Whence the Desire to Close the Universe?

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Abstract. The geometry of the universe is today widely believed to be flat based on combined data obtained during the 2000s. Prior to this, the geometry of the universe was essentially unknown. However, within the relevant literature one finds claims indicating a strong preference for a (nearly) closed universe, based on philosophical and other "non-experimental" reasons. The main aim of this article is to identify these reasons and assess the extent to which philosophical reasoning influenced the establishment of the dark matter hypothesis and the development of models for a closed universe. Building on groundwork laid by de Swart (2020), this study expands the discussion by (a) arguing that opinions on the geometry of the universe during the 1970s and 1980s were more divided than often assumed, (b) uncovering a lesser-known Machian argument for flat geometry proposed by Dennis Sciama, and (c) presenting a fine-tuning argument stemming from the 'coincidence problem' articulated by Robert Dicke. The study provides a nuanced perspective on how philosophical considerations contributed to shaping early views on cosmology and dark matter and highlights the significant role philosophical reasoning can play in guiding scientific inquiry in physics.

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Date: Draft of January 21, 2025.

1. INTRODUCTION

In a seminal article on early dark matter theory, pioneering cosmologists Jeremiah Ostriker, James Peebles, and Amos Yahil open their discussion with the following observation:

> There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more. (Ostriker et al. 1974)

Shortly afterwards, they add this rather notable remark:

> If we increase the estimated mass of each galaxy by a factor well in excess of 10, we increase this ratio by the same amount and conclude that observations may be consistent with a Universe which is "just closed" ($\Omega = 1$) – a conclusion believed strongly by some (cf. Wheeler 1973) *for essentially nonexperimental reasons* (*ibid.*, emphasis added).

The authors then proceed to demonstrate that the calculated masses of spiral galaxies correspond to a local mean mass density of $\Omega = 0.2$, a result indicating that the additional mass required to "close the Universe" is still unaccounted for. In the absence of empirical data capable of definitively determining the geometry of the Universe – data that would only become available several years later (e.g. Spergel et al. 2003) – the authors essentially argue that the most plausible geometry of the Universe, based on philosophical considerations, is a flat one. This conclusion further implies that the mean mass density of the Universe must approximately equal the critical density.

The ultimate goal of the present article is to identify and analyse the 'nonexperimental reasons' that shaped the preference of cosmologists for certain models over others in the 1980s, and assess the extent to which certain philosophical considerations influenced their reasoning. By doing so, we aim to show how the interplay of philosophical arguments with empirical and theoretical developments of the time, significantly shaped certain scientific developments that eventually led to the acceptance of dark matter as a central component of cosmology in the 1980s along with the strong belief that the Universe is closed, or at least flat.

An important first step towards this direction has been made by de Swart (2020) who presents a compelling argument showing how the presumably widespread preference for additional matter to 'close the Universe' in the 1970s largely motivated the establishment of dark matter on a cosmological level. His narrative is focused on the landmark papers by Ostriker et al. (1974) and Einasto et al. (1974) in which the authors argue that the total mass of galaxies is significantly underestimated and that additional mass-energy is required to achieve a total density of $\Omega \geq 1$. According to de Swart (2020, p.268), this preference for a closed geometry was widely shared by many cosmologists at the time and it was closely related to a Machian conception of general relativity of which, as we shall see, John Archibald Wheeler (1962; 1973) was a prime advocate. In de Swart's historical account, the realization that the observed mass density at the time appeared to be much less than the required critical density to close the universe essentially made the observational anomalies of missing mass in clusters and flat rotation curves of stars in galaxies relevant, giving birth to the concept of non-baryonic dark matter as we know it today. The core idea was that the missing additional matter required to have a closed Universe was the same 'dark matter' required to explain the velocities of galaxies and the flat rotation curves of stars.

Our analysis expands on de Swart's work by making three novel contributions. First, we argue that the consensus for a closed geometry was not so profound as de Swart seems to imply and that opinions regarding the geometry of the Universe in the 1970s and 1980s were rather divided (Section 3). Nonetheless, as rightly noted by de Swart, there was at the time a strong consensus that the mean mass density of baryonic matter was insufficient to account for the available observations and that additional matter was required, regardless of the actual geometry. Second, in addition to the Machian argument due to Einstein and Wheeler for a closed Universe (Section 4.1), we bring to the surface a lesser known Machian argument from Dennis Sciama, which nonetheless leads to a different conclusion, namely a preference for a Universe with flat geometry (Section 4.2). Finally, we also present a markedly different argument based on fine-tuning considerations and the 'coincidence problem' due to Robert Dicke, which, in a sense, was the predecessor of the well-known flatness problem in cosmology (Section 4.3).

The analysis in Section 3 and the three philosophical arguments to be presented in Section 4, reveal the diverse motivations behind the scientific exploration of cosmic geometry and the eventual establishment of dark matter as a cornerstone of modern cosmology. They demonstrate how non-empirical considerations, such as commitment to certain principles and scepticism towards fine-tuning, can guide the development of scientific models in the absence of definitive observational data. By examining these arguments in detail, we aim to illuminate the intellectual context in which dark matter emerged as a solution to the challenges posed by cosmological anomalies. Ultimately, this historical perspective highlights the interplay between philosophical reasoning and empirical science and offers significant insights into the broader dynamics of theory choice in physics.

2. The geometry of the Universe

Before we delve into our main analysis, it is useful to present a brief introduction to the concept of the geometry of space and its relation to the mean energy-mass density of the Universe.

Two of the most important features of the Universe are its homogeneity and isotropy when viewed at a sufficiently large scale. Homogeneity means that the spatial distribution of matter is equally distributed and thus the Universe looks the same at each point. Isotropy means that there is no geometrically preferred spatial direction, and thus the universe looks the same in all directions. These two properties, typically referred to as the cosmological principle, lie at the core of the most important equation in cosmology, i.e. the Friedmann equation that describes the evolution of the Universe. This is because the imposed symmetries of the cosmological principle give rise to the Robertson-Walker metric (henceforth RW metric). By plugging the RW metric in the Einstein field equations and taking advantage of the symmetries in this solution, one is left with two independent field equations from which the famous (first) Friedmann equation is obtained:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Here, $H^2 \equiv (\dot{a}/a)^2$ is the Hubble parameter, a is the scale factor of the Universe describing how physical distances change with time, k is the curvature term, ρ is the matter-energy density of the Universe, and G and c are constants representing the gravitational constant and the speed of light respectively. Unlike the scale factor and the matter energy density that are functions of the cosmic time t, the curvature of the universe, k, is taken as a fundamental property that does not evolve over time, but is rather fixed at a unique value by the initial conditions of the Universe. Hence, under the assumption of the cosmological principle, the evolution of the Universe is represented as a time-ordered sequence of three-dimensional space-like hypersurfaces with a fixed geometry, each of which is homogeneous and isotropic. A great advantage of homogeneous and isotropic spaces is that they have the largest possible symmetry group, which in three dimensions comprises three independent translations and three rotations. These symmetries strongly restrict the admissible geometries of such spaces allowing only three possibilities: (a) a flat geometry where $k = 0$, (b) a spherical geometry where $k > 0$, and (c) a hyperbolic geometry where $k < 0$.

Although the human brain cannot visualize a three-dimensional curved space, what really matters is that hypersurfaces in such spaces can have the same properties as their two-dimensional counterparts, and these properties have nothing to do with our (in)ability to grasp and visualize their shape. The best way to visualise and understand these three-dimensional geometries is therefore by considering their two-dimensional counterparts. The mathematical generalization from two to three or more dimensions is relatively straightforward.

A universe with a flat geometry is the three-dimensional counterpart of a two-dimensional plane with no curvature ($k = 0$) and in which the axioms of standard Euclidean geometry hold. In this geometry parallel lines never meet, the angles of a triangle add up to 180° and the circumference of a circle of radius r is $2\pi r$. A flat Universe, insofar as it has a simple topology,[1] must be infinite in extent in order to preserve isotropy: if there are definite 'edges', i.e. boundaries, at the end of this three-dimensional plane, then the Universe will not look the same in all directions to an observer standing on those boundaries.

