Relational Quantum Mechanics Does Not Resolve the Problem of Measurement

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Abstract

Relational quantum mechanics (RQM) explains the world in terms of an ontology of systems and events, where an event consists of a variable of a system taking a value relative to another system. Two strands of RQM may be distinguished depending on whether events are taken to be absolute or relative. The arguments in this paper apply to both. I argue that, in order to solve the problem of measurement, RQM needs to offer a specification of the circumstances in which events occur. Current formulations of RQM claim that events occur whenever interactions occur, without further defining what is meant by 'interaction'. I develop the most plausible ways of understanding the notion of interaction, but I show that they fail to provide a satisfactory specification for the occurrence of events. In light of these failed constructive efforts, I conclude that the prospects for formulating a version of RQM which both satisfies its aims and solves the problem of measurement are dim.

Key words: Relational Quantum Mechanics, Quantum Mechanics, Measurement Problem.

1 Introduction

It's been almost 30 years since the Relational interpretation of Quantum Mechanics (RQM) was introduced by Carlo Rovelli (Rovelli, 1996) and, over time, the interpretation has changed and developed significantly. Still, I claim that current formulations of RQM do not solve the problem of measurement. Moreover, I argue that the prospects of articulating a version of RQM which both meets its own goals and solves the measurement problem are not promising.

In brief, the problem is the following. RQM explains the world in terms of an ontology of systems and events, where an event consists of a variable of a system taking a value relative to another system. In order to solve the problem of measurement, RQM needs to offer a specification of the circumstances in which events occur. Current formulations of RQM claim that events occur whenever interactions occur, without further defining the notion of interaction. I develop the most plausible ways of understanding the notion of interaction in RQM, but I show that they fail to provide a satisfactory specification for the occurrence of events. In light of the failed constructive efforts, I conclude that the prospects for formulating a version of RQM which both satisfies its aims and solves the problem of measurement are dim.

The paper proceeds as follows. I begin in section 2 by listing RQM's key goals, which provide useful boundaries on how the theory may be clarified and developed. The discussion then bifurcates. I first focus on the strand of RQM according to which the occurrence of events is taken to be absolute (Adlam and Rovelli, 2023), which I call ARQM. I outline ARQM in section 3. I then argue that ARQM faces an unresolved problem of measurement in section 4, due to the unclarity concerning the notion of interaction. The absence of a clear characterisation of the notion interaction in the primary literature suggests that the supporters of RQM hope to appeal to the notion of interaction standardly used in quantum theory. Thus in section 5 I attempt to apply such standard notion of interaction to the context of ARQM. Unfortunately, these efforts fail. Incidentally, in this process I also clarify the nature of the dynamics in RQM, uncovering some open questions on the topic. In section 6, I consider the idea that interactions might be defined in terms of correlations or entanglement, but this proposal also faces important objections. Finally, in section 7 I turn to the strand of RQM in which events themselves are taken to be relative, which I call RRQM. I quickly show that the arguments developed in the context of ARQM easily generalise to RRQM, but, sadly, I show that RRQM provides no better solutions.

2 The aims of RQM

First, it's helpful to list the aims of RQM, as they offer useful bounds on how the theory may be made precise and developed. Ultimately, Rovelli and his collaborators hope to offer an interpretation with the following features (Adlam and Rovelli, 2023 p.2, Laudisa and Rovelli, 2019, introduction and section 1.2):

1. RQM gives no special significance to agents, measurements or minds.

2. RQM does not assume a classical/quantum divide.

3. RQM does not require one to modify or add anything to the orthodox mathematical framework of QM.

4. RQM does not posit any hidden variables.

5. RQM is a single-world theory.

6. RQM is compatible with the theory of relativity.

7. RQM is applicable in the context of relativistic QM, quantum field theory and quantum gravity.

The expectations are high, let us see how Rovelli and collaborators intend to meet them.

3 ARQM: relational quantum mechanics with absolute events

3.1 The ontology of ARQM

The world as described by RQM is constituted by two basic elements: *systems* and *events*.[1] Each system is characterised by an algebra of physical quantities (Rovelli, 2021, p.1, 2022, p.1057, Adlam and Rovelli, 2023, p.12). *Whenever* two systems interact, and *only when*[2] they interact, one or more of the quantities of each of the interacting systems takes on a value. A system's quantity taking on a value at an interaction is called an '*event*'. At all other times, quantities of systems do not have determinate values (Adlam and Rovelli, 2023, p.11-12, Rovelli 2022, p.1066).

RQM's key idea is that *events are relative* to the systems involved in the interaction. The scope of the relativity has been subject of controversy and, as already noted, it drives a wedge between two strands of RQM. In this section I will focus on the strand of RQM which takes the *values* obtained at an event to be relative, but the occurrence itself of the event as absolute (Adlam and Rovelli, 2023). I will call this view ARQM.[3] According to ARQM, whenever two systems F and S interact, a quantity $\mathcal{V}$ of S takes a value v

[1] See, for instance, Rovelli (2022, p. 1057). I do not address questions of relative fundamentality and emergence between events and system (Adlam and Rovelli, 2023, p. 12).

[2] 'values have variables only during quantum events.' (Adlam and Rovelli, 2023, p.11, *sic*). This clearly intended to state that *variables* have *values* only during quantum events.

[3] Absolute RQM.

relative to F and a quantity $\mathcal{V}'$ of F takes a value v' *relative to* S.[4] I will denote an interaction between two systems S and F with $S - F$ and I will denote the resulting event in which S's quantity $\mathcal{V}$ takes a certain value v relative to F as $e_S^{(F)}(\mathcal{V})$ or $e_S^{(F)}(\mathcal{V} = v)$.

I will not be concerned with the exact *nature* of the relativity involved in ARQM. I will only assume the following, which clearly follows from the literature:

Event-Experience Link - ARQM: *A conscious observer O can have a first-person experience of a quantity $\mathcal{V}$ of a system S taking a certain value only if the quantity $\mathcal{V}$ takes a value relative to the observer*[5] *(i.e. only if the event $e_S^{(O)}(\mathcal{V})$ occurs).*

After all, if no event $e_S^{(O)}(\mathcal{V})$ occurs, the variable $\mathcal{V}$ does not even have a definite value relative to O, so O certainly cannot have a first-person experience of a value of $\mathcal{V}$.[6]

Now that the basic ontology of ARQM has been laid out, it's time to see how the formalism of quantum theory relates to it.

3.2 Quantum theory:

According to RQM, the formalism of quantum theory serves to offer probabilistic predictions regarding the occurrence of events and it is applied as follows.

Each system is assigned an algebra of operators which represent the physical quantities of the system and whose eigenvalues define the possible values that the quantities may take (Rovelli, 2022, p.1062). Moreover, systems are assigned quantum states *relative to other systems*: given the set of events, a system S may be assigned a quantum state relative to F, provided that there are events of the right kind in which a quantity of S has taken a value relative to F. This condition will be further specified below. From the relativity of events it follows straightforwardly that quantum states are also relative and, in general, one system will have different quantum states relative to different systems. I will denote the quantum state of a system S relative to a system

[4]'a quantum event arises in an interaction between two systems in which the variables of one system take on definite values relative to the other, *and vice versa*.' (Adlam and Rovelli, 2023, p.11, emphasis mine).

[5]Or possibly a subsystem of O. This variation would not affect my arguments, but I will set it aside to avoid further complicating the arguments.

[6]"A quantum event arises in an interaction between two systems such that the values of some physical variables of one system *become definite* relative to another system" (Adlam and Rovelli, 2023, p.2, emphasis mine).

F as $|\psi\rangle_S^{(F)}$. In accordance with the goals stated in section 2, quantum states are not assigned relative to conscious observers only, rather they are assigned relative to any system.

Rovelli stresses that the *quantum state does not represent* the system or the world, rather it is only a useful mathematical tool which allows one to extract probabilities from a collection of relative events (Rovelli, 2018). To make this precise, we may say that the quantum state does not represent the categorical, occurrent properties of a system, although, it may well encode some modal properties of the system it is assigned to, concerning probabilities of events involving the system itself (see the Relative Born Rule below).

