{x}'_m,t) \\ |\vec{x}_1,...,\vec{x}_k\rangle_R\langle\vec{x}'_1,...,\vec{x}'_j|_R\langle\vec{x}_{k+1},...,\vec{x}_n|\vec{x}'_{j+1},...,\vec{x}'_m\rangle_{\bar{R}} \\ = \frac{1}{\sqrt{n!}\sqrt{m!}}\sum_{l=0}^{n\text{ or }m} l!\int_R d^3\vec{x}_1...d^3\vec{x}_{n-l}\int_{\bar{R}} d^3\vec{x}_{n-l+1}...d^3\vec{x}_n\int_R d^3\vec{x}'_1...d^3\vec{x}'_{m-l} \\ \psi^{(n)}(\vec{x}_1,...,\vec{x}_n,t)\psi^{*(m)}(\vec{x}'_1,...,\vec{x}'_{m-l},\vec{x}_{n-l+1},...,\vec{x}_n,t) \\ |\vec{x}_1,...,\vec{x}_{n-l}\rangle_R\langle\vec{x}'_1,...,\vec{x}'_{m-l}|_R\end{aligned} \tag{69}$$

where only the terms where there are the same number of particles outside R ($m-j=n-k$) survive from the first line to the second (due to orthogonality considerations). This count is denoted by l and summed from zero to n or m, whichever is smaller, in the second line. The factor of $l!$ is a combinatorial factor that arises because of the different ways that the particles outside of R might be paired up. In (69), we have assumed that each $\psi^{(n)}$ is symmetric under particle permutations (as must be the case for bosonic wave functions). Together, (68) and (69) give the general form of Newton-Wigner Fock states for regions of space at a time (something that we have not seen presented elsewhere).

To better understand (69) and to set up our discussion of locality, let us derive the reduced density matrix state for a region R when the global state is a general single-particle state (where the only contribution to (55) is $|\Psi^{(1)}(t)\rangle$):

$$|\Psi(t)\rangle = \int d^3\vec{x}\ \psi(\vec{x},t)\ \hat{a}^\dagger_{NW}(\vec{x})|\Omega_{NW}\rangle\ . \tag{70}$$

Applying (69) with $n=m=1$, the reduced density matrix for R at t is

$$\begin{aligned}\hat{\rho}^R(t) = &\int_R d^3\vec{x}\ d^3\vec{x}'\ \psi(\vec{x},t)\psi^*(\vec{x}',t)\ |\vec{x}\rangle_R\langle\vec{x}'|_R \\ &+\int_{\bar{R}} d^3\vec{x}\ \psi(\vec{x},t)\psi^*(\vec{x},t)\ |\Omega_{NW}\rangle_R\langle\Omega_{NW}|_R\ ,\end{aligned} \tag{71}$$

an expression that sums a contribution for the particle being within R and a contribution for it being outside of the region.

With states for regions in hand, we can now discuss relativistic locality. Fleming (2000, pg. 504) observes that "*NW* localized states have a superluminal contribution to their evolution,"[43]

[43]Other authors have also concluded that the Newton-Wigner approach leads to non-locality. Piazza and Costa (2008, pg. 4): "interactions are local in the standard localization scheme but not in the Newton-Wigner one." Falcone and Conti (2024, pg. 2): "The Newton–Wigner scheme in QFT predicts a phenomenon of superluminal spreading . . . that is in contrast with the relativistic notion of causality." (See also Fulling 1989, pg. 55; Pavšič 2018.)

citing the fact that (using Heisenberg picture operators)

$$\langle\Omega|\hat{a}_{NW}(\vec{x},t)\hat{a}^{\dagger}_{NW}(\vec{x}',t')|\Omega\rangle \neq 0 \tag{72}$$

even when $(\vec{x},t)$ and $(\vec{x}',t')$ are space-like separated, something that would not be true if these operators commuted at space-like separation. That is, the Newton-Wigner creation operators violate (43) and thus are not local operators.