A spherical geometry of space is best understood in terms of a two-dimensional surface of a sphere in a Riemannian geometry. In this geometry, the fifth Euclidean axiom stating that parallel lines always remain a fixed distance apart is violated. Consequently, the angles of a triangle on such a surface add up to more than 180° and the circumference of a circle is less than $2\pi R$. The crucial point to note however, is that if these triangles and circles are small compared to the size of the surface, then Euclidean laws become a good approximation, for the same reason that the angles of a drawn triangle on the surface of the Earth add up to 180° with a negligible deviation. Hence, a universe with spherical geometry might actually appear to be flat to an observer making measurements in small scales, and this is precisely one of the main difficulties in measuring the exact geometry of our Universe; the measured region is only a miniscule fraction of the entire Universe. Similarly to the two-dimensional surface of a sphere, a three-dimensional spherical universe (often called a three-sphere) has a finite size but no boundaries, and for this reason it is typically referred to as a *closed* universe. It is a three-sphere in

[1]Roughly speaking, geometry describes the local shape and properties of space, whereas topology concerns the global properties of space on larger scales. General relativity captures very well the way matter dictates the geometry of space, however, it does not say anything about the topology of space on large scales. As we shall see below, it is possible that certain geometries are compatible with more than one topology. For more details on cosmic topology see Lachieze-Rey and Luminet (1995); Reboucas and Gomero (2004) and Luminet (2016).

which light is restricted on its hypersurface and is not allowed to move towards or away from its notional centre. No matter the direction, a light ray travelling in a straight line would eventually arrive back to where it started from the opposite direction.

Finally, the third possibility where k is negative, corresponds to a hyperbolic geometry whose two-dimensional counterpart is usually represented by a saddle-like surface. Unlike a Riemannian geometry where parallel lines converge, in a hyperbolic geometry parallel lines diverge away from one another. Consequently, the angles of a triangle in such a surface add up to less than 180° and the circumference of a circle is greater than $2\pi r$. And similarly to the flat geometry, insofar as the Universe has a simple topology, such a universe must also be infinite in extent to preserve isotropy, and it is thus often referred to as an *open* universe.

Flat and hyperbolic universes are most often understood as being infinite in extent, however, as already mentioned, this is only true if the Universe has a simple topology. If the Universe has a non-trivial topology, then flat and hyperbolic spaces can also be finite, just like a spherical universe. For instance, the simplest topology corresponding to a flat geometry is a three-dimensional surface, however the same geometry is also compatible with a non-trivial topology of a torus, and even more complex topologies such as a Clifford torus and a Poincaré dodecahedral space. This means that flat space may have different properties on large scales depending on whether its topology corresponds to a three-dimensional plane or a more complex topology such as that of a torus. Similarly, a hyperbolic geometry corresponds to a simple topology of a hyperbolic three-space but it is also compatible with several non-trivial topologies such as a hyperbolic three-torus, a Klein bottle and a hyperbolic honeycomb which all render the hyperbolic spaces finite. The upshot is that flat and hyperbolic universes can be both finite or infinite depending on the corresponding topologies. A closed universe is necessarily finite no matter the topology.[2]

Before proceeding to the next section, it is important to clarify the relationship between the curvature of space and the energy-matter density of the Universe to appreciate the strong connection between dark matter and the geometry of the Universe. As can be seen from the Friedmann equation, for a given value of the Hubble parameter H, there is a special value of the matter density

[2]If the geometry of the universe is even slightly non-flat (as allowed by current observations), then, in principle, it is possible to detect the effects of a non-trivial topology, depending on its scale. No effects of non-trivial topologies have been detected so far however.

Curvature	Density parameter	Geometry	Type of universe	
			Simple Topology	Non-trivial Topology
$k > 0$	$\Omega > 1$	Spherical	Closed/Finite	Closed/Finite
$k = 0$	$\Omega = 1$	Flat	Open/Infinite	Closed/Finite
$k < 0$	$0 < \Omega < 1$	Hyperbolic	Open/Infinite	Closed/Finite

TABLE 1. Summary of types and geometries of universe depending on curvature, density and topology.

$$\rho_c(t) = \frac{3H^2}{8\pi G} \tag{1}$$

that renders the geometry of the Universe flat ($k = 0$). This is the so-called critical density ρ_c, which essentially corresponds to the amount of matter necessary to provide enough gravitational attraction to balance the expansion of the Universe. For the current values of the Hubble constant h and the universal gravitational constant G, its value becomes $\rho_c(t_0) = 1.88h^2 \times 10^{26}$ kg m^{-3}. If the actual mean matter density of the Universe ρ is less than the critical density, then the Universe has a negative curvature and expands forever. If it is equal, then the Universe is flat and 'balanced'. If it is larger, then matter eventually halts and reverses the expansion resulting in a closed Universe. These possibilities are typically expressed in terms of the density parameter $\Omega(t) = \rho/\rho_c$ which is a dimensionless quantity describing the ratio between the actual density of the Universe and the critical density.[3] Depending on the actual matter density of the Universe (and regardless of the type of matter), the density parameter can be (i) $0 < \Omega < 1$, (ii) $\Omega = 1$, and (iii) $\Omega > 1$ corresponding to a hyperbolic, flat, and spherical universe respectively. Hence, amongst other things, the Friedmann equation indicates how the amount of matter in the Universe is related to its geometry. And as we shall see in the next section, the requirement for a certain amount of matter to 'close the Universe' played an important role in realizing that the dark matter responsible for the observed galactic anomalies in the 1960s and 1970s might actually be non-baryonic. The main points of our discussion in this section are summarised in Table 1.

3. NON-BARYONIC DARK MATTER AND GEOMETRY

As noted in the introduction, de Swart (2020) provides a convincing argument indicating how a widespread preference to close the Universe in the 1970's

[3]Note that both the critical density ρ_c and the density parameter Ω are functions of time, since the Hubble parameter also changes with time.

and 1980s played an instrumental role in the establishment of the dark matter hypothesis. Before we delve into the details of how dark matter was established in the 1970s and 1980s however, it should be noted that the terminology used by de Swart in terms of a closed geometry (cf. the title of the article: ‘Closing in on the Cosmos’) is somewhat misleading, since, as we shall see in the next section, prominent cosmologists of the time – such as James Peebles and Dennis Sciama – seemed to be in favour of a flat rather than a closed geometry. The widespread desire to ‘close the Universe’ to which de Swart refers, should be understood as a requirement for a total mass-energy density corresponding to $\Omega \geq 1$, i.e. a value of ρ_0 which is *at least equal* to the critical mass and would therefore eliminate the possibility of an open Universe. The non-experimental/philosophical reasons for this preference will be presented in detail in the following section. In this section, we shall argue that not all cosmologists were against the idea of an open Universe.

In particular, we will argue that the establishment of non-baryonic cold dark matter at the cosmological level – and its eventual connection to the observed anomalies at the galactic scales – essentially stemmed from the parallel theoretical developments in Big Bang Nucleosynthesis and large-scale structure formation. These developments jointly indicated that additional non-baryonic mass was needed to align the theory with observational data, regardless of the whether the Universe is open, flat, or closed. In particular, the theory of nucleosynthesis revealed that the total amount of baryonic matter, from all possible sources, was significantly smaller than the critical density. Similarly, the problem of large-scale structure formation demonstrated that baryonic matter alone could not account for the observed large-scale structure of the Universe and, consequently, the nature of the cosmic microwave background (CMB) radiation. Along with a cautious belief that the total mass density of the Universe was of the same order of magnitude as the critical density based on measurements of the Hubble parameter, these developments highlighted the requirement for additional matter to reconcile theoretical predictions with data. In other words, by the early 1980s cosmologists were convinced that there must be some additional amount of non-baryonic matter in the Universe without necessarily endorsing a closed or flat cosmological model, and this was sufficient for the establishment of non-baryonic dark matter.