In RQM, the evolution of relative quantum states essentially follows "textbook" quantum mechanics, but (relative) collapse occurs at relative events, rather than at "measurement". More precisely, the evolution of the quantum state follows two rules. Consider two systems S and F such that S has a pure quantum state $|\psi(t)\rangle_S^{(F)}$ relative to F. $|\psi(t)\rangle_S^{(F)}$ evolves unitarily according to the Hamiltonian as long as S and F do not interact (Rovelli 2021, p.5). The status of the Hamiltonian in RQM is an unresolved matter, which I will address in section 5.2.1. On the other hand, at any interaction resulting in an event $e_S^{(F)}(\mathcal{V} = v)$, the relative quantum state collapses to the relevant eigenstate $|\psi\rangle_S^{(F)} \rightarrow \frac{\Pi_v |\psi\rangle_S^{(F)}}{|\Pi_v |\psi\rangle_S^{(F)}|}$, where Π_v is the projector associated with the value v of the quantity $\mathcal{V}$.[7]

These rules for the evolution of the quantum state indicate that systems do not necessarily have a quantum state relative to all systems. A system S has a quantum state relative to a system F, only if there has been an interaction $S - F$ resulting in an event $e_S^{(F)}(\mathcal{V} = v)$ or if there has been an interaction $R - F$ resulting in an event $e_R^{(F)}(\mathcal{Q} = q)$, where S is a subsystem of R.[8] For simplicity, I will assume that all systems have a quantum state relative to all *other* systems. But note that since events always involve *two distinct* interacting systems, one of which the event is relative to, while the other takes on a value of a certain variable, *there is no self- assignment of quantum states*.[9] In other words, there are no quantum states of the form

[7]'[T]here is collapse in each observer-dependent evolution of probabilities.' (Rovelli, 1996, p.1672). See also, Di Biagio and Rovelli (2022, p.5).

[8]For example, suppose $R = S \cup S'$. Then $e_R^{(F)}(\mathcal{Q} = q)$ defines $\rho_R^{(F)}$, from which $\rho_S^{(F)} = Tr_{S'}(\rho_R^{(F)})$.

[9]The rejection of self-ascription is also implied in the following inference by Adlam and Rovelli: there is no quantum state of the universe because "quantum states are by definition relational, and there is nothing for the quantum state of the whole universe to be relativized to" (Adlam and Rovelli, 2023, p.12). See also Rovelli (1996, p.1672, 2021, pp.4-5).

$|\psi\rangle_S^{(S)}$.

As mentioned above, quantum states are mathematical devices used to derive probabilities of the outcomes of interactions. Probabilities in RQM are derived according to the standard Born Rule (Rovelli, 2022, pp.1057-8, Adlam and Rovelli, 2023, p. 11, 15-16, Di Biagio and Rovelli, 2021, p.30) understood within the context of relative quantum states:

Relative Born Rule - ARQM: *At an interaction between two systems F and S (i.e. $F - S$), the probability relative to F for a quantity $\mathcal{V}$ of a system S to take on the value v relative to F is given by the Born Rule on the quantum state of S relative to F.*

It's worth noting that from the relativity of quantum states it follows that probabilities in ARQM are also relative to systems.

Thus far, I have laid out the basic and uncontroversial elements of the framework of ARQM. However, the attentive reader will note a gaping hole in the account. Given an interaction where a quantity $\mathcal{V}$ of a system F obtains a definite value relative to the system S, we now know how to use the quantum state to derive the probabilities for the possible values of such a quantity. However, it is not yet clear how the theory predicts the circumstances in which interactions occur and which event occurs (i.e. what quantity becomes determinate) in each interaction. This gives rise to RQM's measurement problem, that I expound in the next section.

4 Events, interactions and the problem of measurement

Any plausible interpretation of quantum mechanics must recover all of its successful predictions. For example, suppose a scientist F is performing a spin measurement on a system S, using a measuring apparatus A. Using (orthodox) quantum theory, the scientist is able to predict a set of possible outcomes of the experiment and the probabilities assigned to each outcome. They also predict that they will have a first-person experience of one of the possible outcomes. RQM needs to recover these predictions.

In practice, the scientist might appeal to vague notions such as measurement or a classical/quantum divide. If RQM is to satisfy its own desiderata (see section 2), RQM must be able to recover the scientist's predictions without appeal to such notions. Given the Event-Experience Link - ARQM, accounting for such a prediction involves, at the very least, the prediction of the occurrence of an event $e_A^{(F)}(X)$ in which (relative to the scientist) a certain quantity of the apparatus (e.g. the position X of a pointer) takes a value relative to the scientist. Moreover, ARQM must specify the correct probabilities for each value x_i of X, namely it must specify probabilities for each $e_A^{(F)}(X = x_i)$.[10]

This is the basic requirement. But more is required of ARQM. According to ARQM, events occur not only in experimental situations, but also in all kinds of situations and between all kinds of systems. To justify this claim, the same principles which indicate the occurrence of events in experimental situations must also indicate the occurrence of events in other situations. However, the requirement on ARQM outside of experimental situations is more lax, since ARQM only needs to *justify* the claim that there are events, rather than exactly predict which events occur. It is perfectly acceptable for ARQM to offer only approximate answers in certain regimes.

Hence, ARQM is required to provide a specification of the circumstances for the occurrence of events and for which variable becomes determined in each event, such that, (i) in all experimental applications of quantum theory, it unambiguously predicts the events relevant to the explanation of quantum mechanical predictions and with the appropriate probabilities and (ii) it justifies the claim that events occur between all kinds of systems and outside of experimental contexts. I call the challenge of offering such a specification ARQM's measurement problem, given the similarity between this requirement and Bell's (1990) request for a precise determination of when the Schrödinger evolution is interrupted by collapse.

Prima facie, ARQM offers such a specification: an event occurs between S and F when and only when an interaction $F - S$ occurs. However this only shifts the burden onto the notion of interaction: ARQM is required to provide a specification of the circumstances for the occurrence of interactions which satisfies the conditions just detailed for events. Given the similar requirements, in what follows I might slip between taking the requirement to be about events or interactions.

On the one hand, this problem has not gone completely unnoticed in the literature. Healey (2022, pp.6-7) and Muciño et al. (2022, pp.10-11) raise worries concerning the unclarity of the notion of events. However, they seem mostly concerned with the preferred basis problem and with a requirement to specify the *time* for the occurrence of interactions. Muciño et al. say:

> [RQM] needs for there to be a well-defined moment at which each interaction takes place; otherwise, the proposal becomes vague

[10] Other events will surely be involved in a proper account of the measurement (see, for example, Di Biagio and Rovell (2021, pp.5-6)), but I can set those complications aside for the purposes of my argument.

and loses all strength. (Muciño et al., 2022, p.10)

Adlam and Rovelli reply to this with the claim that 'RQM does not need to insist that events occur at well-defined spacetime locations' (Adlam and Rovelli, 2022, p.17), because one may hold that 'spacetime should be understood to emerge from a background of quantum events' (*ibid*). Note that my requirements above do not ask for an exact time of the occurrence of an interaction. However, it's worth noting that ARQM will have to recover predictions for successive spin measurements, thus it will need to recover at least some facts about the *order* of events.

On the other hand, Oldofredi (2023) claims that RQM "dissolves" the measurement problem. Oldofredi explains that "since in Rovelli's theory ψ is not considered a real object but rather a mere computational tool, nothing physical is literally collapsing in measurement interactions" (*ibid.* p.7). Instead, collapse is just "an information update relative to a certain agent" (*ibid.* p.7). Moreover, he explains that "the exact details about the mechanisms causing the suspension of the unitary dynamics cannot be available in RQM" (*ibid.* p.7), due to a limit of the descriptive capabilities of RQM. For these reasons he claims that in RQM "the collapse postulate does not generate conceptual conundra" (*ibid.* p.7).

As it is clear from the arguments offered above, Oldofredi misses the key issue with RQM's collapse postulate. He is firstly wrong in claiming that collapse is just "an information update relative to a certain agent" (*ibid.* p.7) for the simple fact that quantum states hold relative to any system, not just agents. The collapse of the quantum state does not represent an update in an agent's knowledge about a system, rather it represents a change in an *objective* relation between two systems, since the quantum state is *objectively* determined by the occurrence of relative events. But more importantly, even if the quantum state does not represent a concrete object (or the occurrent categorical properties of an object), the need to specify the circumstances for the occurrence of events is not dispelled, and thus, by proxy, the need for specifying the circumstances in which relative quantum states collapse is not dispelled either.