We can readily see how (72) leads to a violation of relativistic locality from section 2. Revisiting figures 1 and 3, we will show that two global states can initially agree within R and later disagree within R^{-}, because initial differences in $\bar{R}$ can affect what later occurs within R^{-}, (70) can be expressed using Heisenberg picture creation operators as

$$|\Psi(t)\rangle = \int d^3\vec{x}\ \psi(\vec{x})\hat{a}^{\dagger}_{NW}(\vec{x},t)|\Omega\rangle\ . \tag{73}$$

Suppose that $\psi(\vec{x})$ is zero within R. Then, the initial state can be written as

$$|\Psi(0)\rangle = \int_{\bar{R}} d^3\vec{x}\ \psi(\vec{x})\ |\Omega_{NW}\rangle_R \otimes \hat{a}^{\dagger}_{NW}(\vec{x},0)|\Omega_{NW}\rangle_{\bar{R}}\ , \tag{74}$$

where $\hat{a}^{\dagger}_{NW}(\vec{x},0)$ acts only on the vacuum for $\bar{R}$ and not the vacuum for R. At a later time t,

$$|\Psi(t)\rangle \neq \int_{\bar{R}} d^3\vec{x}\ \psi(\vec{x})\ |\Omega_{NW}\rangle_{R^-} \otimes \hat{a}^{\dagger}_{NW}(\vec{x},t)|\Omega_{NW}\rangle_{\bar{R}^-}\ . \tag{75}$$

That expression is not correct because, for $\vec{x} \in \bar{R}$, $\hat{a}^{\dagger}_{NW}(\vec{x},t)|\Omega\rangle$ is not the product of the vacuum within R^{-} with some single-particle state confined to $\bar{R}^{-}$. (If it were, then inner products of the form $\langle\Omega|\hat{a}_{NW}(\vec{y},t)\hat{a}^{\dagger}_{NW}(\vec{x},0)|\Omega\rangle$ would always be zero for $\vec{x} \in \bar{R}$ and $\vec{y} \in R^{-}$, contra (72).) The fact that $\hat{a}^{\dagger}_{NW}(\vec{x},t)$ alters the state within R^{-} even when $\vec{x}$ is in $\bar{R}$ — the fact that $\hat{a}^{\dagger}_{NW}(\vec{x},t)$ is not a local operator — is what leads to the violation of relativistic locality. Returning to our standard for relativistic locality from section 2: two single particle states (73) that initially agree on $\psi(\vec{x})$ within R, and thus agree on the reduced density matrix within R (71), can later disagree on the reduced density matrix within R^{-} because differences in the factors $\psi(\vec{x})$ preceding $\hat{a}^{\dagger}_{NW}(\vec{x},t)|\Omega\rangle$ for $\vec{x}$ in $\bar{R}$ will lead to differences within R^{-}.

We have thus seen that a particle approach to QFT leads to non-locality if we use the Newton-Wigner creation operators. In section 4.4.1, we saw that the standard creation operators do not allow one to assess the locality of the dynamics because they do not yield a way of assigning states to regions. All of this illustrates a significant advantage of the field approach over a particle approach.[44] Only in terms of wave functionals, rather than particle states, do we see relativistic locality. Making use of the fact that the Heisenberg-picture field

[44] Some other advantages are enumerated in Sebens (2022).

operators are local operators that commute at space-like separation (unlike the standard or Newton-Wigner particle creation operators), one can show that the wave functional approach satisfies relativistic locality (section 4.2). It turns out that whether QFT is relativistically local or not depends on whether one uses particles or fields to articulate the fundamental laws and ontology of the theory.

