“the logical sequence of his Principles”: Understanding Du Châtelet on Newton’s law of gravitation in the *Principia*

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Abstract

Émilie Du Châtelet (1706-1749) is perhaps equally well-known for her *magnum opus*, the *Institutions de Physique* of 1740, and for her later French translation of and commentary to Newton’s *Principia* (first published posthumously in 1756, with the corrected edition in 1759). One of the few topics which Du Châtelet addresses in detail in both the *Institutions de Physique* (chapter 15) and the commentary to her translation is Newton’s arguments for his law of gravitation in the *Principia*. To date, however, no systematic comparison of the two has been undertaken (and very little has been said on either of them separately). I reconstruct and compare these two accounts. This offers a new perspective on Du Châtelet’s developing thinking on the justification of Newton’s law of gravitation within the Newtonian system.

1 Introduction

This paper was motivated by a puzzle about Du Châtelet’s account of Newton’s arguments for his law of gravitation (hereafter NGL) given in the *Principia* (Book 3, props. 1-7).[1] Newton’s arguments for NGL are discussed in detail by Du Châtelet in both chapter 15 of the *Institutions de Physique* (*Foundations of Physics*, hereafter *Foundations*) of 1740, and then in the later commentary to her translation of the *Principia* (first published posthumously in 1756, with the corrected edition in 1759).[2] On the face of it, however, these two accounts differ from one another in significant ways. This raises the question: how, exactly, was Du Châtelet thinking about the arguments for NGL within the Newtonian

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[1] Recall that NGL states that the gravitational force of a point particle of masses M on a point particle of mass m is $-GMm/r^2\hat{\mathbf{r}}$, where $\mathbf{r}$ is the vector displacement between them.

[2] See Scriba (1971) for discussion of the two editions.

system at the time of writing of, respectively, the *Foundations* and commentary? And how, if at all, did her thinking on this change?[3]

Both chapter 15 of the *Foundations* and the commentary have only recently been translated into English,[4] and, to date, no systematic comparison of the two texts has been undertaken (and, within the English-speaking world, very little has been said on either of them separately).[5] The aim of this paper is to systematically reconstruct and then compare the accounts of Newton's arguments for NGL given by Du Châtelet in the *Foundations* and commentary. This has two philosophical payoffs. The first is that a comparison of Du Châtelet's accounts of Newton's arguments for NGL in the commentary and chapter 15 of the *Foundations* reveals several important threads in her argument which are difficult to see when the two texts are taken in isolation. In turn, this allows for a deeper and more faithful reconstruction of Du Châtelet's thinking on Newton's arguments for NGL than has been done previously.

Secondly, this paper aims to emphasise how much of Du Châtelet's thinking on the justification of NGL within the Newtonian system was original (a point also made by Smith (2022)). Whilst the arguments Du Châtelet presents are ones she attributes to Newton,[6] her treatment in the commentary of Newton's numerator in the law of gravitation is not one which is found in the *Principia* (nor in *De Mundi Systemate*). Moreover, Du Châtelet's thinking on the justification of NGL within the Newtonian system—how she understood "the logical sequence of his Principles" (Du Châtelet 1759, Introduction, 17)—is a question of historical and philosophical interest in its own right, both from the perspective of Du Châtelet scholarship, and of understanding the reception of Newton's work on the continent at the time.

As such, the plan for this paper is as follows. First, in §2, I present and reconstruct in detail Du Châtelet's accounts of Newton's arguments for NGL in both the *Foundations* (§2.1) and commentary (§2.2). I then, in §3, turn to the comparison of the two. After addressing the question whether such a comparison

[3]Note that these questions are doubly exegetical: they are about how Du Châtelet understood Newton's (or perhaps, an ideal Newtonian's) arguments for NGL. This is important, because in the *Foundations*, Du Châtelet does not see Newton's arguments as sufficient to establish universal attraction as an ultimate explanation for, e.g. Kepler's laws or the fall of bodies on Earth—largely due to her commitment to mechanical explanations, see e.g. Brading (2019, ch. 4).

[4]Chapter 15 of the *Foundations* was translated by Brading et al. (2018); the commentary was translated by Zinsser and Bour (2009).

[5]Smith's (2022) discussion of the commentary is a notable exception, and I will engage with his paper at several points in what follows. Reichenberger (2018) also undertakes a comparison of parts of the *Foundations* and commentary, but her focus is on Du Châtelet's discussion of Newton's laws of motion rather than NGL. Brading (2018, 2019) has some discussion of chapters 15-16 of the *Foundations*, though she gives very little attention to chapter 15, and her aims are importantly different from mine: Brading is interested in Du Châtelet's assessment of the merits and demerits of Newtonian universal gravitation as compared with the alternative vortex theory (favoured in France at the time), cf. fn. 3. See also Chen (2021), Detlefsen (2019), and Hecht (2012) for philosophical discussion of chapter 16 of the *Foundations*, and Emch and Emch-Dériaz (2006), Toulmonde (2022), and Zinsser (2001) for historical discussion of the commentary.

[6]See, e.g. Harper (2002) for details on Newton's own arguments for NGL.

is appropriate, I draw out various similarities and differences between the texts, and propose possible explanations of these differences. §4 concludes.