5 The standard notion of interaction

The primary literature does not offer a detailed discussion of the circumstances in which events occur. In Rovelli's early writings on RQM (Rovelli, 1996, 1997) there are suggestions that correlations in the quantum state of two systems relative to a third system may be the mark of the occurrence of events. However, these suggestions have been recently denied:

> (i) a variable of S has a value with respect to F, and (ii) with respect to W, there is a correlation to be expected between a variable of S and a pointer variable of F. The first implies the second, but the second does not imply the first. (Di Biagio and Rovelli, 2022, p.8).

For completeness I will consider these ideas linking correlations and interactions in section 6. For now, I will instead consider recent suggestions that the key to interactions lies in the dynamics:

> [The happening of events] is partially reflected in the state of the composite system relative to a third system. But only partially. Events cannot be read out of the state. The existence of a correlation between two variables gives indications about events, but in general it is not sufficient to tell which event was or was not realised. To know what event lead to the creation of a correlation, one needs to know more, for example the dynamics that coupled the two systems and, in particular, what variables are involved in the interaction. (Di Biagio and Rovelli, 2022, p.5)

This focus on the dynamics,[11] together with the conspicuous absence of a detailed characterisation of the notion of interaction suggests that Rovelli and his collaborators might be appealing to the standard notion of interaction in quantum theory, which relies on the form of the Hamiltonian. Therefore, in what follows I will explore the prospects of applying the standard notion of interaction to resolve the measurement problem in ARQM.

In the practice of quantum mechanics, interactions between systems are modelled by interaction terms in the Hamiltonian. An interaction term $H_{S\cup F}$ between systems S and F with an associated Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_F$ is a term which cannot be decomposed into terms acting trivially onto one of the factor spaces:

$$H_{S\cup F} \neq H_S \otimes \mathbb{1}_F + \mathbb{1}_S \otimes H_F$$

This standard formalism immediately suggests a way to define interactions: roughly, an interaction between two systems occurs if and only if the Hamiltonian of the systems involves an interaction term between the two systems. Unfortunately, things are not so simple.

Models of quantum theory assign only one (usually time-independent) Hamiltonian to the modelled systems. The proposal above would then define only *one*, *unordered* set of interactions for a system. This clearly won't do. Systems undergo multiple interactions and, in section 4, I have argued

[11] See also Rovelli (2021, p.3).

that RQM is required to define an order between (at least some) events. Moreover, I will argue below that the presence of an interaction term is naturally understood as only a necessary, but insufficient condition for an interaction.

In order to develop a plausible proposal for specifying the occurrence of interactions, a more careful consideration of the notion of the Hamiltonian is needed. I will endeavour to do so in what follows. However, it is convenient to attempt the task of defining interactions first in context of absolute states. Once the ideas are developed in that setting, I will attempt to extend them the context of ARQM.

5.1 Dynamics and interactions in the context of absolute states

I will now develop a natural proposal for specifying the occurrence of interactions through the dynamics, in the context of absolute quantum states. By doing so, I will demonstrate two claims. Firstly, that it may not be taken for granted that it is possible to sensibly define the occurrence of interactions (in RQM's sense) from the standard notion of interaction. As we will see, the proposal I define is promising, but it still has issues which require further work. Secondly, that whether an interaction between systems S and F occurs depends in part on the quantum state of $S \cup F$.

I will start by considering a plausible story about what warrants modelling a system with a certain Hamiltonian. This story is not essential to my two claims, and I will show it can forgotten for the more pragmatically inclined readers. It goes as follows. *Really*, the quantum states of systems are governed by a global, time-independent Hamiltonian of the whole universe. The justification for assigning a certain Hamiltonian to a given collection of systems is that, for the relevant collection of systems, it approximates well the universal Hamiltonian (at that time). Such a universal Hamiltonian will involve an extremely large amount of terms and, for a given interval of time, many of those will have a negligible or null effect on the relevant collection of systems. What is left after disregarding such negligible terms is the Hamiltonian for the quantum model of the given collection of systems in the given interval of time.

This reasoning suggests a method to derive a sequence of interaction terms over time for a single system, at least in principle. Partition time into intervals. Then, roughly: *an interaction between systems S and F is occurring during a time interval if and only if the global Hamiltonian contains an interaction term between S and F which does not have a null effect on the* *system $S \cup F$ in the given time interval.* Let's consider this characterization in more detail.

First, what does it mean for a term in the Hamiltonian to have a non-null effect on a system? The only way to quantify the effect of the Hamiltonian on a system is by looking at its effect on the quantum state of the system. So a term in the Hamiltonian has a null effect iff if the term were removed from the Hamiltonian, then the evolution of the quantum state would remain unchanged. Secondly, why do we only consider interaction terms? Because the evolution of the quantum state of a system depends (in part) on the quantum state of another (disjoint) system *only if* there is an interaction term in the Hamiltonian of the two systems. Hence the criterion for interactions proposed in the previous paragraph is natural because it is equivalent to the following: an interaction between two systems S and F is occurring during a time interval Δt iff the time evolution of the quantum state of S during Δt is affected by the quantum state of F and vice versa.

It's worth getting more quantitative about what it means for a term to have a null effect. It is instructive to first consider an example involving two systems S and F with a Hamiltonian $H = H_0 + H_{S\cup F}$, where H_0 is a free term and $H_{S\cup F}$ is an interaction term:

Non-Interaction (2 systems case): *Two systems S and F have not interacted in the interval of time $[t, t+\Delta t]$ if and only if for all $\delta t \leq \Delta t$:*

$$|\psi(t+\delta t)\rangle_{S\cup F} = e^{-\frac{i}{\hbar}(H_0+H_{S\cup F})\delta t}\,|\psi(t)\rangle_{S\cup F} = e^{-\frac{i}{\hbar}(H_0+\alpha)\delta t}\,|\psi(t)\rangle_{S\cup F}$$

where $\alpha \in \mathbb{R}$.[12]

This condition is equivalent to the quantum state $|\psi\rangle_{S\cup F}$ being an eigenstate of $H_{S\cup F}$. Note that this definition allows for the addition of a global phase $e^{-\frac{i}{\hbar}\alpha\delta t}$ since it makes no physical difference.

The condition may be easily generalised to the realistic case of more than two systems. Consider a collection of systems $\Omega = \{S_i\}$ with a Hilbert space $\mathcal{H} = \otimes_{S_i}\mathcal{H}_{S_i}$. One may express a Hamiltonian defined over such a space as a sum of free and interaction terms:

$$H = H_0 + H_{S_1\cup S_2} + H_{S_1\cup S_3} + ... + H_{S_2\cup S_3} + H_{S_2\cup S_4} + ... + H_{S_1\cup S_2\cup S_3} + H_{S_1\cup S_3\cup S_4} + ...$$

where H_0 is the sum of the free Hamiltonians for each system and $H_{S_i\cup S_j\cup S_k\cup...}$ is the interaction Hamiltonian between systems S_i, S_j, S_k, ... Consider two systems $S, F \in \{S_1, S_2, ..., S_1\cup S_2, S_1\cup S_3, ..., S_1\cup S_2\cup S_3, ...\}$ with associated

[12]Note: the first equality is just explanatory, the second is the only substantive constraint.