5 Branching: Local or Global?

The many-worlds interpretation posits a universal quantum state (a wave function, wave functional, or density matrix[45]) that evolves under unitary dynamics (without collapse). The theory does not have additional postulates dividing this quantum state into worlds or saying precisely when one world branches into many. Worlds are non-fundamental entities that emerge (via decoherence) as convenient ways of carving the quantum state into approximately non-interacting pieces. This is why Wallace (2012) titled his book *The Emergent Multiverse*. Because worlds are merely an emergent higher-level description of the fundamental ontology, we do not see debates about how the wave function should be carved into worlds as challenging the locality of the many-worlds interpretation. We take the locality of the many-worlds interpretation to be established by studying the fundamental ontology and dynamics, as was done in section 4. In this section, we argue that the fundamental locality of the many-worlds interpretation is compatible with a non-local understanding of branching. Here we are responding to an objection that has been raised to the use of a global branching picture in the argument that the many-worlds interpretation gets the quantum probabilities right given by Carroll and Sebens (2014) and Sebens and Carroll (2018)—the Sebens-Carroll derivation of the Born rule.

Let us begin by considering the simple example of a Bohm-EPR setup where two far-separated particles are in a spin singlet state (51) and one is measured before the other. Before that measurement, we have the initial quantum state:

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}}|R\rangle_A|R\rangle_B|R\rangle_{D_1}|R\rangle_{D_2}\Big(|\uparrow_z\rangle_A|\downarrow_z\rangle_B - |\downarrow_z\rangle_A|\uparrow_z\rangle_B\Big)\,, \tag{76}$$

where A is Alice, B is Bob, and D_1 and D_2 are the measurement devices of A and B respectively. They are all in their ready states, denoted by R. The part in parentheses represents the pair of Alice's and Bob's entangled particles in the singlet state.

After Alice makes her z-spin measurement, but before A checks the measurement outcome,

[45] See Chua and Chen (2023).

the state evolves under unitary dynamics to:

$$
\begin{aligned}
|\Psi_f\rangle &= \frac{1}{\sqrt{2}}|R\rangle_A|R\rangle_B|R\rangle_{D_2}\Big(|\text{“}\uparrow\text{”}\rangle_{D_1}|\uparrow_z\rangle_A|\downarrow_z\rangle_B - |\text{“}\downarrow\text{”}\rangle_{D_1}|\downarrow_z\rangle_A|\uparrow_z\rangle_B\Big) \\
&= \frac{1}{\sqrt{2}}\Big(|R\rangle_A|R\rangle_B|R\rangle_{D_2}|\text{“}\uparrow\text{”}\rangle_{D_1}|\uparrow_z\rangle_A|\downarrow_z\rangle_B \\
&\quad - |R\rangle_A|R\rangle_B|R\rangle_{D_2}|\text{“}\downarrow\text{”}\rangle_{D_1}|\downarrow_z\rangle_A|\uparrow_z\rangle_B\Big)\,.
\end{aligned} \tag{77}
$$

We now have decoherence. The states of the macroscopic measuring device D_1 are entangled with the spin of particle A and $\langle\text{“}\downarrow\text{”}|\text{“}\uparrow\text{”}\rangle_{D_1} \approx 0$. Every Everettian agrees that, at this point, a branching of some kind has occurred. But, there is disagreement as to the extent of that branching. Wallace (2012, ch. 8) adopts a local branching picture.[46] He would say the measuring device D_1 has branched, but distant things, like Bob and his measuring device, have not. Alice's measurement locally splits the world and then branching spreads as other things become entangled with the outcome of that measurement (a spreading that happens quickly, but not faster than the speed of light). This is depicted in figure 4.