2 The *Foundations* and Du Châtelet's commentary: a comparison

2.1 The *Foundations*

I will begin with the discussion of the inverse-square in the *Foundations*. On this, Du Châtelet begins by laying out props. 1, 2, and 4, corol. 6 of the *Principia*,[7] and then concludes:

> Thus, Kepler's [second] law[8] [...] enabled Mr. Newton to discover a central force in general, which he called the centripetal force; and the [third] law [...] enabled him to know the law that this force follows [i.e. the inverse-square]. (Du Châtelet 1740, 15.349)

Du Châtelet's account of Newton's argument for the inverse-square here is exceptionally clear, and exactly as given in the *Principia*—namely, that K2L (by prop. 2) implies a centripetal force,[9] and that K3L (by prop. 4, corol. 6, for the special case of concentric circular orbits) implies that this force is inversely proportional to the square of the distance of the body from the sun.

Turning now to Du Châtelet's discussion of Newton's numerator; here, Du Châtelet focuses on arguing that gravitational force is proportional to the mass of the attracted body. She begins by appealing to Newton's pendulum experiments:

> We have seen [...] that pendulums of equal weight make their vibrations in equal times when the wire from which they are suspended

[7]Here, and throughout, I will adopt the convention that reference to Book 1 of the *Principia* is always omitted when mentioning propositions, corollaries, etc. from Book 1 (but not Books 2 or 3).

[8]Here, and throughout, I have amended Du Châtelet's terminology to reflect the modern numbering of Kepler's laws, which are as follows:

K1L: The orbit of a planet is an ellipse with the Sun at one of the foci.

K2L: A line segment joining a planet and the Sun sweeps out equal areas in equal times.

K3L: A planet's orbital period is proportional to the square root of the cube of the length of the major axis of its orbit (equivalently, of its mean distance from the Sun).

[9]Note that, throughout, forces are assumed to be as characterised by Newton's second law of motion. Newton's laws of motion, recall, are as follows:

N1L: Force-free bodies travel with uniform velocity.

N2L: The total force on a body is equal to the product of that body's mass and its acceleration. ($\mathbf{F} = m\mathbf{a}$)

N3L: If two bodies exert forces on one another, then these forces are equal in magnitude but opposite in direction.

I will not comment on this assumption further.

> is equal, no matter what kind of bodies compose them; and consequently the force that makes them fall here on Earth pertains to all the proper matter of bodies, and resides in every part of them, such that in different bodies this force is always directly proportional to the quantity of proper matter that they contain. Therefore since [...] the same force that makes bodies fall toward the Earth keeps the Moon in its orbit, this force resides in the whole body of the Moon, in direct proportion to the proper matter of this Planet, just as it resides here on Earth in the different bodies in direct proportion to their quantity of proper matter. (Du Châtelet 1740, 15.362)

So far, this exactly follows Newton's reasoning in Book 3, prop. 6 of the *Principia*: that the earth attracts bodies near its surface in proportion to their masses is established empirically by Newton's pendulum experiments, and since this same attractive force is responsible for the orbit of the moon about the earth, the earth must also attract the moon in proportion to its mass. However, Du Châtelet then continues:

> Now, the principal Planets, in revolving around the Sun, and the secondary Planets, in revolving around their principal Planet, follow the same laws as the Moon in its revolution around the Earth. Therefore the force that keeps them in their orbits acts on each of them in direct proportion to the quantity of proper matter that they contain. (Du Châtelet 1740, 15.362)

In this passage, Du Châtelet gives an inductive argument: since the earth attracts the moon in proportion to the moon's mass, and the planets and their satellites follow the same laws in their orbits as the moon in its orbit about the earth, gravitational force must be proportional to the mass of the attracted body for all bodies in the solar system. In Book 3, prop. 6 of the *Principia*, Newton does not proceed directly from his pendulum experiments to the proportionality of gravitational force to the mass of the attracted body for the planets and their satellites, but instead gives a separate argument, appealing to K3L. Du Châtelet continues by picking up on this latter argument. First, from K3L applied to the planets in their orbits about the sun, it follows that "at equal distances from the sun the force that carries them [the planets] toward it acts upon them equally" (Du Châtelet 1740, 15.363), just as the gravitational force of the earth does for bodies near its surface. Therefore, since "the force that acts equally upon unequal bodies must necessarily be proportional to the mass of these bodies", it follows that

> [The] force that makes bodies fall toward the earth, and that makes the Planets revolve around their center, is proportional to their different masses; and consequently the weight of each Planet toward the Sun is in direct proportion to the quantity of proper matter that each of them contains. (Du Châtelet 1740, 15.363)

Du Châtelet then concludes her discussion by noting that the same reasoning applies to the satellites of Jupiter and Saturn (Du Châtelet 1740, 15.364), which also obey K3L in their orbits.

Du Châtelet does not discuss the proportionality of gravitational force to the mass of the attracting body in chapter 15, though she does allude to this towards the end of the chapter, where she mentions that in Newton's calculations for the form of the earth, he, unlike Huygens, had assumed "[the law of heaviness] to be different at different places on the earth, and dependent upon the mutual attraction of the parts of matter" (Du Châtelet 1740, 15.379). Insofar as Du Châtelet does say anything explicit about about the proportionality to the mass of the attracting body in the *Foundations*, it comes in chapter 16, where she writes

> Attraction, being regarded by some Newtonians as an essential property of matter, is always assumed to be reciprocal: thus, the Earth in gravitating towards the Sun makes the Sun gravitate towards it, and the Sun and the Earth attract one another reciprocally in direct proportion to their masses. (Du Châtelet 1740, 16.388)

In this passage, the reciprocity of gravitational attraction is presented as an assumption, and to the extent that Du Châtelet says anything to motivate this, it is that gravitational attraction, within the Newtonian system, is sometimes supposed to be an "essential property of matter".[10] In particular, she does not appeal to the argument given in the *Principia*, where the reciprocity of gravitational attraction (and the proportionality to the mass of the attracting body) is derived from N3L (Book 3, prop. 5, corol. 1; prop. 7, corol. 1) via prop. 69. Du Châtelet (1740, 16.388) then goes on to cite a long list of empirical phenomena as possible confirmation of this: the tides, the departures of Saturn's orbit from Kepler's ellipses, the form of the earth, and that the mass proportionality in the numerator of NGL explains why lighter bodies orbit heavier ones, etc.