Hilbert spaces $\mathcal{H}_S$ and $\mathcal{H}_F$. Define the Hilbert space $\mathcal{H}_{\Omega-S,F}$ as the Hilbert space composed by all of the factors spaces in $\mathcal{H}$, apart from $\mathcal{H}_S$ and $\mathcal{H}_F$ (and the factor spaces composing $\mathcal{H}_S$ and $\mathcal{H}_F$).[13] Then:

Non-Interaction Condition: *Consider a collection of systems $\Omega = \{S_i\}$ with a Hilbert space $\mathcal{H} = \otimes_i \mathcal{H}_{S_i}$. Two systems $S, F \in \{S_1, S_2, ..., S_1 \cup S_2, S_1 \cup S_3, ..., S_1 \cup S_2 \cup S_3, ...\}$ have not (directly) interacted in the interval of time $[t, t + \Delta t]$ if and only if for all $\delta t \leq \Delta t$:*

$$\rho_{S\cup F}(t+\delta t) = Tr_{\mathcal{H}_{\Omega-S,F}}(e^{-\frac{i}{\hbar}H\delta t}\rho(t)e^{\frac{i}{\hbar}H\delta t}) = Tr_{\mathcal{H}_{\Omega-S,F}}(e^{-\frac{i}{\hbar}(H-H_{S\cup F})\delta t}\rho(t)e^{\frac{i}{\hbar}(H-H_{S\cup F})\delta t})$$

where $\alpha \in \mathbb{R}$.[14]

Terms of the Hamiltonian which do not satisfy the Non-Interaction Condition will be called *active* terms. Then, given the above characterisation of the occurrence of interactions, *active* terms indicate that an interaction is occurring.

However, this criterion does not yet provide a sparse-flash ontology of interactions in RQM's style, given that it talks of interactions as *occurring.* Some further tweaking is necessary. Partition time into a sequence of intervals ΔT_i. Then define a sequence Γ_S of sets of interaction terms for a system S, where each set contains the terms that are active in a time interval ΔT_i. More precisely: the members of the i^{th} set in the sequence Γ_S are the *active* interaction terms involving S in the ΔT_i time interval: $\Gamma_S = \langle ...\{H_{S\cup S_k}, H_{S\cup S_{k+1}}, ...\}, \{H_{S\cup S_l}, H_{S\cup S_{l+1}}, ...\}...\rangle$. To avoid a dependence of which interactions occur on the arbitrary choice of time partition, this sequence cannot be straightforwardly interpreted as giving the interactions.[15] A more circuitous definition is needed:

Interactions - Absolute Quantum States: *Consider a partition of time into time intervals ΔT_i. Define the sequence Γ_S as per above. Let $\Gamma_S(i)$ be the i^{th} element in the sequence.*

- *if $H_{S\cup F} \notin \Gamma_S(i)$, then, no interaction between S and F occurs in the time interval ΔT_i.*

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[13] More precisely: let $\Omega = \{S_i\}, i \in I \subseteq \mathbb{N}$. Let $I_S, I_F \subseteq I$ such that $S = \cup_{i\in I_S} S_i$, $F = \cup_{i\in I_F} S_i$. Note that the notation is sloppy: since S_i are systems rather than sets, $S_i \cup S_j$ is not a union of two sets but simply the joint system. Then let $\mathcal{H}_{\Omega-S,F} = \otimes_{i\in I\setminus(I_S\cup I_F)}\mathcal{H}_{S_i}$. For example, suppose $\mathcal{H} = \mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_7$ and $\mathcal{H}_S = \mathcal{H}_1 \otimes \mathcal{H}_3$ and $\mathcal{H}_F = \mathcal{H}_6 \otimes \mathcal{H}_7$. Then $\mathcal{H}_{\Omega-S,F} = \mathcal{H}_2 \otimes \mathcal{H}_4 \otimes \mathcal{H}_5$.

[14] As above, the first equality is only explanatory, the second is the only substantive constraint.

[15] Consider splitting a time interval in half. Some of the terms in the original interval might now appear in two time intervals (and hence appear twice).

- *if* $H_{S\cup F} \in \Gamma_S(i)$ *for all* $i \in \Lambda$*, where* Λ *is a set of neighbouring numbers in* $\mathbb{N}$*, then at least one interaction between* S *and* F *occurs in the time interval* $\Delta T = \cup_{i\in\Lambda}\Delta T_i$*.*

This criterion is a promising starting point, but it faces several issues.

Firstly, as already noted by Muciño et al. (2021, pp.9-15, 2022, pp.12-13), interaction terms do not select a preferred basis, since they may be expressed in terms of any basis. Thus, this formalism cannot determine, which variables become determinate at an interaction. One has to hope that the solution of the preferred basis problem will come from somewhere else.[16]

Secondly, note that the definition above considers only *direct* interactions, in the following sense. According to the above criterion, interactions between S and F are determined only by terms $H_{S\cup F}$. However, it's plausible that some kind of *indirect* interaction between S and F occurs if subsystems of S and F interact according to the criterion above, or if "super"-systems of S and F (i.e. systems $S \cup S'$ and $F \cup F'$) interact according to the criterion above. For the purposes of this paper I will set these issues aside.

Thirdly, an *exact* condition like the Non-Interaction Condition will not rule out enough interactions. For example, a great number of realistic interactions (e.g. the Columb potential) have an infinite range, meaning that the corresponding interaction terms will be active for all charged systems *at all times*.[17] That is obviously problematic. Therefore, one will have to resort to an approximate condition, namely, one might need to claim that when the equation in the Non-Interaction Condition *approximately* holds, then the relevant interaction term is not active.

Fourthly, there are cases in which interactions collectively (approximately) screen each other, but individually they might not. Consider, for instance, an electron in hydrogen atom, which is sitting sitting close to an (electrically neutral) large piece of matter. Call the electron in the Hydrogen atom S. Collectively, the Coloumb interactions between S and the electrons and protons in the large piece of matter may well approximately cancel each other out. Thus it may seem appropriate to claim that S is not interacting with all of the charged particles in the piece of matter, rather it might only be interacting with the proton in its own Hydrogen atom. But when considering an individual interaction term $H_{S\cup e}$ where e is an electron in the piece

[16] Adlam and Rovelli (2023, pp. 15-16) sketch a solution based on decoherence theory. Given the relativitiy of quantum states and without a precise characterisation of interactions, it's unclear to me how to understand their proposal.

[17] An appeal to relativistic constraints will help by ruling out space-like separated interactions. However, it is unlikely that will be enough and it is unclear how to exactly spell this out.

of matter, the Non-Interaction Condition might not hold. It's possible that an appeal to approximation might help in this case as well.

Prima facie, appeals to approximation are worrisome. The occurrence of an interaction cannot depend on a subjective decision on what level of approximation is appropriate, if interactions are to play such a fundamental role in the theory. Relatedly, while the approximation limit would be vague, the occurrence of an interaction isn't. I believe such worries can be put to rest. The approximate condition should not be taken to determine whether an interaction occurs or not, but rather it should be viewed as our imperfect attempt to find out which interactions occur. As set out in section 4, as long as it delivers clear answers which allow for correct predictions in the experimental cases and as long as it sufficiently indicates the existence of interactions also outside of experimental situations the condition will be satisfactory. However these worries do point to a more general challenge for RQM: if RQM's interactions need to be identified via approximate conditions then they don't naturally arise from the formalism of quantum theory. A challenge for their postulation to be justified can thus be brought forward from such a consideration. I won't consider this challenge in the present paper.

These problems do not seem to show that the Interactions - Absolute Quantum States and the Non-Interaction Condition are doomed, but at least they show that the latter condition needs to be tweaked. On top of all this, one still needs to argue that the condition *does* deliver a sequence of interactions which explains the verified experimental predictions and, more generally, satisfies the requirements set out in section 4.

Nonetheless, the aim of this section is not to deliver a perfect proposal, rather it is to show that, firstly, it cannot be taken for granted that the standard notion of interaction would deliver a specification for the occurrence of sparse, flash-like interactions and, secondly, that whether an interaction between systems S and F occurs depends in part on the quantum state of $S \cup F$. I believe the arguments in this section are enough to show the first claim. The second claim also follows from what argued here, since whether an interaction term $H_{S \cup F}$ is active depends on the quantum state $|\psi\rangle_{S \cup F}$.