We concede that assessing the exact degrees of preferences for the different models of the geometry of the Universe amongst the scientific community at that time is challenging. Nevertheless, one finds in the literature claims from cosmologists who endorse the idea of additional non-baryonic matter but at the same time advocate in favour of an open cosmological model, and our aim in this section is to acknoweledge this fact. The most characteristic example comes from a rather influential paper of the time by Gott et al. (1974, p.553) who clearly express their preference for an open Universe invoking arguments from simplicity and unification: 'Open models [...] explain simply and readily a wide variety of observations, from the abundance of deuterium to the value of the Hubble constant. If a closed model is to fit the observations, a number of additional (apparently ad hoc) processes must be called into play, which mimic in a complicated fashion the same result one would obtain from a simple open model'. Similarly, in a letter published in Nature in 1981 Lindley (1981, p.391) begins by noting that 'The feeling among cosmologists is that the Universe is open, with Ω perhaps in the range 0.1 – 0.5'. An earlier example comes from Chiu (1967) who considers a set of astronomical observations from the CMB radiation and the intergalactic density of hydrogen to conclude that 'Our Universe can be described by an open cosmological model.' (p.1) and that 'it is unlikely that [it is] described by a closed model.' [p.3].

To fully appreciate the situation in the 1970s and early 1980s, it is useful to assess the state of the art with respect to the matter density and the geometry of the Universe in light of three distinct, but related, issues: (a) the estimation of the actual mean mass density of the Universe at the time, (b) the calculation of baryonic density from primordial abundances of helium and deuterium, and (c) the conception of the structure formation problem with respect to data from the CMB.

As we have seen in Section 2, the mean mass density of the Universe including all forms of matter determines the geometry of the Universe. The current estimates including contributions from baryonic matter, radiation, dark matter and dark energy provide a value of Ω very close to 1 indicating a flat geometry, but this was only determined in the early 2000s mainly by data from the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck satellite. In the 1970s, the actual density of the Universe was essentially unknown, although various studies had already placed limits on the possible contributions to the total matter density from different sources such as galaxies, radiation, intergalactic matter, stars etc. The consensus at the time was that the joint contribution of these different sources was significantly less than the critical mass, leaving all three possibilities for a hyperbolic (open), flat and spherical geometry open.[4] These constraints however, were in sharp contrast with crude estimates from measurements of the Hubble

[4]See, for instance, Field (1972) for a review of the various constraints on different possible contributors to the mass density at the time.

constant at the time pointing towards an actual density close to the value of the critical density. Within the Friedmann model, the ratio of the present density to the critical density was given by

$$\frac{\rho_0}{\rho} = 2q_0$$

where q_0 is the deceleration parameter, a dimensionless measure of cosmic acceleration defined as $q \coloneqq -\ddot{a}a/\dot{a}^2$.

In his celebrated textbook, Weinberg (1972) provides a plausible value for ρ based on the measured value of the Hubble constant at the time, while clearly expressing his hesitations due to the large uncertainties in the determination of the value of H_0. He writes: 'If we give credence to the values $q_0 \simeq 1$ and $H_0 \simeq 75$ km/sec/Mpc deduced from the red shift versus luminosity relation [...], then we must conclude that the density of the universe is about $2\rho_c$' (p.476) which would of course result in a closed Universe. A few pages later, he connects this result with dark matter: 'If one *tentatively* accepts the result that q_0 is of order unity, then one is forced to the conclusion that a mass density of approximately 2×10^{29} g/cm^3 must be found outside the normal galaxies' (p. 478, emphasis added) although with the caveat that these results are provisional. The following passage from Sandage and Hardy (1973, p.754) is also indicative of the reservations about the actual value of q_0 (and consequently ρ_0) at the time: 'The determination of q_0 is clearly a problem for the future. Only the grossest alternative solutions such as $q_0 = -1$ or $q_0 > 3$ can be discarded at this moment'.

The upshot is that in the 1970s the scientific community in cosmology was rather divided over the real value of the total mass density and the geometry of the Universe. On the one hand, crude estimates based on measurements of the Hubble constant indicated that the mass density of the Universe is of the same order of magnitude and possibly larger than the critical density, while on the other hand the calculated contributions from the then known possible sources (excluding non-baryonic dark matter and dark energy) were significantly less than the critical density. Cosmologists such as Ostriker et al. (1974) in favour of a closed model saw the latter results as an indication of the presence of additional mass up to the critical density, whereas Gott et al. (1974) interpreted this as an indication of an open universe with a parameter density in the range of $0.05 \geq \Omega \geq 0.09$.

Gott et al. (1974)'s views were also significantly enforced by recent results on the total density of baryonic matter based on arguments from nucleosynthesis. According to the theory of nucleosynthesis the formation of nuclei in the Universe took place at about one second after the Big Bang and at that stage hydrogen and helium-4 were produced in much larger abundance compared to other primordial elements. The expectation is that about 24% of the matter in the Universe is in the form of helium-4 and that there is one helium-4 nucleus for every 14 hydrogen ones. Measurements of the abundances of these elements could therefore provide the total density of baryonic matter in the Universe, from which these nuclei are composed, since the two quantities are closely related. Based on this kind of arguments, an important study (among many) by Geiss and Reeves (1972) showed that data related to helium and deuterium abundances in the solar system could only be explained if the present baryonic density is $\Omega_b = 0.06$, with the assumption that these abundances in the solar system are primordial. The significance of these results was that they were independent of observations of low luminosity objects such as brown dwarfs and distant galaxies, since the obtained limit applied to all baryonic matter in the Universe, providing strong evidence that the density of baryonic mass is significantly lower than the critical density regardless of other observations.[5]

Based on similar results by Rogerson and York (1973) on the interstellar abundance of deuterium, Gott et al. (1974) presented a summary of different constraints on the total matter density arguing that the satisfaction of these different criteria limits the range of possible values to $0.05 < \Omega < 0.09$. When discussing constraints from the deceleration parameter, Gott et al. (1974, p.545) refer to the calculations by Sandage and Hardy (1973) pointing towards an apparent value of $q_0 = 1.0 \pm 1$ by noting that 'The apparent value must be corrected for the systematic effects of luminosity evolution, which are as yet inadequately understood but are likely to give a true value of q_0 that is smaller by 0.4 to 1.2' and that 'Until the statistical uncertainties and systematic corrections involved in this difficult determination are improved, we feel safe in concluding only that $q_0 < 2.0$' indicating their reservations to accept these measurements as evidence for a closed Universe.

These results were also in line with the third issue at hand, namely the significant developments by Peebles (1965, 1966, 1968) on the structure formation problem in the late 1960s. Following the discovery of the cosmic microwave background radiation in 1965, Peebles showed that the formation of the observed large-scale structure of the Universe requires a relatively large amplitude of the fluctuations in the photon-baryon fluid at the decoupling epoch (i.e. at a redshift of $z \approx 1000$, or

[5]For a review of the constraints on Ω from element abundances in the 1970s see Schramm and Wagoner (1977).

$\sim$ 300 000 years after the Big Bang), which should correspond to comparable fluctuations in the CMB temperature spectrum. This is because structure formation in a homogeneous universe can only happen from gravitational collapse if the density fluctuations are larger than the Jeans length, i.e. the distance travelled by a sound wave during a collapse timescale. Before the decoupling of photons and baryons however, the speed of sound is comparable to the speed of light, and thus the Jeans scale is essentially comparable to an event horizon. As a result, smaller fluctuations which could potentially give rise to galaxies and clusters do not grow, but rather propagate as sound waves. Gravitational collapse can therefore only begin at the decoupling of photons and baryons, and with a relatively large amplitude which should show up in the CMB power spectrum. The apparent problem was that such fluctuations were nowhere to be seen. The deepest, and most important implication which would soon become clearer, was that there must be a significant amount of additional non-baryonic matter in the Universe produced in its early times.