Putting this together, we can summarise Du Châtelet's account of NGL in the *Foundations* as follows. The centripetal force is derived from K2L. The inverse-square is derived from K3L. The proportionality to the mass of the attracted body is established inductively on the basis of both K3L and Newton's pendulum experiments. Finally, the proportionality to the mass of the attracting body is derived from the assumption that gravitational attraction is reciprocal, and the strength of this assumption is to be assessed on its ability to explain phenomena such as the orbital trajectories of celestial bodies, the tides, the form of the earth, etc.

2.2 The commentary

As in the *Foundations*, Du Châtelet begins with Newton's argument for the centripetal force, appealing to K2L and invoking props. 1-2 (Du Châtelet 1759, 2.2). She then turns to the inverse-square law:

[10] Though note that this was *not* Newton's own view; Newton himself was careful to distinguish universal from essential properties, see Newton (1726, p. 796).

> M. Newton demonstrates [(prop. 4, corol. 6)] that, if the periodic times of bodies revolving in circles are in sesquiplicate [3/2 times] ratio to their radii, the centripetal force that carries them toward the center of these circles is in a reciprocal ratio to the squares of these same radii [...]. Now, by Kepler's [third law] that all the planets observe, the times of their revolutions are in sesquiplicate proportion to their distances to their center, so the force that carries the planets toward the Sun decreases in inverse proportion to the square of their distances to this star, supposing that they turn in concentric circles around the Sun. (Du Châtelet 1759, 2.6)

This is exactly the argument given by Newton (1726, Book 3, prop. 2)—namely, concentric circular orbits about the sun and K3L jointly entail the inverse-square. However, Du Châtelet then continues:

> Starting from [Kepler's first law], M. Newton sought the law of centripetal force necessary to make the planets describe an ellipse, and he found in prop. 11 that this force must be inversely proportional to the square of the distances of the body to the focus of this ellipse. (Du Châtelet 1759, 2.8)

As Smith (2022) notes, it is not immediately obvious what to make of this. In Book 3 of the *Principia*, Newton does not infer the inverse-square from K1L, but rather the converse (Book 3, prop. 13). This raises two (related) questions: why did she choose to include this second argument, and how did she see the arguments in §2.6 and §2.8 fitting together?

The interpretation suggested by Smith (2022) is that Du Châtelet was aiming to reconstruct the reasoning which led Newton to the inverse-square (rather than the published arguments of the *Principia*)—and thought that Newton had originally obtained the inverse-square from K1L.[11] There are two difficulties with this. First, if Du Châtelet thought that Newton had originally obtained the inverse-square from K1L, it would be odd for her to begin her discussion of the inverse-square with Newton's results about K3L and circular orbits. Second, in neither Book 3 of the *Principia* nor *De Mundi Systemate* does Newton give any hint of having inferred the inverse-square from K1L.

However, there is another interpretative option here, which is that Du Châtelet saw the argument given in Book 3, prop. 2 of the *Principia* as important motivation for the inverse-square, but thought the derivation of the inverse-square from K1L more convincing—and accordingly, was presenting (and attributing to Newton) what she saw as the best argument for the inverse-square within the Newtonian system at the time. Why might she have thought this? My suggestion is that it had to do with the assumption of concentric circular orbits which goes into Newton's argument in Book 3, prop. 2. To expand on this suggestion, notice that at the end of (Du Châtelet 1759, 2.6) (and unlike in the *Foundations*), Du Châtelet emphasises that *if* one assumes concentric circular

[11] Though note that Newton originally obtained the inverse-square from K3L—see Smith (1999).

orbits about the Sun, one can use prop. 4, corol. 6 to derive the inverse-square from K3L. But the planetary orbits are not concentric circles about the Sun—as Du Châtelet (1759, 2.7) immediately goes on to discuss, the best theory of the planetary orbits at the time of her writing was Kepler's confocal ellipses (i.e. K1L)—and insofar as they are not concentric circles, one might worry that this undermines the argument for the inverse-square given in Book 3, prop. 2.[12]

One can then ask: does the same argument go through for confocal elliptical orbits, i.e. do K1L and K3L jointly entail the inverse-square? The answer is, of course, "yes"—because K1L directly entails the inverse-square. And if so, why not simply derive the inverse-square directly from K1L? My suggestion is that it was exactly this kind of reasoning which motivated Du Châtelet to depart from the published arguments of the *Principia* and emphasise that the inverse-square can be derived from K1L instead. This is supported by the emphasis Du Châtelet puts on the assumption of concentric circular orbits in prop. 4, corol. 6 in her discussion of Book 3, prop. 2, the structure of her argument—with the orbital trajectories of the planets situated immediately after this and immediately before her mentioning that the inverse-square can be derived from K1L—and the fact that Du Châtelet concludes by stating that

> It only remained—in order to be entirely sure that the centripetal force that directs celestial bodies in their paths follows the proportion of the inverse square of the distances—to examine whether or not the periodic times follow the same proportion in ellipses as in circles.
>
> Now, M. Newton demonstrated in prop. 15 that the periodic times in ellipses are in a ratio of one and a half times to their major axis. (Du Châtelet 1759, 2.8)

where she again invokes K3L, this time as confirmation of the inverse-square.

