Between Theory and Experiment: Model Use in Dark Matter Detection

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Abstract

There is a complex interplay between the models in dark matter detection experiments that have led to a difficulty in interpreting the results of the experiments and ascertain whether we have detected the particle or not. The aim of this paper is to categorise and explore the different models used in said experiments, by emphasizing the distinctions and dependencies among different types of models used in this field. With a background theory, models are categorised into four distinct types: theoretical, phenomenological, experimental and data. This taxonomy highlights how each model serves a unique purpose and operates under varying degrees of independence from their respective frameworks. A key focus is on the experimental model, which is shown to rely on constraints from both data and phenomenological ones. The article argues that while theoretical models provide a backdrop for understanding the nature of dark matter, the experimental models must stand independently, particularly in their methodological approaches. This is done via a discussion of the inherent challenges in dark matter detection, such as inconsistent results and difficulties in cross-comparison, stemming from the diverse modelling approaches.

Keywords— Scientific Modelling, Philosophy of Cosmology, Philosophy of Science in Practice, Philosophy of Experiment

1 Introduction

Philosophical discussions around dark matter (DM) tends to focus on the lack of detection, the explanations derived from indirect evidence, or the merits of

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alternatives theories (Kashyap 2023, Merritt 2017, De Baerdemaeker and Dawid 2022, Martens 2021, Martens and King 2023). There is little discussion surrounding the detection experiments themselves.[1] The role different models play in DM detection experiment is crucial, not simply in understanding whether we have detected the particle or not, but also in understanding the dynamics of scientific practice and experimentation, and how models, theories and experiments interact.

These experiments, for example, DAMA, XENON1T, CDMS, CoGeNT, LUX and their various iterations, have been running for decades without finding any conclusive evidence. There is difficulty in interpreting the results and contrasting those from one experiment against another. In the few cases where this has been possible to a certain extent, there appears to be a challenging lack of consistency among the experiments. Though the existence of dark matter has strong indirect evidence (Misiaszek and Rossi 2024), there has been, as of yet, no consensus regarding whether we have experimentally detected it or not.[2] This paper will provide a general description of how the experimental set-ups function and attempt to shed light on where the problem lies by looking at model use in DM detection experiments. Given the complexity of the experiments and the high number of assumptions necessary to interpret the results, we begin by providing a taxonomy of the difference between theories, models and experiments that is useful for this case.[3]

The connection between theory, model and experiment is generally discussed in the philosophy of experiment or in literature on modelling in science. In the former, there are various divisions of the levels between theory and experiment. In the latter, however, models are either distinguished via ill-defined levels of generality (e.g. French (2003)) or are taken to be all on the same level, to be compared with theory or play the role of intermediaries (Morrison and Morgan 1999). In both discussions, the distinctions between levels of generality that are to be found trace their origins back to Suppes' hierarchy and theory-experiment connection (Suppes 1969). In the literature on modelling surrounding experiments in modern physics, there has been a focus on particle physics (Karaca 2013, Antoniou 2021, Beauchemin 2017, Falkenburg 2024).

Taking a cue from that discussion, this article proposes a five fold taxonomy that is inspired from those presented by Karaca (2013) and Antoniou (2021) and is motivated by the view of models as autonomous agents[4] put forth by Morrison and Morgan (1999). These will range from background theory to data models obtained

[1]De Baerdemaeker (2021) presents a discussion, but focusing on the difference with particle physics experiments as being method driven, as opposed to target driven.

[2]See Merritt (2017) for an overview of the problems associated with the existence of DM, both in terms of experimental results and inconsistent observational data. See also Chan (2019) for a rebuttal.

[3]Though it might be possible to generalise the taxonomy to a broader context, we will not do so here.

[4]Though they use the term 'agent', it will not be discussed here. Of importance to us is their autonomy.

in experiments. As will be argued, the relation is neither unidirectional, nor that of direct dependency: there is a constant interplay between the different levels.

This paper will describe the diverse models present in DM detection experiments and attempt to provide a description of the problems from that perspective. To this end we will have to emphasise the role played by autonomous constraints in model construction. For every model in this case, there is some background theory and some constraints, the two of which have to be independently obtained. For example, as we will discuss in section 5, the experimental model will necessarily have a different background theory from the phenomenological one, lest we are threatened by a vicious circularity that is well documented in the literature and will be especially problematic in the case of DM detection (Schindler 2013, Franklin 2015, Beauchemin 2017, Ritson and Staley 2021). In other words, the construction of models is obtained by a constant limiting of the admissible space[5] of a background theory. After sufficiently narrowing the model's parameter space, one can perform an experiment to test the validity of the model. This means that the background and the constraints necessary to construct a model must be independent. The problems plaguing DM detection, as we will show, come from various directions, with problems pertaining to lack of insufficient constraints, indiscriminate experiments, assumptions regarding DM interaction and so on. There is a strong interdependency on the models used in DM detection that make it incredibly difficult to disentangle the various parameters from each other.

The paper proceeds as follows: in section 2, a five-fold distinction based on the notion of constraints in modelling will be put forth, from background theory to data models. Section 3 will argue that what makes models autonomous are the constraints that need to be independently obtained. Taking into account the role complementary and competing models play we will show that, if models are not autonomous, problems will emerge regarding the testable data and the influence of experiments on models and theories. To further provide evidence for the autonomy of models in a more relevant context, section 4 will present a brief history of general relativity (GR) and how the cosmological constant cold[6] dark matter model (ΛCDM) became standard. In section 5, we then motivate the need for and discuss the role experimental models play in DM detection experiments. The difference between such models and other types will be demonstrated by presenting how, given the lack of consistent data from DM detectors, authors in philosophy and physics have attempted to provide alternatives from the different models in the taxonomy. The article concludes with section 6 by summarising what was presented in this article.

[5]The term 'admissible space' will be used to refer generally to both, the parameter space of an empirical model and the field of a mathematical one.

[6]The term 'cold' refers to the non-relativistic nature of the DM particle.

2 A taxonomy of theories and models

Many of the distinctions in modelling are present in the philosophy of experimentation and discussions around theory-ladenness in experiments. This article will take its cue from that discussion by looking at two sources whose distinction are useful for this case: Karaca (2013) and Antoniou (2021).

Karaca presents an important three-fold distinction in modelling to discuss theory-ladenness in high energy physics (HEP) experiments. His article concerns itself chiefly with avoiding circularity in experiments. To do so, he presents a top-down three-fold distinction in HEP theories that can be imported for this discussion. Antoniou's article, though also centred around HEP, clarifies the notion of a data model. To this end, he provides a bottom-up three-fold distinction that is focused on experimentally testable data and data models.

