**Do infinite cardinals really exist?**

**Abstract**

If the philosophy of mathematics wants to be rigorous, the concept of infinity must stop being equivocal (both potential and actual) as it currently is. The conception of infinity as actual is responsible for all the paradoxes that compromise the very foundation of mathematics and is also the basis on which Cantor's argument is based on the non-countability of $R$, and the existence of infinite cardinals of different magnitude. Here we present proof that all infinite sets (in a potential sense) are countable and that there are no infinite cardinals.

This article presents a new argument against the existence of the Platonic world of ideas, the ontological basis for the actual infinity. This allows us to deny mathematical Platonism and adopt a non-subjective psychological realism that explains the effectiveness of mathematics in physics and that can go beyond the scope of mathematics.

**Keywords**: actual infinity, Zeno, Cantor, Platonism, Constructivism.

**1.** **Introduction**

There has been a century-long debate between believers in the existence of actual infinity (Cantorians) and non-believers (constructivist). Does actual infinite exist or not?

Many believe that this debate is illegitimate because it is a matter of schools of thought that should be respected, with each working independently without assuming that only their own perspective holds the truth. The concept of truth has been questioned (Hersh 1997) (Dummett 1978). One may ask: Is the Pythagorean theorem true? Relativists claim that within Euclidean geometry the theorem is true, but within other geometries it is false. However, it must be considered that the notion of a straight line is not defined at the foundation of these geometries. There are different sets of axioms that constitute distinct worlds. In the models of these worlds, it is clear that different concepts of straight lines are being dealt with. Some define the right triangle in one way (with Euclidean straight lines), while others define it differently (with non-Euclidean straight lines). They are not talking about the same thing. The Pythagorean theorem only applies to triangles with Euclidean straight lines. It is always true. Referring to the Pythagorean theorem in other geometries is a misunderstanding, and claiming it is false is a fallacy of equivocation.

The same does not occur when questioning whether the continuum hypothesis is true or not among those who believe in infinite cardinals. Some say yes and others say no. It has been shown that we are dealing with an independent axiom, and each side believes what they prefer, leading to two opposing universes of set theory (Hamkins 2012). However, in this case, Kurt Gödel, for example, argues (Gödel 1947, 515) that the truth lies on one side or the other, as we are speaking about something perfectly defined in the same way in both schools of thought. One of the two must be false. Gödel advocates for an inductive analysis, studying the success of the various theories (Gödel 1994, 109).

The same applies to the issue of the actual infinite. The concept (or pseudo-concept) is quite clear. Some say it corresponds to a reality, while others say it does not. One of the two must be mistaken.

To establish a foundation for a science, we cannot rely on a pseudo-concept, a false concept. This is why it is essential to thoroughly analyze whether the proposed concept is contradictory and whether it entails unsolvable mysteries and paradoxes. It is also necessary to question the existence of the metaphysical world that supposedly supports this concept. This will lead us to a non-Platonic type of mathematical realism.

**2. Is infinity an ambiguous concept?**

When we speak of infinity, do we suspect that we are referring to something with two possible, opposing meanings? I do not believe that any philosopher holds this suspicion, because if that were the case, they would need to stop using this word in order to be rigorous in their reasoning. The concept of infinity is not in question. Everyone knows the etymology of the word, which forms the basis of our conceptions. Perhaps the Italian language offers the greatest clarity on this point, as the word "finito" there means finished, completed, or realized, and, consequently, "in-finito" means not finished, not completed, essentially unrealized; in other words, unfinishable, endless, unrealizable, and precisely because of that, it has no end or limit.

Mathematicians have not defined the concept of infinity, as some claim, but rather the concept of an "infinite set." Therefore, we must analyze their concepts to see if they are consistent and aligned with the idea of infinity that we all have in mind (not something very large and finite, but something that can never stop forming or being constructed).

The Cantorians speak of infinite sets as those whose cardinality or number of elements is not a natural number. But if they speak of a fixed cardinality, they are thinking of something very large but finite. That is why they consider the adjective "infinite" not to have its ordinary and common meaning (as something potential or under construction), but an opposite meaning (something completed that we would ordinarily call finite or finished). They thus oppose the ordinary and etymological definition of infinity. Their definition depends on the existence of transfinite numbers (non-natural), which could be defined as those that measure infinite sets. In principle, if infinity never ends (as is commonly thought), infinite sets should not end either, and we could not speak of a number that measures them but of a variable that grows indefinitely. Therefore, this definition makes infinity an ambiguous concept.

Furthermore, Cantorians, using the property of reflectivity of infinite sets already observed by Galileo, Duns Scotus, and others, but especially by Bernard Bolzano (Bolzano 1854, Section 20), have defined infinite sets as those that are equipotent to a part of themselves. However, this definition (Dedekind's infinite, proposed in 1888 by Richard Dedekind) is only universal if the so-called axiom of countable choice is accepted (Jech 2008). In this definition, it is understood that there is a bijection between the set and a part of it, and that this bijection is defined as an actual infinite set of assignments. And here lies the problem with this definition: if we admit that this infinite set (the bijection) is completed (and not in construction), then we are equating the whole with the part, which goes against the principle of non-contradiction. If we call the whole T, then the part is NOT T (since it is not the whole). Equating the whole with the part is equivalent to saying that T is NOT T. As we can see, accepting Dedekind's definition not only formally but also in its philosophical depth amounts to denying the principle of non-contradiction.

Finite sets of natural numbers express their cardinality (number of elements) always through a natural number. The set of all of them cannot express its cardinality through a natural number, and this is so, as we constructivists think, because such a complete set does not exist; there are always elements that are not there. But the Cantorians, in view of the successions that do not end, fall into a tendency of the mind to make up for what is not there from reality (both in perception and in intellectual judgment) and create the actual infinite sets that they suppose exist in Platonic fantasy worlds as they think their minds are too small to contain them. It has already been noted many times that Kant realized this demand of the mind and expressed it in his Critique of Judgment: "The spirit hears in itself the voice of reason, which for all given magnitudes... demands totality." and, consequently, the understanding in an intuition, and for all these members of an increasing series of numbers... obliges us to conceive it as given entirely (in its entirety)." (Kant, 1790, section 26).

