Can a time varying but spatially uniform electric scalar potential shift the energy levels of a quantum system? A critical look at the new Aharonov-Bohm effect proposed by Chiao et al

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

January 7, 2025

Abstract

Recently Chiao and his collaborators proposed a new version of the electric Aharonov-Bohm effect [Phys. Rev. A 107, 042209 (2023)]. They argued that a quantum system confined in a Faraday cage with a time varying but spatially uniform electric scalar potential can pick up the Aharonov-Bohm phase, and the observable consequence is the energy level shift of the quantum system. In this paper, I argue that Chiao et al's analysis is problematic, and a time varying, spatially uniform electric scalar potential cannot result in observable energy level shift of quantum systems. A possible explanation of this seemingly puzzling result is also given based on the one true gauge principle.

The Aharonov-Bohm effect [1] (AB effect hereafter) is a quantum mechanical effect in which a charged particle confined to a region without electric and magnetic field is affected by the potentials. There are two forms of the AB effect, the magnetic AB effect and the electric AB effect. The magnetic AB effect occurs when the interferometer paths enclose a magnetic flux, and the electric AB effect occurs when there is an electric potential difference between the interferometer paths, and both effects involve a shift of the interference pattern. Recently Chiao et al proposed a novel version of the electric AB effect without using interferometer [2]. They argued that a quantum system confined in a Faraday cage with a time varying, spatially uniform scalar potential can also pick up the AB phase, and the observable consequence is the energy level shift of the quantum system. Since several further studies have been conducted based on this interesting propsal [3-7], including a proposal for a new gravitational AB effect [3], a careful examination of its validity is necessary. In this paper, I will argue that Chiao et al's analysis is problematic, and a time varying but spatially uniform scalar potential cannot result in observable energy level shift of quantum systems, and thus their proposal is not a version of the electric AB effect.

In Chiao et al's propsal [2], the basic set-up consists of a Faraday shell with a time varying voltage on its surface. Inside the Faraday shell, the **E**-field is zero, and there is only a time-varying, spatially uniform scalar potential $V(t) = V_0 \cos(\Omega t)$, where $\frac{\Omega}{2\pi}$ is the frequency and V_0 is the amplitude. The quantum system used to register the effect of this $V(t)$ is a gas of hydrogen-like atoms inside the Faraday shell such as rubidium gas. According to Chiao et al's analysis, the time varying, spatial uniform potential, $V(t)$, will split the energy levels of the quantum system into a series of energy levels, and the observable energy level shift can be used to probe the scalar electric AB effect. Let's see whether this result is valid.

As already pointed out by Chiao et al [2], we can use two gauges for calculation for the above setup. The first gauge is $V(t) = 0$ and $\mathbf{A} = 0$ inside the Faraday shell. The second gauge is $V'(t) = V_0 \cos(\Omega t)$ and $\mathbf{A}' = 0$ inside the Faraday shell (for $t \geq 0$). They are related with each other by a gauge transformation:

$$V' = V - \partial_t \lambda \quad \text{and} \quad \mathbf{A}' = \mathbf{A} + \nabla \lambda \ , \tag{1}$$

where the gauge function $\lambda(t) = -\frac{V_0}{\Omega} \sin(\Omega t)$. Note that the notation for the scalar potentials in the two gauges here is different from that given in Chiao et al's paper.

In the first gauge, the Hamiltonian of the quantum system is the same as its initial Hamiltonian $H = H_0$, for which the solutions to the time-independent Schrödinger equation are known, i.e. $H_0 \Psi_i(\mathbf{r}) = E_i \Psi_i(\mathbf{r})$. In the second gauge, the Hamiltonian of the quantum system will be $H' = H_0 + eV(t)$. By the gauge transformation of the wave function, the corrsponding solutions of the time-dependent Schrödinger equation will be

$$\psi'_i(\mathbf{r}, t) \ = \ \exp\left(i\frac{e}{\hbar}\lambda\right) \psi_i(\mathbf{r}, t) = \Psi_i(\mathbf{r}) \exp\left(-i\frac{E_i t}{\hbar} + i\frac{e}{\hbar}\lambda\right) \ . \tag{2}$$

This means that for a time-dependent Hamiltonian the quantum system has different energy spectrum in different gauges, and the energy eigenvalues are not gauge invariant (see also [7]).

