Nothing Matters

Shelly Yiran Shi

University of California San Diego

Forthcoming in *Philosophy of Science*. Please cite the published version.

Abstract

One challenge to relationism in general relativity is that the metric field is underdetermined by the stress-energy tensor. This is manifested in the existence of distinct vacuum solutions to Einstein's field equations. In this paper, I reformulate the problem of underdetermination as a problem from vacuum solutions. I call this the vacuum challenge and identify the gravitational degrees of freedom (associated with the Weyl tensor) as the "source" of the challenge. The Weyl tensor allows for gravitational effects that something outside of a system exerts on the system. I provide a relationist response to the vacuum challenge.

1 Introduction

Suppose that you're in a room with a lightbulb. A curtain is drawn between you and the lightbulb. The lightbulb cannot be seen, but some light still filters through the curtain into the room. You and your friend Feld have a disagreement. Feld thinks that there can be free (i.e., sourceless) fields, whereas you hold that all electromagnetic (EM) radiation is sourced. Feld claims that the room you're in is evidence that she is correct, for one can model just the room with the curtain as a boundary condition and the representation of that scenario corresponds to a solution of the relevant laws of nature. That argumentative move feels too cheap, you say. All Feld did was replace the source with an incomplete model possessing a special boundary condition: the state of the curtain. That state carries upon it an echo or trace of the real source. The free solution exists as an artifact of modeling, but in the real world radiation may be sourced, or so you think.

In the debates between substantivalists and relationists in general relativity (GR), the existence of distinct vacuum solutions to Einstein's field equations (EFE) is often taken to present a quick and serious challenge to relationism. For example, Minkowski and Schwarzschild spacetimes have different metric fields but are both vacuum solutions with the same matter content. Their existence suggests that the metric field is underdetermined by the matter content, which would conflict with versions of relationism that claim that the metric is always "sourced" by matter. In what follows I will argue that one can respond to this famous threat to relationism in a similar way to how one may respond to the above challenge to the claim that all radiation is sourced. The boundary condition contains suspicious "echoes" of forgotten material sources. This response will not prove that relationism is true any more than the earlier move proved that all radiation is sourced. But it will, I think, deflate one worry about relationism.

To see the general idea, note that the Schwarzschild solution is Ricci-flat but not Riemann-flat. Minkowski is both. What makes Schwarzschild not Riemann-flat is that it has non-vanishing Weyl curvature. The Weyl curvature tensor can be thought to represent the "free" gravitational degrees of freedom whereas Ricci represents the "source" degrees of freedom. Yet, just like in our imagined dialogue above, one might look with suspicion upon Schwarzschild. The Schwarzschild solution is an exact solution of EFE regarding the outside of a spherical mass, often used to model static objects. All we've done is draw a curtain around the matter to make it a vacuum, yet we do this for the sake of modeling. The boundary conditions contain the trace of ignored material sources. Relationists can argue that the underdetermination arises from this modeling.

The paper proceeds as follows. Section 2 makes the challenge posed to relationism from vacuum solutions precise. I call this the vacuum challenge. Section 3 then moves to provide a relationist response to the challenge. Section 3.1 begins with the Schwarzschild solution. In this case, I argue that boundary conditions can be viewed as stand-ins for a source, either a distant star or a singularity. Section 3.2 then moves to what is perhaps the most hostile environment for my view, the Ozsváth-Schücking metric. This solution describes a pp-wave spacetime containing only gravitational radiation. Here I suggest two available responses. First, I show that the dimensional parameters of the model can be viewed as echoes of forgotten matter. Second, I show that one can pose a dilemma based on the Ehlers-Kundt conjecture and a theorem by Penrose to argue that the spacetime is either incomplete or a mere idealization. If one of these responses is successful, then even the most hostile vacuum solution can be made compatible with relationism.