When Cantorians talk about infinity they use this word with two different meanings (potential and actual) since, on the one hand, the successions never end, but, on the other hand, they are already finished.

**3. Mysteries and paradoxes with the actual infinite**

The concept of potential infinity (the ordinary one) does not involve any paradoxes or mysteries. The paradoxes of infinity emerged as soon as philosophers conceived the idea of complete infinite sets of points and defined the continuum as an infinite set of unextended points (Grünbaum 1952, 288). Interestingly, there is currently some theory that questions the lack of extension in points (Ehrlich 2022, 784). Zeno of Elea made it very clear: if there are infinite points on a line, the object moving from one point to another traverses infinite points, performs infinite operations, and finishes the task. Therefore, infinity (the unfinishable) must be considered as something finite (finishable and finished). This is the paradox that sums them all up.

The paradox has no solution as long as we give reality to the concept of the actual infinite. The proof lies in the fact that most mathematicians and philosophers consider the paradoxes of infinity insoluble within ordinary logic. For Bertrand Russell, the state of motion does not exist (Russell 1903, 578), and furthermore, Achilles will catch the tortoise just as Tristram Shandy will be able to finish writing his autobiography, perpetually out of sync with reality, as long as he lives for infinite years (Russell 1903, 442). This "solution" (which is imposed in schools and universities) requires a great deal of faith in the completion of an infinite process, which is precisely what cannot happen.

Henri Bergson's solution, denying the existence of the actual infinite (Bergson 1950, 1062-1064), was so attractive and convincing that attempts were made to discredit it. Adolf Grünbaum took on that task by inventing his famous second Achilles.

This character stops at all the points that the tortoise reaches successively while the first Achilles, in continuous running, strives to reach the points where the tortoise had reached in its previous stage. Thus, we returned to a discontinuous path of an actual infinite number of points. This Grünbaum paradox is resolved as Russell's barber paradox. The existence of such a barber was impossible and the existence of this second Achilles is also impossible. It cannot be a physical engine because it should have infinite power since it reaches accelerations greater than any number, no matter how large. And, as a mental object, it could not follow its path to the end because this would represent reaching the end of a succession by following the path of the succession. But the succession has no end.

The resolution of Grünbaum's problem is so easy (Sanvisens 2021, 99-104; 1992, 19), that it is difficult to understand the need for the complex developments that Grünbaum makes to “explain” Zeno's paradoxes, in which not all authors agree. The entire immense Cantorian bibliography on the solution of paradoxes has been so unconvincing that it has been necessary to resort to the dubious concept of the infinitesimal to address the issue. The interesting thing about the case is that the authors who have tried it (McLaughlin 1995, 65) end up confessing that, since the movement is carried out in unobservable infinitesimal regions, no explanation is necessary. A majority of mathematicians consider that the hard core of Zeno's paradoxes is insoluble because the actual infinity is an unfathomable mystery for the limited human mind. But the Cantorians must remember that they are the ones who have reestablished the paradoxes by assuming that there are finished sets of infinite points without the need for any development or division (Grünbaum 1952, 288). Constructivists, on the other hand, consider that points do not exist until they are created by the mind and thus the paradoxes disappear.

**4. New formal definition of infinite sets**

In order to establish a good set theory, it is necessary to formally define the concept of an infinite set. I propose the following definition: "a set is infinite when all its parts are own." (The proper parts are those that do not constitute the whole). For every part there is always an element of the set that does not belong to it. In fact, this definition fits perfectly with Poincaré's informal definition "when we speak of an infinite collection, we mean a collection to which new elements can be added incessantly" (Poincaré 1913, 77).

The continuum is not an infinite set of points but a space (indefinable concept) whereas many points (positions with respect to an origin) can be indicated as real numbers can be constructed.

An open interval (a, b) on the abscissa axis is a space on the abscissa axis where any point on the abscissa greater than a, but less than b, can be determined exclusively on it.

The infinite sets defined here have no cardinal since they are not finished and cannot be considered as a totality. That is why all its parts are own.

**5. The paradox of the lord of the abscissa**

In 2021 a new paradox appeared (Sanvisens, 2021; 2023 a; 2023 b) that allows us to see (real intuition) the contradiction that the concept of actual infinity brings with it.

It is as follows: each natural number is written on an opaque and thin card and all those cards are placed in order at the abscissa points 1, 1/2, 1/4, 1/8, 1/16, ... At the point of abscissa -1 an observer point is placed which is called the lord of the abscissa. When this lord looks at the entire formation of cards that is in front of him, curiously he cannot see any card showing a certain natural number, because any card has in front of it between it and the lord of the abscissa another infinite number of opaque cards that hide it from the sight of this lord. Mysteriously, these cards cover each other, and the result is the invisibility of the set.

This paradox could be considered unclear since, mathematically, it would be expressed by saying that each element of the set of cards has another one in front of it that hides it, and, although something impossible is intuited, this impossibility is not formally clear.

That is why I have thought it convenient to visibly show the contradiction, adding a ray of light emitted by the same lord of the abscissa. The light always moves towards the cards unless it collides with one in which case it is reflected and returns to the lord, who can see the information of the natural number written on the card.

The mysterious (and impossible) thing is that the light, as it advances, cannot collide with any card because they all have infinitely many opaque cards in front of them that prevent the light from reaching them. But if the light does not hit any card, then it must move indefinitely through the entire set of opaque cards, which it cannot do. The contradiction in the behavior of that light that must advance and not advance at the same time, reflects the contradiction that the existence of an actual infinite set of cards entails.

Another paradox that complements this one occurs to me: suppose that the lord of the abscissa decides to go around and place himself (setting his watch at 0 seconds) at the abscissa point 1, moving from there the card with the number 1. Then in for a time of 1/2 seconds, it is placed at the 1/2 abscissa point, moving card number 2 from there; and so on. When 1 second has passed on the clock, the lord will have displaced all the infinite cards in the collection and will find himself alone at some point on the x-axis. The problem is: at what point? It cannot be found at any point since said point should be the last in the formation, but this infinite succession of points does not have a last point, since the point of abscissa 0 is outside the formation.