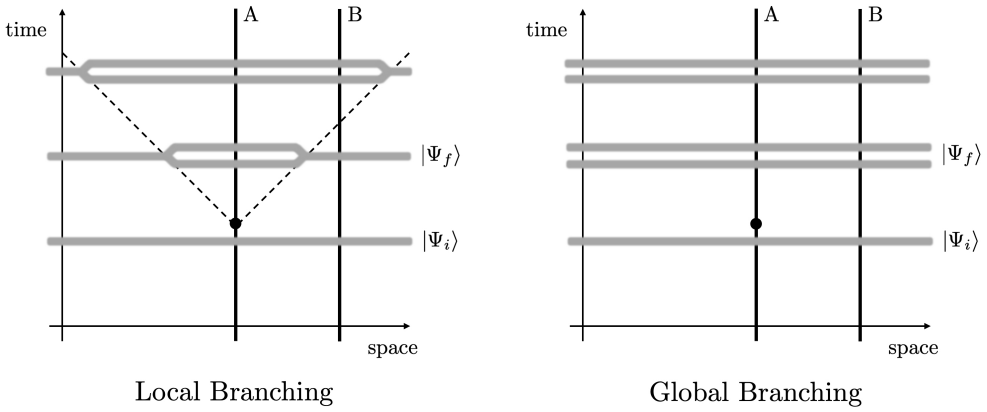

Figure 4: The left-hand image depicts Wallace's local branching where there is a local branching of worlds when Alice makes her measurement (the black dot) and that branching then spreads over time (eventually reaching Bob). This image is essentially figure 8.1 of Wallace (2012). The right-hand image depicts global branching, where the entire world branches when Alice makes her measurement (including Bob).

Wallace's local branching picture fits well with the first way of writing the state in (77), but the second line suggests a different way or understanding the transition that has occurred during Alice's measurement. According to the global branching picture in Sebens and Carroll (2018, sec. 2.2), Alice's measurement causes the entire world to branch. Branching is not something that gradually spreads out over time. It is instantaneous. Bob and his measuring

[46] See also Faglia (2024).

device split into (identical) copies when Alice makes her (distant) measurement.[47] Still, the reduced density matrix describing the physical state of Bob's laboratory (including him, his device, and his particle) has not been changed by the measurement. This is something that the local and global branching accounts can agree on, and it is what plays a central role in the proofs of fundamental locality.

There is much that could be said about the relative merits of the local and global branching accounts.[48] Perhaps the debate can be settled through consideration of these merits or perhaps we should allow both local and global branching as legitimate ways to give a non-fundamental description of the underlying quantum state evolution—arguing that there is not "one true answer" to the question of the scope of branching in the many-worlds interpretation.[49] In any case, our goal here is not to settle the debate between local and global branching. We only aim to defend global branching against the objection that instantaneous branching violates relativistic locality.

Here is how Kent (2015, pg. 214–215) puts the concern:

> "Is this weird pre-observation splitting of observers supposed to happen the instant a measurement interaction is completed? If so, with respect to which reference frame? And are we happy to postulate a story about an underlying objective reality that thus *not only breaks Lorentz symmetry but also proposes that measurement events superluminally cause observer splittings*?"

Along similar lines, McQueen and Vaidman (2019, pg. 9) argue that global branching

> "…goes against the spirit of the many worlds interpretation, which involves removing as much nonlocality as possible. Thus, after removing the nonlocality of collapse, they reinsert a different kind of nonlocality. The new nonlocality involves branching events happening arbitrarily far away triggering branching events here on Earth."

These authors raise a serious worry. As compared to Bohmian mechanics or spontaneous collapse theories, the many-worlds interpretation is the only option positing fundamental dynamics that satisfy relativistic locality. We would not want to lose that appealing feature of the approach. However, global branching does not threaten the fundamental locality of the many-worlds interpretation. It is merely an innocuous case of non-fundamental non-locality, similar to other examples in physics.

Newtonian gravity and classical electrostatics are both exemplary cases of successful non-fundamental physics and both are non-local. Newtonian gravity includes gravitational

[47] Ney (2024) gives a more detailed analysis of the debate between global and local branching. She also discusses a third option, semi-local branching, that she attributes to McQueen and Vaidman (2019). Gao (2024) gives a fourth option that he calls "nonlocal branching."

[48] Sebens and Carroll (2018, sec. 2.2) give only a quick argument in favor of global branching.