2 Does $T_{\mu\nu}$ fully determine $g_{\mu\nu}$?

Spacetime curvature and matter fields in GR are deeply connected yet utterly distinct entities. Einstein analogized the left-hand side of his field equations to fine marble, while considering the right-hand side as low-grade wood. The original non-augmented field equations (without the cosmological constant) read,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}. \tag{1}$$

John Wheeler famously captures the essence of GR with the succinct statement: "matter tells spacetime how to curve, and spacetime tells matter how to move."[1] The metaphysical debate revolves around two conflicting viewpoints concerning the nature of spacetime. Substantivalism claims that spacetime is fundamental, existing as a distinct substance. Relationism posits that only matter is fundamental, with properties of spacetime (such as the metric) being derived from the relational distribution of matter.[2] The debate between substantivalists and relationists traces back to the seventeenth century in the renowned Leibniz-Clarke correspondence, and found a new voice in Mach's 1883 "fixed stars" response. However, it has become significantly more convoluted in response to modern theories of gravity. Some suggest we should refrain from imposing the categories of seventeenth-century metaphysics onto a theory that has outgrown them,[3] while others have proposed alternative formulations of GR to revive Mach's principle.[4]

[1]See Misner, Thorne, and Wheeler (1973, 5) and Wheeler and Ford (2000, 235).
[2]See Lehmkuhl (2018) for a third alternative – super-substantivalism.
[3]See Rynasiewicz (1996, 2000).
[4]Sciama (1953), Barbour and Bertotti (1982), and Raine (1995).

In this section, I revisit and clarify the problem of matter-metric determination in GR. The problem has been articulated by many based on a prevalent formulation of Mach's principle, which requires *the metric field* $g_{\mu\nu}$ *to be fully determined by the stress-energy tensor* $T_{\mu\nu}$.[5] Call this Mach's Principle-T. It is both mathematically and physically correct that GR does not satisfy Mach's Principle-T. In this paper, I reformulate this problem as a problem from vacuum solutions. According to Pooley, relationists in GR grapple with the fact that "the dynamically significant chronometric facts outstrip the chronometric facts about matter, as is most vividly illustrated by the abundance of interesting vacuum solutions" (2013, 578). Call this the vacuum challenge. Vacuum solutions are solutions to EFE with a vanishing stress-energy tensor (i.e., $T_{\mu\nu} = 0$). The vacuum challenge offers a clear and sharp way of formulating the problem of underdetermination: differences in metrics given "the same" matter content.

To delineate the scope of my argument, I begin with two disclaimers. First, I only focus on the problem of non-Minkowski vacuum solutions and restrict my discussion to the non-augmented EFE, which admit the Minkowski solution.[6] Arguably, Minkowski spacetime can take on a relationist interpretation if one adopts a dynamical approach to GR.[7] In this paper, I table the matter of Minkowski spacetime to focus on a more pressing concern. To that effect, my argument is based on what is commonly referred to as "Mach's Principle-3" formulated by Pirani (1956, 199), which says that *spacetime should be Minkowskian in the absence of matter*. In this context, I will henceforth use relationism and Machianism interchangeably. Second, I take it for granted that gauge

[5]See Einsten (1918, 241-42), Weingard (1975, 427), Callender and Hoefer (2002, 176), and Lehmkuhl (2011, 455).
[6]See Earman (2003) for considerations regarding the cosmological constant.
[7]See Brown (2005), Brown and Pooley (2006).

freedom does not cause trouble for relationism. Between 1913 and 1914 when the failure of general covariance in the *Entwerf* theory still bothered Einstein, his commitment to Mach's principle led him to the hole argument and made him believe there was a deeper reason – the metric $g_{\mu\nu}$ could not be fully determined by the stress-energy tensor $T_{\mu\nu}$ in any generally covariant theory.[8] We now understand this underdetermination as a feature not a bug. The Bianchi identity implies that there are four out of ten equations in the EFE that lack second-order time derivatives of the metric. In the 3+1 decomposition, these four equations are constraints (of elliptic type), signifying gauge freedom. We cannot determine the solutions to more than up to diffeomorphism. The failure of the EFE to determine $g_{\mu\nu}$ uniquely is analogous to the failure of Maxwell's equation to determine the electromagnetic potential A_μ uniquely.

Relationists' challenge in GR is associated with another kind of underdetermination manifested in the existence of multiple vacuum solutions. In four or more dimensions, the Ricci tensor $R_{\mu\nu}$ is not sufficient to describe the curvature of the space. The full Riemann tensor $R_{\rho\sigma\mu\nu}$ is needed. On a common view, "matter" only couples with the ten algebraically independent components captured by the Ricci tensor, that is, the trace of the Riemann curvature tensor. The other ten components come from the Weyl tensor $C_{\rho\sigma\mu\nu}$, representing the "free" gravitational field present in vacuum region (not directly tied to the regional distribution of mass-energy). To satisfy the EFE in a vacuum region where $T_{\mu\nu} = 0$, only Ricci-flatness, i.e., $R_{\mu\nu} = 0$ is required. There are Ricci-flat solutions that are not conformally-flat, i.e., solutions where the Weyl curvature is non-vanishing.