Here the potential nature (dynamic and not static) of infinity is clear. The fact that there is no end point is evidence that succession is something dynamic that is mentally constructed without end. The conclusion is that infinity is not something hazy already fully constructed (actual), but something under construction (potential).

The paradox of the lord of the abscissa can also be stated in purely mathematical terms. We are going to call P1, P2, P3, … the points where we have placed the numbered cards. This succession of points approaches the abscissa point 0 so close that there is no space between the point 0 and some point of this succession that does not contain any point of the succession.

If we construct the infinite closed intervals [P1,P2], [P2,P3], [P3,P4], ... we could say (if we admit that there are infinitely many intervals in fact) that the union of all of them must give the entire interval that goes from P1 to point 0 (excluding point 0). This is exactly the semi-open interval [P1,0). Cantorians are forced to conclude that the union of infinite non-disjoint closed intervals is a semi-open interval.

This conclusion (forced by admitting the existence of an actual infinite number of intervals) goes against logic. In effect: when two non-disjoint closed intervals are joined, a closed interval is always necessarily obtained because it contains the extreme points of the first and second interval. If we add a third closed interval to this union, we once again have the union of two closed intervals, which must give a closed interval, and so on indefinitely. No matter how many closed intervals we add, the union will always be closed. But the Cantorians are now forced to say that, if the intervals are infinite, then the union is an open. For a Cantorian, logic fails in infinity, since what always happens in the same way, in infinity happens in the opposite way. The Cantorians attribute this paradox to the mysterious nature of the infinite that far exceeds the finite nature of human understanding. But the truth is very different, because the paradox disappears as soon as we stop believing that there is an actual infinite number of intervals. What exists is a finite number of intervals that can grow indefinitely as the mind conceives new smaller intervals (potential infinity). The truth, which can be stated as an authentic rigorous theorem of mathematics, is that the infinite succession of the unions of the different non-disjoint closed intervals mentioned above tends to a semi-open interval (which is its limit). This is equivalent to saying that the succession of the abscissa of the ends of these unions tends to zero. This is reality, without mysteries or paradoxes.

Another form of this paradox is the “theorem” that says that every string of pearls in a straight line that has finished being constructed has one last pearl at the terminal end.

In fact, if the string is built by setting pearl by pearl successively, at the moment when the construction is finished, the last pearl is set. On the other hand, any string of pearls can be built successively. If the string is supposedly composed of infinite pearls, we can build it successively in a time of two minutes. To do this, we follow the following procedure: in one minute we link the second pearl to the first. In half a minute we set the third pearl; In 1/4 of a minute, we set the fourth; In 1/8 of a minute, we link the fifth, and so on. After two minutes, which will pass inexorably, we will have successively set the infinite pearls on the string. This “theorem” has a corollary that says that if a string of pearls does not have a last terminal pearl it is because it has not finished its construction process. If this process never ends, we have what we call a potential infinite string, lacking the last pearl because its construction has not finished.

6. THE MYSTERY OF HILBERT'S HOTEL

The idea of ​​a theoretical hotel with infinite rooms was proposed by David Hilbert in an unpublished lecture that took place in January 1924. Each room was occupied by one guest. But if each guest moved to the room with consecutive number, room number 1 was empty and one more guest could enter. For Cantorians this does not constitute a paradox, but rather an aporia related to the contemplation of the mystery of the infinite (Sanvisens, 2021, pp. 11-17). In fact, they consider that this hotel is a model of an infinite set and that is why miraculous things happen there, such as the appearance of empty rooms when all of them were full, and the fact is that infinity, for them, is an absolute mystery for the limited human mind.

But true science seeks to clarify mysteries and this one about Hilbert's hotel is very easy to solve. If an empty room appears where there was none before, it is a sign that it has now been built. That is, Hilbert's hotel is under construction; It's not finished. Infinity is something potential, not actual. Infinity ceases to be a religious mystery and enters the realm of science.

But the Cantorians have clung to their dogma of faith in the actual infinity. They blindly believe that all the rooms are already built and that what happens is that the infinite guests have been placed differently, leaving an empty room. We are going to accept (ab absurdum) their interpretation of the facts, and we will reach their ultimate devastating consequences.

Let's admit that the hotel has infinite guests, that they can only move to other rooms without ever leaving the hotel and that, consequently, they are always the same.

The concierge gives an order transmitted to all guests, according to which a transfer 1 will be given in which, in one minute, the guest in room 1 moves to room 2; Then a transfer 2 in which, in 1/2 minute, the guests in room 2 move to room 3; Then a transfer 3 in which, in 1/4 minute, those from 3 go to 4; and so on. After two minutes (a time that will inexorably pass in which infinite transfers of this type will have been made), the concierge will be able to verify that there will be no guests in any of the hotel rooms, since those who were in room n will have left moved from there in transfer number n.

If there are no guests in any room of the hotel and the guests can only be in some room of the hotel, that implies that the infinite guests of the hotel have disappeared, which is contradictory, because we said that the guests are always the same.

If the existence of an actual infinite number of guests, rooms in this hotel and transfers leads to a contradiction, we must conclude that the actual infinite does not exist. If the Cantorians persist in their dogma of faith, they are against science.

**7. The numbering of real numbers**

The argument that Cantor gave to “prove” that real numbers are not countable was based on a false premise: that of the existence of an actual infinite number of mathematical objects. This was already observed by Alexander Zenkin and other authors (Zenkin 2004).

The Cantorians have defended themselves against this accusation by saying that they work in an axiomatic system that contains an axiom of the existence of an infinite actual countable set ($N$) and another of the existence of the parts of this set (equipotent to $R$) also actual. But, as we saw in sections 5 and 6, there is no actual infinity. Both the set of natural numbers and the set of real numbers are potential infinite sets, as defined in section 4. The axiom of infinity to which the Cantorians refer, as we will see in section 8, refers to a potential infinity. Consequently, the Cantorians' claim must be rejected.