Indeed, a few years later Gunn et al. (1978) argued that any heavy, stable and non-interactive particle which is a cosmic relic could dominate the mass density of the Universe and would, in fact, make 'an excellent candidate for the material in galactic halos and for the mass require to bind the great clusters of galaxies' (Gunn et al. 1978, p.1015). Most importantly, their results placed the seeds for a potential solution to the structure formation problem, expressed by a number of different cosmologists in the early 1980s (Peebles 1982; Bond and Szalay 1983; Bond and Efstathiou 1984; Vittorio and Silk 1984). The possible presence of a significant amount of non-baryonic matter in the early Universe meant that the speed of sound in this fluid could be much lower than the photon-baryon fluid and thus fluctuations would not propagate but continue to grow until they collapse before the decoupling epoch. This would in turn allow the formation of large-scale structure at an early stage, making theory compatible with the then available observations of the CMB temperature spectrum. It was at this stage that dark matter as we know it today, i.e. as a (family of) non-baryonic primordial particle(s) at a cosmological level, began to gradually gain widespread acceptance within the scientific community. The exact percentage of the contribution of this non-baryonic field to the total mass density of the universe however, was essentially unknown.[6]

Nevertheless, as rightly argued by de Swart (2020), a number of cosmologists were strongly motivated by independent philosophical arguments in favour of an – at

[6]cf. Bond and Efstathiou (1984, p.L48): 'Cold [dark matter] particle and massive neutrino models with $\Omega = 1$ are not strongly constrained at present'.

least – closed Universe with the seminal work of Jeremiah Ostriker, James Peebles, and Amos Yahil being the most characteristic example. Interestingly, in their 1974 article the three authors appeal to Wheeler (1974) to justify their preference for a Universe that is 'just closed', while in his later work Peebles (1984, p.470) appeals to a co-authored essay with Robert Dicke (Dicke and Peebles 1979) to justify the assumption that $\Omega = 1$ in his proposed model for dark matter and the origin of galaxies. In the next section, we shall trace the origins of these preferences for a flat or closed universe, by presenting what we consider to be the three most important philosophical arguments for the geometry of space in the twentieth century, prior to the experimental determination of the curvature of the universe. As we shall see, although two of these arguments are based on 'Mach's principle', they actually follow entirely different trajectories of reasoning, while the third argument involves an independent line of thought stemming from fine-tuning considerations.

4. Philosophical arguments for cosmic geometries

4.1. **Einstein and Wheeler's closed Machian Universe.** The earliest argument for a particular cosmic geometry in the context of general relativity was put forwards by Albert Einstein (1917[1952]). Einstein's argument is based on his attempts to implement Mach's principle in general relativity, suggesting that the universe ought to be closed. As is well known, along with the principle of equivalence, Mach's principle formed one of the cornerstones of Einstein's reasoning in his early development of the theory of general relativity.[7] In fact, the theory of general relativity (GR) was constructed in part as a means of overcoming the lack of conformity with Mach's principle found in classical mechanics. The exact definition of Mach's principle, it's validity and relevance to GR, are all subject to enduring controversies, however we will not dwell long on these issues here.[8] Instead we will only consider Mach's principle insofar as it has been leveraged in favour of various arguments for particular classes of cosmological models.

The key attribute of Mach's principle that makes it relevant to cosmology is the suggestion of a necessary coupling between local physical systems and their broader environment in the rest of the cosmos. Noticing that epistemically, we can only observe the (rectilinear and uniform) inertial motion of a given body insofar as it is performed with respect to the other bodies of the universe, Mach (1872,

[7]See (Einstein 1996, p.228) for Einstein's specific admission of these motivations.

[8]For some relevant recent literature concerning the controversies surrounding the definition of Mach's principle, it's validity and status in general relativity see for instance: Barbour and Pfister (1995); Norton (1995); DiSalle (2002); Hoefer (1994, 2014); Staley (2021); Fay (2024).

1883) raised the hypothetical suggestion that this so-called 'inertial' motion might in fact be caused by a direct physical influence of those other bodies (i.e. the mass content of the rest of the universe). Through Einstein's equivalence principle, this idea became linked explicitly with the project of unifying inertia and gravity; in particular, Einstein (1912) hypothesised that the inertial properties of matter might be entirely reducible to their gravitational interaction with the rest of the matter in the universe.[9]

As Hoefer (1994) has shown, Mach's principle, for Einstein, would become conflated with the requirement for the covariance of physical laws under general coordinate transformations. Mach's principle was associated with the extension of the relativity principle to arbitrary motions of the reference system, since this would allow one to eliminate the classical privileging of 'inertial' reference frames. If absolute space has no physical reality, then only the relative configuration and relative motion of bodies should be physically relevant, thus the laws of physics should be invariant under any transformations of our reference frames which maintain this relative configuration.

In a move which would be criticised by later Machians such as Sciama and Barbour (Sciama 1951; Barbour 2010), Einstein — following the advice of his colleague Marcel Grossmann — sought a generalisation of the relativity of motion in the principle of general covariance under the full diffeomorphic group of arbitrary coordinate transformations. However, it has since become well understood that Einstein's approach did not generalise the relativity of motion in the sense that Mach intended. Hoefer (1994, p.292) offers some insight into the cause of Einstein's confusion by stating that the temptation to view the four dimensional coordinate systems as aligning with the relative spaces of Einstein's earlier work and to assume that generally covariant GR as standard formulation applicable in any such relative space, explains why Einstein claimed that general covariance guarantees the general relativity of motion. Einstein initially expected that his generally covariant field equations would automatically implement Mach's principle, or at least that appropriate Machian boundary conditions would easily be found. Between 1915 and 1917, he sought these boundary conditions, but these efforts culminated in his famous cosmological paper (Einstein 1917) in which an alternative solution to the problem was proposed.

[9]Note that the first to link Mach's principle to the unification of gravity and inertia was Immanuel Friedlaender (Friedlaender and Friedlaender 1896), however it was Einstein's work that made this a subject of focused research.

In this paper, Einstein formulates Mach's principle as the following criterion on a cosmological model of general relativity:

> In a consistent theory of relativity there can be no inertia *relatively to "space,"* but only an inertia of masses *relatively to one another* (Einstein 1952, p.180, emphasis in the original).

From this he reasons that the line element must become singular at infinity, i.e. that (supposing the universe as an *island of matter* in space) the spatial components of the metric should go to zero and the temporal component should go to infinity. However, with the help of the mathematician Jakob Grommer and the astronomer Willem de Sitter, Einstein was led to the realisation that such boundary conditions are not feasible.[10] Finally, Einstein presents his solution: the need for boundary conditions can be avoided by postulating that the universe is of constant positive curvature and thus closed and finite:

> ...if it were possible to regard the universe as a continuum which is finite (closed) with respect to its spatial dimensions, we should have no need at all of any such boundary conditions (Einstein 1952, p.183).

As is well known, to ensure that his spherical universe did not collapse upon itself, Einstein further modified his field equations by introducing the cosmological constant λ. However, as rightly noted by Hoefer (1994) the introduction of the constant λ into the field equations in effect undermines Mach's principle since λ does not have a physical interpretation other than as a primitive property of spacetime, whereas Mach's principle expresses the goal of reducing the physical properties of spacetime to relations between material things – an issue first raised by de Sitter. Moreover, Hoefer also notes that Einstein's solution does not guarantee that the metric actually is fully determined by the material distribution in Einstein's closed universe.

The difficulties with Einstein's reasoning did not stand in the way for John Archibald Wheeler however, who was one of the most prominent advocates of the idea that Mach's principle implied a closed universe in later decades.[11] Although Wheeler's reasoning for a closed Universe was similar to Einstein's, it is somewhat

[10] Interestingly, William de Sitter commenting on Einstein's theory in 1917 attributes a similar attempt to Paul Ehrenfest: 'The idea to make the four-dimensional world spherical in order to avoid the necessity of assigning boundary conditions, was suggested several months ago by Prof. Ehrenfest, in a conversation with the writer. It was, however, at that time not further developed' (De Sitter 1917, p.1219).