The gravitational potential $g_{\mu\nu}$ and the stress-energy tensor $T_{\mu\nu}$ are analogous to the

[8] See Einstein's letter to de Sitter, 4 November 1916 (CPAE 8, Doc. 273). See also Norton (1984, 1993), Maudlin (1990), and Hoefer (1994).

electromagnetic potential A_α and the charge-current vector J_α in electromagnetism (EM), respectively. Both $g_{\mu\nu}$ and A_α represent the potential, whereas both $T_{\mu\nu}$ and J_α can represent the source. On the other hand, the Weyl tensor $C_{\rho\sigma\mu\nu}$, in analogy to the trace-free Maxwell field tensor $F_{\alpha\beta}$, describes the trace-free part of the Riemann tensor. It captures the curvature in the Riemann tensor but with the "Ricci part" removed:

$$C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} - g_{\rho[\mu}R_{\nu]\sigma} + g_{\sigma[\mu}R_{\nu]\rho} + \frac{1}{3}g_{\rho[\mu}g_{\nu]\sigma}R. \tag{2}$$

Mathematically, the Riemann tensor is decomposed into three irreducible parts (the Ricci scalar, the trace-removed Ricci tensor, and the Weyl tensor). Physically, the Weyl tensor is the only surviving part in source-free regions. It encapsulates the *gravitational degrees of freedom*. While $R_{\mu\nu}$ and $T_{\mu\nu}$ are algebraically related through (1), the relation between $C_{\rho\sigma\mu\nu}$ and $T_{\mu\nu}$ is less explicit. It involves a first-order differential equation obtained from the Bianchi identity and the EFE:

$$\nabla^\rho C_{\rho\sigma\mu\nu} = \kappa\Big(\nabla_{[\mu}T_{\nu]\sigma} + \frac{1}{3}g_{\sigma[\mu}\nabla_{\nu]}T\Big). \tag{3}$$

One can see a close resemblance to EM by comparing (3) with Maxwell's equation $\nabla^\mu F_{\nu\mu} = J_\nu$. Indeed, (3) can describe the propagation of gravitational waves. The freedom in the choice of $C_{\rho\sigma\mu\nu}$ means that given $T_{\mu\nu}$, multiple solutions exist, each specified by particular boundary conditions.[9] Not all components in $C_{\rho\sigma\mu\nu}$ contribute to dynamical degrees of freedom though. In linearized gravity, the gauge and residual gauge conditions cut down $4 \times 2 = 8$ degrees of freedom from 10 equations, leaving 2 degrees of

[9]See Carroll (2004, 169-70).

freedom for a propagating spin-2 field.[10]

Without the later terminology from Weyl's (1918) work available to him, Einstein was aware of the problem of boundary conditions in his Machian pursuit. In a letter to Lorentz in January 1915, Einstein claims that centrifugal and Coriolis forces are determined by the boundary conditions and field equations. He interprets the boundary conditions as a stand-in for the distant stars supposedly epistemically inaccessible to us, while admitting it's "awkward that the boundary conditions must be picked out suitably."[11] He was concerned about the arbitrariness involved in stipulating spacetime properties at infinity. His goal between 1916 and 1919 was to find Machian boundary conditions that are compatible with observations of the actual universe.

Now we are ready to state the problem of underdetermination in GR more precisely. Similar to how degrees of freedom in electromagnetic potential A_α can be broken down into gauge, source (J_α), and field ($F_{\alpha\beta}$) components, degrees of freedom in gravitational potential $g_{\mu\nu}$ can also be decomposed into gauge, source ($R_{\mu\nu}$), and gravitational ($C_{\rho\sigma\mu\nu}$) components (Table 1). While the gauge degrees of freedom based on the hole argument might help to reject the existence of spacetime as a fundamental and distinct substance, the gravitational degrees of freedom associated with the Weyl tensor seem to require it. I will show that the partial determination of $g_{\mu\nu}$ from the source (after gauge fixing) does not automatically favor substantivalism, that the question remains as to what else is needed to determine $g_{\mu\nu}$. Is the extra ingredient "source from elsewhere" or "substantival spacetime"? It follows from my analysis that Mach's Principle-3 might be upheld in GR even though Mach's Principle-T fails.

