Kinematical Equivalence and Cosmic Conspiracies

Caspar Jacobs & Eleanor March

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Abstract

Discussions of theoretical equivalence typically only concern a theory's dynamically possible models. Recently, however, March (2024) has shown that a theory's kinematically possible models are also relevant to questions of theoretical equivalence. We apply March's notion of kinematic equivalence to the difference between reduced and sophisticated theories introduced by Dewar (2019). Although Dewar claims that these are equivalent, Jacobs (2024) has argued that only sophisticated theories can explain what are otherwise 'cosmic conspiracies'. We show that this is a consequence of reduced and sophisticated theories' kinematical inequivalence. Furthermore, we use Caulton's (2024) 'downwards Hume's dictum' to show that kinematically inequivalent are also ontologically inequivalent.

1 Introduction

It is widely acknowledged that the symmetries of a theory may indicate superfluous structure (Earman, 1989; Ismael and van Fraassen, 2003; Dasgupta, 2021). For example, the boost symmetries of Newtonian mechanics indicate the redundancy of a standard of rest. There are broadly two ways to remove such redundant structure: *reduction* and *sophistication* (Dewar, 2019). In brief, the aim of the former is to formulate a theory that is *invariant* under the relevant symmetries, whereas the aim of the latter is to formulate a theory whose symmetries are *isomorphisms*.

On the one hand, sophisticated and reduced theories are often seen as theoretically equivalent. Dewar (2019) proves that reduced and sophisticated versions of various theories, including electromagnetism, are categorically equivalent, and conjectures that this is generally the case. On the other hand, there seems to be an explanatory difference between such theories. Dewar notices that certain relations are automatically satisfied in sophisticated theories but have to be postulated by hand for reduced theories. For example, the Gauss-Faraday law is a mathematical theorem of a sophisticated theory formulated in terms of a vector potential, but requires an additional posit in a reduced theory formulated in terms of the Faraday tensor. Jacobs (2024a) has further extended this observation by proving that reduced theories (under certain plausible conditions) inevitably entail such 'cosmic conspiracies'.

Dewar's own explanation of these facts is that "the two theories are equivalent in terms of their *intensional ontology*, in terms of the kinds of structures that they postulate as present in any world aptly described by them; but they differ in their *explanatory* structure." The latter difference is further clarified in terms of fundamentality. Although both kind of theories are committed to the same structures, Dewar believes, they posit different structures as fundamental. This enables them to offer different explanations.

We believe that this contains the seed for a deeper explanation of the explanatory difference between sophisticated and reduced theories. In order to provide such an explanation, however, we contend that one has to zoom out from the way in which theories are usually presented, namely as a class of solutions or 'dynamically possible models' (DPM), and also take into account the theory's 'kinematically possible models' (KPMs). We will then find that sophisticated and reduced theories are not fully equivalent. This is made precise by March's recent proposal of 'kinematical categorical equivalence' ('kinematical equivalence' for short) (March, 2024b). We will show that sophisticated and reduced theories are *not* kinematically equivalent.

We then use this result to explain in more detail why reduced theories involve cosmic conspiracies. The idea in brief is that sophisticated theories reduce conspiratorial *dynamical* patterns in the instantiation of quantities to necessary *kinematical* patterns. This means that no additional postulates are necessary to explain such relations: they follow simply from the theory's fundamental posits.

The paper will proceed as follows. In §2, we present a few examples of reduced versus sophisticated theories and their explanatory differences. In §3, we show that those theories are indeed kinematically inequivalent. §4 discusses the implications of this result with respect to the metaphysical and explanatory differences between reduced and sophisticated theories. We finally consider a number of objections in §5.

2 Reduction vs. Sophistication

In this section we first rehearse the distinction between reduction and sophistication and then present a number of examples of the explanatory difference between reduced and sophisticated theories.

It is by now a well-known fact that when a theory, T_O, possesses symmetries, its symmetry-related models (SRMs) are often empirically equivalent. This means that the quantities that vary under those symmetries are unmeasurable. All things considered, a theory without such unmeasurable quantities is preferable. If one is an *interpretationalist*, then one simply declares that the SRMs of T_O represent the same state of affairs. If, on the other hand, one is a *motivationalist*, one demands some story about how those SRMs represent the same state of affairs (Møller-Nielsen, 2017). It is here that the distinction between reduction and sophistication comes into play: these are two different ways to satisfy the motivationalist's demand.

Both offer, in some sense, a new theory to replace T_O. Every model of the old theory corresponds to an empirically equivalent model of this new theory. The reductionist's theory, T_R, is such that SRMs of T_O correspond to *numerically identical* models of T_R. In other words, for each equivalence class of SRMs of T_O there is a unique model of T_R. This clearly rids the theory of its undetectable structure. The sophisticator's theory, T_S meanwhile, is such that the SRMs of T_O correspond to *isomorphic* yet numerically *distinct* models of T_S. How this move rids T_O of its undetectable structure requires some explanation. The idea is that it is easy to interpret isomorphic models as representations of the same state of affairs, since such models only differ by a permutation of set-elements. If those set-elements are taken to represent individuals, such as space-time points or quantity magnitudes, then an anti-haecceitist metaphysics entails that such permutations are distinctions without a difference. They are merely distinct representations of the same state of affairs. The SRMs of some theories, such as GR, are *already* isomorphic: such theories are 'born' sophisticated. The adoption of anti-haecceitism is then sufficient to rid the theory of its undetectable structure.

We will now discuss three examples of reduction versus sophistication:

2.1 Handedness

Our first example is the 'handedness' theory from Dewar (2019). This is a first-order theory H in the signature $\{L(x), R(x)\}$ with two axioms:

1. $\forall x(L(x) \vee R(x))$
2. $\forall x \neg(L(x) \wedge R(x))$

The predicates L and R can be thought of as 'left-handed' and 'right-handed', so that H says that everything is either left- or right-handed but not both. This theory is symmetric under a permutation of the predicate symbols L and R, in the sense that the map $L(x) \to R(x)$, $R(x) \to L(x)$ maps the axioms of H to logically equivalent ones.

Dewar suggests both reduced and sophisticated handedness theories to replace H. For the reduced theory H_R, we replace the predicates L and R with a single 'congruence' relation $C(x, y)$ that satisfies four axioms:

- $\forall x C(x, x)$
- $\forall x \forall y(C(x, y) \to C(y, x))$
- $\forall x \forall y \forall z((C(x, y) \wedge C(y, z)) \to C(x, z))$
- $\forall x \forall y \forall z((\neg C(x, y) \wedge \neg C(y, z)) \to C(x, z))$

Intuitively, these axioms say that the congruence relation is an equivalence relation with at most two equivalence classes. It is straightforward to see that dynamically possible models of H related by the map $L(x) \to R(x)$, $R(x) \to L(x)$ correspond to the same dynamically possible model of H_R under the relation $\forall x \forall y (C(x, y) \leftrightarrow ((L(x) \wedge L(y)) \vee (R(x) \wedge R(y))))$.