The discussion on DM detection experiments, though following similarly to HEP experiments, is nonetheless different. As we will see, the initial conditions of the particle to be detected are unknown, or at least given an estimated guess and, as such, there is a stronger reliance on the theoretical background. Given this, we will provide a five-fold distinction based on Karaca's and Antoniou's with some clarifications specific to this case study. The levels will be divided as follows:

- Background Theory: This is the theory that provides various abstract relations and the properties required for their interpretation. These relations are conditions imposed on the mathematical structure used.
- Theoretical Model: This model is one that is bound by the background theory but introduces a domain of objects that provides constraints on the background theory.
- Phenomenological Model: This model is one that introduces a particular state of affairs and specifies more precisely the interaction given the situation at hand. It provides the causal narrative of the particle.
- Experimental Model: This model is based on the specifics of an experimental setup whose interactions are known. It is based on the initial conditions of the experimental setup and its purpose is to look at the final conditions to provide insight into the interactions taking place. This model does not have the same background theory as the phenomenological model, but rather uses parameters from the phenomenological model to constraint it sufficiently so as to have testable results.[7]

[7] The difference in background theory between the phenomenological model is important in the case of DM detection experiments. This may not be the same for experiments in particle physics. See Falkenburg (2024) for an example of how such models can have the same background theory as the phenomenological model yet can nonetheless avoid circularity.

- Data models: This model is one that is constructed from the experimental results in a form that can communicate with the experimental model, either by providing a contrast against the constraints provided by the phenomenological ones or by providing constraints on the experimental one.

To better highlight the case study, we will explore the taxonomy in a manner relevant to DM detection experiments. As will be discussed in section 5.3, the failure of the experiments to produce a verification or falsification of the existence of DM, has to led to attempts to make sense of the results, or lack thereof. These have taken the form of alternatives to the background theory, theoretical model and/or phenomenological model, or of model independent methods that attempt to reduce the influence of either the experimental model or phenomenological model in the analysis of the results.

The background theory for the assumption of the existence of DM is GR, which dictates the constraints on the admissible space by specifying a relation between mass-energy and the shape of space it occupies. At this level however, one still does not have a domain of physical objects. We remain at the level of a quasi-mathematical structure, with an added assertoric relational condition in addition to the mathematical definitional ones[8].[9] This means that such a system has no internal means to provide direct theoretical results of experiments or methods for model construction. GR, which can admit diverse theoretical models, possesses no internal means to ascertain which is the true representation of physical space. There is no *a priori* way to distinguish true from false theoretical models.[10] Only via experimentation, *a posteriori*, can we choose which one better represents the physical space of the universe. This lack of internal decidability mechanism is what points to the idea that empirical models are autonomous and their constructions as necessarily independent (Cartwright et al. 1995).

The theoretical model is one that is based off Karaca's 'model theories' but with an added condition: a domain of objects. He describes this type of model as one that provides "interaction specific features"(Karaca 2013, p. 100). The addition of the domain of objects is there to provide constraints on the background theory so that it is able to better account for these specific features. For DM detection, the theoretical model would be the big bang model[11] (BB), the standard model

[8]See Hellman and Shapiro (2018, Ch. 2) for the difference between assertoric and definitional axioms.

[9]Framing it as such we are following Hilbert, who, in a letter to Frege, calls definitional axioms relations and conditions (Hilbert in Frege 1980, p. 51)

[10]This is quite similar to the case of 19th century mathematical practice, where, with the introduction of non-Euclidean geometry, there became no *a priori* method to distinguish the 'true' model of physical space thereby rendering the field 'tainted' by experience. See Folina (2012), Torretti (2021).

[11]This is also sometimes called the big bang theory, however the naming convention is unimportant. As Bailer-Jones (2002), notes, there are widely diverging views amongst physicists regarding the differences between models and theories.

of particle physics (SM), and the ΛCDM model in conjunction. Note that the inclusion of SM, even though it is arrived at independently of GR, is included to account for the observational inconsistencies necessary that led to the supposition of the existence of DM. Namely, that DM is assumed to exist from cosmological observations because the SM possesses no taxonomy of a particle that accounts for the phenomena.

There are two key points to take into account. First, as will be discussed in section 4, ΛCDM is a specification of BB and the two can be considered a single model[12]. Second, there is a key difference between BB, SM, and ΛCDM: SM specifies a taxonomy of particles (and an added assumption of their existence), ΛCDM proposes the existence of a type of particle, DM, and the cosmological constant, BB assumes the existence of a particular object, the original singularity of the universe. The uniqueness of the universe that BB models also means it provides more specific constraints than the taxonomy of properties and objects of the latter two that render these models more general. In other words, BB can provide direct testable results, acting as a phenomenological model, when experimenting on some cosmological objects and/or phenomena, e.g. detecting the cosmic microwave background (CMB), but not for the specific case of DM detection.

The phenomenological model in the case study is that of the particle's causal narrative in our galaxy. However, it is important to note the difference in this term between physical and philosophical literature: "For the physicist, unlike the philosopher, the distinction between theoretical and phenomenological [...] separate[s] laws which are fundamental and explanatory from those that merely describe" (Cartwright 1983, p. 2). We will use it in the sense specified in physics, as one providing a description of the physical state of affairs from the point of view of the theoretical model. This means that the phenomenological model in DM detection experiments is generally taken to be the Standard Halo Model (SHM), which proposes a Gaussian velocity distribution of DM particles in the galaxy.

The experimental model is one that is based on the specifics of an experimental setup, using the phenomenological model to provide constraints. Simplistically, one can think of the difference between the phenomenological and experimental model as the difference between a description of the particle and that of the detector: the phenomenological model provides a description of the initial (and possibly final) conditions of the particle under study, whereas the experimental model provides the initial and final conditions of the detector. The final conditions of both will be, in our case, the same, or rather, the final conditions of the detector will necessarily be influenced by the various parameters of the particle provided by the phenomenological model. They remain independent in that two similar setups might use the same phenomenological model, but the differences in particular objects and

[12]BB has had a longer history than ΛCDM, with various iterations of BB emerging with different time-evolutions. As such, BB can be considered a class of models. For the purposes of this paper, however, we will deal with the two as separate.

devices used in setups would introduce unique conditions. Antoniou (2021) calls this 'model of experiments' and it is a model "that facilitates the completion of a measurement process in an experiment and allows the connection between the final data model and theoretical hypothesis" (Antoniou 2021, pp. 100-101). To be more specific for our case study, it is the type of model which can communicate with or be constrained by the data models and the phenomenological ones. It important to reiterate that, in our case study, the experimental model is not constructed from the phenomenological model, but rather, the latter provides constraints for the former. This also means that the experimental model might not be based on the same background theory as the phenomenological model, as we will soon see. The experimental model can allow one to see whether there is a conflict if the constraints from the phenomenological model fall outside the parameter space provided by the data model and vice versa. There is no unidirectional approach: the experimental model can be contrasted against the data model in various ways, of which the above is only one.