Of course, whether she would have been correct to see the situation *viz-à-viz* the inverse-square this way is a different matter, and Smith (2002a,b, 2014, 2016, 2022) makes a compelling case that she would not have been. In brief, the problem is that prop. 4, corol. 6 goes through if 'concentric circles', '3/2 power of r', and 'inverse-square' are replaced with, respectively 'very nearly concentric circles', 'very nearly 3/2 power of r', and 'very nearly inverse-square' (prop. 45),[13] whereas props. 7 and 10 strongly suggest that the same does not hold if one replaces 'confocal ellipse' with 'very nearly a confocal ellipse' in prop. 11 (at least for orbits with low eccentricity). In other words, the argument from K3L has an additional degree of perturbative robustness which the argument from K1L lacks, and this, Newton realised, was exactly what was

[12] As further motivation for thinking that this may well have been Du Châtelet's reasoning, note that the assumption of concentric circular orbits in Book 3, prop. 2 would have been particularly troubling for Du Châtelet if Smith (2022) is right in suggesting that she held to a standard of exact solutions, and saw perturbational approaches as inadequate. In this case, prop. 45 and its corollaries would not have helped to allay the worry about the assumption of concentric circular orbits; see the subsequent discussion.

[13] As is the situation for for most of the planets in our solar system.

needed if his argument for the inverse-square was to survive the perturbative corrections arising from the universality of gravitational attraction (i.e. the fact that the planetary orbits about the sun are not exactly confocal ellipses).

Finally, we come to Du Châtelet's discussion of Newton's numerator. Here, Du Châtelet begins with an argument for universal gravitation:

> Since it is proved by observation and by induction that all planets have attractive force inversely proportional to the square of the distances, and that by the [third] law of motion the action is always equal to the reaction, it must be concluded, with M. Newton, that all planets gravitate toward one another, and that, just as the Sun attracts the planets, it is reciprocally attracted by them. For since the Earth, Jupiter, and Saturn act on their satellites in inverse proportion to the squares of the distances, there is no reason to believe that this action does not operate at all distances in the same proportion. So the planets must mutually attract, and the effects of this mutual attraction can be observed in the conjunction of Jupiter and Saturn.
>
> Analogy leads us to believe that the secondary planets are, in every respect, bodies of the same sort as their principal planets. It is very probable that they also have attractive force, and that, consequently, they attract their principal planet in the same way that they are attracted to it, and that they attract each other, which is confirmed again by the attraction of the Moon on the Earth, the effects of which become perceivable in the tides and in the precession of the equinoxes, as will be seen in what follows.
>
> One can then conclude that the attractive force belongs to all celestial bodies, and that it acts throughout our planetary system according to the inverse proportion of the square of the distances. (Du Châtelet 1759, 2.17-18)

The beginning of this passage closely follows the arguments of Book 3, prop. 5 and its corollaries: by induction, all planets have attractive force, and since the sun attracts the planets, then by N3L it must be reciprocally attracted by them.

Du Châtelet then gives two further inductive arguments, which are worth dwelling on in some detail. First, since the earth, Jupiter, and Saturn attract their satellites in accordance with the inverse-square law, "there is no reason to believe that this action does not operate at all distances in the same proportion", and hence they must attract one another in accordance with this law, which is confirmed by its observable effects in the "conjunction of Jupiter and Saturn". This line of inference appears to be a straightforwardly inductive one: from the premise that a particular body attracts some bodies to the conclusion that it must attract all bodies alike.

Second, there is the argument about the secondary planets. Here, Du Châtelet begins with a remark about the "analogy" between the principal and secondary planets which “leads us to believe that” the two are “in every respect, bodies of the same sort”. The “analogy” Du Châtelet is referring to here is, presumably, the fact that the satellites of the earth, Jupiter, and Saturn all obey Kepler’s laws in their orbits about these planets, just as the planets obey Kepler’s laws in their orbits about the sun. She then draws an inductive conclusion: since the principal planets have attractive force, it is “‘very probable” that the secondary planets do as well. Finally, since the secondary planets also have attractive force, they must attract the principal planets and one another, which is confirmed by its observable effects “in the tides and in the precession of the equinoxes”. Again, this line of inference appears to be a straightforwardly inductive one: that the attractive force of any body affects all other bodies alike.

Having said this, it is not immediately obvious what additional work the appeal to N3L at the beginning of this passage is supposed to be doing. Newton, apparently, needed the reciprocity of gravitational attraction in Book 3, prop. 5 (in particular, of the moon and the earth) so as not to beg the question against the Tychonic model in his argument that the earth is attracted by the sun in proportion to its mass in Book 3, prop. 6 (see Stein (1990) for discussion). This could not have been Du Châtelet’s reasoning, however—she is clear in chapter 1 that she is taking the Copernican model as given throughout the commentary (Du Châtelet 1759, 1.2).

But is Du Châtelet’s reasoning really as straightforwardly inductive as I suggested above? It certainly appears to be—but we would do well to take a closer look. If Du Châtelet’s second argument for universal gravitation was simply that (i) by induction, all planets have attractive force, and (ii) by induction, the attractive force of any planet affects all other bodies alike, then this would have as a conclusion that each planet attracts all other celestial bodies, including the other planets, their satellites, and the sun. The first of these is stated explicitly by Du Châtelet. But she does not draw the corresponding conclusion for the attraction of the sun by the planets, nor for the attraction of the sun by the secondary planets. In fact, the attraction of the sun by the planets is the one point in her argument where Du Châtelet appeals to N3L, but does not argue for the same conclusion directly on inductive grounds.