[10]See Wald (1984, ch.10) for counting degrees of freedom in the initial value formulation.
[11]CPAE 8, Doc.47, translated by Ann Hentschel.

Physical Interpretation	**EM**	**GR**	**Degrees of Freedom**
Source	J_a	$T_{\mu\nu}$ ($R_{\mu\nu}$)	Source degrees of freedom
Trace-free part	F_{ab}	$C_{\mu\nu\lambda\sigma}$	Field degrees of freedom
Potential	A_a	$g_{\mu\nu}$	Gauge + field + source degrees of freedom

Table 1: Degrees of Freedom in EM and GR

3 Echoes of Hidden Matter

When Machianizing boundary conditions at infinity turned out to be trickier than Einstein initially thought, he came up with an alternative solution: eliminate infinity. He introduced the cosmological constant and advocated a closed spherical universe, later known as the Einstein universe (1917). He defended Machianism in his extensive correspondence with de Sitter between 1916 and 1919, only to relinquish it in his later years. However, as Hoefer (1994) suggests, the historical intricacy of Einstein's Machian pursuit presents various possibilities for implementing Mach's ideas in GR, rather than justifying Einstein's own reason for abandonment. The purpose of this section is to provide such a possibility.

Two exact solutions to the EFE are particularly apt for the purpose – Schwarzschild metric and Ozsváth-Schücking metric. The historical significance of Schwarzschild spacetime lies in Einstein's quest to establish Machian boundary conditions as a replacement for the Minkowskian boundary used in the Schawarzschild solution. I demonstrate that this transition is not a major concern. Another brilliant yet less well-known example of vacuum solutions was proposed by Ozsváth and Schücking (1962) in their paper titled "An Anti-Mach-Metric". It describes a singularity-free spacetime with gravitational waves. The Ozsváth-Schücking metric may seem to pose the biggest challenge to relationism. Nevertheless, I show that relationists have two available responses.

3.1 Schwarzschild Spacetime

Solving the EFE is difficult, but simple solutions can be found if symmetry conditions eliminate some degrees of freedom. A symmetric metric tensor written in a matrix form has 10 slots to be determined in the 4×4 matrix. In Schwarzschild spacetime, isotropy and staticity simplify the metric so that after applying harmonic gauge in polar coordinates (t, r, θ, ϕ), the line element becomes $A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. Only two slots $A(r)$ and $B(r)$ need to be determined. One piece of information comes from the Newtonian limit about the mass of the central object, which solves $A(r)$. The other comes from the Minkowski limit regarding the asymptotic behavior far away from the central object. As r approaches infinity, the metric approaches the Minkowskian limit. That is, $A(r) \to -1$ and $B(r) \to 1$ as $r \to \infty$. These two pieces of information together with the EFE give us the Schwarzschild solution in its familiar form:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{4}$$

where G is the gravitational constant, M is the mass of the central object.

The standard interpretation is that the M in the Schwarzschild metric manifests a geometric property instead of the existence of matter, because the solution can model the gravitational field of a star as well as a black hole. One can always curtain a source and replace it with boundary conditions, but "this is purely a matter of taste" ("um eine reine Geschmacksfrage"), to borrow Einstein's words.[12] While de Sitter preferred to leave inertia unexplained, Einstein preferred to explain inertia via matter.[13] The standard geometrical interpretation may be appealing to substantivalists, but for relationists, Mach's explanans are better – there are effects "which masses *outside* of the system exert on its parts" (1893/1919, 288 emphasis in original).

In fact, it was the Minkowskian limit rather than the Newtonian limit that bothered Einstein, since singularities could be viewed as a "placeholder" or "surrogates for matter".[14] Einstein attributed the non-Machian characteristics in Schwarzschild spacetime to the fact that Minkowskian boundary conditions are stipulated in a specific coordinate system, hence not generally covariant. To find boundary conditions that are invariant under coordinate transformations meant to find a metric whose components are either 0 or ∞. According to de Sitter, Einstein found the metric that takes on degenerate values at infinity ($g_{i4} = g_{4i} = \infty$ for $i = 1, 2, 3$, $g_{44} = \infty^2$, and all other components equal to zero) but eventually abandoned it because it conflicted with observations.[15]

From a modern perspective, since gauge fixing is always required to express any solution, it is interesting, if not puzzling, that Einstein required Machian boundary conditions to be generally covariant. A change in perspective may circumvent the worry about boundary conditions. Instead of requiring the metric to approach the Minkowskian limit at infinity, we can find source-carrier parameters that, if vanished, result in the vanishing of the Riemann tensor. Vishwakarma (2015b) proposed a