A data model is independent from the rest of the taxonomy: it is a "representation of a measurement outcome into canonical form that allows [...] the comparison of experimental data with the hypothesis under investigation." (Antoniou 2021, p. 100) This is a model obtained from the experimental results for comparison that takes place at the level of experimental models. Constraints on the latter can come from the data model and/or phenomenological model which provides comparison as mentioned.

The common feature present throughout the taxonomy is that of constraints. These are placed on a model from various means. This implies that said constraints must necessarily be independent of the background theory, thus rendering the models autonomous. However, it is important to note that notion of constraints used is general. As we will see constraints for different parameters are provided by different models in various ways.

3 Autonomous constraints

Framing background theories as pseudo-mathematical systems with assertoric constraints added to the definitional axioms of a mathematical structure will invite a necessary comparison: such constraints have their basis from outside mathematics. We do not know if these constraints hold or if analysis on them is reliable. In other words, unlike definitional axioms which define the mathematical structures, these constraints are *independent* of, or *autonomous* from the structure. Moving from theories to models means we add such autonomous constraints to construct models in various ways.

The notion of autonomous models is present in the literature. Their autonomy is ascertained via two important elements: first, "theory does not provide us with an algorithm from which the model is constructed and by which all modelling decisions determined" (Morrison and Morgan 1999, p. 16). In our vernacular, a theory provides a relational connection between two properties and constrains the admissible space. Second, models are constructed via a mixture of elements some of which are external to the theory. In other words, a theory does not and cannot provide all the necessary constraints for a model. They have to be external, because "When we look at the way these models are constructed, it appears that the stories not only help to legitimise the model after its construction, but also play a role in both selecting and putting together the bits of physical theories involved." (Morrison and Morgan 1999, p. 14)

However, Morrison and Morgan go on to say that "There is no *logical* reason why models should be constructed to have these qualities of partial dependence" (Morrison and Morgan 1999, p. 17). I beg to differ. There is indeed a logical reason why: framing models in terms of the addition of constraints means that they inform only of what is admissible in the parameter space and what is not. Any model that proposes interaction and/or phenomena within a theory's constraints is admissible, regardless of its form.

If the constraints, and therefore the models, are completely dependent on the theory then any model constructed within the admissible space will have to be either sufficiently broad so as to discuss the entire space making it indistinguishable from theory, or the scientific model used will be equivalent to a mathematical one. However, the fact that one can have contradictory models from the same background theory excludes this idea. For example, Beauchemin (2017, p. 299) notes that there are often competing models about charged hadron-detector interaction. A similar case of contradictory models of GR will be discussed in the next section. Contradictory models are problematic in complete dependency on theory. In addition, such a dependency on theory would imply that a falsification of a model would also falsify the theory, similarly to how a contradiction present in one mathematical model would be problematic for the mathematical theory and axiomatic system, which is evidently not the case in scientific models.[13]

On the other hand, models need to be partially independent of the phenomena, the causal processes, otherwise, a one-to-one correspondence with physical phenomena would mean the model would have to incorporate all aspects of the physical process, including time. The more accurate one wished to be, the more time the model would take to play out. Namely that a complete dependence on a phenomenon would, be too specific, and contain all the necessary dimensions, including the temporal one. That would mean that a model of the universe would have to be as big as the universe and the time evolution would take just as long. Indeed, without some preconceived notion of what properties are functionally

[13]There is, however, a crucial difference between empirical and mathematical models: an axiomatic system with contradictory models might not pose a problem for the system due to Gödel's incompleteness theorem. However, in the mathematical case, that is because the consistency of the system cannot be demonstrated from within the system itself. Given that the constraints are autonomous and must come from outside the background theory or model then this would not apply here.

required and need to be abstracted, which would remove complete dependency on the phenomena, we would not have access to the mathematical representations necessary. As Cartwright points out, “a model [...] is employed whenever a mathematical theory is applied to reality, and I use the word ‘model’ deliberately to suggest the failure of exact.” (Cartwright 1983, p. 158)

Any account that construes models as not autonomous will face serious problem when discussing competing and complementary models. It would miss the crucial role these two types play in scientific practice.

3.1 Complementary and competing models

An important distinction needs to be made between these two types. Competing models provide contradictory consequences, whereas complementary models, can either be independent, such as SM and BB[14], or they can be about similar phenomena providing grounds for abstraction via invariance. A prime example of the latter is Einstein’s thought experiment which became the basis for the equivalence principle. In recounting the process, Einstein writes that:

> *for an observer in free-fall from the roof of a house there is during the fall* — at least in his immediate vicinity — *no gravitational field*. Namely, if the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature. The observer, therefore, is justified in interpreting his state as being “at rest” [...] On the other hand, one can also start with a space that has no gravitational field. A material point in this space, when sufficiently distant from other masses, behaves free of acceleration relative to an inertial system K. However, if one introduces a *uniformly accelerated* coordinate system K' relative to K (uniformly accelerated parallel translation), then K' is no inertial system in the sense of classical mechanics or the theory of special relativity. Every mass point sufficiently distant from others is uniformly accelerated relative to K'. When seen from K, the acceleration of the system K' is of course the cause of the relative acceleration of the mass point relative to K'; and on the basis of classical mechanics, as understood up to the present day, it is the only possible interpretation. However, we can also view K' as an admissible system (“at rest”) and attribute the acceleration of masses relative to K' to a static gravitational field that fills the entire space that is under consideration. This interpretation again is possible based upon the experimental fact that in a gravitational field (such as that relative to K') all bodies fall in the same manner. (Einstein 1920, p. 21)

[14] They do not have to be completely independent, only arrived at independently. Any model from a sufficiently comprehensive theory will have some dependency on other models that will either render them competing or dependently complementary.

This led him to the equivalence principle, first formulated in 1907: "we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system" (Einstein 1907, p. 302). We see an important element here, namely that the two models, K and K' are complementary in that they show sufficient structural similarity so as to extract a principle, even if their respective causal explanations vary.

Competing models, on the other hand, provide grounds for application via contradiction, for eliminating one in favour of the other, rendering its own constraints as better suited to account for the phenomena. Assume we have two competing models M and M' with contradictory consequences c and $\neg c$. We then, for whatever reason, assume M is true, or at least more valid than M' and formulate an experiment to ascertain its truth. If the experiment 'fails' then M is refuted.[15] That does not mean, however, that M' is true.[16] Another experiment to ascertain the truth of M' must be then constructed.