Of course, Du Châtelet may have simply decided to leave the conclusion that the planets attract the sun implicit—but then it is not clear what additional work the appeal to N3L and pairwise reciprocity is doing in her argument. So suppose, instead, that we take this omission seriously. In that case, we would have to conclude that Du Châtelet saw her inductive arguments as sufficient to establish that the planets (and their satellites) attract one another, but not that they attract the sun—and accordingly, the appeal to N3L and pairwise reciprocity is needed to reach this last conclusion. What could have been the reason for this?

Here, a comparison with Newton is helpful. Newton does not explicitly appeal to the idea that the attractive force of a body affects all other bodies alike in the *Principia*, but he does in *De Mundi Systemate*, for the attraction of the moon by the sun:

> [By] the action of a force so great [i.e. the attractive force of the sun] it is unavoidable that all bodies within, nay, and far beyond, the bounds of the planetary system must descend directly to the sun, unless by other motions they are impelled towards other parts: nor is our earth to be excluded from the number of such bodies; for certainly the moon is a body of the same nature with the planets, and subject to the same attractions with the other planets, seeing it is by the circumterrestrial force that it is retained in its orbit. (Newton 1728, Article 27)

But Newton stops short of pursuing this line of reasoning in full: he does not apply the same argument directly to the earth, but instead appeals to the fact that "the earth and moon are equally attracted towards the sun" to establish that the sun attracts the earth.

As Stein (1990) notes, from a modern perspective, this inference is easy to accept; for Newton (and his contemporaries) it was evidently more difficult. What the source of this difficulty was is unclear, but Newton's final clause in the above-quoted passage gives a possible indication. Here, by way of argument for his claim that the moon is "subject to the same attractions with the other planets", Newton writes that "it is by the circumterrestrial force that [the moon] is retained in its orbit". In other words, what licences the inductive inference from the planets to the moon *vis-à-vis* the gravitational force of the sun is the fact that the moon has already been established to be subject to *some* gravitational force (namely, that of the earth) in virtue of its orbit about the earth, and therefore is the kind of body which is subject to gravity. But this assumption—that *if* a body is subject to some gravitational force, it is subject to all gravitational forces—is of course weaker than the assumption that all bodies are subject to all gravitational forces. Newton's worry, accordingly, appears to be about the possibility of bodies which are not subject to *any* gravitational forces—the obvious candidates for which are precisely those bodies in the solar system which do not orbit any other bodies (namely, the sun in the Copernican model, or the earth in the Tychonic model).[14]

If this was Newton's worry, we can ask: could it also have been Du Châtelet's? The answer, I think, is "quite possibly", given her assumption of Copernicanism—but the situation is not straightforward. By way of positive evidence for this, we have what has already been noted: namely, that Du Châtelet states her inductive conclusions for the attraction of the planets by one another, and the attraction of the planets by their satellites, but not for the attraction of the sun by the planets or their satellites (where she instead invokes N3L). A further piece of textual evidence comes much earlier in the commentary, in her

[14] This possibility, by the way, is explicitly acknowledged in the corresponding discussion of the attraction of the earth by the sun in the *Principia*, where Newton infers first that "the weights of the moon and earth toward the sun are either nil or exactly proportional to their masses" before concluding that "they do have some weight, according to prop. 5, corols. 1 and 3" (Newton 1726, p. 808). That this may in fact have been Newton's worry is also consonant with the line taken by Stein (1990, pp. 215–216) on Newton's argument that the earth is subject to the attractive force of the sun, which I lack the space to discuss here.

presentation of the solar system

> The Sun appears to be of an entirely different nature from the planets; we do not know if it is composed of solid or fluid particles. We know only that its particles shine; that they heat, and that they burn when they are gathered together in a sufficient quantity. Thus, in all likelihood, the Sun is a fiery body roughly similar to fire on Earth, since its rays produce the same effects. (Du Châtelet 1759, 1.6)

So Du Châtelet is clear, here, that the sun is not a body of the same kind as the planets. But if the sun is not a body of the same kind as the planets, then it is not obviously subject to the inductive inference based upon the planets. This provides further reason to think that Du Châtelet invokes N3L in her argument because the sun cannot be assumed to be the kind of body which is subject to gravitational force.

The main complicating factor is a passage which comes right at the end of Du Châtelet's discussion of Newton's numerator:

> Experiments and observations thus lead us to conclude that the attraction of celestial bodies is proportional to mass, both in the attracting bodies and in the bodies attracted; that mass determines that one body turns around another; that all bodies can be regarded as attracting and attracted. Finally, it can be concluded that attraction is always reciprocal between two bodies, and that it is the proportion between their masses that decides whether this double attraction can be perceived. (Du Châtelet 1759, 2.31)

The difficulty, of course, is that the pairwise reciprocity of gravitational attraction is presented here as a conclusion, whereas if Du Châtelet thought that the assumption of pairwise reciprocity was necessary for her argument that gravitational attraction is universal, we would of course expect it to feature as a premise instead.