That is why we can say that this argument is a fallacy due to the falsity of a premise. There is no complete collection of natural numbers, nor of real numbers, nor of infinite fractional digits in any real number. Irrational numbers do not have infinite fractional digits, but rather they have none: they cannot be represented by fractional digits. On the other hand, its potentially infinite approximations, yes.

The supposedly infinite fractional digits of an irrational number are, in reality, (and this is well known) the last fractional digits of each of the infinite approximations to this number. And there are never actually infinite approximations, but rather they are potentially infinite. This is not new. Gauss already thought this way (Dauben 1983, 85).

But, if we get a little closer to Cantor's arguments, we will see that they all make the same error: that of considering that random (that is, non-algorithmic) successions can have a limit. Let's see why this is a mistake.

The limit of a succession is determined solely by the algorithm that describes it. The terms of the succession do not determine any limit, since they never end. They could only determine a limit if they ended, since the last term would be the limit. But that is impossible because the succession of terms never ends. If these terms are governed by an algorithm, it is that algorithm that determines the limit, which does not belong to the succession. If there is no algorithm (as in random) then nothing determines the limit, and it does not exist.

Cantorians only have one possible escape and that is to believe that there are infinite imps with different personalities, that they are the ones that determine the infinite terms of these random sequences, and that this “personality” is what determines the limit. Unfortunately, this “personality” would be nothing more than an algorithm unknown to us, but an algorithm, after all; That is, the sequences would be algorithmic and not random.

The reason why they have no limit is because randomness does not define a succession but an infinite set of them. Each of these successions does not have a total existence, but rather a semi-existence. They are successions that are made without any determination and are never defined (fully determined); There is always an infinite path to determine.

We find these random successions in Cantor's argument based on nested intervals (Cantor 1874) and in the famous diagonal argument (Cantor 1891). In the latter, the random successions would be constituted by the successive numbers formed by the changed digits of the diagonal. Thus, if the digits on the diagonal are d1, d2, d3, ... and the changed digits: d1', d2', d3', ... the sequence I am referring to would be the one formed by the numbers: 0,d1'; 0,d1'd2' ; 0,d1’d2’d3’ ; …

Only if the table of real numbers were ordered by means of an algorithmic function, then the sequence would not be random, and its limit (the diagonal number with changed digits) would exist, and Cantor's argument would be true. But if the table were ordered according to a random function (not algorithmic), then there would be no limit to the mentioned succession of numbers formed from the diagonal digits, and therefore the diagonal number would not exist, in which case the Cantor's argument would fail. Thus, if we randomly order the real numbers we can number them without any contradiction.

In fact, already in 1908, at the IV International Congress of Mathematics held in Rome, Émile Borel said that all sets are countable, only that some are effectively countable (without possible ambiguity in the assignment of elements) and others are not. (Borel, 1931, 254).

The ordering according to non-algorithmic (random) functions may seem strange, but it is something fully accepted within cantorism. We see it in the argument of Alan Turing (1937, 260) when he puts the natural numbers in one-to-one correspondence with the computable real numbers, of which he demonstrated their enumerability. This author realized that this correspondence could not be established by means of any algorithmic function, because then the diagonal number would be computable and there would be a contradiction. That is why the bijection between $N$ (natural) and$ Rc$ (computable real) had to be a non-algorithmic (random) function. In this case the diagonal number would be the limit of a random sequence and would not exist, so there would be no contradiction in assuming the existence of a bijection. Unfortunately, Alan Turing concluded that there was a limit to this random sequence, but that this limit was a non-computable number and that is why the contradiction also disappeared. This false conclusion of Turing has been responsible for the general acceptance of the existence of non-computable numbers, denied by Émile Borel (1898, 161) and many other authors. In fact, the Cantorians already needed such numbers because Turing had shown that computable numbers were countable. That is why non-computable numbers had to exist to account for the supposed non-countability of $R$. But non-computable numbers must be random, that is, they should be the limit of random successions. As we have already seen, such a limit does not exist.

Non-algorithmic functions could only be fully determined by making an independent choice within the codomain (or arrival set) of them for each element of the starting set. That is, this determination depends on infinite independent choices, that is, not linked by an algorithm. In each election, only one value is chosen through the activity of an elector. Each of these elections must be made successively by a single elector, since if they were made simultaneously by infinite independent electors (not linked by an algorithm and with ignorance of the assignments made by the others) we could not guarantee the obtaining a function, since several elements of the arrival set could be assigned to a single element of the starting set, but, above all, because there are not simultaneously infinite elements of the arrival set (that would be an actual infinity, which is something non-existent. The only infinite that exists is potential, as we have seen).

But if the choices are made successively then the process is never finished because it is a potential infinity. The function is never fully determined; It does not exist strictly speaking, but we can call it semi-existent.

Algorithmic functions do fully exist since the choice is not made element by element, but rather an algorithm is chosen at once that potentially determines all assignments.

**8. The axiom of infinity**

This is an axiom of the ZFC system (of Zermelo-Fraenkel with Choice axiom) and goes like this: "There exists a set x that contains the empty set y that is such that, if y belongs to x, then the union of y with {y} is also in x" (Cohen 1967, 243). The meaning of x (potential or actual) must be analyzed according to the statement of the axiom (Sanvisens 2023 a, 78-80). The key to delving deeper into this axiom is in the statement that says that, if y belongs to x, then the union of y with {y} also belongs to x.

The axiom constructs the set x step by step. First it puts the empty set inside. Then the union of the void with the set whose only element is the void; then the union of this set union with the set whose only element is this union; and so on. The number of elements of this set is always finite, but it can always increase by constructing the union of the last set “y” obtained with the set whose only element is this “y”. Consequently, the set x constructed in the axiom is a potential infinite set, not an actual one. The fact that the construction of all the elements of this set is not simultaneous (or generic) is because, given any set “y”, the set {y} has to be constructed mentally and followed by a mental algorithm consisting of performing “the union of y with {y} in each case” which is what the axiom actually does. The ontological dependence of {y} with respect to y has already been noted by other authors (Ferrier, 2019, 5).

This means that there is an obvious but surreptitious contradiction between the axiom of infinity and the theorem (in ZFC) that says that every set (finite or infinite) has a cardinal, since, if infinite sets are potentially infinite and not actually infinite, then they cannot have a cardinality, since that are not complete, are not finished.