[11] cf. Tipler (1977, p.500): 'John Wheeler is the most ardent proponent of the one-cycle closed universe.'

simpler, seemingly coming from a heuristic comparison of GR with his and Feynman's 'absorber condition' in electrodynamics (Blum and Brill 2020). Wheeler and Feynman's absorber condition posits that all photons are emitted and absorbed by particles. This allowed for an action-at-a-distance reformulation of electrodynamics which sought to resolve various difficulties such as the presence of divergences in quantum electrodynamics. By considering every photon as a direct interaction between an emitter and an absorber, Wheeler and Feynman could circumvent the need for a field and consider electrodynamics as an action-at-a-distance theory between particles.

Over the course of the 1940s, Wheeler became increasingly involved with the attempt to extend this action-at-a-distance program to gravitation, which meant the elimination of the concept of spacetime in favour of an ontology consisting of particle worldlines and lightcones. By the late 1950s, Wheeler had become aware of the connection between this program and Mach's principle, which thereafter became a central research question for him. Mach's principle was the means by which Wheeler's action-at-a-distance (AAD) gravity could make contact with general relativity. However, since not all models of GR satisfy Mach's requirement, Wheeler wanted to restrict his attention to those that do. Wheeler concluded that a gravitational analogue of his and Feynman's absorber condition in electrodynamics would only be possible if the universe was spatially closed, otherwise gravito-inertial influences would be able to escape to infinity (Blum and Brill 2020).

Another argument which Wheeler leveraged in support of his closed universe cosmology involves a widely misunderstood connection between the strength of gravity and the size of the universe which has for quite some time been recognised as a key condition associated with Mach's principle. This idea will be discussed in more detail in the next section since it played a key role in Dennis Sciama's thinking concerning the geometry of the universe. For our purposes here it suffices to say that a strong reading of Mach's principle requires that in the earth's frame of reference the plane of a Foucault pendulum must be pulled around by the gravity of the distant stars. As Thirring (1918, 1921) had shown in the early days of GR, such an effect (now known as the Lense-Thirring effect) does exist in Einstein's theory, however the universe would require a very specific constitution in order to ensure that this condition holds. At least as early as 1953, Wheeler was aware of a relation of the form: $\frac{G}{c^2}\sum\frac{m_{\mathrm{i}}}{r_{\mathrm{i}}} \sim 1$, where m_i and r_i are the masses of the universe and their distance to the given point respectively.[12] However this expression remained vague due to difficulties integrating over the influences of distant stars, since at long distances "the curvature of space begins to introduce substantial corrections into the calculation of Thirring and Lense" (Misner et al. 1973, p.548). In Misner et al. (1973, §21.12.), Wheeler briefly derives an expression of this form and comments that it "is a characteristic feature of the Friedman model and other simple models of a closed universe", however the vagueness of the result does not allow him to specifically endorse any particular cosmological model. As we shall see in the next section, the characteristic vagueness of this Machian condition in the context of GR continued to impede the development of a specifically Machian cosmological theory, however, a closer analysis of this condition and the reasons for it do suggest that there is a deep connection between Mach's principle and the geometry of the cosmos, but that this has very little to do with Einstein's initial idea that a Machian universe should be closed to avoid the problem of finding boundary conditions.

4.2. **Sciama's Machian argument for a flat universe.** In his doctoral work of the early 1950's, the British physicist Dennis Sciama sought to re-examine the question of Mach's principle by going back to the basics (Sciama 1953a,b).[13] Sciama's aim was to construct a theory of inertia which was in full conformity with Mach's principle (that inertia should have a material origin) as well as Einstein's insight that the material origin of inertia should come from the gravitational influence of the universe's masses. However, rather than require the covariance of physics under the general diffeomorphic group as Einstein did, Sciama restricted his analysis to shape-preserving transformations which do not alter the universe's internal geometry, i.e. shape (Sciama 1953b, p.50). For this reason, his initial work on a simplified model of inertia and gravitation does not involve positing curved space-time; instead Sciama drew from Rosen (1940) and Weyl (1944) to justify his use of a Euclidean background spacetime. For illustrative purposes, in the first half of his thesis, which would become published in a paper (Sciama 1953a), Sciama constructed a simplified model of gravity and inertia which, unlike GR, would incorporate Mach's principle by construction by employing an analogy with electromagnetism. The

[12] This expression was given in a talk he gave in Tokyo in September 1953 (Blum and Brill 2020). A relation of this form has widely been associated with Mach's principle, especially due to Brans and Dicke (1961) and Sciama (1953a), however it has also been defended on purely dimensional grounds (Sciama 1969) as well as in connection with the "famous numerical coincidences between atomic and cosmological quantities" (Sciama 1971, p.100) which were first pointed out by Dirac (1938).

[13] We are thankful to the family of Dennis Sciama for allowing us to recover his PhD thesis from the Cambridge archives and digitise it.

gravito-inertial forces would follow from the single four-vector potential:

$$\Phi = -\int_V \frac{\rho}{r} dV, \qquad \bar{A} = -\int_V \frac{\bar{v}\rho}{r} dV.$$

Instead of the charge and current density of surrounding matter as sources of Φ and $\bar{A}$ however, the sources of this gravito-inertial force are now the mass ρ and momentum $\rho\bar{v}$ densities of the relevant surrounding matter. However, since, unlike charge, mass is always positive, distant matter is no longer neutral and the integration must be performed over the entire observable universe V, where r is the distance of the matter density being integrated from the relevant point at which we are calculating the potentials. From this simple model, Sciama was able to derive both inertial and gravitational effects simultaneously, however the correct centrifugal and Coriolis forces are only produced if the strength G of this gravito-inertial coupling is given by the expression:

$$G\Phi = -c^2. \tag{2}$$

Therefore, inertia could only be explained entirely in terms of the gravitational influence of distant matter if this equation holds exactly. Reflecting on the consequences of his expression, Sciama (1953b, p.40) notes that the equation implies that the total energy (inertial and gravitational) of a particle at rest in the universe is zero, a conclusion which would later connect these ideas to the Einstein-de Sitter model of cosmology.

Unlike in general relativity, where, as Wheeler noted, the curvature of spacetime prevents any precise calculation of the contribution of very distant masses to inertial effects, Sciama was here able to derive a precise expression. Assuming a homogenous and isotropic universe with a Hubble expansion parameter H, Sciama (1953a) obtained: $\Phi = -2\pi\rho c^2 H^{-2}$, which when plugged into Eq.2 gives:

$$2\pi G\rho = H^2$$

As one may notice, Sciama's condition is very similar to the standard flatness condition of the Friedmann equation (Eq.1) derived in Section 2:

$$\frac{8}{3}\pi G\rho = H^2$$

with a slight difference of a factor of 4/3, which can be plausibly attributed to the differences in the assumptions of the two models. It is natural to ask whether Sciama's condition on the gravitational constant is in fact identical to the critical density condition in cosmology. However, at the time Sciama was writing his doctoral dissertation, the question of the geometry of the Universe was not among his major concerns. As a young physicist in the 1950s, Sciama was operating within the steady-state model of cosmology under the influence of steady-state theorists Hermann Bondi and Thomas Gold.[14]

Following the mounting experimental evidence against the steady-state cosmology in the 1960's, Sciama would humbly renounce his support for this theory and simultaneously abandon much of his early career work. However, he did continue to endorse the centrality of Mach's principle for cosmology, and held on to the hope that the constraints which Mach's principle places on cosmology might some day enable the derivation of a unique cosmological theory. For instance, in his "Physical foundations of general relativity" (Sciama 1969), Sciama reveals his continuing interest in the Machian expression discussed earlier by including a brief dimensional argument in support of of this relation (Sciama 1969, p.56):

> On dimensional grounds alone we expect to find a relation of the form: $G\rho R^2/c^2 \sim 1$, as given by the linear theory. Only the value of the right-hand side is in doubt, and presumably the exact theory would throw up a factor like, say, $3/8\pi$.