[49] The situation here can be compared to questions about the number of branches with a given outcome, where there is not a unique correct way to count branches (Wallace 2012, pg. 100.)

forces that take the form $F = G\frac{m_1 m_2}{r^2}$ where the force on one body depends on the masses and locations of other distant bodies at that moment. Similarly, in electrostatics the Coulomb force on one body depends on the charged and locations of other distant bodies at that moment. These theories are not fundamental, but they describe real forces with impressive accuracy in the appropriate regimes. Electrostatics is a non-local theory that emerges as an approximation to our paradigm case of a local theory: electromagnetism. Newtonian gravity is a non-local theory that emerges as an approximation to another paradigm case of a local theory: general relativity. What these two examples demonstrate is that it is perfectly fine to have non-fundamental non-locality emerge from underlying local physics. This is what we believe is happening when global branching is applied as a non-fundamental way to carve the quantum state into worlds. Because the fundamental dynamics are local, the many-worlds interpretation is local (even if branching is non-local).

With those general remarks made, let us address a particular way of putting the worry about non-locality and global branching along the lines of Kent, McQueen, and Vaidman: The Sebens-Carroll derivation of the Born rule (Sebens and Carroll 2018, sec. 2.2) assumes that after Alice's measurement (77) Bob can wonder whether he is the copy of himself on the branch where the result recorded on D_1 was up or the branch where the result is down. But, at the particular point in space and time where Bob is supposed to be doing that wondering, it may be a frame-dependent fact whether Alice has conducted her measurement or not. Is it really then legitimate for Bob to wonder in that way? We think that it could be. Bob is free to look at the situation from a frame in which he has branched and perform his non-fundamental carving of the universal quantum state into branches relative to that frame (in which case he has himself split and he can assign probabilities to being in different branches), or, to look at the situation from a frame in which he has not branched and perform his carving relative to that frame (in which case he has not split and cannot assign true self-locating probabilities to being in different branches).

Ney (2024), Ney (n.d., this volume) proposes a different way of defending the global branching picture from the charge of non-locality. She argues that, even at the emergent level where we are talking about people and worlds, there is no non-locality. She would say that Bob's change from being one person to two (or more) after Alice's measurement is not an intrinsic change in Bob himself (whose reduced density matrix remains unaffected by Alice's measurement) but merely an extrinsic change capturing Bob's relation to what is happening in Alice's laboratory. Ney compares this case to a wife becoming a widow when her husband dies at a distant location (an extrinsic change that is not a change in the wife herself). Because Alice's measurement does not cause an intrinsic change in Bob, there is no violation of relativistic locality here.[50]

Ney has made a clever move, but we find it odd to claim that Bob had not undergone an

[50] See also Faglia (2024, p. 48).

intrinsic change when he branches from one Bob into many. Adopting a global branching picture, we would prefer to say that: at the emergent level of worlds and people, Bob has changed (and that is a non-local effect of Alice's measurement), whereas, at the fundamental level, Bob's fundamental physical state has not changed and there is no non-locality. Here we are taking an option that Ney (2024) anticipates, arguing that "global branching is unobjectionable because it is ultimately just an emergent manifestation of a more fundamental ontology that is entirely local." Ney responds to this objection as it might be pursued within Carroll and Singh's "Mad Dog Everettianism,"[51] but in this article we have taken a different tack: building on Wallace and Timpson's spacetime state realism and assigning states to regions of space using either particle or field approaches to QFT (section 4).

6 Conclusion

In this paper we have argued that the unitary evolution of the quantum state in QFT without collapse (Everettian QFT) is local. We began by putting forward a single standard for relativistic locality that can be applied across different theories and then showed explicitly that electromagnetism, the Klein-Gordon equation, and the Dirac equation meet that standard. For QFT, one can debate whether it is better to take a field (wave functional) or a particle (Fock space) approach. Here we have focused our attention on one advantage of the field approach, showing (in a rough way that could be made more rigorous) that a field approach to QFT achieves relativistic locality whereas a particle approach to QFT is either non-local or fails to assign states to regions. Finally, we argued that local state evolution at the fundamental level is compatible with a non-local branching of worlds at the non-fundamental level. Throughout our discussion of different theories, we have focused on the evolution of the fundamental ontology and not on any interventions or measurements that might be conducted by observers. If the underlying dynamics are local, such interventions cannot violate that locality.