As already noted at the beginning, some may think that there is no such thing as a universal Hamiltonian and that one should not appeal to such a fiction. They will be thus unconvinced by the above proposal. This would certainly be more evidence of my claim that it cannot be given for granted that the standard notion of interaction will provide a suitable specification of the occurrence of interactions. Moreover, I argue that for any approach to specifying interactions my second claim will be true, namely the claim that whether an interaction occurs depends on the quantum state of the systems involved. The reason is the following. Any method for selecting the relevant interaction terms must agree with the Hamiltonians used in the practice of physics. Our practice of assigning Hamiltonians to quantum models takes into account the quantum state of the systems involved in the model (or at least, it accounts for information which is correlated with quantum states of the systems). For example, consider a proton and an electron. If their wavefunction shows that they are sharply localised in position representation at opposite ends of the Earth, one would not model them with the Hydrogen Hamiltonian (in fact, one might not know how to model them at all). If, instead, their wavefunction shows they are close enough (and far away enough from other particles), the hydrogen Hamiltonian may be appropriate. Therefore, all methods for determining the occurrence of an interaction will agree that whether an interaction between systems S and F occurs depends on the quantum state of $S \cup F$.

Now that we have explored the possibilities in the simpler scenario of absolute states, it's time to explore whether these ideas generalise appropriately to the context of ARQM.

5.2 Dynamics and interactions in the context of ARQM

5.2.1 Hamiltonians and sequences of interaction terms in ARQM

The Hamiltonian is key to the standard notion of interactions in quantum theory. However, the complexities of the definition and status of the Hamiltonian in ARQM have been left unaddressed by the primary literature. Thus, the first task will be to carefully consider such complexities and clarify ambiguities concerning the Hamiltonian in ARQM. I claim that there are open questions on how to understand Hamiltonians in ARQM, but notwithstanding such open questions, it is clear that they should be understood as *relative*. This, together with the relativity of quantum states, implies that sequences of interaction terms should be understood as *relative*, rather than absolute.

As above it is helpful to start under the assumption that one is looking for a universal Hamiltonian and then proceed to relax this assumption. Which terms are contained in the universal Hamiltonian depends on the features of systems. If a system S has a certain mass m_S, then the universal Hamiltonian will contain the corresponding term, if it has an electric charge, the universal Hamiltonian will include the relevant electromagnetic interaction terms with all other charged systems, and so on. As noted by Healey (2022, p.5) and Dorato and Morganti (2022, p.2), the primary literature stresses the relativity only of the quantities which are associated with operators. Other quantities, such as the quantities used in determining the terms in a universal Hamiltonian (charge, mass, spin, ...) are not taken to be relative.[18] Therefore, there is a sense in which one might speak of a universal, absolute Hamiltonian H_g on the universal Hilbert space $\mathcal{H}_g$ for all systems. However, a brief reflection on the role and use of the Hamiltonian within ARQM shows that such universal H_g cannot perform the role of a Hamiltonian.

The Hamiltonian is the operator which determines the evolution of relative quantum states, when no interactions occur. As noted in section 3, there are no self-assigned quantum states, namely no states of the type $|\psi\rangle_S^{(S)}$. Thus, quantum states relative to a system S_i evolve on a Hilbert space $\mathcal{H}^{(S_i)}$ containing the Hilbert spaces of all systems *apart from the Hilbert space of* S_i *itself.*[19] Hence, the global Hamiltonian H_g cannot evolve the quantum states relative to any system S_i, since it acts on the different Hilbert space $\mathcal{H}_g$.

One might hope to get around this problem by appealing to the partial trace. In standard quantum theory, when a joint system $S \cup F$ evolves under the unitary $U_{S\cup F}(t)$, one may obtain the evolution of the density operator of one of the two systems (say S) by tracing out the other Hilbert space:

$$\rho_S(t) = Tr_F(\rho_{S\cup F}(t)) = Tr_F(U_{S\cup F}(t)\rho_{S\cup F}(0)U^\dagger_{S\cup F}(t))$$

Similarly, one might hope to use the global hamiltonian H_g acting on the global quantum state $\rho_g \in \mathcal{H}_g$ to define the evolution of relative quantum states $\rho^{(S_i)}_{g-S_i} \in \mathcal{H}^{(S_i)}$ via the partial trace. Of course, this cannot work, because ρ_g is not defined.

The plausible way out of this problem is to define relative Hamiltonians $H^{(S_i)}$ which act on the Hilbert spaces $\mathcal{H}^{(S_i)}$ relative to the systems S_i. For example, one could construct the relative Hamiltonians $H^{(S_i)}$ as if S_i itself were not to exist. In such a case, $H^{(S_i)}$ would contain all the terms in H_g which act trivially on $\mathcal{H}_{S_i}$, namely the terms that can be written as $O = \mathbb{1}_{S_i} \otimes O_{g-S_i}$.

Let us turn to interactions now. From such relative (quasi-global) Hamiltonians $H^{(S_i)}$ one may define a sequence of sets of interaction terms for a system S, following the steps in Section 4.3. It is evident that such a sequence will be relative to S_i, for two reasons. Firstly, $H^{(S_i)}$ is itself relative to S_i. Secondly, as argued at length in the previous section, which interaction terms are active depends on the quantum state of the relevant systems, which is relative in ARQM. Hence *sequences* $\Gamma^{S_i}_S$ *of sets of interaction terms for a system* S *may be defined only relative to a another system* S_i. In particular, whether a term $H_{S\cup F}$ is included in a sequence $\Gamma^{S_i}_S$ will depend on

[18]These quantities would typically be state-dependent in relativistic quantum field theory, and thus relative within the conceptual scheme of RQM. This would seem to complicate things even further for RQM, thus, I will focus on non-relativistic QM here.

[19]In the notation of section 5.1: $\mathcal{H}^{(S_i)} = \mathcal{H}_{g-S_i} = \mathcal{H}_{\Omega-S_i}$.

the quantum state $|\psi\rangle^{(S_i)}_{S\cup F}$. Note that, since since $H^{(S_i)}$ cannot contain any interaction terms involving S_i and since there are no quantum states of the type $|\psi\rangle^{(S_i)}_{S\cup S_i}$,[20] *any sequence of interaction terms* $\Gamma^{S_i}_S$ *relative to a system* S_i *won't contain interaction terms involving* S_i and *sequences of the type* $\Gamma^{S_i}_{S_i}$ *are not defined.*

Just like in the previous section, some might be unconvinced by my appeal to quasi-global Hamiltonians $H^{(S_i)}$. For the purposes of my arguments, what matters is that my conclusion concerning the relativity of Hamiltonians, and thus relativity of sequences of sets of interaction terms, follows regardless of such an assumption. The reason being that whichever conditions justify the assignment of a certain Hamiltonian to a collection of systems for a given interval of time, they must result in assignments of Hamiltonians which coincide with the ones assigned in the tested and experimentally verified quantum models. In the practice of physics, the assignment of quantum models takes into account the quantum state of the modelled systems. For instance, as already noted, it would be wrong to assign the Hydrogen Hamiltonian to a proton and an electron whose wavefunction shows that they are sharply localised on opposite sides of the Earth. However, if they are sufficiently close then it might be appropriate to use a Hydrogen Hamiltonian (granted a myriad of other factors). The quantum state is relative in RQM, therefore, one will find that Hamiltonians must also be conceived as relative in RQM. In turn, the sequence of interaction terms $\Gamma^{S_i}_S$ for a system S must be understood as *relative to* another system S_i, and, in turn, whether an interaction term $H_{S\cup F}$ is included in such a sequence will depend on the quantum state$|\psi\rangle^{(S_i)}_{S\cup F}$ of $S \cup F$ relative to S_i.

Now that the details of the formalism of ARQM have been ironed out, let's see if and how it can provide a sensible specification of the occurrence of interactions.

5.2.2 Interactions - ARQM

In ARQM, the occurrence of interactions is absolute, while the sequences Γ^F_S are relative. Thus, one cannot straightforwardly define the *absolute* occurrence of interactions from them. One needs to find a way to extract absolute events involving a system S from a multitude of relative sequences ($\Gamma^{F_1}_S$, $\Gamma^{F_2}_S$, ...), without arbitrarily choosing a preferred "reference" system, since no system is "special" in RQM (see section 2).

I can see two avenues to do so. One might take interactions involving

[20]Because this quantum state would imply a definition of a self assigned quantum state $\rho^{(S_i)}_{S_i}$.

a system S to be predicted exclusively by the sequences *relative to* S itself. In other words, to predict which interactions a system S is involved in, one will have to check the sequence Γ_S^S. Unfortunately, as noted in the previous section, there are no such sequences. This proposal is a non-starter.