The constructivist position, which requires some type of mental construction to guarantee the existence of an object, is criticized because it is considered that this implies dispensing in certain cases with the principle of the excluded middle. This was true in the case of Luitzen E.J. Brouwer (1918) and other distinguished intuitionists such as Arend Heyting (1930), but not in the case of other constructivists such as Andrei A. Markov Jr. (Manin 1981, 248-249). Erret Bishop (Bishop, 1967) and others. It must be kept in mind that many times the mathematical object is not constructed directly but through certain conditions. In Brouwer's fixed point theorems themselves, the perfectly defined conditions of the continuous transformation determined the fixed point, although they did not explicitly indicate it. The non-existence of the fixed-point leads to a contradiction; Therefore, such a point must exist even if it has not been constructed. This breaks intuitionistic logic (a branch of constructivism). That is why it was decided, in this school, to sometimes dispense with the resource of reduction ab absurdum (which is the application of the principle of the excluded middle). But it was not necessary, because, as seen in the example given, the fixed point had already been constructed by giving the conditions of the continuous transformation, just as the center of the circle is implicitly defined when determining the circle.

**9. Theorem of good transitable order**

Based on the axiom of choice (which is perfectly acceptable in constructivism, in a potential sense when it comes to infinite sets) Ernst Zermelo (1904; 1908) proved the good order theorem, according to which every set (M) can be well ordered.

Following an analogous but much simpler methodology it is possible to demonstrate that every set can have a good traversable order. A set has a good traversable order when not only does every subset have a minimal element, but every element has an immediate successor and antecedent except the first element, which has no antecedent. The proof is as follows: Applying the axiom of choice to the case of a set formed by a single set, we can deduce that it is always possible to choose any element (without using any algorithm) from any set, forming a set with that element.

From the set M we choose the element m1. We obtain {m1} and M - {m1}

From M - {m1} we choose m2. We obtain {m2} and M - {m1, m2}

From M - {m1, m2} we choose m3. We obtain {m3} and M - {m1, m2, m3}, etc.

We (potentially) repeat this choice process with the tendency to exhaust all the elements of M, and thus we potentially obtain the set M potentially ordered according to the traversable chain {m1, m2, m3, …}.

It is then possible to establish a bijection between the set of subscripts ($N$) and the set M, thus proving that every set is countable. In effect: It is not possible to indicate any element of M that cannot be the image of some natural number (an index), since that element could be the image of any natural number since image attribution does not follow any algorithm. Therefore, given any element of M we can ensure that it will be the image of some natural number since the natural numbers are potentially infinite.

**10. A contradiction in cantorian arithmetic**

Let's imagine that we start from a point located on the abscissa axis. Suppose that the points multiply like bacteria, creating two points each at each stage and then separating from each other so that the distances between the points are maintained.

In a multiplication stage we will obtain 2 points; in two stages 22; in three stages 23, …; When we reach the alef 0 stage (and this will necessarily happen according to the Cantorians, if the duration of each stage follows an infinite temporal succession that tends to zero), then we will have 2alef 0 points separated from each other on the abscissa axis. A collection of infinitely many separate points is a discrete set and, therefore, its cardinal is alef 0. Therefore, we can say that: 2alef 0 = alef 0.

This result contradicts the fundamental formula of Cantorian arithmetic which says:

2alef0 = $c$ (cardinal of $R$) = alef 1 (in the continuum hypothesis).

The conclusion is that Cantorian arithmetic is contradictory and must be rejected. The truth is that infinite sets have no cardinal. There is no aleph 0 or aleph 1 or large cardinals, nor anything that constitutes Cantor's paradise.

**11. The myth of the world of ideas**

Where do the infinite elements of an actual infinite set exist? For example, where do infinite even numbers exist?

For Cantorians (generally Platonists) they exist in a metaphysical world called by Plato the world of ideas. All of them constitute an infinite totality. To make this position credible, cantorism has modified the ontological status of numbers. For them a number is not an idea in someone's mind (a form of the mind constructed in and by the mind), but an idea existing in a world where all possible ideas actually exist simultaneously. These ideas are captured, intuited, “perceived” by minds, but their existence does not depend on any mind.

The Cantorians assure that it is the same set as concept. The concept of an even number is the same as the set of all even numbers. Nobody denies that the concept of an even number exists. That is why no one can deny (for Cantorians) the existence of the set of all even numbers. But this is a new fallacy due to falsehood of one of the premises. In effect: concept is not the same as set. A concept is a series of properties or a formula that generates elements. A concept is not a list or grouping of elements (a set).

Cantorians think that the existence of all even numbers is implicit in the formula 2n (even number concept). The truth is that a formula is not a number. To find even numbers you have to act on the formula, each time providing a value for the variable n. If we give n the value 1, then we mentally construct the even number 2. When we give n the value 2, we mentally construct the even number 4, and so on. Even numbers only exist when they are constructed from the formula. The formula does not contain them, but rather serves to generate them, but some mind must generate them, or they would not exist.

There is a possible objection to the constructivist position that I have just stated, and it is the case, for example, of the set of the first hundred thousand even numbers. Nobody denies that this set exists and yet in what mind are these hundred thousand numbers? In principle there is no mind that is thinking about one hundred thousand even numbers.

To answer we must propose an algorithmic model of the mind's action. Everything the mind considers is the result of an algorithm. A number is a form of the mind corresponding to an enumeration algorithm. If several numbers are related to each other by means of an algorithm generating all of them, then all these numbers are potentially in the mind in the form of this algorithm, and, in fact, the mind can update them one by one and finish the process, although he doesn't usually do it. But, to do this, the mind must have two basic pieces of information: the initial number and the final number. Otherwise, the algorithm could not finish, and the mind could never have all the numbers actually, and therefore not potentially either.

The first hundred thousand even numbers are in the mind potentially as an algorithm whose termination occurs with the number 200.000. On the contrary, there is no possible algorithm that can generate all pairs, since the termination data would be missing. In other words, the 2n algorithm generates any even number, but alone it is not used to generate a specific pair well another algorithm would be needed to determine the value of n, and this algorithm would never finish generating the set of all of them. That is why the mind cannot contain infinite even numbers, but it can contain any finite number of them.