The sophisticated theory H_S has dynamically possible models $\langle D, \mathbf{2}, \chi \rangle$, where D_H is a domain of objects, $\mathbf{2}$ is a two-element set, and $\chi : D \to \mathbf{2}$ is a function. The isomorphisms of such models correspond to pairs (f, g) where $f : D \to D'$ and $g : \mathbf{2} \to \mathbf{2}'$ are bijections. It is straightforward to see that models of H related by the map $L(x) \to R(x)$, $R(x) \to L(x)$ correspond to isomorphic models of H_S under a choice of bijection $\{L(x), R(x)\} \to \mathbf{2}$.

To investigate the relationships between the dynamically possible models of reduced and sophisticated theories, it is helpful to introduce the notion of *categorical equivalence*:

Categorical equivalence: theories T_1 and T_2 are equivalent just in case there is an equivalence of categories between their associated categories of models $\mathbf{T}_1$ and $\mathbf{T}_2$ (which preserves empirical content).[1]

The claim that categorical equivalence tracks equivalence of theories is motivated by the fact that the collection of models of a theory often can be given the structure of a category. One straightforward way to do this is to take the objects of the category to be the models of the theory and the arrows of the category to be maps between models which preserve physical content.

We can associate categories of models to H_R and H_S as follows:

$\mathbf{H}_R$: objects are models of H_R; arrows are model isomorphisms.

$\mathbf{H}_S$: objects are models $\langle D, \mathbf{2}, \chi \rangle$ of H_S; arrows are model isomorphisms (f, g).

We can then show that $\mathbf{H}_R$ and $\mathbf{H}_S$ are categorically equivalent. Let F_H be the functor that takes each object $\langle D, \mathbf{2}, \chi \rangle$ in $\mathbf{H}_S$ to an object $\mathfrak{M}$ in $\mathbf{H}_R$ such that $D_{\mathfrak{M}} = D$, and for any $a, b \in D_{\mathfrak{M}}$, $\langle a, b \rangle \in C$ iff $\chi(a) = \chi(b)$; and each arrow (f, g) to f. Then

Proposition 1. *F_H is an equivalence of categories.*

Proof. See Dewar (2019). □

However, whilst H_R and and H_S are categorically equivalent, there nevertheless appears to be an important explanatory difference between them. For in H_R one has to posit as a brute fact that the congruence relation C is an equivalence relation which partitions objects into at most two equivalence classes—every object is either left- or right-handed but not both—whereas if we define congruence such that $\langle a, b \rangle \in C$ iff $\chi(a) = \chi(b)$ in H_S, this same fact comes out as a theorem. Even if one is not worried by the need to stipulate that C is an

[1] Recall that an equivalence of categories $\mathbf{C}$ and $\mathbf{D}$ is a functor $F : \mathbf{C} \to \mathbf{D}$ which is full, faithful, and essentially surjective.

equivalence relation, the fact that C has at most two equivalence classes seems like a cosmic conspiracy: the equivalence classes behave exactly *as if* they are in fact determined by a pair of monadic predicates.

2.2 Absolutism vs. comparativism about mass

Our second example comes from the theory of Newtonian point-particle mechanics. This theory, which we will call M, has dynamically possible models $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), \mathbb{R}^+, m(i) \rangle$, where $\mathcal{M} = \langle M, t_a, h^{ab}, \nabla, \xi^a \rangle$, ϕ is a scalar field which represents the gravitational potential, $\mathcal{B}$ is a (structured) set of particles, the $\gamma(i) : \mathcal{B} \times \mathbb{R} \to \mathcal{M}$ are (smooth, future-directed) timelike curves which represent particle worldlines, and $m(i) : \mathcal{B} \to \mathbb{R}^+$ is an assignment of mass values to particles.

Suppose that uniform mass scalings—transformations of the form $m(i) \to \psi \circ m(i)$, where ψ is a bijection on the domain R^+ of $\mathbb{R}^+$ which preserves the relation $\leq$ and the operation $+$ of addition—are dynamical symmetries of M. Since uniform mass scalings do not preserve the relation $\times$ on R^+, they are not automorphisms of the mass value space $\mathbb{R}^+ = \langle R^+, \leq, +, \times \rangle$. This means that models of M related by a uniform mass scaling are not isomorphic: M has redundant structure in the form of the $\times$ operation.

We can again construct reduced and sophisticated versions of M.[2] In the reduced theory M_R, we replace the assignment of monadic mass properties $m(i)$ for each particle with an assignment of mass ratios $m(i,j) : \mathcal{B} \times \mathcal{B} \to \mathbb{R}^+$ for each pair of particles. The dynamically possible models of M_R are $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), \mathbb{R}^+, m(i,j) \rangle$, where $m(i,j)$ satisfies the condition $m(i,j)m(j,k) = m(i,k)$. Since $m(i,j)$ is invariant under uniform mass scalings, symmetry-related models of M correspond to the same model of M_R.

In the sophisticated theory M_S, meanwhile, we modify the definition of mass value space. Rather than a complete positive ordered semi-field $\mathbb{R}^+$, it is now an additive extensive structure $\langle D_m, \leq, \circ \rangle$, where $D_m = R^+$. Since uniform mass scalings are automorphisms of $\langle D_m, \leq, \circ \rangle$, they are also isomorphisms of the dynamically possible models $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i) \rangle$ of M_S.

Just as before, we can associate categories of models to M_R and M_S:

$\mathbf{M}_R$: objects are models $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), \mathbb{R}^+, m(i,j) \rangle$ of M_R; arrows are pairs (χ, f) consisting of a diffeomorphism $\chi : M \to M'$ and a bijection on the domain of $\mathbb{R}^+$ which jointly preserve $\mathbb{R}^+$, $\mathcal{M}$, ϕ, $\gamma(i)$ and $m(i,j)$.

$\mathbf{M}_H$: objects are models $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i) \rangle$ of M_S; rrows are pairs (χ, f) consisting of a diffeomorphism $\chi : M \to M'$ and a bijection on D_m which jointly preserve $\mathcal{M}$, $\leq$, $\circ$, ϕ, $\gamma(i)$ and $m(i,j)$.

We can again show that these are categorically equivalent. Let F_M be the functor that takes each object $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i) \rangle$ in $\mathbf{M}_S$ to the object

[2] We will focus on only two options here; for a more complete discussion of reduced and sophisticated formulations of Newtonian point-particle mechanics, see Jacobs (2023c) and March (2024c).