Acknowledgements

We would like to thank David Baker, Jacob Barandes, Alexander Blum, Francisco Calderón, Mario Hubert, Nick Huggett, Dustin Lazarovici, Alyssa Ney, Noel Swanson, Roderich Tumulka, and David Wallace for helpful comments and discussions.

[51] See Carroll and Singh (2019) and Carroll (2022).

References

Adlam, Emily. "Should Everettians Aspire to Locality?" In: *Everett and Locality (ed. Alyssa Ney)*. Forthcoming. Oxford University Press.

Baker, David John. "Against Field Interpretations of Quantum Field Theory". In: *The British Journal for the Philosophy of Science* 60.3 (2009), pp. 585–609.

Bohm, David and Basil J. Hiley. *The Undivided Universe: An ontological interpretation of quantum theory*. Routledge, 1993.

Calderón, Francisco. "The Causal Axioms of Algebraic Quantum Field Theory: A diagnostic". In: *Studies in History and Philosophy of Science* 104 (2024), pp. 98–108.

Carroll, S. M. "Reality as a Vector in Hilbert Space". In: *Quantum Mechanics and Fundamentality*. Springer, 2022, pp. 211–224.

Carroll, S. M. and Charles T. Sebens. "Many Worlds, the Born Rule, and Self-locating Uncertainty". In: *Quantum Theory: A Two-Time Success Story, Yakir Aharonov Festschrift*. 2014, pp. 157–169.

Carroll, S. M. and Ashmeet Singh. "Mad-Dog Everettianism: Quantum mechanics at its most minimal". In: *What is Fundamental?* Springer, 2019, pp. 95–104.

Chua, Eugene Y. S. and Eddy Keming Chen. *Decoherence, Branching, and the Born Rule in a Mixed-State Everettian Multiverse*. arXiv:2307.13218 [physics, physics:quant-ph]. 2023.

Clifton, Rob et al. "Superentangled states". In: *Physical Review A* 58.1 (July 1998). Publisher: American Physical Society, pp. 135–145.

Deckert, Dirk-André, Michael Esfeld, and Andrea Oldofredi. "A Persistent Particle Ontology for Quantum Field Theory in Terms of the Dirac Sea". In: *The British Journal for the Philosophy of Science* (Sept. 2019). Publisher: The University of Chicago Press.

Duncan, Anthony. *The Conceptual Framework of Quantum Field Theory*. Oxford University Press, 2012.

Dürr, Detlef, Sheldon Goldstein, et al. "On the Role of Density Matrices in Bohmian Mechanics". In: *Foundations of Physics* 35.3 (2005), pp. 449–467.

— "Trajectories and particle creation and annihilation in quantum field theory". In: *Journal of Physics A: Mathematical and General* 36.14 (2003), pp. 4143–4149.

Dürr, Detlef and Dustin Lazarovici. *Understanding Quantum Mechanics: The world according to modern quantum foundations*. Springer, 2020.

Earman, John and Giovanni Valente. "Relativistic Causality in Algebraic Quantum Field Theory". In: *International Studies in the Philosophy of Science* 28.1 (2014). Publisher: Taylor & Francis, pp. 1–48.

Evans, Lawrence C. *Partial Differential Equations*. American Mathematical Society, 1998.

Faglia, Paolo. "Non-Separability, Locality and Criteria of Reality: A Reply to Waegell and Mcqueen". In: *Studies in History and Philosophy of Science* 106.C (2024), pp. 43–53.

Falcone, Riccardo and Claudio Conti. “Localization in quantum field theory”. In: *Reviews in Physics* 12 (2024), p. 100095.

Fleming, Gordon. “Reeh-Schlieder Meets Newton-Wigner”. In: *Philosophy of Science* 67.3 (2000). Publisher: University of Chicago Press, S495–S515.

Fleming, Gordon and Jeremy Butterfield. “Strange positions”. In: *From Physics to Philosophy*. Ed. by Constantine Pagonis and Jeremy Butterfield. Cambridge: Cambridge University Press, 1999, pp. 108–165.