Otherwise, one may preserve the equality of all systems by giving equal status to all sequences. Then, roughly speaking, an absolute interaction occurs between systems S and F if and only if there is a relevant interaction term in a sequence Γ_S^W or Γ_F^W relative to any third system W. More precisely:

A1: *Consider two distinct systems S, F. Given a partition of time into intervals ΔT_i, define series of interaction terms Γ_S^W and Γ_F^W relative to a third distinct system W. Then: Let $\Gamma_S^W(i)$ and $\Gamma_F^W(i)$ be the i^{th} element in the sequence.*

- *if $H_{S\cup F} \notin \Gamma_S^W(i), \Gamma_F^W(i)$, then, no interaction between S and F occurs in the time interval ΔT_i.*
- *if $H_{S\cup F} \in \Gamma_S^W(i)$ or $H_{S\cup F} \in \Gamma_F^W(i)$ for all $i \in \Lambda$, where Λ is a set of neighbouring numbers in $\mathbb{N}$, then at least one interaction between S and F occurs in the time interval $\Delta T = \cup_{i\in\Lambda}\Delta T_i$.*

Note that, as expected from the discussion in the context of absolute states, A1 does not address the preferred basis problem. But this proposal is not just lacking, it is problematic.

For one and the same system S there are many sequences of interaction terms relative to different systems ($\Gamma_S^{W_1}$, $\Gamma_S^{W_2}, ...$). These sequences all denote *absolute* interactions involving S according to A1, even though they may well be different. This proposal will thus face difficulties in ensuring that the pattern of absolute events leads to an empirically adequate picture of the world. In fact, a basic thought experiment shows that A1 will not offer an empirically adequate picture of the world.

Consider the following physical situation. Consider a spin-$\frac{1}{2}$ system S, and three spin-z measuring apparatus A, $B_\uparrow$ and $B_\downarrow$. The measuring apparatus are set up in such a way that depending on the outcome of the measurement on S by A, S goes on to be measured by either $B_\uparrow$ or $B_\downarrow$. Within ARQM, this may be accounted as follows.

There is an $S-A$ interaction in which the spin-z of S becomes determinate. After the $S-A$ interaction, say at a time t_1, the quantum state $|\psi(t_1)\rangle_S^{(A)}$ of S relative to A will be either $|\uparrow, +z\rangle$ or $|\downarrow, -z\rangle$, where the spin-z of the particle has become perfectly correlated with its spatial wavefunction. Then one may assume that if $|\psi(t_1)\rangle_S^{(A)} = |\uparrow, +z\rangle$, the interaction term $H_{S\cup B_\uparrow}$ is *active* (relative to A) while $H_{S\cup B_\downarrow}$ is not (relative to A) and vice versa if

$|\psi(t_1)\rangle_S^{(A)} = |\downarrow, -z\rangle$. Therefore if the outcome of the $S - A$ interaction is $\uparrow$ relative to A, then the sequence Γ_S^A specifies an interaction with $B_\uparrow$ and vice versa. Given A1, whichever of the two interactions occurs, it occurs absolutely.

Suppose that there is a third system W such that, after the $S - A$ interaction, the quantum state of $S \cup A$ relative to W is the following:

$$|\psi(t_1)\rangle_{S\cup A}^{(W)} = \alpha\, |\uparrow, +z\rangle_S\, |A\uparrow\rangle_A + \beta\, |\downarrow, -z\rangle_S\, |A\downarrow\rangle_A$$

where $|A\uparrow\rangle$ and $|A\downarrow\rangle$ correspond to A witnessing, respectively, $\uparrow$ and $\downarrow$. This is not an unwarranted assumption. Indeed, the primary literature on RQM repeatedly stresses that, after a measurement, the quantum states of the measured system and the observer (in this case the measurement apparatus) relative to a third system must be an entangled superposition predicting a correlation between the measured variable and the pointer variable.[21] I do not address such arguments in this paper. I only assume the weak claim that *it is possible that there is a third system* W relative to which the quantum state of $S \cup A$ is the above.

Consider now the quantum states $|\psi(t_1)\rangle_{B_\uparrow}^{(W)}$ and $|\psi(t_1)\rangle_{B_\downarrow}^{(W)}$. To ensure ARQM delivers an empirically adequate picture of the world, it seems necessary to assume that such quantum states are sufficiently similar to $|\psi(t_1)\rangle_{B_\uparrow}^{(A)}$ and $|\psi(t_1)\rangle_{B_\downarrow}^{(A)}$, for the following reason. Suppose, instead, that $|\psi(t_1)\rangle_{B_\uparrow}^{(W)}$ was wildly different from $|\psi(t_1)\rangle_{B_\uparrow}^{(A)}$. Then, $\Gamma_{B_\uparrow}^W$ won't specify the occurrence of an interaction $B_\uparrow - S$, but may instead specify interactions with wildly different systems (e.g. a tree outside the lab, a rock on Mars, or else). One would therefore be hard pressed to show how ARQM forms an empirically adequate picture of the world.

Assume instead that $|\psi(t_1)\rangle_{B_\uparrow}^{(W)}$ and $|\psi(t_1)\rangle_{B_\downarrow}^{(W)}$ are sufficiently similar to $|\psi(t_1)\rangle_{B_\uparrow}^{(A)}$ and $|\psi(t_1)\rangle_{B_\downarrow}^{(A)}$. Even so, A1 cannot be saved. Under such an assumption, both terms $H_{S\cup B_\uparrow}$ or $H_{S\cup B_\downarrow}$ will be active relative to W, and therefore, they will both figure in the sequence Γ_S^W. According to A1 then, both interactions $S - B_\uparrow$ and $S - B_\downarrow$ would occur, absolutely. In other words, S interacts with both experimental apparatus $B_\uparrow$ and $B_\downarrow$.

This is implausible. More importantly, similar scenarios in which conscious observers are involved may be surely devised, seemingly leading to a contradiction with scientists' first-person experience and thus undermining

[21] "(i) a variable of S has a value with respect to F and (ii) with respect to W, there is a correlation to be expected between a variable of S and a pointer variable of F'. The first implies the second' (Di Biagio and Rovelli, 2022, p.8). See also Rovelli (1996, p.1643, 1654).

the empirical basis of QM. For example, consider a scientist F performing an experiment with a measuring apparatus A. Suppose that, depending on the outcome of the experiment, the scientist will move to one of two rooms, in which they will interact with another measuring apparatus, respectively, $B_\uparrow$ or $B_\downarrow$. What is assumed is surely possible. Relative to a third observer W, the scientist will be in a superposition, and thus the sequence Γ_F^W will determine that F interacts with *both* $B_\uparrow$ and $B_\downarrow$. However, the scientist's first-person experience is of interacting with only one of $B_\uparrow$ or $B_\downarrow$. It seems that A1 would lead to an empirically inadequate picture of the world.

Therefore, the standard notion of interaction does not offer a solution to the problem of specifying interactions for ARQM.

6 Interactions from correlations

As already noted above, Rovelli sometimes suggested a connection between correlations and the occurrence of interactions. For completeness I will now explore this idea.

Consider, for example, the following statements:

> it should be possible to understand what is the physical meaning of "v has a value relative to O" by considering the description that P [i.e. a third system] gives (or could give) of the $S - O$ system. (Rovelli, 1996, p.1653)

> the fact that q has a value relative to O means that q is correlated with the pointer variable in O. [...] By "O has information about q" we mean "relative to O, q has a value" and also "relative to P [i.e. a third system], there is a certain correlation in the S and O states."(Rovelli, 1996, p.1654)

See also Rovelli (1996, p.1652).