$\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i,j)\rangle$ such that $m(i,j) = m(i)/m(j)$ in $\mathbf{M}_R$ and each arrow (χ, f) to $(\chi, \mathrm{id}_{\mathbb{R}^+})$. Then

Proposition 2. *F_M is an equivalence of categories.*

Proof. F_M is essentially surjective and full by construction, since $\mathrm{id}_{\mathbb{R}^+}$ is the unique isomorphism which preserves $\mathbb{R}^+$ (this is because any two complete ordered positive semi-fields are uniquely isomorphic to each other). For faithfulness, let $\mathfrak{M} = \langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i)\rangle$, $\mathfrak{M}' = \langle \mathcal{M}', \phi', \mathcal{B}, \gamma(i)', D_m, \leq, \circ, m(i)'\rangle$ be any two objects in $\mathbf{M}_S$ and suppose that there exist distinct arrows $(\chi, f), (\chi, f') : \mathfrak{M} \to \mathfrak{M}'$. Then $m(i)' = f \circ m(i) = f' \circ m(i)$, and hence $f^{-1} \circ f' = \mathrm{id}_{D_m}$ and $f = f'$. Therefore, F_M is faithful. □

As with Dewar's handedness example, M_R and M_S are categorically equivalent. However, also as with the handedness example, we can see that the reduced theory M_R posits brute facts which are explained in M_S. In this case, the apparent brute fact is the condition $m(i,j)m(j,k) = m(i,k)$ on the assignment of mass ratios. If mass ratios are defined from monadic mass properties as in M_S, then this is a mathematical identity, whereas in M_R, this appears to be a cosmic conspiracy: the mass ratios behave exactly *as if* they are in fact constructed from monadic mass properties.

2.3 Electromagnetism

Our final example is the theory of electromagnetism in Minkowski spacetime, formulated in terms of a gauge potential. This theory, which we will call EM, has dynamically possible models $\langle M, \eta_{ab}, A_a, J^a\rangle$, where η_{ab} is a flat Lorentzian metric on M, A_a is a one-form, and J^a represents the four-current which satisfies $2\nabla_n \nabla^{[n} A^{a]} = J^a$. As is well-known, $U(1)$ gauge transformations $A_a \to A_a + \varphi_a$, where φ_a is exact, are dynamical symmetries of this theory. The value of $A_a(x)$ at any point x is undetectable.

One option here is to reduce the theory. Whilst a variety of reduced formulations of electromagnetism have been considered in the literature (for example, Healey (2007) takes the holonomies of A_a as primitive), here we will focus on the simplest case: move to the Faraday tensor formulation of electromagnetism. This theory, which we will call EM_R, has dynamically possible models $\langle M, \eta_{ab}, F_{ab}, J^a\rangle$, where F_{ab} is a two-form which satisfies $\mathrm{d}_a F_{bc} = 0$ and $\nabla_n F^{na} = J^a$.

Alternatively, we can sophisticate EM by taking the $U(1)$ gauge transformations of EM as arrows in our category of models.[3] Call this theory EM_S. Together, this gives us the following two categories:

$\mathbf{EM}_R$: Objects are models $\langle M, \eta_{ab}, F_{ab}, J^a\rangle$ of EM_R; arrows are diffeomorphisms which preserve the metric, Faraday tensor, and four-current.

[3] This is an example of what Dewar calls *external* sophistication. There is also the option of *internal* sophistication here, which requires the fibre bundles picture (Jacobs, 2023b). Note, however, that the theory of electromagnetism on a $U(1)$ principal bundle is categorically *inequivalent* to $\mathbf{EM}_R$ (Weatherall, 2018).

EM$_S$: Objects are models $\langle M, \eta_{ab}, A_a, J^a \rangle$ of EM$_S$; arrows are pairs (χ, φ_a), where φ_a is exact and χ is a diffeomorphism which jointly preserves the metric, (gauge-transformed) vector potential $A_a + \varphi_a$, and four-current.

These are once more categorically equivalent. Let F_{EM} be the functor which takes each object $\langle M, \eta_{ab}, A_a, J^a \rangle$ in $\mathbf{EM}_S$ to the object $\langle M, \eta_{ab}, \mathrm{d}_a A_b, J^a \rangle$ in $\mathbf{EM}^R$ and each arrow (χ, φ_a) to χ. Then

Proposition 3. *F_{EM} is an equivalence of categories.*

Proof. See Weatherall (2015). □

Furthermore, just as with the previous examples, EM$_R$ posits brute facts which are explained by EM$_S$. Here, the apparent brute fact is the Gauss-Faraday law $\mathrm{d}_a F_{bc} = 0$. This is a mathematical identity if F_{ab} is defined as $F_{ab} = 2\mathrm{d}_a A_b$ in EM$_S$, but in EM$_R$ it seems a cosmic conspiracy: the Faraday tensor behaves exactly *as if* it is in fact the exterior derivative of an electromagnetic one-form.

Summing up the morals of this section, in each case:

1. The reduced and sophisticated theory are categorically equivalent;
2. The sophisticated theory can explain brute facts of the reduced theory;
3. They present different metaphysical pictures in terms of which quantities are fundamental (in particular, reduced theories typically have a comparative ontology where sophisticated theories have an absolutist one).

Although this series of examples does not constitute a proof, they show that reduced theories often imply cosmic conspiracies despite their categorical equivalence to sophisticated theories. Indeed, Jacobs (2024a) shows that if the fundamental quantities of a reduced theory are *relational*, *invariant* functions of the old theory's fundamental quantities, such 'cosmic conspiracies' occur whenever the symmetry in question has a free and transitive action. This is the case for all of the examples considered above, as well as a number of other examples discussed by Dewar and Jacobs.[4]

3 Kinematical Equivalence

Roughly, then, it looks like reduced and sophisticated theories are 'the same but different'. They are the same insofar as they are categorically equivalent, and different in that they enable different explanations. But there is a tension: if the two theories are equivalent—that is, if they 'say the same' about what the world is like—then how can they explain one fact in different ways? Moreover, while the notion of sameness is made formally precise in terms of categorical

[4] The case of Newton-Cartan theory versus Maxwell Gravitation, which Jacobs (2021) identified as an example of reduction versus sophistication, is somewhat tricky; see Jacobs (2023a) and especially March (2024b,a) for more discussion.

equivalence, the difference relies on more opaque notions such as fundamentality and explanatory power.

The aim of this section is to make the difference between reduced and sophisticated theories equally precise. In particular, we conjecture that reduced and sophisticated theories are not *kinematically equivalent*. This novel concept of equivalence was introduced by March (2024b). Unlike previous discussions, which solely consider the dynamically possible models (DPMs) of a theory, kinematic equivalence also takes into account the broader space of the theory's kinematically possible models (KPMs).[5] Broadly, the space of KPMs of a theory consists of models that contain the right kind of objects. For example, if some theory posits a relativistic spacetime and scalar field, then the KPMs of that theory are models of a scalar field in a relativistic spacetime. The DPMs of a theory form a subspace of its KPMs, namely those KPMs that satisfy some dynamical equations of motion. In the example, the DPMs are those models in which the scalar field behaves in the right way. Roughly, then, we can think of the space of KPMs as delineating the kind of systems a theory is able to model appropriately, before we can ask whether or not the theory is true or false of those systems (i.e. whether or not the dynamical equations of motion are satisfied). This idea is articulated clearly by Curiel:

> [It] is satisfaction of the kinematical constraints that renders meaning to those terms representing a system's physical quantities in the first place, even before one can ask whether or not the system satisfies the theory's equations of motion. (Curiel, 2016)

Whilst we don't agree with everything Curiel has to say about kinematical possibility, we concur completely with this statement. (See March (2024b) for a more thoroughgoing comparison of Curiel's notion of kinematical possibility and the one we will introduce below.)