The proposed equation would therefore be $G\rho R^2/c^2 = 3/8\pi$ which is conspicuously the exact flatness condition that, as we saw, can be derived from the Friedmann equation (since $H^2 = c^2/R^2$). Although Sciama does not explicitly mention this connection in his book, it serves as a clear nod to his support for flat geometry of the cosmos. Some years later, in an interview for the American Institute of Physics, Sciama reveals why he had a preference for a flat universe, confirming its connection to his early work on Mach's principle (Weart and Sciama 1978):

> [**Sciama:**] Sometimes I've had a preference for the universe that just expands forever with the velocity tending to zero, the Einstein-de Sitter model, I once hoped, but that hope has not been realized in our work up to date, that Mach's Principle would lead to the unique model of the Universe, and then the hope was that it would be the one I've just described. [...] the Einstein-de Sitter model is the one where the total energy of the universe is zero, the kinetic energy and the negative gravitational potential energy just balancing. Well if you think that kinetic energy manifesting inertia is due to gravitation, then you might intuit that the most Machian way of having one made by the other would be if there's equal amount of energy, which would

[14] In fact it was Thomas Gold who was chiefly responsible for stimulating Sciama's interest in Mach's principle (Sciama 1969).

> give you uniquely the Einstein-de Sitter model, I still have a secret hope that that might turn out so, but it may well not.

As we saw earlier in this section, this balance between the energy of matter and the gravitational potential leading to a universe with zero total energy was an exact prediction of Sciama's 1953 Machian model of inertia and gravity. This interview by the American Institute of Physics therefore demonstrates that his continued endorsement for the flat Einstein-de Sitter cosmology was deeply connected to his early work on Mach's principle. However the pressure to conform to empiricist norms of research forced him to be somewhat secretive about these prejudices; indeed a few years later, Sciama would deliberately omit any mention of these kinds of speculations in his 1993 textbook "Modern cosmology and the dark matter problem" (Sciama 1993).[15]

Throughout his career, Dennis Sciama served as a supervisor and mentor to numerous students who would go on to make highly significant contributions to physics, including John D. Barrow, David Deutsch, George F. R. Ellis, and Stephen Hawking. Sciama's preference for flat cosmology, grounded in Mach's principle, likely influenced his students and peers to some extent. Notably, Roger Penrose has often recounted how Sciama's enthusiasm for Mach's principle during the 1950s helped inspire him to switch from mathematics to cosmology. However, Sciama's characteristic humility, the growing importance of experimental cosmology during that era, and the occasionally diverging views of his students ensured that, unlike John Wheeler, he did not foster a rigid dogma around Mach's principle within his circle.

4.3. **Dicke's coincidences and the flatness problem.** The third and final philosophical argument for the geometry of the Universe in the 1970s and 1980s comes from a fine-tuning problem mainly due to Robert Dicke, the supervisor and long-term collaborator of James Peebles at Princeton. Although not widely discussed in the literature of the time, this issue was, according to Peebles (1993, p.365), 'part of the standard lore for at least a decade' within the discussions of Dicke's research group at Princeton, until it was first published in a series of lecture notes by Dicke (1970).[16] Dicke's coincidence arguments served as the precursor to the well-known

[15] Indeed much of the material in this book is taken directly from his previous 1971 textbook "Modern Cosmology" (Sciama 1971). These include almost all sections from chapter chapter 8 of the original book apart from those containing Sciama's philosophical remarks on cosmology, Mach's principle and the 'large numbers' coincidences of Dirac.

[16] cf. also (Peebles 2020, p.59): 'This coincidence argument may have occurred to many who did not bother to publish, because it was not at all clear what to make of it. I remember its discussions in the early 1960s in meetings of Dicke's Gravity Research Group.'

flatness problem, which led amongst other things, to the development of the theory of inflation by Alan Guth in 1981.[17] The underlying idea is that if the curvature term or the cosmological constant term in the Friedmann equation are taken to be zero at the early universe then galaxies, the solar system and therefore we, as observers, begin to form right after the very special epoch at which the deceleration of the expanding universe turns into an acceleration, i.e. when the energy-mass density ceases to be the dominant term.

For a more concrete example, consider the Friedmann equation in terms of the different fractional contributions to the expansion rate:

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_r H_0^2 \left(\frac{a_0}{a(t)}\right)^4 \left[1 + \frac{\Omega_m}{\Omega_r}(\frac{a}{a_0} + \frac{\Omega_k}{\Omega_r}\left(\frac{a}{a_0}\right)^2 + \frac{\Omega_\Lambda}{\Omega_r}\left(\frac{a}{a_0}\right)^4\right]$$

Here, the parameter $\Omega_r \sim 10^{-4}$ is the fractional contribution to the present expansion rate by the thermal radiation and massless neutrinos and Ω_m is the matter density parameter, which based on observations is about 0.1. At the epoch of nucleosynthesis where light elements begin to form, i.e. at redshift $z = a_0/a \sim 10^{10}$, we may consider two possible scenarios: one in which there is no cosmological constant and $\Omega_k = 0.9$, and one in which there is no space curvature and the cosmological constant contributes $\Omega_\Lambda = 0.9$ to the present rate of expansion. In both cases, the contributions from the curvature ($\sim 10^{-16}$) and Λ ($\sim 10^{-36}$) to the expansion rate at the time of nucleosynthesis are practically negligible compared to the contribution of the mass density ($\sim 10^{-7}$). However, in both cases, it is a curious coincidence that galaxies, stars and observers like us begin to form at the very special epoch where the curvature and cosmological constant, which would have been extremely small at the epoch of nucleosynthesis, become the dominant contribution to the expansion rate. If both terms are taken to be non-zero at the epoch of nucleosynthesis then the situation becomes even more puzzling since in that case one of the terms would dominate the expansion much earlier leading to either an open or closed universe or an accelerated expansion at times that do not match the observational data from the early universe.

[17] Dicke mentions in an oral interview to the American Institute that Alan Guth, who was a postdoc at Cornell at the time, attended one of his lectures on these strange coincidences (Lightman and Dicke 1988). This fact is also reported by Alan Guth in his short autobiography for the 2014 Kavli Prize in Astrophysics: 'On Monday (November 13 [1978]) I happened into a lecture given by Bob Dicke, a well-known cosmologist from Princeton, who described a feature of the conventional big bang theory called the flatness problem. The problem concerned the extreme amount of fine tuning that is needed to make the conventional big bang theory work. [...] I did not understand the calculations behind what Dicke was saying, but I was very impressed by the conclusion, and tucked it away in the back of my brain.'(Guth 2014).

In other words, the fine-tuning worry occurs from the fact that we live in the special epoch when one of these two terms (Ω_k and Ω_Λ) which had an almost negligible effect in the early Universe, have just recently grown enough to dominate the expansion rate. And the fact that we happen to observe the universe at this distinguished and rare period when either curvature or Λ has just become important was considered by Dicke and his collaborators a 'suspicious coincidence'. This curious coincidence however, can be avoided if both space curvature and Λ are negligibly small, corresponding to an Einstein-de Sitter Universe with flat geometry. In this case, the universe evolves from radiation-dominated to matter-dominated and the mass density remains constant at a value very near the critical density (Ω_m=1) throughout the entire cosmic history.

The well-known flatness problem in cosmology is a special case of these curious coincidences and its first clear formulation goes back to a co-authored chapter by Dicke and Peebles (1979) in an edited collection of essays on General Relativity. In this chapter, written in a rather jaunty and conversational style, Dicke and Peebles consider a number of 'enigmas' in cosmology, including the flatness problem, and examine various possible solutions. The flatness problem in this context arises from a basic consideration of the Friedmann equation with $\Lambda = 0$. Dicke and Peebles begin by noting that the (then) present relative values of the curvature term and the mass density are poorly known 'because the mean mass density, ρ, is so uncertain' (p.506). They then note that tracing the expansion back in time, at the time of nucleosynthesis, we find that the mass term is 14 orders of magnitude larger than the curvature term which means that the expansion rate has been fine-tuned to agree with the mass density to a great accuracy. Their conclusion is that this precise initial balance of the effective kinetic energy of expansion and the gravitational potential energy must apply to each separate part, otherwise the inhomogeneities would produce large and irregular space curvature leading to black holes of all sizes, and essentially a chaotic universe which would look much different than the one we observe. The upshot is that these considerations show that regardless of the (unknown) mass density at the time, it must have been arbitrarily close to the critical value at the time of the Big Bang.