[12] CPAE 8, Doc. 273.
[13] CPAE 8, Doc. 272.
[14] See Lehmkuhl (2017, 212) and Norton (2023, §10.4).
[15] See de Sitter (1916, fn.2) and Hoefer (1994, 304-16).

systematic way to define the source of curvature in vacuum solutions in terms of dimensional parameters. These parameters correspond to observable quantities such as energy, momentum, angular momentum and their densities. Based on this formulation, by requiring gravitational field to reduce to Newtonian potential in the limit, the source-carrier parameter L in Schwarzschild spacetime can be determined by the source mass: $L = -\frac{2GM}{c^2}$, so (4) can be rewritten as

$$ds^2 = -\left(1 + \frac{L}{r}\right) c^2 dt^2 + \left(1 + \frac{L}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{5}$$

If L vanishes, the Riemann curvature also vanishes, reducing the solution to Minkowski spacetime. The idea aligns naturally with the perturbation theory. Minkowski spacetime as a Riemann-flat solution has no gravitational field. Introducing some matter to it alters the metric, and this deviation is described by the source-carrier L.

The Schwarzschild solution is only valid outside a spherically symmetric mass. The boundary at the Schwarzschild radius (just like the state of the curtain) depends on the property of the source, be it a singularity or a star. The Minkowski boundary assumes effectively that there are no other masses in spacetime, or if there are, they are too distant to make a difference. The boundary conditions in Schwarzschild spacetime are not arbitrary but effective, determined by the external source (or its absence).

3.2 Ozsváth-Schücking Spacetime

In 1962, Ozsváth and Schücking proposed an ultimate anti-Machian metric. They adopted Pirani's formulation of Mach's Principle-3, which says spacetime should be Minkowskian in the absence of matter, where they interpreted "the absence of matter" as the absence of any form of source-matter, including singularities (1962, 339). The Ozsváth-Schücking (O-S) metric is an exact vacuum solution that is singularity-free and not asymptotically flat. The line element is given by

$$ds^2 = -(dx_1)^2 + 4x_4 dx_1 dx_3 - 2dx_2 dx_3 - 2(x_4)^2(dx_3)^2 - (dx_4)^2. \quad (6)$$

Unlike the Schwarzschild solution, it doesn't approximate Minkowski spacetime at infinity. Rather, it describes a pp-wave (plane-fronted waves with parallel propagation) spacetime filled up with gravitational radiation but nothing else.

This is different from the weak-field case, where gravitational waves are produced by a massive spinning object or a compact binary system. Under weak-field assumptions, the far zone allows the gravitational radiation to be meaningfully calculated just like EM radiation.[16] While the background metric is determined by $T_{\mu\nu} = 0$ in the far zone, the stress-energy tensor of the source in the near zone is non-vanishing. Since the waves in the far zone are effectively treated as plane waves, the information about the location of source is hidden. Curtaining the lightbulb doesn't mean light is sourceless though. The dynamics is determined by the vacuum background *and* the non-trivial stress-energy tensor in the near zone.

In weak-field approximations, the source appears once the curtain that separates the far zone and the near zone is removed. But the O-S solution, being geodesically complete, does not describe a scenario where a source is hidden by a curtain. Rather, it describes a case where there is light in the room, the curtain is removed, and you can see everywhere yet there is still no lightbulb. How is that possible?

[16] See Gomes and Rovelli (2023).

Vishwakarma (2015b) suggests one way to find the hidden matter using dimensional parameters and identifies a family of vacuum solutions, in which the angular momentum density sources the curvature. Among them there is the O-S spacetime, sourced by a particular value of a parameter l, related to the angular momentum density $\mathcal{J}$ by $l = \frac{G\mathcal{J}}{c^3}$. The O-S solution (6) can be rewritten in new coordinates as

$$ds^2 = \left(1 - \frac{l^2x^2}{8}\right) c^2dt^2 - dx^2 - dy^2 - \left(1 + \frac{l^2x^2}{8}\right) dz^2 + lx(cdt - dz)dy + \frac{l^2x^2}{4}cdtdz, \quad (7)$$

where $l = 2\sqrt{2}$. Similar to the Schwarzschild case, if l vanishes, the solution reduces to Minkowski spacetime. Thus, it is the source-carrier l, or the angular momentum density, which explains the non-vanishing Riemann tensor. While the stress-energy tensor cannot fully represent the source in the spacetime, dimensional parameters can reveal the hidden matter. Vishwakarma argues that his method Machianizes Ozsváth and Schücking's anti-Mach metric – "if the presence of source in the vacuum solutions is defined by the presence of such parameters...all the solutions become Machian!"(2015b, 1109)