Following March (2024b) and Caulton (2024), our guiding principle for the construction of the space of KPMs of a theory will be a version of the principle of free recombination.[6] This principle says that if the models of a theory contain a collection of quantities X_i then arbitrary combinatorial arrangements of values for the X_i count as KPMs of the theory. The principle of free recombination is a consequence of what is known as Hume's dictum:

Hume's dictum: There are no necessary connections between wholly distinct entities.

However, care is needed in identifying what the relevant quantities are that are subject to the principle of free recombination. March (2024b) identifies three reasons for which the base units for free recombination may differ from the naïve version of free recombination outlined above. The first reason is that we

[5] Wolf and Read (2023) consider theoretical equivalence in light of 'boundary possible models', a subset of KPMs in which certain boundary conditions are satisfied but which are not necessarily DPMs.

[6] However, our points about explanation may not require any such modal principle; see §5.2.

may not wish to regard all the objects in the models of a theory as ontologically or conceptually independent. For example, in full Newtonian spacetime $\langle M, t_a, h^{ab}, \nabla, \xi^a \rangle$, the connection ∇ is definable from the metrics and standard of rest. The second two reasons are related to the fact that some of the objects in the models of a theory may have some of their properties by kinematical, rather than dynamical necessity. March (2024b) suggests two ways in which this can happen. One is that some properties might be part of the definition of an object in the theory's models. For instance, perhaps part of what we mean by Galilean spacetime is that it is flat. This would mean that the base units for free recombination for theories set on Galilean spacetime are not an arbitrary connection, but a flat connection. The other is that some of the properties of objects in the theory's models may express domain restrictions, that is, restrictions on the kind of physical systems the theory is able to treat, such as the metric signature in general relativity. This would mean that the base units for free recombination for relativity theory are not arbitrary metrics, but Lorentzian metrics. Fortunately, these complications do not seem to play a role in our examples.

With the kinematics-dynamics distinction on the table, we can now introduce March's (2024b) criterion of kinematical equivalence. This is a straightforward generalisation of categorical equivalence. First, given a theory T, we can define a theory T^k whose models take the same form as T and whose axioms are just the kinematical constraints of T. In other words, T^k denotes T's space of KPMs. Then we can associate a category of models $\mathbf{T}^k$ to T^k in much the same way as we could for T, subject to the conditions that:

- If T has an associated category of models $\mathbf{T}$, then $\mathbf{T}$ is a full subcategory of $\mathbf{T}^k$.
- If $\mathfrak{M} \in \text{ob}(\mathbf{T})$ and $\mathfrak{M}' \in \text{ob}(\mathbf{T}^k) \backslash \text{ob}(\mathbf{T})$ then $\hom_{\mathbf{T}^k}(\mathfrak{M}, \mathfrak{M}') = \hom_{\mathbf{T}^k}(\mathfrak{M}', \mathfrak{M}) = \varnothing$.

The first condition simply states that the category of DPMs is a subcategory of the category of KPMs. The second condition states that no arrow of $\mathbf{T}^k$ can take a mere KPM to a DPM or vice versa. This makes sense if models related by arrows are taken as physically equivalent. See March (2024b) for further discussion.

We can now state the criterion of kinematical categorical equivalence ('kinematical equivalence' for short):

Kinematical categorical equivalence Let T_1, T_2 be theories, and let T_1^k, T_2^k be their associated kinematical theories. Let $\mathbf{T}_1^k$, $\mathbf{T}_2^k$ denote their associated categories of models. Then T_1, T_2 are kinematically categorically equivalent just in case there is an equivalence of categories $F : \mathbf{T}_1^k \to \mathbf{T}_2^k$ such that for all $M_1^k \in \text{ob}(\mathbf{T}_1^k)$, $F(M_1^k) \in \text{ob}(\mathbf{T}_2)$ iff $M_1^k \in \text{ob}(\mathbf{T}_1)$ (which preserves empirical content).

With this criterion in hand, let us now reconsider our examples from §2. We will show that on plausible choices for the kinematical constraints of those theories, they are *not* kinematically equivalent.

3.1 Handedness

Beginning with H_R, the dynamical constraints of this theory are the sentences $\forall x C(x,x)$, $\forall x \forall y (C(x,y) \rightarrow C(y,x))$, $\forall x \forall y \forall z ((C(x,y) \wedge C(y,z)) \rightarrow C(x,z))$, $\forall x \forall y \forall z ((\neg C(x,y) \wedge \neg C(y,z)) \rightarrow C(x,z))$. Which of these, if any, are kinematical constraints?

Here, we suggest that there are two plausible choices. The first option is to implement a naïve version of Hume's dictum, and take the kinematical theory of H_R as the empty theory. This would amount to taking the fundamental unit for free recombination to be *some* two-place relation, but not to place any constraints on what kind of relation this is. However, there is an alternative option, which is to notice that the sentences $\forall x C(x,x)$, $\forall x \forall y (C(x,y) \rightarrow C(y,x))$, $\forall x \forall y \forall z ((C(x,y) \wedge C(y,z)) \rightarrow C(x,z))$ state that C is an equivalence relation. The second options then takes the fundamental units for free recombination in H_R to be an equivalence relation. The fourth axiom, $\forall x \forall y \forall z ((\neg C(x,y) \wedge \neg C(y,z)) \rightarrow C(x,z))$, is a dynamical constraint. We will focus on this option in what follows.[7]

H_S, on the other hand, has no dynamical constraints, and therefore no kinematical constraints—every KPM of H_S is a DPM of H_S.

Together, this gives us the following kinematical categories:

$\mathbf{H}^k_R$: objects are $\Sigma_C = \{C(x,y)\}$ structures that satisfy $\forall x C(x,x)$, $\forall x \forall y (C(x,y) \rightarrow C(y,x))$, $\forall x \forall y \forall z ((C(x,y) \wedge C(y,z)) \rightarrow C(x,z))$; arrows are model isomorphisms.

$\mathbf{H}^k_S$: objects are structures $\langle D, \mathbf{2}, \chi \rangle$ where $\chi : D \rightarrow \mathbf{2}$; arrows are model isomorphisms (f, g).

Let F^k_H be the functor which takes each object $\langle D, \mathbf{2}, \chi \rangle$ in $\mathbf{H}^k_S$ to an object $\mathfrak{M}$ in $\mathbf{H}^k_R$ such that $D_{\mathfrak{M}} = D$, and for any $a, b \in D_{\mathfrak{M}}$, $\langle a, b \rangle \in C$ iff $\chi(a) = \chi(b)$; and each arrow (f, g) to f. Then

Proposition 4. *F^k_H is not an equivalence of categories; it is full and faithful, but not essentially surjective.*

Proof. F^k_H is full and faithful by the proof of proposition 1. But it is not essentially surjective. To see this, consider any object in $\mathbf{H}^k_R$ that does not satisfy $\forall x \forall y \forall z ((\neg C(x,y) \wedge \neg C(y,z)) \rightarrow C(x,z))$ (such exist). By construction, this is not the image of any object in $\mathbf{H}^k_S$ under F^k_H. □

Therefore, the reduced and sophisticated theories of handedness are not kinematically equivalent. (Moreover, the fact that the spaces of KPMs and DPMs of H_S coincide in fact entails a stronger result: only when the dynamical constraints of H_R are just those of H_R are the theories kinematically equivalent, because otherwise any equivalence functor between the two theories will map mere KPMs of H_R to DPMs of H_S.)