The Platonic conception of a world with all possible ideas has been questioned by many authors (Linnebo 2023). Émile Borel's (1927) criticism is famous according to which, if Platonism were true, there should be a number of truth where all the knowledge capable of being explained in words would be codified.

 Within a block of marble there are not infinite interpenetrated forms. Michelangelo's “Moses” was a creation of this sculptor and not a discovery.

The objections by Benacerraf (Benacerraf and Putnam,1983) or Field (Nutting 2020) to the mathematical platonism of Gödel, Wigner, Dummet, Steiner, Penrose, Quine, Putnam, etc. are well known. One of the main problems is its epistemological access. I am not going to discuss the various types of non-Platonic realism. I am going to approach the criticism of Platonism in another way.

I'm going to try the nonexistence of the Platonic world of ideas. Ideas can be considered sets in Cantorian nomenclature, and, outside of it, they can be considered assigned to sets. A world of ideas is equivalent to a world of sets. The world of ideas must therefore be equivalent to the world of sets (of all sets). Within cantorism the impossibility of the existence of a set of all sets is demonstrated (outside cantorism, too, although in a different way). We conclude that the world of all sets (the world of ideas) cannot be a set.

This is an inconsistent multiplicity. The logical thing would be to think that not all ideas exist; for example, there are no natural numbers greater than 1010.000. But that would be creating an unacceptable arbitrariness. If they don't all exist, none should exist. Thus, the nonexistence of the world of ideas would be demonstrated. But the Cantorians, logically, have not wanted to admit this. Instead, they have created a new concept to accommodate inconsistent multiplicities. In Von Neumann-Bernays-Gödel set theory, these types of multiplicities are called own classes. In this theory, the primitive notion is that of class, which is defined as a collection of objects (more or less as the set was defined before). Sets are, in this theory, classes that are contained in some other class. Classes that are not contained in other classes are own classes. Obviously, the class of all own classes cannot exist, since then these classes would be contained in other classes and would not be own. But then we have that a certain collection of objects (the own classes themselves) cannot be a class. We see, then, that the definition of a class cannot be a collection of objects. So, what is a class? Something similar already happened when trying to define a set. Every definition conflicted with some result of the theory.

 To demonstrate the nonexistence of the world of ideas we will have to delve into the various hierarchical types of inconsistent multiplicities beyond the class of alephs.

We will start from the class of the alephs, which we will call Cl. Although it is not a set, it has parts. Let's call Ƥ(Cl) the class of the parts of Cl. Let's call ƤƤ(Cl) the class of the parts of the parts of Cl. Then there would come ƤƤƤ(Cl), ƤƤƤƤ(Cl), etc.

All these own classes cannot be grouped within a class, but, if they all exist, they would have to be grouped within what we can call superclass: SCl.

This superclass will, in turn, have parts, and we will have the superclass of the parts of SCl, which we will call Ƥ(SCl.) and then ƤƤ(SCl.) and ƤƤƤ(SCl.), etc. Since these super classes cannot be grouped into super classes, we must group them into hyperclasses of a higher hierarchical degree. We will thus have the Hyperclass of the super classes of parts of SCl., which we will call HCl. Then will come the hyperclass of the parts of HCl., that is, Ƥ(HCl.), and ƤƤ(HCl.) and ƤƤƤ(HCl.).

To all these super classes, hyperclasses, etc. what we find we will call Summums infinitus (S.I.) of different order (S.I. first order, S.I. second order, S.I. third order, etc.). These orders are infinite and, if we follow Cantorian logic, they all exist. Consequently, it is necessary to create a Summum Absolutum (S.A.) that contemplates them all. But unfortunately, this Summum Absolutum cannot be absolute, since we could talk about the parts of S.A, which would be a Summum called Ƥ(S.A.), and then ƤƤ(S.A.), and ƤƤƤ(S.A.), etc. And we are back to the same as at the beginning.

We conclude that no S.A. can exist that gathers and updates all the Summums. Consequently, not all Summums can exist (if they all existed, they could be reunited in a higher- level group, which would be outside of all existing summums, which is impossible). But, if not all of them exist, then some do not exist. The arbitrariness that this represents forces us to think that none exists. The world of ideas does not exist anywhere metaphysically. Ideas only exist in minds and always in a finite increasable number (potential infinity). The actual infinity is non-existent.

Within the iterative and hierarchical conception of sets, which is currently predominant, the inconsistency has already been noted, since in it all possible sets and only sets are generated, with which, since all the members of the set universal exist, such a set must exist since the existence of a set only depends on the existence of its members, but then there is a contradiction with the foundation axiom that prohibits sets that contain themselves (Ferrier, 2019, 8) . To solve this type of problem, it has been proposed that the hierarchy of sets should be treated as potential, not as actual (Linnebo, 2010, 155).

**12. From finitism to mentalism**

If the actual infinite does not exist anywhere, nor the world of ideas that sheltered it, we must accept that only the finite exists (finitism). Within finitism, the potential infinite is admitted, because it is nothing more than something that is always finite but is never finished. There is currently a return to the idea of ​​potential infinity (Scamber 2021: Sewell 2023; Reese 2022; Linnebo 2019).

But, if mathematical objects are never infinite and are not living in Plato's world, then where do they live? The answer is immediate: where we know them, that is, in our minds. Mathematical objects are forms of the mind and only exist in minds (psychological realism). It is very strange that this position, so natural, so evident, is today a minority position. Platonic idealists have convinced many of the leading representatives of science that mathematical objects are eternal metaphysical entities that can be grasped by minds through intuition. For platonic realism the effectiveness of mathematics in physics is a miracle. But if we adopt the opposite position, which denies reality to the world of ideas and considers that ideas are mental forms (which we should call the mentalist position or psychological realism), then how can we explain that mathematical thinking can interpret the physical world?

The simplest solution to this problem is the one that admits that there is an intimate relationship between the mind and the physical world. This position has two aspects: materialism and mentalism (known as subjective idealism represented by G. Berkeley).