Roughly, the idea seems that correlations in the quantum state of two systems relative to a third system indicate the occurrence of an interaction. Without further qualifications, this idea is not promising. Consider any two systems S and F which have a joint, pure quantum state $|\psi\rangle_{S\cup F}$. Thanks to the Schmidt decomposition theorem (Peres, 1993), one can always find orthonormal bases $\{|A_i\rangle\}$ and $\{|B_i\rangle\}$ of $\mathcal{H}_S$ and $\mathcal{H}_F$ such that:

$$|\psi\rangle_{S\cup F} = \sum_i \alpha_i |A_i\rangle |B_i\rangle$$

In other words, for any pure quantum state of any bipartite system, there always are perfectly correlated sets of variables of each system. This fact, together with the rough idea implies that any two systems are always interacting. This is obviously incompatible with RQM's picture of a sparse-flash ontology of events (Adlam and Rovelli, 2023, p.11).[22]

Thus it seems that a more promising approach is to consider *the establishment of correlations* between systems as an indication of the occurrence of an interaction. Quantum theory comes with a natural structure defining correlations between systems: entanglement. Thus, I will explore the idea that *interactions are indicated by the establishment of entanglement between two systems.*

It's easy to see that ARQM encounters a similar problem as the ones encountered in the previous section. Entanglement (and the establishment thereof) is a feature of quantum states. But quantum states are relative, while interactions are absolute in ARQM. One needs to find a way to extract absolute events involving a system S from a multitude of relative quantum states ($|\psi\rangle_S^{F_1}$, $|\psi\rangle_S^{F_2}$, ...), without arbitrarily choosing a preferred "reference" system, since no system is "special" in RQM (see section 2). Once again there are two prima facie plausible options. One way would be to consider only the self-ascribed quantum state, but there is no such quantum state in ARQM. The other option, is to give equal status to all quantum states. The following proposal is in the spirit of this second option. For simplicity, consider a step-wise dynamics of the quantum state: assume that the quantum state is defined at discrete times $\{t_0, t_1, t_2...\}$ and that it evolves in between these times. Then:

A2: *An (absolute) interaction between two systems S and F occurs at a time t_n if an only if there is a quantum state $|\psi\rangle_{S\cup F}^{(W)}$ relative to a third system W which is factorisable at a time t_{n-1}, and it is not factorisable at a time t_n. More precisely there are $|\psi(t_{n-1})\rangle_S^{(W)}$ and $|\psi(t_{n-1})\rangle_F^{(W)}$ such that:*

$$|\psi(t_{n-1})\rangle_{S\cup F}^{(W)} = |\psi(t_{n-1})\rangle_S^{(W)} \otimes |\psi(t_{n-1})\rangle_F^{(W)}$$

and there are no $|\psi(t_n)\rangle_S^{(W)}$ and $|\psi(t_n)\rangle_F^{(W)}$ such that:

$$|\psi(t_n)\rangle_{S\cup F}^{(W)} = |\psi(t_n)\rangle_S^{(W)} \otimes |\psi(t_n)\rangle_F^{(W)}$$

At such interaction, events $e_S^{(F)}(\mathcal{V})$ and $e_F^{(S)}(\mathcal{V}')$ occur if and only if the $|\psi(t_n)\rangle_{S\cup F}^{(W)} = \sum_i \alpha_i |v_i\rangle |v_i'\rangle$, where v_i and v_i' are possible values of $\mathcal{V}$ and $\mathcal{V}'$.

[22] Faglia (2023, chapter 4) explores in detail options closely based on this simpler idea, with negative conclusion.

In other words, an interaction $S - F$ occurs at a time t_n if the quantum state of $S \cup F$ (relative to *any* third system W) turns from unentangled to entangled at t_n. The variable that take a value at the event are the variables that $|\psi(t_n)\rangle^{(W)}_{S\cup F}$ predicts will have a perfect correlation.

One prima facie attractive feature of A2 is that it addresses the preferred basis problem, by determining which variable becomes determinate at an interaction. However, this solution relies on the approximation of taking time as discrete. Removing the approximation reveals that entanglement of the quantum state of $S \cup F$ relative to W does not have an exact beginning and end, but rather is a continuous process during which the correlated variables continuously change. Without a further rule, it's not clear how to determine which of the different variables that are perfectly correlated at different times of the evolution are the ones which become determinate.[23] Moreover, the perfectly correlated variables, which are the variables chosen by the Schmidt decomposition, are not usually the appropriate variables to become determinate, as work done on modal interpretations has shown (for example, see (Bacciagaluppi, 2000)).

Moreover, A2 inherits similar problems to A1: the events are absolute, but their occurrence is determined by a multitude of relative, different quantum states. For example, if S and F_1 become entangled relative to W_1, and S and F_2 become entangled relative to W_2, then there will be two absolute interactions $S - F_1$ and $S - F_2$. Therefore, in order to obtain a distribution of events which leads to an empirically adequate picture of the world, the relative quantum states need to be carefully coordinated. Demonstrating that there is a way of coordinating quantum states in such a way is no trivial task, particularly if one is to avoid introducing a global quantum state.[24]

This task is made even more difficult by the fact that A2 is bound to predict some rather odd interactions. Consider once again the situation described in section 5.2.2, in which a spin-$\frac{1}{2}$ system S is measured by a

[23] Pienaar (2021) and Brukner (2021) also point out that Schmidt decompositions are not unique, thus even with a discretisation of time, the selection of variables might not be unique.

[24] Note that this problem is exacerbated by the Cross-Perspective Links principle in Adlam and Rovelli (2023). Although this goes beyond the scope of the paper, it's worth exploring briefly. Suppose a scientist F performs a large number of measurements on spin-$\frac{1}{2}$ particles S_i with the quantum state $|\psi\rangle^{(F)}_{S_i} = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ relative to F. According to Cross-Perspective Links, if any third observer W were to measure F in the pointer variable after each interaction, they would obtain the same value F obtained in their measurement. Thus, for the frequencies to agree with the (Relative) Born Rule, it would seem that the quantum state of $S_i \cup F$ relative to W ought to be: $|\psi\rangle^{W}_{S_i\cup F} = \frac{1}{\sqrt{2}}(|\uparrow\rangle |F, \uparrow\rangle + |\downarrow\rangle |F, \downarrow\rangle)$, where $|F, \uparrow\rangle$ and $|F, \downarrow\rangle$ correspond to F seeing $\uparrow$ and $\downarrow$. This would seem to hold for all third systems W.

spin-z measuring apparatus A and, depending on the outcome, it will be measured again by either spin-z measuring apparatus $B_\uparrow$ or $B_\downarrow$.

According to A2, the first interaction between S and A at a time t_1 will be accounted by the quantum state of $S \cup A$ relative to a third system W becoming entangled in the right basis:

$$|\psi(t_1)\rangle^{(W)}_{S\cup A} = \alpha\,|\uparrow, +z\rangle_S\,|A\uparrow\rangle_A + \beta\,|\downarrow, -z\rangle_S\,|A\downarrow\rangle_A$$

Assume that the quantum states of $B_\uparrow$ and $B_\downarrow$ relative to W correspond to the measuring apparatus being ready to measure, namely $|B_\uparrow, \text{ready}\rangle$ and $|B_\downarrow, \text{ready}\rangle$. Then the following is a plausible time evolution of the quantum state of $S \cup A \cup B_\uparrow \cup B_\downarrow$ relative to W:

$$|\psi(t_2)\rangle^{(W)}_{S\cup A\cup B_\uparrow\cup B_\downarrow} = \alpha\,|\uparrow, +z\rangle\,|A, \uparrow\rangle\,|B_\uparrow, \uparrow\rangle\,|B_\downarrow, \text{ready}\rangle + \beta\,|\downarrow, +z\rangle\,|A, \downarrow\rangle\,|B_\uparrow, \text{ready}\rangle\,|B_\downarrow, \text{ready}\rangle$$

and

$$|\psi(t_3)\rangle^{(W)}_{S\cup A\cup B_\uparrow\cup B_\downarrow} = \alpha\,|\uparrow, +z\rangle\,|A, \uparrow\rangle\,|B_\uparrow, \uparrow\rangle\,|B_\downarrow, \text{ready}\rangle + \beta\,|\downarrow, +z\rangle\,|A, \downarrow\rangle\,|B_\uparrow, \text{ready}\rangle\,|B_\downarrow, \downarrow\rangle$$

This time evolution shows that, relative to W, first $S \cup A$ becomes entangled $B_\uparrow$, and then $S \cup A \cup B_\uparrow$ becomes entangled with $B_\downarrow$. Thus, according to A2 there would be $S \cup A - B_\uparrow$ and $S \cup A \cup B_\uparrow - B_\downarrow$ interactions. Evidently, it is difficult to make sense how these interactions could figure in an empirically adequate picture of the world. It is even more difficult to understand how ARQM with A2 could make sense of these simple experimental scenarios.