[7] As we will see shortly, this does not make a difference to the kinematical inequivalence results, since if H_R and H_S are kinematically inequivalent under this second choice of kinematical constraints, they will also be inequivalent under the first choice.

3.2 Absolutism vs. comparativism about mass

When we turn to our second example, the obvious option for the fundamental units for free recombination for M_R are $\mathcal{M}$, ϕ, $\gamma(i)$, and the mass ratios $m(i,j)$ (here, as before, we set aside issues of the dynamics for the $\gamma(i)$). This would make the condition $m(i,j)m(j,k) = m(i,k)$ a dynamical law. For M_S, we take the fundamental units for free recombination to be $\mathcal{M}$, ϕ, $\gamma(i)$, and the mass values $m(i)$. This gives us the following kinematical categories:

$\mathbf{M}_R^k$: objects are structures $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), \mathbb{R}^+, m(i,j)\rangle$; arrows are pairs (χ, f) consisting of a diffeomorphism $\chi : M \to M'$ and a bijection on the domain of $\mathbb{R}^+$ which jointly preserve $\mathbb{R}^+$, $\mathcal{M}$, ϕ, $\gamma(i)$ and $m(i,j)$.

$\mathbf{M}_S^k$: objects are structures $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i)\rangle$; arrows are pairs (χ, f) consisting of a diffeomorphism $\chi : M \to M'$ and a bijection on D_m which jointly preserve $\mathcal{M}$, $\leq$, $\circ$, ϕ, $\gamma(i)$ and $m(i)$.

Let F_M^k be the functor which takes each object $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i)\rangle$ in $\mathbf{M}_S^k$ to the object $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), D_m, \leq, \circ, m(i,j)\rangle$, $m(i,j) = m(i)/m(j)$ in $\mathbf{M}_R^k$ and each arrow (χ, f) to $(\chi, \mathrm{id}_{\mathbb{R}^+})$. Then

Proposition 5. *F_M^k is not an equivalence of categories; it is full and faithful, but not essentially surjective.*

Proof. F_M^k is full and faithful by the proof of proposition 2. But it is not essentially surjective. To see this, consider any object $\langle \mathcal{M}, \phi, \mathcal{B}, \gamma(i), \mathbb{R}^+, m(i,j)\rangle$ in $\mathbf{M}_R^k$ such that $m(i,j)m(j,k) \neq m(i,k)$ (such exist). By construction, this is not the image of any object in $\mathbf{M}_S^k$ under F_M^k. □

Therefore, M_R and M_S are not kinematically equivalent.

3.3 Electromagnetism

Finally, consider our third example: electromagnetism. The dynamical constraints of EM_R are $\mathrm{d}_a F_{bc} = 0$ and $\nabla_n F^{na} = J^a$. If the fundamental units of EM_R for free recombination are (arbitrary) two-forms and four-currents,[8] then neither $\mathrm{d}_a F_{bc} = 0$ nor $\nabla_n F^{na} = J^a$ are kinematical constraints. Similarly, if the fundamental units of EM_S for free recombination are (arbitrary) one-forms and four-currents, then $2\nabla_n \nabla^{[n} A^{a]} = J^a$ is also not a kinematical constraint.

Again, we can use this to define kinematical categories for EM_R and EM_S:

$\mathbf{EM}_R^k$: objects are structures $\langle M, \eta_{ab}, F_{ab}, J^a\rangle$ where η_{ab} is flat and F_{ab} is a two-form; arrows are diffeomorphisms which preserve the metric, Faraday tensor, and four-current.

$\mathbf{EM}_S^k$: objects are structures $\langle M, \eta_{ab}, A_a, J^a\rangle$ where η_{ab} is flat and A_a is a one-form; arrows are pairs (χ, φ_a), where φ_a is exact and χ is a diffeomorphism which preserves the metric, (gauge-transformed) vector potential $A_a + \varphi_a$, and four-current.

[8] What if we take the relevant units to be *closed* two-forms? We discuss this option in §5.2.

Let F^k_{EM} be the functor which takes each object $\langle M, \eta_{ab}, A_a, J^a \rangle$ in $\mathbf{EM}^k_S$ to the object $\langle M, \eta_{ab}, \mathrm{d}_a A_b, J^a \rangle$ in $\mathbf{EM}^k_R$ and each arrow (χ, φ_a) to χ. Then

Proposition 6. *F^k_{EM} is not a kinematical categorical equivalence; it is full and faithful, but not essentially surjective.*

Proof. F^k_{EM} is full and faithful by the proof of proposition 3. But it is not essentially surjective. To see this, consider any object $\langle M, \eta_{ab}, F_{ab}, J^a \rangle$ in $\mathbf{EM}^k_R$ such that $\mathrm{d}_a F_{bc} \neq 0$ (such exist). By construction, this is not the image of any object in $\mathbf{EM}^k_S$ under F^k_{EM}. □

In the same way in which Dewar (2019) conjectures on the basis of a range of examples that reduced and sophisticated theories are generally categorically equivalent, then, these results lead us to conjecture that, in general, reduced and sophisticated theories are kinematically *in*equivalent. In particular, it seems plausible to us that there always exists some natural choice of functor between the sophisticated and the reduced theory that is not essentially surjective. It would require further research, however, to show that this is invariably the case.

4 (In)equivalences

We will now use these results to comment on the (in)equivalence between reduced and sophisticated theories with respect to ontology and explanatory strength.

4.1 Ontology

We believe that the above shows a sense in which reduced and sophisticated theories are *not* equivalent. In our opinion, KPMs are a crucial part of a theory. For one, it is impossible even to specify the DPMs before one has specified the theory's kinematical structure, namely its KPMs. Moreover, considerations of KPMs play an important role in symmetry-based inferences. Earman (1989) famously posited that a theory's external dynamical symmetries should match its spacetime symmetries. Hetzroni (2019), Jacobs (2024b) and others have recently expanded this principle to include internal symmetries. The expanded principle says that a theory's dynamical symmetries—external *and* internal—should match its kinematical symmetries, where the latter are the automorphisms of the theory's kinematical structures. This includes a theory's spacetime, but also internal value spaces such as a principal fibre bundle. These principles are highly useful in determining the minimal and maximal structure a theory is committed to, but they require consideration of KPMs as well as DPMs. Given that KPMs are just as much part of a theory as DPMs, then, we believe that kinematically inequivalent theories are not theoretically equivalent.