Dicke and Peebles then proceed to note that this model represents an Einstein - de Sitter universe (with $k = \Lambda = 0$) which, according to them, was motivated back in 1932 merely because of its simplicity, in lack of any observational data.[18]

[18]cf. Einstein and de Sitter (1932, p.214) '...we must conclude that at the present time it is possible to represent the facts without assuming a curvature of three-dimensional space. The curvature

Interestingly, Dicke and Peebles find the argument from simplicity rather weak, and conclude their short discussion of the flatness problem by noting that the observed large-scale clumping of matter in the universe requires that the balance between expansion and gravity in the early universe was extremely accurate though not exact, leaving open the possibilities of both a closed and an open – but nevertheless nearly flat – Universe.[19]

It is not clear whether Ostriker, Peebles and Yahil (1974) had these complications in mind when they expressed their preference for a universe which is "just close" for "nonexperimental" reasons, although their instant reference to Wheeler (1973) suggests that this was probably not the case. Nevertheless, in later work, Peebles (1984) presented a model for dark matter and the structure of galaxies and star clusters based on a set of 'particularly simple and attractive assumptions' (p.470). The first of the two assumptions which are relevant to our discussion, was that the cosmological constant is negligibly small since according to Peebles 'if Λ did play an important role it would be a coincidence that the mass density associated with Λ is comparable to the present density of matter, which is not attractive' (*ibid.*). The second was that the density parameter is $\Omega = 1$, i.e. that we live in a flat Universe, with Peebles noting that the motivation for this assumption is the same as for the previous one, citing Dicke and Peebles (1979). It is therefore evident that, at least for Peebles, the preference for additional baryonic matter corresponding to a flat geometry was mainly driven by his (and Dicke's) reluctance to accept that galaxies – and the observers within them – coincidentally came into existence at a special point in cosmic history.

5. Concluding remarks

The establishment of dark matter as a non-baryonic primordial particle in the 1980s was the culmination of a complex process that lasted several years. This process involved the observation of anomalies at galactic and cosmological scales, theoretical advancements in understanding physical processes across early and late cosmic history, and philosophical considerations by prominent cosmologists of the

is, however, essentially determinable, and an increase in the precision of the data derived from observations will enable us in the future to fix its sign and to determine its value'.

[19]In a recent review of the history of the flatness problem, Helbig (2021, p.6) rightly notes that the Einstein - de Sitter model is best understood merely as a practical and simple suggestion by the authors and that there is no clear indication in the article that Einstein or de Sitter were indeed in favour of a flat universe due to its simplicity. We agree, however, what is important for the purposes of our discussion is that Dicke and Peebles were not convinced by the argument from simplicity although it is, in their own words, 'attractive'. The flatness of the universe was for them a necessary condition to explain the current structure of the universe.

time. In this article, we argued that a critical milestone in the acceptance of dark matter as we know it today was the realization that baryonic matter constituted significantly less than the critical density, and that the observed large-scale structure formation and the temperature spectrum of the cosmic microwave background radiation could not be explained by the gravitational collapse of a photon-baryon fluid in the early Universe. Alongside these scientific developments, the philosophically motivated preference of many cosmologists for an 'at least closed' geometry (i.e., a flat or closed Universe with $\Omega \geq 1$) laid the groundwork for hypothesizing the existence of a non-luminous form of matter. This additional matter, which by far exceeded the amount of baryonic matter, also provided a potential explanation for the observed mass discrepancies in galaxies and galaxy clusters.

As we have seen, in the absence of direct observational data about the geometry of the Universe, cosmologists of the time relied on philosophical and non-experimental reasoning to propose different models of the cosmos. This article has identified and presented three such arguments: (a) Einstein and Wheeler's Machian argument for a closed Universe, (b) Sciama's Machian argument for a flat Universe, and (c) Dicke and Peebles' coincidence argument for a flat Universe. Despite their differing motivations and methodologies, all three arguments converge on the conclusion that additional mass is required to achieve or exceed the critical density, aligning with the idea of non-baryonic dark matter as a necessary component of the Universe.

The first two arguments, from Einstein/Wheeler and Sciama, are rooted in their proponents' shared desire to implement and validate Mach's principle, which posits that the inertial motions of objects are determined by the gravitational influence of all other matter in the Universe. The critical distinction between these two arguments lies in their treatment of Mach's principle: while Einstein and Wheeler regard it as a guiding principle for the selection of cosmological models within the framework of general relativity, Sciama adopts it as a genuinely constitutive principle. That is, for Einstein and Wheeler, general relativity is taken as a given foundation, leading them to argue that only a closed Universe can satisfy Mach's principle. In contrast, Sciama's commitment to Mach's principle as a constitutive tenet drives him to construct a truly Machian theory, from which he deduces that a flat Universe is the natural outcome under the critical density condition. The deeper motivations behind the urge to implement Mach's principle in a theory of gravitation for Einstein, Wheeler and Sciama remain an open question which goes beyond the scope of this article, although this issue has already received some attention in the existing literature.

Finally, the third argument due to Dicke and Peebles, takes a markedly different approach, originating from their reluctance to accept that galaxy and star formation in the Universe occured at a very special epoch when the deceleration of the expanding universe turns into an acceleration. This apparent fine-tuning of parameters, necessary to permit the emergence of observers at such a precise moment in cosmic history, was seen by many physicists of the time as an implausible and suspicious coincidence. The natural way out of this fine-tuning problem for Dicke and Peebles was to hypothesize a flat Universe. Thus, while Einstein, Wheeler, and Sciama were motivated by the belief that Mach's principle should shape the fundamental structure of gravitational theory, Dicke and Peebles were driven by their desire to eliminate what they considered an improbable and unattractive coincidence in the cosmic timeline.

References

Barbour, J. (2010). The definition of mach's principle. *Foundations of Physics 40*(9), 1263–1284.

Barbour, J. B. and H. Pfister (1995). *Mach's principle: from Newton's bucket to quantum gravity*, Volume 6. Springer Science & Business Media.

Blum, A. S. and D. Brill (2020). Tokyo wheeler or the epistemic preconditions of the renaissance of relativity. *The renaissance of general relativity in context*, 141–187.

Bond, J. R. and G. Efstathiou (1984). Cosmic background radiation anisotropies in universes dominated by nonbaryonic dark matter. *The Astrophysical Journal 285*, L45–L48.

Bond, J. R. and A. S. Szalay (1983). The collisionless damping of density fluctuations in an expanding universe. *Astrophysical Journal, Part 1 (ISSN 0004-637X), vol. 274, Nov. 15, 1983, p. 443-468. 274*, 443–468.

Brans, C. and R. H. Dicke (1961). Mach's principle and a relativistic theory of gravitation. *Physical review 124*(3), 925.

Chiu, H.-Y. (1967). A cosmological model for our universe. i. *Annals of Physics 43*(1), 1–41.

De Sitter, W. (1917). On the relativity of inertia. remarks concerning einstein's latest hypothesis. *Proc. Kon. Ned. Acad. Wet 19*(2), 1217–1225.

de Swart, J. (2020). Closing in on the Cosmos: Cosmology's Rebirth and the Rise of the Dark Matter Problem. In A. S. Blum, R. Lalli, and J. Renn (Eds.), *The Renaissance of General Relativity in Context*, pp. 257–284. Springer International Publishing.

Dicke, R. H. (1970). *Gravitation and the Universe: Jayne Lectures for 1969.* American Philosophical Society.

Dicke, R. H. and J. Peebles (1979). The big bang cosmology: Enigmas and nostrums. In S. W. Hawking and W. Israel (Eds.), *General Relativity: An Einstein Centenary Survey*, pp. 504–517. Cambridge University Press.

Dirac, P. A. M. (1938). A new basis for cosmology. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 165*(921), 199–208.