The situation leading up to the famous Michelson-Morley experiment is just that. After Thomas Young's double slit experiment, that demonstrated that light is a wave, it was assumed that there was a medium, called aether, in which light propagates. Two models of aether were proposed, Fresnel's stationary aether and Stoke's model of aether drag. The latter was excluded due to experimental observations. This, however, did not indicate that the former was true. In 1880, Michelson and Morley conducted their experiment to test Fresnel's model of stationary aether.[17]. They concluded that aether does not exist.[18] In other words, though the two models of aether were contradictory, the refutation of one did not mean the truth of the other, even if they exhausted the possibilities between them. Both were false given that their proposed medium did not exist. One cannot infer the truth of competing models by refuting the other. By positing an object ontology, there is always a need for another test of their truth.

Another example is that despite the excellent success of GR over a century, experiments, such as LIGO, were nonetheless conducted to detect the existence of gravitational waves. This means that, "unlike theoretical accounts, which can

[15]It is another point altogether whether the failure of the experiment justifies the falsity of M. For the sake of this example, assume failure means refutation.

[16]An equivalent idea if found in mathematics. In an attempt to provide a constructive consistency proof for classical arithmetic, Gödel discusses the law of excluded middle by stating that: "the negation of a universal proposition was to be understood asserting the existence (in the sense specified) of a counter example." (Dawson 2005, p. 157) In this context, Gödel defines existence as "abbreviations for actual constructions". However, by taking the notion of existence from physics, namely that for an object to exist it must detectable, then the mentioned quote holds: negating that M exists, via an experiment, asserts the existence of a counter example. However, M' is not such a counterexample given that the underlying assumption itself might be problematic.

[17]See Laymon (1988) for a discussion on the experiment's reliance on Fresnel's model and what that shows.

[18]For a more in-depth account of this history, see Swenson (1972).

be justified only by an inference to the best explanation, causal accounts have an independent test of their truth: we can perform controlled experiments to find out if our causal stories are right or wrong." (Cartwright 1983, p. 82) More precisely, it is not only the causal story that is tested, but also the existence of the underlying objects that enter into the dynamics.

Though the ontology of objects plays a role, the issue at hand deals with the epistemology of models. One of two pieces of knowledge are needed to test competing models: either one needs to ascertain the existence of the object types in the theoretical model before any of the competing phenomenological models can be construed as viable, or, as in the case of aether, the competing phenomenological models needs to provide sufficient knowledge of the object's parameters so as be able to experimentally test for its existence. DM detection experiments require an assumption of their existence as well as assumptions regarding their dynamic so that one can construct a phenomenological model. The dual role the experimental setups play in DM detection, that as the detector of the particle and detector of the mass of the particle[19], adds confusion as to whether we have detected the particle or not. That is because, in a DM experiment, the assumption is that, like the case with aether, the interaction of the particle with the setup is evidence of its existence. Unlike the case with aether, however, without sufficient knowledge that could constrain the parameters of the object under consideration, it is difficult to ascertain whether one has a positive detection even if there is some interaction in the experiment. Either one has already established the existence of the object by some means to test a model's dynamics against another, or one has sufficient knowledge of the dynamics, obtained from the models, that will be used to test for the existence of the object.

Simply, if background theories provide relational constraints on the space with no mention of object ontology, then no independent test can be performed to ascertain its validity. Only with the introduction of models (the different levels), of some object domain, can we begin to formulate an experiment as an "independent test of their truth". The domain of objects in a model, cannot be assumed to exist based solely on the relations, as is generally the case with mathematics.[20] An experiment based on the model's proposed dynamics must be performed, to detect their presence, one that establishes that objects of a certain type exist.

Complementary and competing models play a crucial role in ascertaining that models are autonomous and provide the logical reasons for such autonomy. This autonomy is obtained from constraints external to the models, such as from experiments or observations. Competing models that rely on the existence of a certain objects require a test for the existence of said objects, one that is sufficiently constrained by the dynamics of the proposed models.

[19]See section 5.

[20]The notion of an objects' existence itself is different between mathematics and physics (See Linnebo 2008, p. 68).

4 From GR to ΛCDM

The distinction presented and the need for autonomy imply that the application of a theory to an experiment is a rather fluid method. To obtain a theory given a mathematical structure, we can abstract some physical relations, formulate them mathematically and they become the initial constraints on a mathematical space to obtain a theory. Given that we have placed the background of DM detection experiments in the taxonomy and motivated the need for autonomous constraints, a brief discussion of the history of the theory and models is required. This would serve to both, better highlight the autonomous nature of the parameter constraints that have arisen from tensions amongst different models, and to clarify how such constraints are arrived at, which would explain the exceptionally large parameter space present in the model.

A very simple example to demonstrate the constraints presented by the background theory is that of Newtonian Gravity. In Euclidean Geometry, there is no axiomatic constraints on how two points can behave: assuming some time evolution of the points, their motion in any direction, with any speed and acceleration is admissible. More in line with geometry: in lieu of time evolution, one can always present it as two lines in Euclidean space, lines that, physically speaking, could represent the trajectory of particles. The properties of those lines are given by the axioms that provide no further constraints. Adding the condition $F = (Gm_1m_2)/(r^2)$ means that the particles trajectory, if they possess an additional property, mass, is constrained. This formula is not an infallible truth, therefore not an axiom in the assertoric sense, merely a condition that functions as one.

At the level of these kinds of laws, however, there is, as of yet, no mention of object ontology, domain and so on. If an ontology is present, it is implicit, because "existence is an internal characteristic of causal claims. There is nothing similar for theoretical laws" (Cartwright 1983, p. 93). In fact, "The fundamental laws of physics [...] do not tell what the objects in their domain do" (Cartwright 1983, p. 54), because theoretical laws have no non-mathematical objects in their domain. The only non-mathematical property introduced by this formula is mass. The force in this case is not a property. The formula could equally be written as $am_1 = (Gm_2)/(r^2)$, where acceleration is defined mathematically in relation to position, which is itself a mathematical property in a metric space. The constant G can also be construed as a non-mathematical addition given it possesses units, however, it has no physical counterpart in that it is a constant of proportionality. What is also important to note is that this condition is a relation between two masses. In other words, assign a mass to each point and our space becomes constrained.

The abstraction of relations and their introduction into a theory can be clearly seen in the introduction of the cosmological constant (Λ) in the Einstein Field Equations (EFE). The general form of the EFE is:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu},$$

where $G_{\mu\nu}$ is the Einstein tensor, a representation of the curvature of spacetime, Λ is the cosmological constant, $g_{\mu\nu}$ is the metric tensor, κ is the gravitational constant and $T_{\mu\nu}$ is the stress energy tensor. GR was published prior to the knowledge that the universe is expanding and was assumed to be stable, and, as such, the term $\Lambda g_{\mu\nu}$ was not initially present.