Materialism does not explain that matter, governed by vector forces, can generate spirit, governed by meanings that have nothing vector. Mentalism considers the mind to be the ultimate reality and the ontic foundation of all reality. Consider that individual minds participate in the universal mind, its laws and statutes. Matter is a projection of the universal mind that became independent, but that communicates with it through energetic processes (relationship between spiritual energy and physical energy and laws of resonance). This position explains well why the mind can interpret the physical world since said world is nothing more than a manifestation of it, and the laws that govern it are nothing more than statutes or programs generated by the mind. Individual minds participate in the general mind, but do not know everything about it. That is why they must empirically study matter to discover its laws. The laws of physical causality cannot be obtained by studying logic and mathematics. They are laws programmed by the universal mind and must be found out through empirical research, as empiricists rightly think.

The existence of matter is subject to the existence and action of the universal mind, not to the existence of individual minds. The mentalist position, basically that of G. Berkeley, should not be called subjective idealism, because in it, reality does not depend on any subject, but on the universal mind. Currently, physicists who study quantum mechanics are increasingly favorable to Berkeley's position.

**13. The principle of causality is equivalent to the principle of nonexistence of the actual infinity**

The principle of causality could be stated by saying that every being that begins to exist has a cause of its existence (a reason for being outside of itself). It is considered a self-evident truth (per se notum) within Thomism or as a principle without demonstration in other philosophical systems. Sanvisens (1995) realized that this principle is equivalent to the nonexistence of actual infinity.

To understand this equivalence, we will use an example related to cosmology. Infinite universes are thinkable, different from each other with respect to the laws that govern them (with parameters with continuous variation). If no cause were needed to explain their existence, all of them could appear simultaneously and we would then have a cosmos populated by an actual infinite number of universes.

Since actual infinity does not exist, these universes cannot appear without a cause. The appearance of each universe requires a cause, and that guarantees that there is no cosmos with an actual infinite number of universes. And what we say about the universes we can extend to any beings we can think of.

In no way does quantum physics oppose this principle. Quantum indeterminacy does not refer to the lack of causation, but to the simultaneous nonexistence of complementary magnitudes, such as position and velocity. In quantum physics, observation (whatever it is) becomes a fundamental element to determine (cause) reality. That is why I referred in section 12 to the obvious connection between this science and the mentalist conception of G. Berkeley.

**14. The foundation of mathematics**

Mathematics will not be able to have a good foundation until the system can, by itself, resolve all the paradoxes without the need to clumsily add new axioms. Paradoxes do not only affect philosophy, as Gödel thought (Gödel, 1947/1981, 424-426), referring to those of set theory, but especially the paradoxes of Zeno and the paradox of the Lord of the abscissa affect the center itself of the mathematics of infinity, and therefore they must be resolved in the only possible way: by eliminating the current false concept of actual infinity that contaminates, complicates and ruins the current non-constructivist foundation systems.

As finitists think, potential infinity is enough to account for all of mathematics, because it can go anywhere that the non-existent actual infinity would go. The belief in actual infinity and in the continuum as an actual infinite set of points is an unjustified act of faith and was already duly criticized by Hermann Weyl (1918) and by many other ancient and modern authors. Weyl's work has been updated (Avron, 2020).

In a lecture held in Münster for the German Mathematical Society on June 4, 1925, David Hilbert uttered a famous phrase: «Aus dem paradies, das Cantor uns geschaffen, soll uns niemand vertreiben könen.» ("No one will be able to expel us from the paradise that Cantor has created for us") (Hilbert, 1926; Piñeiro, 2017, 122; Sanvisens, 2021, 19.).

Mathematicians have been on guard against philosophers who want to seek the truth about infinity (Boffi 2021), because they believe that they will impede the progress of their science that has achieved such high levels. And that is because they have not noticed that there are two different mathematics today, since the time of Cantor: a mathematics that encompasses analysis, algebra, topology, probability, etc. and which constitutes what we could call real mathematics, and other mathematics that deals with transfinite and infinitesimal numbers, and which we could call Cantor's paradise, according to Hilbert's expression. This other mathematics has become very complicated, motley, almost terrifying.

But everything that has been done with infinite cardinals in the Cantorian school since Zermelo established in 1930 the cumulative hierarchy of sets in different states indexed by ordinals and supported by Platonism, is disqualified by what we have seen (that there are no infinite cardinals).

Many axioms of infinity have been created, constituting a hell of infinite cardinals, which, in order of strength, can be classified according to Solomon Feferman as: «inaccessible, Mahlo, weakly compact, indescribable, subtle, ineffable, Ramsey, measurable, strong, Woodin, super strong, strongly compact, supercompact, almost huge, huge and super huge.» (Feferman et al., 2000, 11). Even Gödel and Tarski objected to the need to introduce strongly compact cardinals (Feferman, 2000, 10-11). A multitude of complex concepts, such as limit ordinals, club sets, etc. They have come to organize and regulate all this phantasmagoria.

All this mathematics is founded on a false concept (a pseudo-concept), that of actual infinity, and acts, not “as if” it existed, but with its existence as the basis of all operation and all consideration. The building that has been obtained by mathematicians with very privileged minds is a monstrosity of enormous dimensions totally disconnected from the real world. Mathematicians like Stephen G. Simpson, who have been investigating this expected link for decades, have realized that the axioms of the great cardinals are far removed from useful mathematical practice in the world (Simpson, 2009/2014, 9).

Unfortunately, at present real mathematics and the building of Cantor's paradise are enmeshed within the same system (or systems), and a joint effort is needed to unravel this tangle. In fact, the ZFC axiomatic system with an axiom of the existence of a potential infinite set, and with the axiom of potential choice that we have talked about, with a first-order logic, with the support of real intuition, which not even David Hilbert wanted to eliminate completely (Tasic, 2001, 108), and with some definitions that we will see now, it is enough to base all mathematics and account for all the progress achieved in it.