Folland, Gerald B. *Introduction to Partial Differential Equations*. 2nd. Princeton University Press, 1995.

Fraser, Doreen. “Particles in Quantum Field Theory”. In: *The Routledge Companion to Philosophy of Physics*. Ed. by Eleanor Knox and Alastair Wilson. Routledge, 2022, pp. 323–336.

— “The Fate of ‘Particles’ in Quantum Field Theories with Interactions”. In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 39.4 (2008). Publisher: Elsevier, pp. 841–859.

Fulling, Stephen A. *Aspects of Quantum Field Theory in Curved Spacetime*. London Mathematical Society Student Texts. Cambridge: Cambridge University Press, 1989.

Gao, Shan. *Understanding Branching in the Many-worlds Interpretation of Quantum Mechanics*. Preprint. July 2024.

Greiner, Walter. *Relativistic Quantum Mechanics: Wave equations*. 3rd. Springer, 2000.

Greiner, Walter and Joachim Reinhardt. *Field Quantization*. Springer-Verlag, 1996.

Halvorson, Hans. “Reeh-Schlieder Defeats Newton-Wigner: On Alternative Localization Schemes in Relativistic Quantum Field Theory”. In: *Philosophy of Science* 68.1 (2001). Publisher: University of Chicago Press, pp. 111–133.

Hatfield, Brian. *Quantum Field Theory of Point Particles and Strings*. Perseus Books, 1992.

Healey, Richard A. “Holism and Nonseparability”. In: *The Journal of Philosophy* 88.8 (1991). Publisher: Journal of Philosophy, Inc., pp. 393–421.

Henley, Ernest M. and Walter Thirring. *Elementary Quantum Field Theory*. McGraw Hill Book Company Inc., 1962.

Holzhey, Christoph, Finn Larsen, and Frank Wilczek. “Geometric and renormalized entropy in conformal field theory”. In: *Nuclear Physics B* 424.3 (1994), pp. 443–467.

Huang, Kerson. *Quantum Field Theory: From Operators to Path Integrals*. Wiley, 1998.

Jackiw, Roman. “Schrödinger Picture Analysis of Boson and Fermion Quantum Field Theories.” In: *Mathematical Quantum Field Theory*. Canadian Mathematical Society, 1987, pp. 1–27.

Kent, Adrian. “Does it Make Sense to Speak of Self-Locating Uncertainty in the Universal Wave Function? Remarks on Sebens and Carroll”. In: *Foundations of Physics* 45.2 (2015), pp. 211–217.

Lange, Marc. *An Introduction to the Philosophy of Physics: Locality, Fields, Energy, and Mass*. Blackwell, 2002.

Malament, David. “In Defense of Dogma: Why There Cannot Be a Relativistic Quantum Mechanical Theory of (Localizable) Particles”. In: *Perspectives on Quantum Reality*. Ed. by R. Clifton. Kluwer Academic Publishers, 1996, pp. 1–10.

Maudlin, Tim. “What Bell did”. In: *Journal of Physics A: Mathematical and Theoretical* 47.42 (2014), p. 424010.

McQueen, Kelvin J. and Lev Vaidman. “In Defence of the Self-Location Uncertainty Account of Probability in the Many-Worlds Interpretation”. In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 66.C (2019), pp. 14–23.

Myrvold, Wayne C. “Lessons of Bell’s Theorem: Nonlocality, yes; Action at a distance, not necessarily.” In: *Quantum Nonlocality and Reality – 50 Years of Bell’s Theorem*. Ed. by Shan Gao and Mary Bell. 2015.

Ney, Alyssa. “Branching Everywhere and All at Once”. In: *Everett and Locality (ed. Alyssa Ney)*. Forthcoming. Oxford University Press.

— “The Argument From Locality for Many Worlds Quantum Mechanics”. In: *Journal of Philosophy* (). Forthcoming.

Norsen. *Foundations of Quantum Mechanics: An exploration of the physical meaning of quantum theory*. Springer, 2017.