DiSalle, R. (2002). *Reading natural philosophy: essays in the history and philosophy of science and mathematics*, Chapter 7: Reconsidering Ernst Mach on space, time, and motion, pp. 167–191. Open Court, Chicago.

Einasto, J., A. Kaasik, and E. Saar (1974). Dynamic evidence on massive coronas of galaxies. *Nature 250*(5464), 309–310.

Einstein, A. (1912). Is there a gravitational effect which is analogous to electrodynamic induction. *Vierteljahrsschrift fur gerichtliche Medizin und offentliches Sanitatswesen 44*, 37–40.

Einstein, A. (1917). Kosmologische betrachtungen zur allgemeinen relativitätstheorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 142–152.

Einstein, A. (1952). Cosmological considerations on the general theory of relativity. In A. Einstein, H. Lorentz, H. Weyl, and H. Minkowski (Eds.), *The principle of relativity*, pp. 175–1988. Dover Publications.

Einstein, A. (1996). *The Collected Papers of Albert Einstein, Volume 4: The Swiss Years: Writings, 1912–1914*, Volume 15. Princeton University Press.

Einstein, A. and W. de Sitter (1932). On the relation between the expansion and the mean density of the universe. *Proceedings of the National Academy of Sciences 18*(3), 213–214.

Fay, J. (2024). Mach's principle and mach's hypotheses. *Studies in History and Philosophy of Science 103*, 58–68.

Field, G. B. (1972). Intergalactic matter. *Annual Review of Astronomy and Astrophysics 10*(1), 227–260.

Friedlaender, B. and I. Friedlaender (1896). *Absolute oder relative Bewegung?* Leonhard Simion.

Geiss, J. and H. Reeves (1972). Cosmic and solar system abundances of deuterium and helium-3. *Astronomy and Astrophysics, Vol. 18, p. 126 (1972) 18*, 126.

Gott, J. R., J. E. Gunn, D. N. Schramm, and B. M. Tinsley (1974). An unbound universe? *Astrophysical Journal 194*, 543–552.

Gunn, J., B. Lee, I. Lerche, D. Schramm, and G. Steigman (1978). Some astrophysical consequences of the existence of a heavy stable neutral lepton. *Astrophysical Journal 223*, 1015–1031.

Guth, A. (2014). The quest to discover the fundamental laws. *https://www.kavliprize.org/alan-guth-autobiography*.

Helbig, P. (2021). Arguments against the flatness problem in classical cosmology: a review. *The European Physical Journal H 46*(1), 1–23.

Hoefer, C. (1994). Einstein's struggle for a machian gravitation theory. *Studies in History and Philosophy of Science Part A 25*(3), 287–335.

Hoefer, C. (2014). Mach's principle as action-at-a-distance in gr: The causality question. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics 48*, 128–136.

Lachieze-Rey, M. and J.-P. Luminet (1995). Cosmic topology. *Physics Reports 254*(3), 135–214.

Lightman, A. and R. Dicke (1988). Aip oral history interview of robert dicke.

Lindley, D. (1981). Why is the universe open? *Nature 291*(5814), 391–392.

Luminet, J.-P. (2016). The status of cosmic topology after planck data. *Universe 2*(1), 1.

Mach, E. (1893/1883). *The Science of Mechanics: A Critical and Historical Account of Its Development* (1 ed.). The Open Court Publishing Company.

Mach, E. (2014/1872). *History and Root of the Principle of the Conservation of Energy*. Cambridge University Press.

Misner, C. W., K. S. Thorne, and J. A. Wheeler (1973). *Gravitation*. Macmillan.

Norton, J. D. (1995). *Mach's principle: from Newton's buckets to quantum gravity*, Chapter Mach's principle before einstein, pp. 9–55. Birkhäuser.

Ostriker, J., P. Peebles, and A. Yahil (1974). The size and mass of galaxies and the mass of the universe. *Astrophysical Journal 193*, L1–L4.

Peebles, P. (1968). Recombination of the primeval plasma. *Astrophysical Journal 153*, 1–11.

Peebles, P. J. E. (1965). The black-body radiation content of the universe and the formation of galaxies. *Astrophysical Journal 142*, 1317.

Peebles, P. J. E. (1966). Primordial helium abundance and the primordial fireball. *Astrophysical Journal 146*, 542.

Peebles, P. J. E. (1982). Large-scale background temperature and mass fluctuations due to scale-invariant primeval perturbations. *Astrophysical Journal 263*, L1–L5.

Peebles, P. J. E. (1984). Dark matter and the origin of galaxies and globular star clusters. *Astrophysical Journal, Part 1 (ISSN 0004-637X), vol. 277, Feb. 15, 1984, p. 470-477. 277*, 470–477.

Peebles, P. J. E. (1993). *Principles of physical cosmology*, Volume 27. Princeton university press.

Peebles, P. J. E. (2020). *Cosmology's century: An inside history of our modern understanding of the universe*. Princeton University Press.

Reboucas, M. J. and G. Gomero (2004). Cosmic topology: a brief overview. *Brazilian journal of physics 34*, 1358–1366.

Rogerson, J. B. and D. G. York (1973). Interstellar deuterium abundance in the direction of beta centauri. *The astrophysical Journal 186*, L95–L98.

Rosen, N. (1940). General relativity and flat space. i. *Physical Review 57*(2), 147.

Sandage, A. and E. Hardy (1973). The redshift-distance relation. VII. Absolute magnitudes on the first three ranked cluster galaxies as functions of cluster richness and Bautz-Morgan cluster type: the effect of q_0. *Astrophysical Journal 183*, 743–758.

Schramm, D. N. and R. V. Wagoner (1977). Element production in the early universe. *Annual Review of Nuclear Science 27*(1), 37–74.

Sciama, D. (1951). Letter to einstein.

Sciama, D. W. (1953a). On the origin of inertia. *Monthly Notices of the Royal Astronomical Society 113*(1), 34–42.

Sciama, D. W. (1953b). *On the origin of inertia*. Ph. D. thesis, University of Cambridge.

Sciama, D. W. (1969). The physical foundations of general relativity.

Sciama, D. W. (1971). *Modern cosmology*. CUP Archive.

Sciama, D. W. (1993). *Modern cosmology and the dark matter problem*. Number 3. Cambridge University Press.

Spergel, D. N., L. Verde, H. V. Peiris, E. Komatsu, M. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, et al. (2003). First-year wilkinson microwave anisotropy probe (wmap)* observations: determination of cosmological parameters. *The Astrophysical Journal Supplement Series 148*(1), 175.

Staley, R. (2021). Mother's milk, and more: On the role of ernst mach's relational physics in the development of einstein's theory of relativity. *Interpreting Mach*, 28–47.

Thirring, H. (1918). Über die wirkung rotierender ferner massen in der einsteinschen gravitationstheorie. *Physikalische Zeitschrift 19*, 33.

Thirring, H. (1921). Berichtigung zu meiner arbeit:" über die wirkung rotierender massen in der einsteinschen gravitationstheorie". *Physikalische Zeitschrift 22*, 29.

Tipler, F. J. (1977). Black holes in closed universes. *Nature 270*(5637), 500–501.

Vittorio, N. and J. Silk (1984). Fine-scale anisotropy of the cosmic microwave background in a universe dominated by cold dark matter. *Astrophysical Journal 285*, L39–L43.

Weart, S. and D. Sciama (1978). Aip oral history interview of dennis sciama.

Weinberg, S. (1972). *Gravitation and cosmology: principles and applications of the general theory of relativity.* Wiley.

Weyl, H. (1944). How far can one get with a linear field theory of gravitation in flat space-time? *American Journal of Mathematics 66*(4), 591–604.

Wheeler, J. A. (1962). The universe in the light of general relativity. *The Monist*, 40–76.

Wheeler, J. A. (1973). From relativity to mutability. In J. Mehra (Ed.), *The physicist's conception of nature*, pp. 202–247. Springer.

Wheeler, J. A. (1974). Perspectives: The universe as home for man. *American Scientist 62*(6), 683–691.