The fundamental concept of mathematics is that of existence, which has three modalities: Something really exists (real existence) when it is formed in some mind, even if only as an algorithm. That which is contradictory (like a round square) cannot be formed in the mind (it only exists in the mind as words that mean nothing). Something potentially exists (possible existence) when it is not contradictory, such as the points of a circle. These points become truly existing when they are conceived one by one by a mind directly or through equations and algorithms. Something has semi-existence (incomplete existence) when only a part is formed in a mind that can grow indefinitely without ever becoming complete. This is the case of infinite sets and non-algorithmic (random) functions.

As a concept of set we can accept the one that Cantor gave in 1895: «We understand by set any grouping into a whole “M” of certain well-differentiated objects “m” of our intuition or our thought (called elements of M)» (Cantor, 1895, section 1). No contradiction will arise with this natural and intuitive concept as long as we consider infinite sets to be semi-existent.

This definition allows us to distinguish set from concept and allows us to understand that numbers are not sets, since they are not groupings, but rather concepts that express a relationship between a multitude and the unit.

The concept of infinity is not ambiguous and foggy, but clear and natural: something is infinite when it cannot be finite (finished, finished, complete) and therefore has a semi-existence.

Although the continuum is not a set of points, but a space whereas many points as we can conceive of real numbers can be indicated, we can work in mathematics “as if” it were a potentially infinite set of points. Thus, all the theorems of current mathematics are preserved intact. The concept of a straight line must also be defined intuitively. I have only found such a definition in (Sanvisens, 2021, 113-117). It may be useful, however, to define it as a space where the points that can be pointed out in it comply with the condition of the equation of the line. The problem with this type of definitions is that they are circular since the measurements of the abscissa and ordinate of the points must be made according to straight lines.

The classical definition of algorithmic functions can be accepted as a correspondence between variables defined by an algorithm that determines a set of ordered pairs. These functions exist; the sets (if they are infinite) have a semi-existence. Non-algorithmic functions can be considered as sets of ordered pairs with no algorithm defining them. That is why they are semi-existent if the sets are infinite.

Mathematical objects are forms of the mind (see section 12) and have been created by minds, and therefore, are finite. The analysis of mathematical objects is objective and discovers hidden truths in the objects created by the mind. The laws of the universal mind in which human minds participate are capable of these discoveries.

This explains why mathematics is effective in real life, in the physical world, as I have also explained. Philosophy is freed from the dark yoke of the actual infinite, with its mysteries and insoluble paradoxes and can access the principle of causality and a mentalist conception of the world (not just mathematics) that includes the concepts of morphogenetic fields (Sheldrake 1981, 1988), that govern the laws of causality, with explanations for cosmic and biological evolution.

**15. Conclusion**

The actual infinity does not exist anywhere; That is why intuition, the instrument to supposedly access the world of ideas, is not capable of capturing it. Gödel's incompleteness theorems distance us from formalism, from the conception of mathematics as the syntax of language, and lead to realism, but not even Gödel was satisfied with Platonic realism. There is no inaccessible metaphysical world of ideas. Aristotelian mathematical realism is not true either. What remains is the psychological one, which has been misinterpreted, as if it were subjective. When a universal mind is conceived as the primordial reality, in which individual minds participate, subjectivity disappears. Minds create concepts, which are forms of the mind, and then discover properties in these forms. It is a constructivist realism close to that of Karl Popper in 1994 (Harada 2005), but immune to sociological criticism because the discovery is made on identical real forms in all minds and by processes that govern the universal mind, the basis of all reality.

There is a strong resemblance between Platonic set theories and religion. Their position towards skeptics is the same, since they avoid any contrast with the facts of reality that could be conflictive (Simpson, 2009/2014, 13). The axioms of infinity are like religious dogmas: you have to believe them, when there are many highly prestigious mathematicians who do not believe in infinite cardinals because they reasonably think that Cantor's theorem is false. They prefer to spend decades studying the false problem of the continuum hypothesis rather than sit down and think about whether there are really uncountable sets.

Saint Augustine says: "Thus, far be it from us to doubt that every number is known to Him whose intelligence, as the Psalm sings, has no number." (Augustine of Hippo, 1964, 693.) In a letter to Ch. Hermite in 1895 Cantor says: «...such numbers (the integers) exist both separately and in their actual infinite totality as divine ideas in intellectu Divino in the highest degree of reality. » (Cabada, 2009, 677).

But the worst of all is the desire to proselytize the new Platonic conjunctist faith. There is an entire world organization for the inculcation of faith in actual infinity in the young, as if this faith were the absolute truth that will save science from the obscurantism of the skeptics. The epistemological and didactic obstacles who oppose faith in actual infinity are investigated. A multitude of articles in all types of prestigious magazines attest to this fact. I cite a few examples: (Dubinsky et al., 2013; Roa Fuentes et al, 2014; Villabona et al., 2022; Arrigo et al, 2004; Tsamir, 2000).

 All the preachers of this faith are convinced that following an infinite succession it is possible to reach the limit by a kind of miraculous leap hidden in the mist. This is the Cantorian faith. This is the faith that must be instilled and made to appear in youth. In the article by Dubinsky et al, for example, they proclaim the dogma of faith that 0.999... is a number that is equal to 1. They have fallen into Cantor's trap. But 0.999... is a condensed way of expressing a succession (something variable, not a number); the succession 0.9, 0.99, 0.999, 0.9999, ... Precisely the number 1 is the limit of this succession; Just as the number 1/3 is the limit of the sequence: 0.3, 0.33, 0.333, …

Saying that 0.999... is a number is a lack of mathematical rigor, because it is a way of saying that by following the path of the succession you can reach the limit; This is false because there is always a difference between the limit and any term of the succession.

They make the same mistake when analyzing Zeno's paradoxes. They say that the limit of the succession is reached by a kind of miracle incomprehensible to the finite human mind. “Only God can understand these things,” they say. But that is false. The human mind can perfectly understand how the limit of a succession is reached by leaving the succession (which is purely mental, not real, because there are no points on the continuum unless a mind stops at them). The Cantorians have fallen squarely into the traps of Zeno, and believe they are in possession of the absolute truth about the infinite, which will always be a religious mystery for them. No. The actual infinity does not exist. There are no infinite cardinals of any kind.

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