What is the physical difference between such theories? We mention two: kinematical possibilities and fundamental quantities. As for the former, kinematically inequivalent theories countenance different counterfactuals. For example, consider a world with three objects, A, B and C, such that A and B are opposite-handed, and B and C are same-handed. By the dynamical constraints of the reduced theory of handedness, it follows that A and C are also opposite-handed. Consider now the counterfactual scenario in which A and C are same-handed. All else equal, this would violate the dynamical constraints of H_R. On the other hand, in H_S, handedness relations supervene on handedness properties. If A and C were same-handed, then either A or C would have a different handedness. In either case, this would automatically affect the handedness relations between A and B and B and C respectively such that they satisfy the constraints. Put differently, in a reduced theory the counterfactual would take us outside of the space of DPMs, whereas in the sophisticated theory it stays within the space of DPMs. Kinematically inequivalent theories treat counterfactuals differently.

The difference in fundamental quantities relies on a version of Hume's dictum. This principle enables one to infer a theory's ontology from its space of KPMs. The core idea is that the theory's fundamental quantities span the space of KPMs, that is, for any combination of values of those quantities there is a model in the space of KPMs. Given a set of fundamental quantities, then, one can construct the space of KPMs. This is the 'Upwards Hume's Dictum' (Caulton, 2024). We take this as definitional of kinematical possibility: the space of kinematical possibilities simply consists of all possible ways in which to combine the theory's fundamental quantities. Conversely, given a space of KPMs, one can infer the fundamental quantities, namely any set of quantities that is sufficient to span the space of KPMs.[9] This is the 'Downwards Hume's Dictum'. We fully subscribe to Caulton's two-fold use of Hume's dictum. We should thus expect kinematically inequivalent theories to have inequivalent ontologies.

In the case of handedness, for example, handedness-relations are *not* fundamental quantities of the sophisticated theory, since they generate too many kinematical possibilities. One way to combine handedness-relations is such that A and B are same-handed, B and C are same-handed yet A and C are opposite-handed. If the handedness-relation is fundamental, then this should correspond to some KPM—as it does in the reduced theory. But there is no KPM of the sophisticated theory—that is, no distribution of handedness *properties* to A, B and C—that corresponds to this distribution of handedness-relations. The other examples work the same way: comparative mass relations overgenerate KPMs, so they are are not fundamental quantities of a sophisticated theory of mass; Faraday tensors overgenerate KPMs, so they are not fundamental quantities of electromagnetism.

It is true that one can define the sophisticated ontology from the *dynamically* possible models of the reduced theory. For example, for any DPM of the handedness theory, there is an assignment of handedness properties consistent with it. This assignment is not unique: if some assignment is consistent with

[9]This leaves it open that there are multiple choices of 'basis' of fundamental quantities. This strikes us as correct: for example, one can describe possibilities in terms of {'green', 'blue'} or in terms of {'grue', 'bleen'}. Mere inspection of the space of KPMs will not rule in favour of either.

the theory's model, then another assignment in which each left-handed object is right-handed and vice versa is also consistent. There is, then, a sense in which the advocate of the reduced theory is committed to handedness properties. But this is not a commitment to handedness properties as part of the theory's fundamental ontology. It is quite clear that in this case, handedness properties supervene jointly on the distribution of handedness relations *and* the dynamical laws. The ontological equivalence in this case is merely extensional. We thus part ways with Dewar's claim that reduced and sophisticated theories have the same 'intensional ontology'.[10]

4.2 Explanation

Next: explanatory inequivalence. Here we concur with Dewar that reduced and sophisticated theories have different explanatory structures, although we think this is just a consequence of their different ontologies. The difference is that sophisticated theories explain certain relations at the level of kinematics, whereas reduced theories explain them at the level of dynamics.

Consider, again, the claim that handed objects are partitioned into at most two equivalence classes. On the sophisticated theory, this is true in any KPM. There is no way to assign objects handedness properties in a way that does not partition them into at most two equivalence classes. The partitioning thus follows from the theory's kinematical posits, namely a pair of handedness properties. On the reduced theory, on the other hand, there are KPMs in which objects are not partitioned into same-handed equivalence classes, or are partitioned into more than two equivalence classes. However, the dynamics of the reduced theory are such that in any DPM the objects are partitioned into at most two equivalence classes. The reduced theory could either include a specific axiom to that effect, or it could follow as a theorem from other axioms. In either case, the explanation here is dynamical rather than kinematical.

The same pattern occurs in our other examples. For example, the transitivity of mass ratios ($m(i,j)m(j,k) = m(i,k)$) is satisfied in any KPM of the absolutist theory—it simply follows from that theory's kinematical posits—but not in all the KPMs of the comparativist theory. It *is* satisfied in any DPM of the comparativist theory, so it may have a dynamical explanation. The same is the case for Gauss-Faraday law too.

We believe that not only is there this explanatory difference between reduced and sophisticated theories, but also that the latter's explanations are clearly better. We offer three reasons for this:

Firstly, kinematic possibility is broader than dynamic possibility in that any dynamic possibility is also a kinematic possibility but not vice versa. So, the explanation offered by sophisticated theories is modally more robust. For example, on the reduced theory the fact that objects are partitioned into at most two handedness equivalence classes could fail to hold if the laws were different.

[10] Indeed, Caulton (2024) shows how one can use KPMs to define an intensional semantics for theory.

On the sophisticated theory, on the other hand, this would hold *even if the laws were different*—as long as the ontology remains the same. In Lange's (2007) terms, such explanations are *nomically stable*.

Secondly, sophisticated explanations are often local, whereas reductionist explanations are non-local. For example, the postulate that objects are partitioned into handed equivalence classes requires non-local correlations between the handedness relations of different objects. If A and B are same-handed, and B and C are also same-handed, then A and C are forced to be same-handed too—no matter how far apart A, B and C are! In the sophisticated theory, on the other hand, A, B and C simply possess an intrinsic handedness property, and the properties possessed by A and B do not constrain the one possessed by C at all. It is true that these objects stand in non-local handedness relations, but these merely supervene on local, intrinsic properties.

Thirdly, the sophisticated explanation is more insightful. Suppose one is truly surprised by the fact that objects are partitioned into at most two equivalence classes of same-handed objects. We submit that, if one is subsequently told that every object in fact possesses one of two monadic handedness properties, this is a satisfactory explanation. On the other hand, if one is told that this is just a (consequence of some) law, then one has not explained much. The request was exactly for the explanation of a noticeable regularity, but to say that that regularity is a law does not offer much more. The explanation that 'F is true, because F is a law' barely illuminates. In Taylor's (2023) terms, there is insufficient 'explanatory distance' between the explanandum and the explanans.

To sum up our verdict on the equivalence of reduced and sophisticated theories: such theories are both ontologically and explanatorily inequivalent in virtue of their kinematic inequivalence. Moreover, the explanatory inequivalence favours sophistication over reduction.