Representing the source of curvature in terms of angular momentum density may be taken with a pinch of salt. While Vishwakarma (2015a) suggests that (7) results from a rotating matter distribution, the attribution of angular momentum density remains unclear. At the very least, this approach offers an alternative interpretation that circumvents reliance on the geometric property to explain the metric. If shoe-horning dimensional parameters into sources seems unconvincing, one can also represent sources by constructing an effective stress-energy tensor (known as the Bel-Robinson tensor) from the Weyl tensor. Goswami and Ellis (2018) showed that such a construction is always possible for radiation-like Petrov type N spacetimes, which include the O-S spacetime.

There is yet another response that relationists can offer. The short version simply asserts that the situation is "unphysical" if there is light in the room and when the curtain is removed, there is no lightbulb. The longer version appeals to a conjecture posed by Ehlers and Kundt (1962, 97) that the plane waves are the only geodesically complete pp-waves[17], in combination with Penrose's (1965) theorem that the plane waves are not globally hyperbolic. Considering plane waves as mathematical idealizations, the Ehlers-Kundt conjecture suggests that any other pp-wave spacetime is incomplete, with the source left out in modeling.

Ozsváth and Schücking regard their solution as a real threat to Machianism precisely because it is complete. Since complete spacetimes are inextendible, we are not just curtaining the part of the spacetime that contains a source. However, the O-S solution is a plane-wave solution. Penrose (1965) shows that embedding a plane wave globally within any hyperbolic normal pseudo-Euclidean space is impossible due to the absence of an adequate spacelike hypersurface in the spacetime for the global specification of Cauchy data. The absence of global hyperbolicity alone may not render a spacetime as unphysical, provided it can be extended, repairing its global hyperbolicity. However, if the spacetime is also complete, it is inextendible. Roche et al. (2023), following Flores and Sánchez (2020), regard the *conjunction* of being complete and not globally hyperbolic as indicative of the spacetime being "unphysical" (or a mere idealization).[18] The essence of the Ehlers-Kundt conjecture, as interpreted by Flores and Sánchez (2020), is that source-free dynamics fall under one of two possibilities: either the

[17]No counterexample to this conjecture has been identified.
[18]See Manchak (2011) for different views on "physically-reasonable" spacetimes.

spacetime is incomplete, or it represents a mere idealization. Accordingly, relationists can either remove the curtain to reveal the source, or claim that it is unphysical.

Finally, it isn't surprising that GR fails to satisfy Mach's Principle-T, because $T_{\mu\nu}$ has limitations in describing the global matter distribution. It can represent uniform and homogeneous matter distribution globally but falls short when matter clusters, as in the early universe. Penrose (1981, 1986) proposed a cosmological initial condition $C_{\rho\sigma\mu\nu} \approx 0$, known as the Weyl curvature hypothesis.[19] The freedom to stipulate vanishing Weyl curvature would prima facie bother relationists. However, the worry removes itself if we follow Penrose's description of the early Universe, in which Ricci curvature represents the uniform matter and Weyl curvature represents "the clumped matter" (2007, 766). When cosmic structures form, the stress-energy tensor represents the "regional source", whereas the Weyl tensor models the "external source" such as stars and blackholes. The degrees of freedom in the Weyl tensor simply allow "the external" to affect "the regional". This is compatible with what Barbour (2010) called Mach's dictum of *the All*, which takes into consideration "things which for the time being we left out of account" (Mach 1883/1919, 235).

