A No-Go Theorem for ψ-ontic Models? No, Surely Not!

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

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Abstract

In a recent reply to my criticisms (Found Phys 55:5, 2025), Carcassi, Oldofredi and Aidala admitted that their no-go result for ψ-ontic models is based on the implicit assumption that all states are equally distinguishable, but insisted that this assumption is a part of the ψ-ontic models defined by Harrigan and Spekkens, and thus their result is still valid. In this note, I refute their argument again.

Last year Carcassi, Oldofredi and Aidala (COA hereafter) argued that the ψ-ontic models framework (OMF) defined by Harrigan and Spekkens [1] cannot be consistent with quantum mechanics (QM) [2]. This is a surprising result. Later, I presented a critical analysis of this no-go result [3]. I argued that in order to derive their result, COA implicitly assume that all ontic states can be distinguished by experiments with certainty. Moreover, I pointed out that this assumption is not a part of OMF, and it is not consistent with QM either. In a recent reply to my criticisms [4], COA admitted that their no-go result is based on the implicit assumption that all states are equally distinguishable. However, they insisted that this assumption is a part of OMF, and thus their result is still valid. In the following, I will argue that this is not the case.

First of all, OMF nowhere assumes that all ontic states can be distinguished by experiments with certainty. OMF has two fundamental assumptions [5]. The first assumption says that if a quantum system is prepared such that QM assigns a wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. Then, a wave function or a pure state corresponds to a probability distribution

$p(\lambda|P)$ over all possible ontic states when the preparation is known to be P, and the probability distributions corresponding to two different wave functions may overlap or not, depending on whether the ψ-epistemic view or the ψ-ontic view is true. Besides, a mixture of pure states $|\psi_i\rangle$ with probabilities p_i is represented by $\sum_i p_i p(\lambda|P_{\psi_i})$.

The second assumption of OMF says that when a measurement is performed, the behaviour of the measuring device is determined by the ontic state of the system, along with the physical properties of the measuring device. More specifically, the framework assumes that for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of QM then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P. It is obvious that neither of these two assumptions claims that all ontic states can be distinguished by experiments with certainty.

Next, the distinguishability assumption cannot be derived from the above assumptions of OMF either. This is also obvious (except for COA). In their reply [4], COA claim that the statement that "a mixture of pure states $|\psi_i\rangle$ with probabilities p_i is represented by $\sum_i p_i p(\lambda|P_{\psi_i})$" implies the distinguishability assumption. Their (very brief) argument can be formulated as follows. The above statement means that in OMF, "statistical mixtures are modeled as classical probability distributions." (p.3) "As it is well-known, classical probability already assumes that all elements are equally distinguishable." (p.3) Thus OMF implies the distinguishability assumption.

Can anyone make sense of this argument? Hardly, I think. Maybe COA did not intend to give an argument. Anyway, in a quantum theory consistent with OMF, statistical mixtures of pure states are still modeled as the probability distributions defined by the above quoted statement, but the distinguishability assumption is not true, since non-orthogonal states in QM cannot be distinguished with certainty.

The key point here is that although all ontic states are equally distinguishable *in ontology* (i.e. they are different ontic states),[1] they may be not distinguishable *in experiments*. Whether two ontic states are distinguishable in experiments is determined by the dynamics for the ontic states during the measuring interaction, but the above statement of OMF about statistical mixture clearly does not specify the dynamics, and thus it cannot imply the distinguishability assumption, no matter whether the involved probability is regarded as classical or not.

Third, if OMF does include or imply the distinguishability assumption, then it will be obviously inconsistent with QM, according to which non-

[1] As I understand, this may be what COA really mean when they say that all states are equally distinguishable.

orthogonal states cannot be distinguished with certainty. (It is virtually impossible that this simple fact has been ignored by all people in quantum foundations except COA.) If this is indeed the case, then we will have a much simpler proof of COA's result without a further analysis of information entropy or something else. But COA did not even refer to this new proof in their reply. Maybe they did not really think that the distinguishability assumption is a part of OMF.

Finally, it is worth emphasizing again that since OMF with its two fundamental assumptions does not include the dynamics for the ontic states, it is not a complete theory (as already noted in my previous criticisms [3]). As a result, one cannot calculate the information entropy directly on an epistemic state in OMF, since the information entropy depends on whether the ontic states in the epistemic state are distinguishable, which is determined by the dynamics. On the other hand, once we know the dynamics for the ontic states such as the Schrödinger dynamics for the state vectors in the Hilbert space (which represent the ontic states), then we can calculate the information entropy as given by the von Neumann entropy, which is just what we do in QM.

To sum up, COA's reply fails to answer my criticisms. In order to prove their no-go result, i.e. that the ψ-ontic models or OMF is inconsistent with QM, COA resorts to an additional distinguishability assumption, which is neither a part of OMF nor consistent with QM. COA have no choice but to admit that their no-go result is false.

References

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