Oldofredi, Andrea. “Particle Creation and Annihilation: Two Bohmian Approaches”. In: *Lato Sensu: Revue de la Société de Philosophie des Sciences* 5.1 (2018). Publisher: Société de Philosophie des Sciences, pp. 77–85.

Pavšič, Matej. “Localized States in Quantum Field Theory”. In: *Advances in Applied Clifford Algebras* 28.5 (Nov. 2018), p. 89.

Peskin, Michael E. and Daniel V. Schroeder. *An Introduction to Quantum Field Theory*. Perseus Books, 1995.

Piazza, Federico and Fabio Costa. “Volumes of Space as Subsystems”. In: *Proceedings of From Quantum to Emergent Gravity: Theory and Phenomenology*. Trieste, Italy: Proceedings of Science, 2008.

Redhead, Michael. “The Vacuum in Relativistic Quantum Field Theory”. In: *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* 1994 (1994). Publisher: University of Chicago Press, pp. 77–87.

Schwartz, Matthew Dean. *Quantum Field Theory and the Standard Model*. New York: Cambridge University Press, 2014.

Schweber, Silvan S. *An Introduction to Relativistic Quantum Field Theory*. Harper & Row, 1961.

Sebens, Charles T. “The Fundamentality of Fields”. In: *Synthese* 200.5 (2022), pp. 1–28.

Sebens, Charles T. and S. M. Carroll. “Self-locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics”. In: *The British Journal for the Philosophy of Science* 69.1 (2018). Publisher: The University of Chicago Press, pp. 25–74.

Strauss, Walter A. *Partial Differential Equations: An Introduction*. John Wiley & Sons, 2008.

Struyve, Ward. “Pilot-Wave Theory and Quantum Fields”. In: *Reports on Progress in Physics* 73.10 (2010), p. 106001.

Susskind, Leonard and James Lindesay. *An Introduction to Black Holes, Information and the String Theory Revolution: The holographic universe*. Hackensack, NJ: World Scientific, 2005.

Swanson, Noel. “How to Be a Relativistic Spacetime State Realist”. In: *The British Journal for the Philosophy of Science* 71.3 (2020), pp. 933–957.

Thaller, Bernd. *The Dirac Equation*. Berlin, Heidelberg: Springer, 1992.

Tong, David. *Lectures on Quantum Field Theory*. 2007.

Tumulka, Roderich. *Foundations of Quantum Mechanics*. Springer International Publishing, 2022.

Vaidman, Lev. “The Many-Worlds Interpretation of Quantum Mechanics is the Only Way to Avoid Action at a Distance”. In: *Everett and Locality (ed. Alyssa Ney)*. Forthcoming. Oxford University Press.

Wald, Robert M. *Advanced Classical Electromagnetism*. Princeton University Press, 2022.

— *General Relativity*. University of Chicago Press, 1984.

Wallace, David. *The Emergent Multiverse: Quantum Theory according to the Everett Interpretation*. Oxford University Press, 2012.

— “The Quantum Theory of Fields”. In: *The Routledge Companion to Philosophy of Physics*. Ed. by Eleanor Knox and Alastair Wilson. Routledge, 2022.

Wallace, David and Christopher Gordon Timpson. “Quantum Mechanics on Spacetime I: Spacetime State Realism”. In: *British Journal for the Philosophy of Science* 61.4 (2010). Publisher: Oxford University Press, pp. 697–727.

Weinberg, Steven. *The Quantum Theory of Fields. 1: Foundations*. Cambridge: Cambridge University Press, 2005.

Wharton, Ken B. and Nathan Argaman. “Colloquium: Bell’s theorem and locally mediated reformulations of quantum mechanics”. In: *Reviews of Modern Physics* 92.2 (May 2020), p. 021002.

Williams, Porter, John Dougherty, and Michael E. Miller. “Cluster Decomposition and Two Senses of Isolability”. In: *Philosophy of Physics* 2.1 (July 2024), p. 12.

Zachmanoglou, E. C. and Dale W. Thoe. *Introduction to Partial Differential Equations with Applications*. Williams & Wilkins, 1976.