It was shown by Friedmann that "the correct treatment of Einstein's basic equations leads to a class of expanding and contracting universes" (Gamow 1955, p. 25) and therefore Einstein, begrudgingly[21] included the term '$\Lambda g_{\mu\nu}$'. Yet, as was later shown by Hubble, the universe is in fact expanding and as such "the introduction of cosmological constant became superfluous" (Gamow 1955, p. 27), leading Einstein (and physicists from then on) to use the field equations with the assumption that $\Lambda = 0$. In 1998, however, it was discovered that the universe's expansion is, in fact, accelerating, necessitating a positive Λ (Peebles and Ratra 2003).

Even with these constraints, GR, as background theory, can admit various theoretical models that can be brought in from elsewhere. For example, though initially proposed as idealised models, there are three models that represent how a background theory can accommodate different model succinctly: de Sitter, Minkowski and anti-de Sitter spaces. They are highly simplified models that assume an empty universe but with a positive, zero and negative Λ respectively. They can even be considered toy models, given that these simplifications permit one to find exact solutions to the EFE, a complex and often impossible task, and thus, more often than not, appear as exercises in the classroom. Assuming that these models may represent our universe, they are contradictory. Yet, their solutions, separately, are valid in GR. More interestingly, another 'idealised model'[22] that was also proposed was the Einstein-de Sitter universe. It became the most widely accepted model of the universe until its replacement by the current one, ΛCDM, following the discovery of the accelerated expansion.[23]

Moving away from such models, after Hubble's discovery and formulation of Hubble's law, that the speed of galaxies moving away from the earth is proportional to their distance, two competing models emerged. The first, BB, put forth by

[21]"In a letter to Lemaître of 1947 he made it clear that his objections were aesthetically based: 'Since I have introduced this term I had always a bad conscience. But at that time I could see no other possibility to deal with the fact of the existence of a finite mean density of matter. I found it very ugly indeed that the field law of gravitation should be composed of two logically independent terms which are connected by addition. About the justification of such feelings concerning logical simplicity it is difficult to argue. I cannot help to feel it strongly and I am unable to believe that such an ugly thing should be realized in nature'" (Kragh 1996, p. 54).

[22]The term is used in this context specifically because, as O'Raifeartaigh et al. (2021) note, it is not a model of the universe that takes the big bang into account, despite it later becoming the standard big bang model.

[23]For a more in-depth look at the history of the models of the universe proposed by Einstein and others after Hubble's discovery, see O'Raifeartaigh et al. 2015.

Georges Lemaître in 1931 in a paper titled 'L'expansion de l'espace', proposed the existence of an (unspecified) time evolution of the universe from a singularity, later called by him, 'l'atom primitif', the primaeval atom. The second, steady-state model, was presented by Fred Hoyle, Hermann Bondi and Thomas Gold. It states that, though the universe is expanding, observationally speaking, it does not change over time (Halvorson and Kragh 2021). It postulates an unchanging observational universe; matter density remains constant in an expanding universe due to matter creation. The detection of uniformity in the CMB excluded the steady-state model, given that the model provided no explanation about the phenomena, thereby solidifying BB as standard.

BB, though it is a theoretical model, can still be further specified. Two such examples are the aforementioned ΛCDM and Einstein-de Sitter universe. Even the former, though now standard, admits a massive weight range for DM particles. Proposal for DM candidates are generally taken to be weakly interacting massive particles (WIMP), but can range from fuzzy DM (10^{-21}eV) to WIMPzilla (10^{21}eV). The range itself is little constrained due to the fact that we are looking for a particle not described by SM yet available in sufficient quantities to account for the various observational data.

Even within these alternatives, the interpretation of Λ, that is common amongst all, is uncertain. This is problematic in the case of DM detection, given that it is uncertain to what extent Λ itself can interfere with the results of the experiments, as we will see in the next section. Though Λ began as a mathematical artefact to stabilise the dynamical universe, it is now construed as the physical representation of vacuum energy, or dark energy, necessary for the universe's expansion. Yet, Λ's exact physical representation is still unclear given that, as vacuum energy, it stands in contrast to the vacuum energy from quantum field theory giving rise to the cosmological constant problem: "Quantum field theory predicts a very large energy density for the vacuum, and this density should have large gravitational effects. However, these effects are not observed, and the discrepancy between theory and observation is an incredible 120 orders of magnitude." (Adler et al. 1995, p. 620)

A caveat, however, needs to be pointed out. These distinctions are divisions of a continuous process. Though BB is a theoretical model, ΛCDM and its competing alternatives can be construed as an addition to BB as a theoretical model. As noted, BB initially provided neither a domain of objects, nor specified a time-evolution. Just as GR, with or without the term $\Lambda g_{\mu\nu}$, is a background theory, so is BB, including or excluding ΛCDM, a theoretical model. From this theoretical model (BB with ΛCDM) one can now construct phenomenological models, such as the SHM. Sometimes such models can be competing, such as those that propose the existence of clumps or substructures in the DM halo (Kimball and Budker 2023, p. 21), other times independent of each other. An example of the latter is one where people in two different galaxies would necessarily have different phenomenological models for their dark matter detection experiments even if the experimental setup is identical and they both rely on ΛCDM. The construction of a phenomenological model from a theoretical one for experimental testing will change based on the causal narrative of the particle one is trying to detect.

5 The experimental model in dark matter detection experiments

The distinction between phenomenological and experimental model however, needs to be more sharply defined. In the case of HEP experiments, the need for experimental models has already been highlighted (Antoniou 2021). With DM detection experiments there is a stronger necessity for this distinction. There is a clear difference between DM detection experiments and that of particle accelerators due to knowledge one possesses about the relevant particle: in the latter we know the initial conditions of both the particle and the detector, and those are contrasted against the final results of the detector. In the former the only knowledge we have is that of the initial and final conditions of the detectors. The DM particle is of unknown mass and unknown velocity and as we will see, these are not only insufficiently constrained, but given so many unknowns, these uncertain parameters are also present in or inform others. In fact, the case of DM detection is sufficiently different from that of particle physics in that that "it is widely accepted that dark matter and dark energy are fundamentally different from any particle or entity that high-energy physics or atomic physics have studied in the past, it is puzzling that methods from these respective disciplines are employed to learn more about their properties" (De Baerdemaeker 2021, p. 125).