The resolution of this difficulty, I think, is to be found in Du Châtelet's discussion of the proportionality of gravitational force to the mass of the attracting body, which comes next in her argument. On this, Du Châtelet begins by asking "what causes a body to turn around another? Why, for example, if the earth and the moon reciprocally attract each other [...], does the earth not turn around the moon, instead of the moon turning around the earth?" (Du Châtelet 1759, 2.19). This is significant for two reasons. First, because Du Châtelet is clear, here, that she has invoked N3L and pairwise reciprocity by this point in her argument. Second, because she is no less clear in drawing attention to what she sees this argument as leaving to be explained: namely, the reason why some bodies in the solar system orbit others, rather than the converse. And indeed, immediately following her argument that gravitational force must be proportional to the mass of the attracting body (which I will return to in a moment), Du Châtelet goes on describe, following Book 3, prop. 8 and its corollaries, how to determine the comparative masses of the celestial bodies that have satellites. This puts Du Châtelet in a position to answer the question she posed at the beginning of her discussion, why one body orbits another:

> The mass of the planets being known, one sees that the bodies that have the least mass turn around those that have more, and that the greater the mass of a body, the more attractive force it has, all things being equal. Thus all the planets turn around the sun because the sun has much more mass than any planet, for the mass of the sun is to that of Jupiter and Saturn, nearly as 1 to 1,100, and 3,000 respectively. These two planets being those of our system with the most mass, it follows that the sun must be the center of the motion of our system. (Du Châtelet 1759, 2.25)

In brief, then, my suggestion about Du Châtelet's comments in §2.31, and how she saw the status of N3L and pairwise reciprocity, is just this. Du Châtelet does indeed invoke N3L in her argument for universal gravitation. But the application of N3L and pairwise reciprocity to gravity has not yet been justified—and it is not justified until it has been shown that a system of pairwise reciprocal attractions can account for the reason why one body orbits another rather than the converse. In other words: Du Châtelet's appeal to N3L in her argument for universal gravitation is made *in anticipation of* her argument for the mass proportionality in Newton's numerator, and the resulting explanation of why lighter bodies orbit heavier ones. This is why it is the fact that "the attraction of celestial bodies is proportional to mass, both in the attracting bodies and in the bodies attracted; that mass determines that one body turns around another; that all bodies can be regarded as attracting and attracted" which allows her to conclude that "attraction is always reciprocal between two bodies".

Two comments are in order. First, this is consistent with my suggestion that Du Châtelet does not inductively infer the attraction of the sun by the planets, and instead appeals to N3L, because the sun has not yet been shown to be the kind of body which is subject to gravitational force. If the sun were not attracted by the planets, there would be a straightforward explanation of the fact that the planets orbit the sun, but the sun does not orbit any other body in the solar system. Accordingly, Du Châtelet's discussion of the proportionality of gravitational force to the mass of the attracting body is framed by the need to explain—within a system of pairwise reciprocal attractions—what causes one body to orbit another rather than the converse, and hence why the sun is at the centre of the solar system.

Second, if I am right that Du Châtelet's appeal to N3L in her argument for universal gravitation is made in anticipation of her argument for the mass proportionality in Newton's numerator, then we should of course expect this latter argument *not* to presuppose N3L. This will take us right up to the end of this section, so let me just preface this by noting that indeed, this is exactly what we find.

We can now turn to Du Châtelet's argument for the proportionality of gravitational force to the mass of the attracting body itself. On this, Du Châtelet writes:

> Since attraction of all celestial bodies on the bodies that surround them follows the inverse ratio of the square of the distances, it is very probable that the parts of which they are composed attract in the same proportion.
>
> The overall attractive force of a planet is composed of the attractive force of its parts, for if one knew that many little planets unite to form a big one, the force of this large planet would be composed of the force of all these little planets. And M. Newton has proved [(props. 74-76)] that if the particles of which a sphere is composed attract one another in inverse proportion to the square of the distances, these spheres in their entirety will attract bodies exterior to them, however distant, in this same inverse proportion of the square of their distances. (Du Châtelet 1759, 2.20)

There are several comments to make here. First, what Du Châtelet appears to be relying on here is that prop. 74, corols. 1-2 establish that if, towards each point in a homogeneous sphere, there are directed equal inverse-square forces,[15] then bodies exterior to the sphere will be subject to a force proportional to the mass of the sphere and the inverse-square of their distance to the centre of that sphere.[16] As noted above, prop. 74, corols. 1-2, prop. 75, corol. 1, and prop. 76, corols. 1-4 do not presuppose N3L. However, this does draw attention to the gap between, on the one hand, the assumptions Du Châtelet is making in this argument, and on the other hand, the antecedents of the authorising props. 74-76—which require in addition that the spheres be homogeneous (or at least spherically symmetric), and that the inverse-square forces towards (like) particles of the sphere be equal.[17] Du Châtelet does not say anything in support of either of these assumptions.

Second, as Smith (2022) notes, Du Châtelet's argument inverts the reasoning in Book 3, prop. 7 of the *Principia*, where Newton first argues that gravitational force is proportional to the mass of the attracting body, and then concludes that the attractive force of a body is composed of the attractive force of its parts. That said, I will just note that Du Châtelet's inference from the premise that "attraction of all celestial bodies on the bodies that surround them follows the inverse ratio of the square of the distances" to the conclusion that "the parts of which they are composed attract in the same proportion" immediately calls to mind Newton's discussion of his rule 3 for the study of natural philosophy:

> The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise from the extension, hardness, impenetrability,

[15] "Equal", here, is in the sense that the force directed towards each such point is the same at the same distance from those points.

[16] Prop. 75 and its corollaries extend this argument to the attraction of one homogeneous sphere by another, and prop. 76 and its corollaries to spheres which are inhomogeneous but spherically symmetric.

[17] This, of course, was one of the objections Cotes raised against Newton—see Biener and Smeenk (2012) for discussion.

> mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy. (Newton 1726, pp. 795-796)

Whilst this is not the place for a detailed discussion of the various competing interpretations of rule 3 that have been offered in the literature,[18] I will just note that this raises an interesting question: if Du Châtelet did have rule 3 in mind, which, if any, of these interpretations is most representative of the way she was thinking about it?