5 Objections and Responses

We can see two ways in which the reductionist might respond: broaden the class of kinematical possibilities of sophisticated theories, or restrict the class of kinematical possibilities of reduced theories. In either case, reduced and sophisticated theories become kinematically equivalent. In the former case, both theories will face cosmic conspiracies; in the latter, neither does.[11]

5.1 Broaden KPMs of sophisticated theories

The reductionist could respond that the KPMs of the sophisticated theory are arbitrarily restricted. To see this, consider another example of a cosmic conspiracy not yet discussed in this paper: the triangle inequality (Maudlin, 2007;

[11] This parallels Jacobs' (2023a) distinction between two ways make a theory with redundant structure satisfy Earman's principles (that a theory's dynamical symmetries match its kinematical symmetries): broaden the class of kinematical symmetries (i.e. weaken the kinematical structure) or narrow the class of dynamical symmetries (i.e. constrain the dynamics).

Jacobs, 2024a). For any three points A, B and C, the distance r_{AB} between A and B added to the distance r_{BC} between B and C must equal or exceed the distance r_{AC} between A and C: $r_{AB}+r_{BC} \geq r_{AC}$. For a theory set on Euclidean space, this is a consequence of the theory's kinematical structure. It is simply impossible to locate three objects within Euclidean space such that the triangle inequality is not satisfied. For a theory in which distances are fundamental, on the other hand, the triangle inequality is not true in every KPM but must follow from the dynamics. The case thus runs parallel to those we have already discussed.

But the relationist could retort that the absolutist's explanation relies on space being Euclidean, and why should we hold *that* fact fixed across KPMs? If we were to drop this assumption, then the triangle inequality is not satisfied in every KPM of the absolutist theory either. The same holds for the examples discussed above: for instance, the comparativist about mass could say that mass value space need not have an additive structure. Caulton (2024) believes that the space of KPMs is inclusive in this sense. After all, there are ways to combine the absolutist's fundamental ontology, which includes spacetime points and distances between them, in such a way that space is not Euclidean.

We depart from Caulton here. Firstly, we think that on a broadly structuralist account of metaphysics this move would trivialise the space of kinematical possibilities. For if no none of the theory's properties and relations are subject to kinematical constraints, then the kinematical possibilities will contain every possible extension for those properties and relations. The only way in which the space of KPMs can differ, then, is if their domains have a different cardinality or if they posit a different number of fundamental n-place relations for some n. This would mean, for example, that any theory that posits an infinite four-dimensional spacetime over which a two-place distance relation is defined has the same space of KPMs—no matter whether it is classical or relativistic. We would like to preserve a contentful notion of KPMs that is compatible with structuralism.

Secondly, we think the relationist's response is unfair. *Any* theory has to posit some kinematical structure. The absolutist posits an Euclidean space, but the relationist likewise has to posit a 'distance space' in which pairs of objects take their value. This distance space certainly has a non-trivial structure, namely that of the positive real numbers (perhaps quotiented by scale transformations (Martens, 2024)). This structure is necessary to explain (for example) the fact that if r_{AB} is smaller than r_{BC}, and r_{BC} is smaller than r_{CD}, then r_{AB} is also smaller than r_{CD}. The situation is thus rather as follows: any theory has to posit a set of kinematic (value) spaces and a set of functions between those spaces. Hume's dictum dictates that the space of KPMs contains any combination of values for those functions: the theory's quantities can have any value at any point in spacetime. But it does not dictate that the value spaces can have any structure whatsoever. This restriction of Hume's dictum is neutral between reduction and sophistication, but it upholds our result that reduced theories are kinematically inequivalent to sophisticated ones.

5.2 Restrict KPMs of reduced theories

Conversely, the reductionist could try to restrict their space of KPMs. In particular, they could try to re-introduce the above-discussed cosmic conspiracies as kinematic constraints. This defines a new theory, T_R^+, with a smaller space of KPMs. It is easy to see that T_R^+ *is* kinematically equivalent to T_S.

We think this is a cheat. The kinematical possibilities by their very definition are unrestricted: they consist of all possible ways to combine the fundamental quantities in question. It is not possible to arbitrarily impose constraints on them. Put differently, if we take the space of KPMs of T_R^+, then by Caulton's downwards Hume's Dictum this theory's fundamental quantities are *not* the relational quantities, but the absolute ones. That is because the absolute quantities span this restricted space of KPMs, while the relational ones do not. The reductionist who constrains their KPMs to enforce kinematical equivalence with the sophisticated theory betrays their reduction.

Of course, this response does assume the truth of Hume's dictum. Someone who rejects Hume's dictum might find ways to impose further kinematic constraints. This option is discussed by Dasgupta (2020), who writes that "you might propose that principles like triangle inequality restrict which ways of reorganizing fundamental matters are genuinely possible. I'll call these 'metaphysical principles', though I leave open whether there are any and what they might be." Such 'metaphysical principles' flatly contradict Hume's dictum.

We lack the space here to discuss Dasgupta's proposal—to which he himself does not clearly subscribe—in detail. Suffice it to say that it is rather unclear what such metaphysical principles are and how they would come to have the modal force they are proposed to have. Indeed, it strikes us that even if the space of KPMs were modally restricted by such metaphysical principles, this would not allow the reductionist to *explain* those principles in any way. This point is reminiscent of Dasgupta's (2011) broader claim that modal theses are irrelevant to many debates about symmetries: even if only the actual world were metaphysically possible, this would not by itself explain why the world is as it is. Likewise, a mere modal restriction on the KPMs does not increase the reduced theory's explanatory strenght.

We think that explanations of cosmic conspiracies in terms of relative fundamentality, such as offered by the sophisticator, are much clearer. Herein we follow Ismael and Schaffer (2020):

> Grounding Inference [their term for certain inferences similar to those based on the downwards Hume's dictum] simply says that all else being equal, in the kind of epistemic setting in which we have no direct access to the grounding substructure of a collection of objects, a theory that explains constraints on their modal covariation by reference to a common ground is better than one that regards it as a brute modal connection between distinct existences. If one looks at any theory that gives a non-trivial account of what the fundamental entities are (i.e., if it says that not everything is fundamental),

> some constraints on mutual variation of non-fundamental entities in the world will turn out to be emergent from grounding substructure Grounding Inference expresses a preference for theories that trace modal connections to common grounds over ones that don't.

We share this preference and believe that it is satisfied by explanations of cosmic conspiracies in terms of kinematical posits.

This takes us to a broader final point. Whilst the bare formalism of a theory may leave us with some flexibility as to how the kinematical constraints of that theory are chosen, one cannot choose those kinematical constraints independently of one's interpretation of the theory. Thus, while it might be open to the proponent of, say, the Faraday tensor formulation of electromagnetism (FTEM) to insist that the Gauss-Faraday law is a kinematical constraint, this does not mean that it is open to them to insist both that the Gauss-Faraday law is a kinematical constraint *and* that the Faraday tensor is fundamental.