5.1 Motivating the experimental model

Most DM detection experiments involve nuclear-scattering (and in more recent experiments, electron-scattering), where a particle is detected by the photon emitted from the scattering off of the target nucleus and deposited in the detector. The detector itself is generally constructed in such as way so as to minimise, or eliminate, external interference. The experimental setup in this case plays a dual role: it provides the parameters for the experimental model to find the mass of the particle but it is also the sole detector of the particle. Epistemologically speaking, there is no other means by which one can distinguish what particle is responsible for the scattering that results in the photon being deposited in the detector. If a mass is detected, so goes the rationale, then it is that of a DM particle.[24] In other words, there is no means to cross-check whether the detection is from a DM particle or not.

The only means to ascertain this is via the supposed removal of other constraints: the phenomenological model informs us that DM particles will have a mass of

[24]Of course if a mass is detected it is one in the range of a possible DM particle given that the experiment is designed to detect masses in a limited part of the range.

a certain, albeit large, range and will reach us with an unknown non-relativistic velocity. The experimental model functions under the assumption that no other particle *could* cause a detection. There is, however, no certainty that the above assumption holds. Though one could contend that the annular modulation signal, the distribution of the signal per year, could be an indication, this is not so clear. A case in point is that one interpretation of the initial results from the DM detection experiment, XENON1T, claimed that dark energy could be a possible candidate for the results (Vagnozzi et al. 2021).

Due to these uncertainties and the necessity of the experimental model, it is difficult, not only to interpret the results, but to also compare them across different experimental setups. The high number of astrophysical uncertainties present in detection means that one is required to make several assumptions regarding the parameters reinforcing the dependency on an experimental model. The dependence is sufficiently entrenched that the team running one of the experiments said that "As regards comparisons, we recall that no direct model independent comparison is *possible* in the field when different target materials and/or approaches are used; the same is true for the strongly model dependent indirect searches" (Bernabei, Belli, Cappella, Caracciolo, Cerulli, et al. 2018, p. 2)(My emphasis). This lack of model-independent comparison, which we will discuss shortly, means that there is no cross-check on the experimental results from one experiment with another.

In addition, though the weight range of the particle is constrained by ΛCDM, it remains nonetheless sufficiently large so that experiments will have trouble accommodating it in its entirety. As mentioned, proposals about their weight range have been put forth to further constrain the domain. Simply, model use in the discussion is sufficiently varied that one needs to specify its type to better understand its function. Different models will complement and compete each and therefore, talk about models is not functional unless we specify what type of model it is.

5.2 Constructing the experimental model

It is important to note that the situation to be presented here regarding the experimental setup is quite simplified: assumptions such as the distribution of the dark matter in the galaxy (which itself would affect the expected event rate by month), that of inelastic scattering and inclusion, or exclusion, of spin dependence/independence also factors in. The main formula used in the experimental model, the differential event rate, is based on standard non-relativistic quantum mechanics (QM) with an added assumption, which is "to consider some kind of weak-interaction-like framework [...][that] is couched in the language of 'WIMP searches'" (Cooley 2022, p. 4). In fact, there is always some assumption of non-gravitational interaction involved in DM searches (De Baerdemaeker 2021, p. 139). Further complications also arise if one decides to go beyond basic QM to include an effective field theory expansion of the scattering cross section, which, though helpful in eliminating some possible measurement interactions, would require a drastic reformulation given further uncertainties emerging from the use of such techniques.[25]

Regardless, the inclusion of experimental models is important. The phenomenological model of such experiments would be that of the dark matter distribution in the local neighbourhood, and the velocity with which it reaches earth. This is generally taken to be the SHM, which assumes an isotropic Gaussian velocity distribution of DM particles in the galaxy.

The experimental model is produced from the differential event rate, which is given by

$$\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2m_\chi \mu^2} \sigma(E_R) g(v_{min}),^{26}$$

where m_χ is the mass of the DM particle, μ is the dark matter-nucleus reduced mass (also dependent on m_χ), σ is the scattering cross section, ρ is the local density of DM and $g(v_{min})$ is the velocity integral given by

$$g(v_{min}) = \int_{v_{min}}^{\infty} dv \frac{f(v,t)}{v},$$

that is dependent on v_{min}, the minimum velocity required for the particle to produce recoil energy E_R and deposit a photon in the detector. It is given by

$$v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}},$$

Notice that there are three main parameters that depend on astrophysical uncertainties: m_χ is the mass of the particle we are trying to detect, ρ the density of DM particles in our local neighbourhood, and $f(v,t)$ is the speed distribution function of DM in the galaxy. What we see however, is that m_χ is present in two parameters: σ and v_{min}, the latter of which is constrained by the phenomenological model to be less that the galactic escape velocity, under 320m/s in our local frame. ρ and $f(v,t)$ are also dependent on the phenomenological model one takes. The two other parameters are N_T, which is the number of target scattering sites per kg of the target material, and M_T, which is the atomic mass of the target nuclei.

To construct an experimental model, we input the specific details of the experimental setup (mass of target nucleus, N_T and M_T) and provide the constraints from the phenomenological model, in this case, m_χ, ρ and $f(v,t)$. Given the velocity distribution, we can produce an m_χ-σ plot. The major issue is that "it is not a single velocity that contributes to the scattering rate at a particular E_R. Rather *all* particles with velocities greater than v_{min} will contribute, making it impossible

[25]See Cooley (2022) for an overview of the complexities involved with the detection experiments.

[26]Taken from Fox et al. (2011). Equivalent formulation can be found elsewhere, such as in Green (2017) and Cooley (2022).

to map rates into velocity space" (Fox et al. 2011, p. 4). In other words, the velocity of any single particle is irrelevant; the experiment can only distinguish between particles of velocity greater than or lower than the minimum. This means that the setup is indiscriminate; the higher the mass, the less velocity required to produce E_{min}. The indistinguishability of the velocity of incident particles means that the same results can be produced by two particles of different masses and corresponding velocities. This fact, combined with the large range for possible DM weights, provides difficulty in both, reaching a consensus regarding detection and identifying the mass of the particle.

In addition, given that different target nuclei have different E_{min} by virtue of the M_T being present in the minimum energy calculation, every E_{min} is unique to an experiment, which necessitates a unique experimental model. Therefore, one cannot have a velocity space to compare experiments with different target nuclei given that for each setup, with its own particular target nucleus, various pairs of both the speed and the mass can be construed as a detection.