Du Châtelet's last step is to argue that gravitational force is proportional to the mass of the attracted body, which she does following Newton's argument in Book 3. First, "[we] know that all bodies here on Earth fall equally fast toward the Earth, air resistance being discounted" (Du Châtelet 1759, 2.27), citing the results of Boyle's free-fall experiments and Newton's pendulum experiments, from which she concludes "the attractive force of our Earth is proportional to the mass of the bodies it attracts, and at the same distance it only depends on their mass" (Du Châtelet 1759, 2.28). Next, appealing to K3L applied to the satellites of Jupiter and Saturn, "[supposing] the satellites of Jupiter [...] to be all placed at the same distance from the center of this planet, and all deprived of their projectile motion, they would all fall toward Jupiter and would reach the surface simultaneously" (Du Châtelet 1759, 2.29), and hence that Jupiter and Saturn attract each of their satellites in proportion to its mass. And finally, applying K3L to the planets in their orbits about the Sun "[in] the same way it is proved [...] that this star acts on each [planet] in proportion to its mass" (Du Châtelet 1759, 2.30).

Again, putting this all together, we can summarise Du Châtelet's account of NGL in the commentary as follows. The centripetal force is derived from K2L. The inverse-square is derived from K1L, but motivated and confirmed by K3L. The universality of gravitational attraction is derived from N3L and induction, and is confirmed empirically by phenomena such as the tides, the departures of Saturn's orbit from Kepler's ellipses, etc. The proportionality to the mass of the attracting body is derived from the assumption that all the parts of a body have inverse-square attractive force, together with the assumption that the gravitational attraction towards a body is composed of the the gravitational attraction towards its parts. And the proportionality to the mass of the attracted body is established inductively on the basis of K3L, Newton's pendulum experiments, and Boyle's free-fall experiments.

3 Discussion

Before I turn to the relationship between Du Châtelet's discussions in the commentary and the *Foundations*, let me address an important worry. Why think

[18] See e.g. Belkind (2017) for a recent overview and discussion.

that Du Châtelet was addressing the same kind of questions in her discussions of NGL in the *Foundations* and the commentary? If Du Châtelet's aims in chapter 15 of the *Foundations* and the commentary were different, then my bringing the two together will seem incongruous: the differences between the two texts have a straightforward explanation, namely that Du Châtelet was simply addressing two different questions.

However, we would do well to think carefully about just what difference in aims this could be. Certainly, there would be a straightforward explanation of the differences between the *Foundations* and commentary if Du Châtelet's intention in one text was to provide a completely faithful reconstruction of the arguments of the *Principia*, but not the other. The primary difficulty with this is that both the *Foundations* and commentary diverge from the official line taken in the *Principia*: Du Châtelet's treatment of Newton's numerator in the *Foundations* makes no mention of N3L, and her treatment of the inverse-square in the commentary appeals to K1L as a justification for the inverse-square (unlike Book 3 of the *Principia*). Secondly, if only one text was intended to provide a faithful reconstruction of the arguments of the *Principia*, one would expect this to be the commentary—whereas, as we have seen, it is the *Foundations* and not the commentary which more closely follows the arguments of the *Principia*. Whilst it is possible that Du Châtelet intended to keep her own reconstruction of Newton's reasoning entirely out of one text but not the other, this is difficult to reconcile with the fact that both texts depart from the official line of the *Principia*.

As far as Du Châtelet's own statement of her aims goes, she is explicit about this in both the *Foundations* and commentary:

> My aim here is to make known to you the way in which Mr. Newton explains the same Phenomena by attraction, and how the path of the Stars enabled him to discover that all celestial bodies tend toward the center of their revolution by the same cause that makes heaviness on Earth. (Du Châtelet 1740, 15.344)

> In [Book 3] M. Newton applies the propositions of [Book 1] to the explanation of celestial phenomena. It is in this application that I will try to follow M. Newton and show the logical sequence of his principles, and how easily they explain astronomical phenomena. (Du Châtelet 1759, Introduction, 17)

In other words, in both the *Foundations* and commentary, Du Châtlet's aim is both to show how Newton's theory (and in particular, NGL) explains various celestial phenomena, and to reconstruct the sequence of reasoning leading from the mathematical results of Book 1 to his law of gravity in Book 3.

With that out the way, let me now turn to the remaining similarities and differences between the two texts. On the inverse-square, the main difference between the *Foundations* and the commentary is the weight which Du Châtelet attaches to K1L as an argument for the inverse-square in the commentary, which does not feature in this capacity in the *Foundations*. I do not have a knock-down explanation for this, but let me suggest just one possibility. Suppose that I am right that Du Châtelet's decision to focus on K1L rather than K3L as an argument for the inverse-square in the commentary was motivated by the assumption of concentric circular orbits in prop. 4, corol. 6. In the *Foundations*, Du Châtelet does not mention the assumption of concentric circular orbits in her presentation of prop. 4 corol. 6, but she does draw attention to it later when discussing corol. 9 of the same proposition in relation to Newton's calculations for the gravitational force of the earth on the moon, in a footnote:

> The second observation is that the demonstration of Messrs. Huygens and Newton is for a circle, and that the Planets make their revolutions in ellipses, of which some are not even regular ellipses, like the one described by the Moon, for example.
>
> But Mr. Huygens demonstrated that each curve in any one of its parts has the same curvature as a certain circle, called the osculating circle, because in this region there is a part common to the curve and the circle. By considering this circle, for which Mr. Huygens discovered how to find the radius for each point on the curve, one can find the expression for the centripetal force in all curves, and compare this force, not only for each point on the same curve, but also from curve to curve. (Du Châtelet 1740, 15.358, fn. 7)