The gauge invariant quantity related to the energy of the quantum system is $-(\partial_t S + eV) = -(\partial_t S' + eV') = E_i$ for the energy eigenstate $\psi_i(\mathbf{r}, t)$ or $\psi'_i(\mathbf{r}, t)$, where S and S' are the phases of the corresponding wave functions multiplied by $\hbar$. In other words, the energy spectrum of the quantum system that one can measure is always E_i without shift, independently of the choice of gauge. In fact, gauge invariant quantities are not changed by a gauge transformation by definition. Since the adding of a time varying, spatially uniform scalar potential is equivalent to a gauge transformation, it cannot result in the change of any gauge invariant quantity.

However, it seems that there is still a puzzle that needs to be answered. When a time varying voltage is added to the Faraday shell, there will be both a time varying $E(t)$-field and time varying scalar potential $V(t)$ outside the shell. Moreover, there will be also a time-varying scalar potential $V(t)$ inside the shell, although the E-field will be zero there. Then, it is natural to expect that something inside the Faraday shell must be also changed when a time varying voltage is added to the shell. This might be the main reason why Chiao et al thought that the quantum system inside the shell such as its energy levels should be affected by the added voltage. But if the physical reality is required to be gauge invariant, then nothing physical inside the Faraday shell is changed by the added voltage, as the above analysis demonstrates.

A possible way to solve this puzzle is to assume that there is one true gauge in which the potential represents the actual physical state, although it cannot be measured.[1] When a time varying voltage is not added to the Faraday shell, the true gauge potential inside the shell is $V(t) = 0$ and $\mathbf{A} = 0$. While a time varying voltage is added to the Faraday shell, the true gauge potential inside the shell is $V'(t) = V_0 \cos(\Omega t)$ and $\mathbf{A}' = 0$. Then, the adding of the time varying voltage indeed results in the change of the physical state inside the shell, which is represented by the potential in the true gauge. However, due to the minimal coupling rule and the local gauge invariance of laws of motion, neither the potential nor the wave function alone is gauge invariant and measurable, and only certain combining properties of the quantum system and the potential (besides the probability density) are gauge invariant and can be measured, such as $\nabla S - e\mathbf{A}$ and $\partial_t S + eV$.

To sum up, I have argued that a time varying but spatially uniform scalar potential cannot result in observable energy level shift of quantum systems, and thus Chiao et al's proposal of a novel version of the electric AB effect is not valid. This result applies not only to the electric potential, but also to the gravitational potential (cf. [3]).

References

[1] Y. Aharonov and D. Bohm, Phys. Rev. 115, 484 (1959).

[2] R. Y. Chiao, H. Hart, M. Scheibner, J. Sharping, N. A. Inan, D. A. Singleton, and M. E. Tobar, Phys. Rev. A 107, 042209 (2023).

---

[1] This assumption is often called the one true gauge principle. In my view, it is the only sensible way to understand gauge potentials [8].

[3] R. Y. Chiao, N. A. Inan, M. Scheibner, J. Sharping, D. A. Singleton, and M. E. Tobar, Phys. Rev. D 109, 064073 (2024).

[4] M. E. Tobar, M T. Hatzon, G. R. Flower and M. G. Goryache. Appl. Phys. Lett. 125, 094002 (2024).

[5] R. Y. Chiao, N. A. Inan, M. Scheibner, J. Sharping, D. A. Singleton, and M. E. Tobar, J. Phys.: Conf. Ser. 2889, 012062 (2024).

[6] R. Y. Chiao, N. A. Inan, D. A. Singleton, and M. E. Tobar, Int. J. Mod. Phys. D 2441021 (2024).

[7] C. Cohen-Tannoudji C., B. Diu and F. Laloe. Quantum mechanics. Volume I (1977, Wiley). p.236.

[8] S. Gao. Explaining the Aharonov-Bohm Effect: From Unitary Gauge to One True Gauge (2025). In preparation.