For example, one experiment, DAMA, used sodium-iodide as the target nucleus in its first phase (Bernabei, Belli, Bussolotti, et al. 2008). Constructing a data model from the results to compare against an experimental model would require the input of those particular parameters (E_{min}) into the models. CDMS, another such experiment, used silicon as targets, which would require a different E_{min} and therefore a different minimum velocity of the incident particle. Given the formulation of the differential event rate, it is difficult to compare the results of two detectors. Any results obtained from the experiments that attempted to provide a data model that is independent of the experimental setup (for cross-experimental comparison) was beset with all the aforementioned astrophysical uncertainties. These models are too specific: analysis on the results via an m_χ-σ plot meant that the graphs obtained from the results of two detectors with two different target nuclei were incomparable. First we must fix the (unknown) velocity to get such a plot and second, the mass of the target nuclei is a factor in σ, in the other axis of the plot. At best, each data model from each experiment can be compared against the experimental model of the same experiment, but two data models from two experiments were incomparable. A change to the experimental models or phenomenological models had to be found that would allow a direct comparison.

One such a reformulation was given early on by Fox et al. and it "makes manifest what the relationships between the different experiments are in terms of what v_{min}-space is probed, and shows (for a given mass) whether tensions exist." (Fox et al. 2011, p. 19). In other words, the experimental model was altered in such a way that velocity need not be fixed to produce results: instead of $g(v_{min})$, Fox et al. uses a rescaled velocity integral $\tilde{g}(v_{min}) = (\rho\sigma_p^{SI}/m_\chi)g(v_{min})$. This is the more current way of formulating the rescaled velocity integral (see Green 2017). Fox's original formulation was done directly via a relation to the other experiments. He does so by specifying a lower and upper range for both E_{min} and v_{min} based on two experiments and relates them to each other via the velocity integral. In the original formulations the comparison of two experiments is used to provide constraints on the parameter space. Since different experiments would probe different ranges of the speed distribution in $\tilde{g}(v_{min})$, we could check for overlap. However, this was limited by a comparison of only two setups and comparison with experimental model constrained from the phenomenological one was difficult given the large range of possible DM mass.

Another drawback, however, is that one now has to fix the mass, the parameter that is to be found, to produce a velocity space plot. However, given the large weight range, this requires the production of various plots per mass and per experimental model to detect any overlap. Nonetheless, data from experiments can now be compared via the experimental model: "Thus, unlike m_χ-σ plots, which have a tremendous amount of processing in them, this [reformulation] provides a direct comparison of experimental results on the same plot." (Fox et al. 2011, p. 20)[27] The comparability, however, is still between two different detectors. It gets increasingly complex to compare a higher number of experiments and add the phenomenological model's constraints.

With such a reformulation, experimental results were, in theory, more comparable. Yet, the results from the various detectors were inconsistent, at least in the probed weight range. Whereas DAMA and CDMS obtained detection, two other detectors, SuperCDMS and LUX did not.[28] What this meant was uncertain and could point to very different problems. Any of the models in the taxonomy could be the culprit and there is no way to properly know from the experiments. The dual role of the experiment, as detector of the particle and detector of the mass of the particle, coupled with the given the number of uncertainties makes knowing where the problem lies more difficult. The possibilities are many: the particle does not exist and something else is causing detection, e.g. dark energy, DM exists but is not weakly interacting, DM exists but is self-interacting, implying that the SHM is wrong, and so on.

In addition it is important to note that SHM is based on models of galactic formation, which are themselves dependent on ΛCDM. The phenomenological model itself might be too dependent on the theoretical model, thereby entrenching uncertainties from the theoretical one in the phenomenological one without sufficient external constraints. Many alternatives have been proposed at different levels of the taxonomy.

This leads to a crucial point, namely that comparison happens on the level of an experimental model, where one now has constraints coming from the data models, as well as that from the phenomenological one. The inclusion of data models from various experiments could, in theory, provide some cross-check regarding the weight, given the unknown velocity and the amount of assumption made in the

[27] The processing in this context is regarding the astrophysical uncertainties present.

[28] See Bernabei, Belli, Cappella, Caracciolo, Castellano, et al. (2013), Agnese, Ahmed, et al. (2013), Agnese, Anderson, et al. (2014), and Akerib et al. (2014) for the results of the experiments.

SHM. Instead of cross checks, it could also possibly provide further parameter constraints by eliminating possible mass ranges. Given the indiscriminate nature of the experiments with regards to the energy deposited, however, it becomes difficult in determining what to eliminate: a range of velocities, a range of masses or some combination thereof. It is therefore even difficult to exclude part of the parameter space due to this fact. In addition, to compare more than two data models with the phenomenological one remains highly complex, given the high number of astrophysical uncertainties that is coupled with the indiscriminate nature of the experiments with regards to velocity.

As mentioned at the end of section 3.1, either one has ascertained that the object type exists prior to testing the dynamics of competing models, or the dynamics are sufficiently understood so that the events in the experimental setup can be interpreted. In this case, given that the detector plays a dual role, the mass and velocity are not well constrained, assumptions are made regarding DM interaction, and assumptions are made in the construction of SHM, then the dynamics of the experimental interactions cannot be properly interpreted as providing a detection. Given these issues, then any attempt to provide cross-checks with other experiments will end up entrenching the dependency on the experimental model, which is unique to every experiment. Competing models and theories on various levels in the taxonomy have emerged as a result.

5.3 Alternatives of different models

Milgrom, for example has provided an alternative to GR itself, Modified Newtonian Dynamics (MOND). More recently, however, it is considered rather a supplement to Newtonian dynamics and GR (Milgrom 2020) and is more prevalent in philosophical circles, yet not without opposition (De Baerdemaeker and Dawid 2022).[29]

Alternatives to the theoretical model, ΛCDM have also been proposed e.g. hot (relativistic) DM, ultra bosonic, primordial black holes (Kimball and Budker 2023, pp. 15-20), superfluid DM (Berezhiani and Khoury 2015), and others. The two most prevalent alternatives are ultra bosonic DM, that are on the lower end of the mass range, which would require different experimental setups (such as those involving electron scattering), and primordial black holes. The latter posits black holes that formed in the early universe without the need for supernovae compression or particle model of DM (Bird et al. 2023, p. 4). These alternatives are supposedly sufficient to provide causal explanations for the galactic phenomena that necessitated the existence of DM and can supposedly account for the lack of definitive detection.

There are also alternatives to the phenomenological model, such as the prevalence of various substructures - clumps and streams - or self-interactions amongst DM particles leading to the formation of large structures, such as DM stars (Kimball

[29]See Kashyap (2023) for more details on the philosophical differences between GR and MOND.

and Budker 2023, p. 21). SHM is calculated on the basis that DM particles are non-self interacting. Therefore, though SHM would admit clumps and streams, it cannot admit large substructures. More constraints on SHM are being added with results from various observations and experiments (Evans et al. 2019).