4 Conclusion

Many successful defenses of relationism have been put forth in the context of Newtonian physics and Special Relativity, but the same strategies are known to face significant challenges when extending to GR due to the problem of underdetermination. In this

[19]It parallels Ritz's hypothesis, which eliminates electromagnetic degrees of freedom by postulating the cosmological initial condition $F^{\alpha\beta} = 0$. For Zeh (2007, 138), the Weyl hypothesis is analogous to $A^{\alpha}_{\text{incoming}} = 0$, but I use $F^{\alpha\beta}$, because $F_{\alpha\beta}$ is analogous to $C_{\rho\sigma\mu\nu}$.

paper, I reformulated the problem of underdetermination as the vacuum challenge. While some relationists have addressed this challenge by excluding "unphysical" vacuum solutions by fiat, I offered a richer story by recognizing the modelling artifacts involved. The metric field is underdetermined by the stress-energy tensor because an extra ingredient, the Weyl tensor, is needed. Decomposing the degrees of freedom in the metric into gauge, "source", and gravitational degrees of freedom, I identified the gravitational degrees of freedom (associated with the Weyl tensor) as the "cause" of the problem of underdetermination.

There is nothing intrinsically substantival about the Weyl tensor. It allows for gravitational effects that *something outside of a system exerts on the system*. On my interpretation, the vacuum challenge originates from the assertion that there *are* gravitational effects from *nothing outside*. The boundary problem arises when we rely on boundary conditions to do the heavy-lifting of sourcing without permitting anything to exist on the opposite side of the boundary. I then proposed a relationist response: "free" gravitational degrees of freedom are only free modelling artifacts. I did not, however, argue that relationsim is correct. There are further interpretive questions regarding the nature of the propagating degrees of freedom in the context of energy conservation in GR. But at least for the vacuum challenge, relationists can respond: nothing matters because in nothing we find matter.

Acknowledgement

I thank Craig Callender for his invaluable guidance and continual support. I thank Nancy Cartwright, Eddy Keming Chen, Eugene Chua, and Kerry McKenzie for their thoughtful feedback. I thank Julian Barbour, Raphael Flauger, Samuel Fletcher, Henrique Gomes, and Carl Hoefer for helpful conversations. Finally, I thank organizers and audiences at the 2024 Caltech Philosophy of Physics Reading Group, ILMPS Conference (London), BJPS Annual Conference (York), and PSA Biennial Conference (New Orleans) for their generous engagement with the project.

References

Barbour, J. (2010). The Definition of Mach's Principle. *Foundations of Physics 40*(9-10), 1263–1284.

Barbour, J. B. and B. Bertotti (1982). Mach's principle and the structure of dynamical theories. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences 382*(1783), 295–306.

Brown, H. R. (2005). *Physical relativity: Space-time structure from a dynamical perspective.* Oxford University Press.

Brown, H. R. and O. Pooley (2006). Minkowski space-time: A glorious non-entity. *Philosophy and Foundations of Physics 1*, 67–89.

Callender, C. and C. Hoefer (2002). Philosophy of space–time physics. In *The Blackwell Guide to the Philosophy of Science*, pp. 173–198. Blackwell.

Carroll, S. (2004). *Spacetime and Geometry : An Introduction to General Relativity.* Addison Wesley.

CPAE 8: Schulmann, R., A. Kox, M. Janssen, and J. Illy (1998). *Collected Papers of Albert Einstein Vol. 8. The Berlin Years: Correspondence, 1914–1918*. Princeton, New Jersey: Princeton University Press.

De Sitter, W. (1916). On the relativity of rotation in einstein's theory. In *Proceedings of the Royal Academy of Sciences Amsterdam*, Volume 19, pp. 527–532.

Earman, J. (2003). The cosmological constant, the fate of the universe, unimodular gravity, and all that. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 34*(4), 559–577.

Ehlers, J. and W. Kundt (1962). Exact solutions of the gravitational field equations. In *Gravitation: An Introduction to Current Research*. John Wiley & Sons.

Einstein, A. (1917). Cosmological considerations on the general theory of relativity. In H. A. Lorentz, H. Weyl, and H. Minkowski (Eds.), *The Principle of Relativity*. Trans. W. Perrett and G. B. Jeffery. Dover (1952).

Einstein, A. (1918). Prinzipielles zur allgemeinen relativitätstheorie. *Annalen der Physik 360*(4), 241–244.

Flores, J. L. and M. Sánchez (2020). The Ehlers-Kundt conjecture about gravitational waves and dynamical systems. *Journal of Differential Equations 268*(12), 7505–7534.

Gomes, H. and C. Rovelli (2024, Jan). On the analogies between gravitational and electromagnetic radiative energy. *Philosophy of Physics 2*(1), 1–25.

Goswami, R. and G. F. Ellis (2018). Transferring energy in general relativity. *Classical and Quantum Gravity 35*(16), 165007.