To make this point, it is helpful to consider in a little more detail what it would mean to take the Gauss-Faraday law as a kinematical constraint (rather than a dynamical law). Here, it is helpful to recall one of the ideas from §3: kinematical constraints delineate the kind of physical systems which can be appropriately modeled by the theory. If the Gauss-Faraday law is a dynamical law, then if this law were to fail to hold of some physical system, FTEM would be false. But it would still make sense to try to apply FTEM to that system. By contrast, if the Gauss-Faraday law is a kinematical constraint, then if this law were to fail to hold of some physical system, it would not even make sense to think of FTEM as *applicable* to that system, much less as true or false of it. For example, on the first view, it makes sense to say that if there were magnetic monopoles, then FTEM would be false. On the second view, however, if there were magnetic monopoles then FTEM would not be false *per se*, but simply inapplicable (and hence questions of truth or falsity inappropriate). For in that case, the theory's basic kinematical structure would not fit the actual world.

It is the first option that is in accordance with physics practice. So, how are we to make sense of the second option? A comparison with gauge potential electromagnetism (GPEM) is helpful here. GPEM has a clear story about why it would not make sense to talk of EM as true or false of physical systems in which magnetic monopoles exist, because such systems are ones in which electromagnetic forces cannot be represented by a one-form field at all. Thus one cannot sensibly ask of such a physical system whether electromagnetic forces, as represented by a one-form field, couple to matter currents in the right kind of way. By contrast, it is much less obvious that one can tell a similar story for FTEM. After all, as long as one has electromagnetic forces, one can define the Faraday tensor, and once one has the Faraday tensor it makes sense to ask whether it is closed or how it couples to matter currents.

Unless, that is, FTEM is really committed to the gauge potential as the fundamental quantity in terms of which the Faraday tensor is defined. Then it would of course *not* make sense to ask whether electromagnetic forces, as represented by the Faraday tensor, couple to matter fields in the right kind of way in a physical system in which the Gauss-Faraday law does not hold, because electromagnetic forces are represented by the Faraday tensor in virtue of being represented by the gauge potential. Thus one sees: it makes sense to take the Gauss-Faraday law as a kinematical constraint if and only if one interprets FTEM as committed to a fundamental gauge potential.

Of course, one way to make this point is via the downwards Hume's dictum, but one need not be committed to an unrestricted version of Hume's dictum to make it. It is entirely legitimate to say that there are some entities which satisfy an unrestricted version of Hume's dictum and others—such as background spacetime structure—which don't. It is also possible to insist that perhaps it is just part of what we mean by the Faraday tensor (or a 'metaphysical principle' about the Faraday tensor) that it is closed. One issue with this is the one we pressed above—that the relevant sense of 'metaphysical principle' is obscure and unexplanatory. Another issue is that such a principle, shorn of commitment to the gauge potential, is ill-motivated—as indicated by physicists' discussions of cases in which the Gauss-Faraday law does not hold. But more importantly, even if one does grant for the sake of argument that such 'metaphysical principles' are clear and well-motivated, there is an obvious sense in which they cannot help. The question whether some equation-like statement is a kinematical constraint or a dynamical law ultimately is a question about what kind of physical situations it is sensible to model with the theory. The example of the Gauss-Faraday law shows that the answer to this question is not independent from one's ontological commitments.

References

Caulton, A. (2024). Hume's dictum as a guide to ontology. https://philsci-archive.pitt.edu/24004/.

Curiel, E. (2016). Kinematics, Dynamics, and the Structure of Physical Theory. *arXiv:1603.02999 [physics]*.

Dasgupta, S. (2011). The Bare Necessities. *Philosophical Perspectives*, 25(1):115–160.

Dasgupta, S. (2020). How to Be a Relationalist. In *Oxford Studies in Metaphysics Volume 12*. Oxford University Press, Oxford.

Dasgupta, S. (2021). Symmetry and Superfluous Structure: A Metaphysical Overview. In Knox, E. and Wilson, A., editors, *The Routledge Companion to Philosophy of Physics*. Routledge.

Dewar, N. (2019). Sophistication about Symmetries. *The British Journal for the Philosophy of Science*, 70(2):485–521.

Earman, J. (1989). *World Enough and Spacetime*. MIT press.

Healey, R. (2007). *Gauging What's Real: The Conceptual Foundations of Contemporary Gauge Theories*. Oxford University Press, Oxford, New York.

Hetzroni, G. (2019). Gauge and Ghosts. *The British Journal for the Philosophy of Science*, (axz021).

Ismael, J. and Schaffer, J. (2020). Quantum holism: Nonseparability as common ground. *Synthese*, 197(10):4131–4160.

Ismael, J. and van Fraassen, B. C. (2003). Symmetry as a Guide to Superfluous Theoretical Structure. In Brading, K. and Castellani, E., editors, *Symmetries in Physics: Philosophical Reflections*, pages 371–92. Cambridge University Press.

Jacobs, C. (2021). *Symmetries As a Guide to the Structure of Physical Quantities*. D.Phil Thesis, University of Oxford.

Jacobs, C. (2023a). Are Dynamic Shifts Dynamical Symmetries? *Philosophy of Science*, 90(5):1352–1362.

Jacobs, C. (2023b). The metaphysics of fibre bundles. *Studies in History and Philosophy of Science*, 97:34–43.

Jacobs, C. (2023c). The Nature of a Constant of Nature: The Case of G. *Philosophy of Science*, 90(4):797–816.

Jacobs, C. (2024a). Comparativist Theories or Conspiracy Theories? *Journal of Philosophy*, 121(7):365–393.

Jacobs, C. (2024b). Symmetries As a Guide to Physical Ideology. https://philsci-archive.pitt.edu/24197/.

Lange, M. (2007). Laws and meta-laws of nature: Conservation laws and symmetries. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 38(3):457–481.

March, E. (2024a). Are Maxwell Gravitation and Newton-Cartan Theory Theoretically Equivalent? *The British Journal for the Philosophy of Science*.

March, E. (2024b). Categorical equivalence and the kinematics-dynamics distinction. *The British Journal for the Philosophy of Science*.

March, E. (2024c). What is the value in an intrinsic formalism? https://philsci-archive.pitt.edu/23220/.

Martens, N. C. M. (2024). *Philosophy of Physical Magnitudes*. Elements in the Philosophy of Physics. Cambridge University Press, Cambridge.

Maudlin, T. (2007). Suggestions from Physics for Deep Metaphysics. In *The Metaphysics Within Physics*. Oxford University Press.

Møller-Nielsen, T. (2017). Invariance, Interpretation, and Motivation. *Philosophy of Science*, 84(5):1253–1264.

Taylor, E. (2023). Explanatory Distance. *The British Journal for the Philosophy of Science*, 74(1):221–239.

Weatherall, J. O. (2015). Are Newtonian Gravitation and Geometrized Newtonian Gravitation Theoretically Equivalent? *arXiv:1411.5757 [gr-qc, physics:physics]*.

Weatherall, J. O. (2018). Erratum: Understanding gauge. *Philosophy of Science*, 85(2):325–325.

Wolf, W. J. and Read, J. (2023). Respecting boundaries: Theoretical equivalence and structure beyond dynamics. *European Journal for Philosophy of Science*, 13(4):47.