Finally, there has also been an increase in methods called model, or halo-independent[30] in the physics literature that, instead of directly comparing two experiments, attempts to bypass the uncertainties altogether by providing formulations that are independent of a model. Many such model-independent methods have been introduced.[31]

In our terminology, we can better specify the 'independent' aspect of these methods. For example, Kavanagh and Green (2013) provide a model-independent velocity distribution function by finding a parametrisation from which one could recreate the main phenomenological models, i.e. SHM and its main alternatives. In our terminology, this would be independent of the phenomenological model. The implication is that even if one changes the experimental setup, as long as no new alternatives are considered viable, the parametrisation would still hold.

Others would be considered, in our terminology, experimental model-independent. One such example is by Gelmini et al. (2017), where the authors attempt to provide a model-independent formulation by taking into account the experimental response in detectors (nuclear scattering) to various speed distributions. In other words, they attempt to bypass the phenomenological model via a reliance on the experimental one. In theory, the initial conditions of the particle could be deduced from the results. This means that a change in the experimental setup, or at least, mechanism, requires non-trivial modifications which would render cross-experimental comparison difficult.[32].

The mentioned problems, possible solutions and model-independent methodologies implies that there is a need for a clearer model division: when such experiments fail to provide either a positive or negative detection, it becomes uncertain what in the background assumptions has failed. This is especially problematic given that the SHM is already calculated on the basis of non-self interacting DM (which is arrived at from the theoretical model with little independent constraints and added assumptions) and the necessarily heavy reliance on an experimental model whose constraints are obtained from data models and phenomenological models, all three of which are arrived at quasi-independently of each other.

5.4 Data models

All the models so far discussed have been derived from one or more theoretical frameworks to, loosely speaking, represent or predict the state of affair present in

[30]The term 'halo' refers to the SHM.

[31]See Green (2017) for an overview.

[32]See Chen et al. 2023 for a modification of the methods proposed by Gelmini et al. 2017 for electron-scattering.

experiments. Therefore a brief discussion of data models is required.

The model-independent methods mentioned have been independent of one or another parameter in the experimental or phenomenological model. However, the use of the term 'model-independent' is drastically different in the experimental literature and can point to a crucial difference in terminology. In one of the earliest published papers on experimental results from the detectors, the authors note that their results are "model independent [which means that] no other experiment whose result can be directly compared with this one is available so far in the field of Dark Matter investigation." (Bernabei, Belli, Cappella, Montecchia, et al. 2006, p. 125) Given this definition, after the results from other detectors were published, the same authors claimed, as we have already quoted them above, that "As regards comparisons, we recall that no direct model independent comparison is *possible* in the field when different target materials and/or approaches are used; the same is for the strongly model dependent indirect searches" (Bernabei, Belli, Cappella, Caracciolo, Cerulli, et al. 2018, p. 2)(My emphasis). Authors elsewhere claim that "The annual-modulation effect measured in DAMA experiments is model-independent. In other words, the annual modulation of the event rate is an experimentally established fact, independent on theoretical interpretations of the identity of dark matter and specifics of its interactions." (Addazi et al. 2015, p. 2)

Therefore it is important to distinguish this notion and the difference between data and experimental models. Whereas the latter utilises theoretical constraints to provide a model using the differential event rate, the construction of data models assumes model-independence by the very nature of experimentation and its supposed sole reliance on the data, on what is obtained from the phenomena.

This mode of thought, however, "makes it hard to differentiate data from [data] models, given that [data] models are themselves typically conceptualised as representations – though what they represent can vary from (parts of) the material world to highly abstract concept." (Leonelli 2019, p. 4) As Harris points out "in many cases [in physics] the data that has traditionally been referred to as raw is in fact a data model." (Harris 2003, p. 1511) Putting aside the term 'raw'[33], it becomes apparent in the quotes from the DM detection experimental results, what is being presented as model-independent is ambiguous without such classificatory distinction. Leonelli presents the view that data models themselves are built from the data according to the scientists needs, which would the render the model-independence about the data, not the data model itself. Couple that with the fact that there is an underlying assumption that only DM particles could possibly provide a detection, and one can clearly see the need for autonomy of data models from the data itself. Given that the data model should be able to communicate with the experimental one, the difference between data models and the data themselves become more evident.

[33]Of course the distinction between 'processed' and 'raw' data might not be too evident in the case of HEP and DM detection experiments. See Morrison (2015), Parker (2017), Leonelli (2019), Bokulich (2021).

6 Conclusion

Looking at the use of models in the ongoing search for dark matter detection and subsequent analysis on them provides invaluable insight into the problems plaguing the detection and the interplay of models in scientific practice. The inconsistent results, difficulty in their interpretation and lack of cross-comparison from the point of view of the five-fold distinction in modelling coupled with autonomous constraints shed light on the problems present in said experiments.

Given the taxonomy presented, which is based on the methodology and epistemology of scientific practice, we can summarise the discussion as follows: the background theory, whose causal relations are abstracted, provides the initial constraints. To construct a theoretical model, one needs to include a domain of objects, which function as external constraints, and possibly other external constraints from observations. To construct a phenomenological one, we also require external constraints, but these can come from the particular properties of the state of affairs under consideration. In the case study, the constraints provided are insufficient, providing a range of mass and velocity too large for any one experiment.

In the move from the phenomenological model to the experimental one, there is a shift in the background theory. In the differential event rate, we have mentioned the reliance on both GR and QM. However, the reliance on each background theory is different: ΛCDM and its associated phenomenological models provide constraints on the differential event rate in terms of mass, velocity and distribution. These parameters are, however, highly interdependent; deriving the velocity and distribution depends on the SHM, which in turn is based on ΛCDM model, with constraints being added from other observations and experiments. The experimental model itself, however, is independent of GR; the differential event rate uses QM as its background theory. This means that the experimental model for DM detection is not dependent on GR for its parameter space. If DM detection experiments pointed to the lack of existence of DM particles, then the experimental model is unaffected, except via the lifting of constraints, but the phenomenological and theoretical models would be falsified.

This shift in background theory from the phenomenological model to the experimental one allows us to avoid the circularity present in HEP experiments. This, however, comes with a different set of problems: the results of the experiments require interpretation so that both the data model and the phenomenological model can communicate with the experimental one. When the experiment fails to verify or falsify the intended model, ΛCDM in this case, then it becomes extremely difficult to pinpoint where the failure lies. In addition, the lack of sufficient constraints in the transition from ΛCDM to SHM, coupled with the assumption, or educated guesses necessary to construct SHM further compound the problem.

This has led, as was discussed, to a plethora of alternatives. From the point of view of the taxonomy presented, one is better positioned to understand the confusion and difficulty in the interpretation of said experiments and what the various alternatives are trying to modify and rectify.

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