Evidently, it is far from clear how an appeal to correlations or entanglement might help to specify the circumstances for the occurrence of interactions.[25] The ARQM's measurement problem is unresolved and, given the failed efforts outlined above, the prospects for addressing it are dim.

7 RQM with relative events

One might to resolve ARQM's problems by relativising events themselves. The relativisation of events is sometimes suggested in the primary literature:

> the fact that a certain quantity q has taken a value with respect to O is a physical fact; as a physical fact, its being true, or not true, must be understood as relative to an observer, say P. (Rovelli, 1997, p.8).

[25]Note moreover that disentangling interactions won't count as interactions according to A2.

See also Smerlak and Rovelli (2007, p.419) and Rovelli (2018, p.9). I will call this interpretation RRQM. Unfortunately, it's easy to show that the problems of ARQM generalise to RRQM and no better solutions are available in RRQM.

7.1 RRQM: what changes

In RRQM the occurrence of events itself is relative to a system. More precisely, according to RRQM, whenever an interaction between F and S occurs *relative to a system* W, *relative to* W, a quantity $\mathcal{V}$ of S takes a value v relative to F and, *relative to* W, a quantity $\mathcal{V}'$ of F takes a value v' relative to S.[26] I denote an interaction between two systems S and F which occurs relative to W as $[S-F]^W$ and an event relative to W in which S's quantity $\mathcal{V}$ takes a value v relative to F as $[e_S^{(F)}(\mathcal{V})]^W$ or $[e_S^{(F)}(\mathcal{V}=v)]^W$.

Once again, I won't dwell on the metaphysics of this relativity, but I will assume a connection between relativity and first-person experience, which flows naturally from claims in the primary literature. According to RRQM, a quantum event "is only real in relation to a specific observer" (Smerlak and Rovelli, 2007, p.429). Since one cannot have an experience of something which is not real,[27] an observer O can only have experience about events which *occur relative to* O. Moreover, for the same reasons behind the Event-Experience Link - ARQM, an observer O can have experience of a value of a quantity only if it has *taken a value relative to* O. Therefore, in RRQM, the following holds:

Event-Experience Link - RRQM: *A conscious observer O can have a first-person experience of a quantity $\mathcal{V}$ of a system S taking a certain value only if the quantity $\mathcal{V}$ takes a value relative to O, relative to O (i.e. only if the event $[e(\mathcal{V})_S^{(O)}]^O$ occurs).*

The changed ontology also suggests a change in the understanding of quantum theory. The further relativisation of events logically leads to a further relativisation of quantum states, since if an event $[e_S^{(F)}(\mathcal{V})]^W$ occurs *only relative to* W, then the quantum state of S relative to F should update *only relative to* W. A notation for doubly relativised quantum states may be introduced: $[|\psi\rangle_S^{(F)}]^W$ denotes the quantum state of S relative to F, relative to W. The Relative Born Rule may then be suitably adapted to claim that,

[26] Riedel (2024) interestingly proposes that relativity should iterate indefinitely. Although this is an option certainly worth exploring, it's not yet clear how to understand first-person experience and theory-confirmation in such a theory. Therefore I set it aside.

[27] Setting aside hallucinations, which are not relevant here.

given an event $[e_S^{(F)}(\mathcal{V})]^W$, the probabilities for values v of $\mathcal{V}$ obtaining at the given event may be derived via the born rule applied to $[|\psi\rangle_S^{(F)}]^W$.

However, the primary literature only ever refers to quantum states being relativised once, and I believe for good reasons. Indeed, due to the Event-Experience Link - RRQM, quantum states $[|\psi\rangle_S^{(F)}]^W$ for which $W \neq F$ seem pointless. Quantum states are used to make predictions concerning the probability of events, and a quantum state $[|\psi\rangle_S^{(F)}]^W$ makes predictions concerning events of the type $[e_S^{(F)}(\mathcal{V})]^W$. But, following the Event-Experience Link - RRQM, the first-person experience of W will be affected only by events of the type $[e(\mathcal{V})_S^{(W)}]^W$, i.e. events which occur relative to W in which the variable takes a value relative to W. Therefore, the only useful quantum states would seem to be quantum states of the type $[|\psi\rangle_S^{(W)}]^W$. Moreover, since it's not clear if anyone could have any experiential (and thus ultimately empirical) access to events of the type $[e_S^{(F)}(\mathcal{V})]^W$, if $F \neq W$, then it's not clear if anyone would be able to know quantum states of the type $[|\psi\rangle_S^{(F)}]^W$, if $F \neq W$. In line with the primary literature, I will ignore quantum states $[|\psi\rangle_S^{(F)}]^W$ such that $F \neq W$, and instead define once relativised quantum states as $|\psi\rangle_S^{(F)} := [|\psi\rangle_S^{(F)}]^F$.

7.2 RRQM's problem of measurement

It is clear that the relativisation of the occurrence of events by itself does nothing to alleviate RQM's problem of measurement described in section 4. For RQM recover the predictions of orthodox quantum theory, it needs to predict that experimenters have a first-person experience of the outcome of an experiment and, in order to do so, it needs to specify the occurrence of certain events. In particular, given the Event-Experience Link - RRQM, RRQM needs to specify the occurrence *relative to a scientist* F of at least some events of the type $[e_A^{(F)}(X)]^F$ in which a certain quantity of the apparatus (e.g. the position X of a pointer) takes a value relative to the scientist F. The requirement for a specification of the circumstances for the occurrence of events for RRQM is subject to the same qualifications detailed in section 4.

7.3 Interactions in RRQM

Since RRQM and ARQM share the same formalism of relativised quantum states, the considerations detailed in section 5.2.1 regarding the dynamics hold in RRQM as well. In particular, the Hamiltonians ought to be considered as relative and, consequently, for a system S one may define sequences of interaction terms Γ^F_S only relative to another system F.

Prima facie, this structure of relative sequences seems to lend itself to specifying the relative occurrence of events: roughly speaking, *the interaction terms present in a sequence of interactions Γ^F_S specify the interactions involving S which occur relative to F.* Unfortunately this proposal does not work.

As noted above, RRQM is required to predict events specifically of the type $[e^{(F)}_S(\mathcal{V})]^F$, which arise from interactions of the type $[S-F]^F$. According to the proposal under consideration, the occurrence of these interactions would be predicted by the presence of a term $H_{S\cup F}$ in the sequence Γ^F_S or Γ^F_F. Unfortunately, since the quantum state $|\psi\rangle^F_F$ is not defined, Γ^F_F is not defined. Moreover, for the same reason, the Hamiltonian relative to F cannot contain interaction terms of the type $H_{S\cup F}$ and, therefore, Γ^F_S cannot contain such terms either (see section 5.2.1 for more details). Hence, it is not clear how to successfully apply the standard notion of interaction in RRQM either.

It's also easy to ascertain that correlations and entanglement won't save RRQM either, for a similar reason. Consider again the intuition that interactions are indicated by the establishment of entanglement between two systems. Since entanglement is a feature of quantum states, and quantum states are relative, once again this idea would seem appropriate to define relative occurrence of events: roughly, if the quantum state $|\psi\rangle^W_{S\cup F}$ of $S\cup F$ relative to W becomes entangled, then an interaction $[S-F]^W$ between S and F occurs relative to W. However, once again, due to the absence of self ascribed quantum states $|\psi\rangle^F_{S\cup F}$, this proposal cannot predict the occurrence of interactions of the form $[e^{(F)}_S(\mathcal{V})]^F$.

Therefore, RRQM fares no better than ARQM with respect to specifying the circumstances of the occurrence of events. The measurement problem is left unresolved.

8 Concluding remarks

Current versions of relational quantum mechanics leave unresolved key questions regarding the occurrence events. Consequently, they provide no solution to the measurement problem. Unless a solution is provided, relational quantum mechanics cannot be a viable interpretation of quantum theory. However, the natural ways to address the problem fail. The prospects for finding a solution are not promising.

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