Du Châtelet's point here is that the assumption of concentric circular orbits does not undermine the arguments given in the *Principia*, precisely because Newton had shown (prop. 6, corol. 5; prop. 7, corol. 3) that once one specifies a different orbital trajectory for a body about some centre (other than concentric circles), one can use his results for concentric circular orbits to determine the distance proportionality of the (instantaneous) centripetal force acting on that body *directly* from its trajectory. But this, of course, is just the reasoning which I have attributed to Du Châtelet in her decision to focus on K1L rather than K3L as an argument for the inverse-square in the commentary. In other words, the above line of reasoning was already present in Du Châtelet's thought at the time of writing of the *Foundations*—and accordingly, I suggest, her decision to focus on K1L rather than K3L as an argument for the inverse-square in the commentary should be viewed as a natural development of this idea, applied consistently across her account of the justification of the inverse-square.

On the numerator of NGL, there are two significant differences between the *Foundations* and commentary. The first is the detailed argument which Du Châtelet gives for universal gravitation in the commentary, the second is her detailed argument for the proportionality of gravitational force to the mass of the attracting body in the commentary—both of which have no counterpart in the *Foundations*.

Again, however, behind these surface-level differences, there is an important underlying continuity of thought. To begin with, at each point in her argument for universal gravitation in the commentary, Du Châtelet is careful to emphasise the empirical phenomena—the tides, the precession of the equinoxes, the departure of Saturn's orbit from Kepler's ellipses, etc.—which confirm this. This suggests that the difference between the *Foundations* and commentary in this respect is not that Du Châtelet sees the universality of gravitational attraction as a hypothesis in the *Foundations* but not the commentary, but rather that in the commentary, Du Châtelet is explicit in fleshing out the kind of reasoning which might lead one to make this hypothesis. Secondly, as I have argued, Du Châtelet's use of N3L and pairwise reciprocity in the commentary is consonant with her seeing this as an assumption, which is exactly how she presents the pairwise reciprocity of gravitational attraction in the *Foundations*.

Third, Du Châtelet's argument for the proportionality of gravitational force to the mass of the attracting body begins from the assumption that all the parts of bodies have inverse-square attractive force, and in particular, that the attractive force of a body is composed of the attractive force towards its parts. In other words, Du Châtelet's point is that once one accepts these two assumptions, one can argue for, rather than hypothesise, the proportionality of gravitational force to the mass of the attracting body. Seen in this light, Du Châtelet's argument for the proportionality of gravitational force to the mass of the attracting body in the commentary simply represents a natural elaboration of her claim in the *Foundations* that the reciprocity of gravitational attraction is motivated by thinking of attractive force as a universal (because "essential") property of matter.

This does, however, draw attention to a more subtle difference between the two texts. As we have seen, in the *Foundations*, Du Châtelet does not draw a clear distinction between universal and essential properties when it comes to gravitational attraction[19]—motivating the reciprocity of gravitational attraction with the idea that Newtonian attraction is sometimes supposed to be an "essential" property of matter. By contrast, her discussion of universal gravitation in the commentary does not mention essential properties—and in her introduction, she is careful to distinguish attractive force from claims about the cause of this attractive force (such as essential gravity):

> I here declare, as M. Newton himself did, that in using the word attraction, I only take it to mean the force that makes bodies tend toward a center, without claiming to assign the cause of this tendency. (Du Châtelet 1759, Introduction, 18)

Of course, part of the explanation for this is probably rhetorical, given that much of chapter 16 of the *Foundations* is devoted to arguing that attractive force cannot be an essential property of matter at all. However, it would have been all too easy for Du Châtelet, in the *Foundations*, to present the universality of gravitational attraction as a separate hypothesis, with essential gravity a proposed Newtonian explanation for this. This raises the question, why did she not?

[19] See Chen (2021) for further discussion on this point.

My own suggestion is that this is where Du Châtelet's having translated the *Principia* between her writing the *Foundations* and commentary comes in—and in particular, the resulting familiarity she would have had with Newton's distinction between universal and essential properties in is discussion of rule 3. This is suggested by the close resemblance between this discussion and Du Châtelet's own discussion of the proportionality to the mass of the attracting body in the commentary, and the fact that in chapter 16 of the *Foundations*, Du Châtelet does not point out that part of her argument against essential gravity—namely that gravitational force varies with distance from the attracting body—was also one that Newton had given in his discussion of rule 3. But how famililiar Du Châtelet was with rule 3 at the time of writing of the *Foundations* is an open question; whether this was in fact the reason remains to be seen.

4 Close

In this paper, my aim has been to reconstruct and then bring together Du Châtelet's discussions of the justification of NGL in the *Foundations* and commentary, to draw out the similarities and differences between the two, and then to propose explanations of these differences. As we have seen, a number of *prima facie* differences between the two texts do not survive on a closer inspection. This is all the more reason to think of the commentary as providing a fuller exposition of Du Châtelet's developing thinking on the justification of NGL, which she had begun in the *Foundations*.

Several questions remain. For example, as noted in §1, one other area where fruitful comparisons have been drawn between the *Foundations* and commentary is Reichenberger's (2018) discussion of the laws of motion, but one might wonder if there are others still. Secondly, whilst chapter 15 of the *Foundations* is written from the perspective of Newton's physics, it would be of interest to explore the extent to which (a version of) the account in chapter 15 goes through if one replaces Newton's laws of motion with Du Châtelet's (which differ from Newton's). Those projects, however, will have to wait for another day.

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