Hoefer, C. (1994). Einstein's struggle for a Machian gravitation theory. *Studies in History and Philosophy of Science Part A 25*(3), 287–335.

Lehmkuhl, D. (2011). Mass–energy–momentum: Only there because of spacetime? *The British journal for the philosophy of science 62*(3), 453–488.

Lehmkuhl, D. (2017). Literal versus careful interpretations of scientific theories: The vacuum approach to the problem of motion in general relativity. *Philosophy of Science 84*(5), 1202–1214.

Lehmkuhl, D. (2018). The metaphysics of super-substantivalism. *Noûs 52*(1), 24–46.

Mach, E. (1893). *1919. The Science of Mechanics.* Trans. Thomas McCormack. Chicago, London: Open Court.

Manchak, J. B. (2011). What is a physically reasonable space-time? *Philosophy of Science 78*(3), 410–420.

Maudlin, T. (1990). Substances and space-time: What Aristotle would have said to Einstein. *Studies in History and Philosophy of Science Part A 21*(4), 531–561.

Misner, C. W., K. S. Thorne, and J. A. Wheeler (1973). *Gravitation.* Macmillan.

Norton, J. D. (1984). How Einstein found his field equations: 1912-1915. *Historical studies in the physical sciences 14*(2), 253–316.

Norton, J. D. (1993). General covariance and the foundations of general relativity: eight decades of dispute. *Reports on progress in physics 56*(7), 791.

Norton, J. D. (2024). Einstein against singularities: Analysis versus geometry. *Philosophy of Physics 2*(1), 1–73.

Ozsváth, I. and E. Schücking (1962). An Anti-Mach-Metric. In *Recent developments in general relativity*, pp. 339–350. Pergamon Press.

Penrose, R. (1965). A remarkable property of plane waves in general relativity. *Reviews of Modern Physics 37*(1), 215–220.

Penrose, R. (1981). Time Asymmetry and Quantum Gravity. In R. Penrose, C. J. Isham, and D. W. Sciama (Eds.), *Quantum gravity 2: A second Oxford symposium*, pp. 244–272. Oxford: Clarendon Press.

Penrose, R. (1986). Gravity and state vector reduction. In R. Penrose and C. J. Isham (Eds.), *Quantum Concepts in Space and Time*, pp. 129–146. Clarendon Press.

Penrose, R. (2007). *The Road to Reality.* Penguin Random house.

Pirani, F. A. (1956). On the definition of inertial systems in general relativity. *Mercier and Kervaire 29*(4), 198–203.

Pooley, O. (2013). Substantivalist and relationalist approaches to spacetime. In R. Batterman (Ed.), *The Oxford Handbook of Philosophy of Physics.* Oxford University Press.

Raine, D. J. (1995). The integral formulation of mach's principle. In *Mach's Principle: From Newton's Bucket to Quantum Gravity.* Birkhäuser Boston.

Ritz, W. (1908). Recherches critiques sur l'électrodynamique générale. In *Annales de Chimie et de Physique*, Volume 13, pp. 145.

Roche, C., A. B. Aazami, and C. Cederbaum (2023). Exact parallel waves in general relativity. *General Relativity and Gravitation 55*(2), 40.

Rynasiewicz, R. (1996). Absolute versus relational space-time: An outmoded debate? *The Journal of Philosophy 93*(6), 279–306.

Rynasiewicz, R. (2000). On the distinction between absolute and relative motion. *Philosophy of Science 67*(1), 70–93.

Sciama, D. W. (1953). On the origin of inertia. *Monthly Notices of the Royal Astronomical Society 113*(1), 34–42.

Vishwakarma, R. G. (2015a). A Machian Approach to General Relativity. *Int. J. Geom. Meth. Mod. Phys 12*, 1550116.

Vishwakarma, R. G. (2015b). A new solution of Einstein's vacuum field equations. *Pramana 85*, 1101–1110.

Wald, R. M. (1984). *General Relativity.* University of Chicago Press.

Weingard, R. (1975). On the ontological status of the metric in general relativity. *The Journal of Philosophy 72*(14), 426–431.

Weyl, H. (1918). Reine infinitesimalgeometrie. *Mathematische Zeitschrift 2*, 384–411.

Wheeler, J. A. and K. Ford (2000). *Geons, black holes and quantum foam: a life in physics.* W. W. Norton & Company.

Zeh, H. D. (2007). *The Physical Basis of the Direction of Time.* Springer Verlag.