Wave Function Collapse, Lorentz Invariance, and the Third Postulate of Relativity

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The changes that quantum states undergo during measurement are both probabilistic and nonlocal. These two characteristics complement one another to insure compatibility with relativity and maintain conservation laws. Nonlocal entanglement relations provide a means to enforce conservation laws in a probabilistic theory, while the probabilistic nature of nonlocal effects prevents the superluminal transmission of information. In order to explain these measurement-induced changes in terms of fundamental physical processes it is necessary to take these two key characteristics into account. One way to do this is to modify the Schrödinger equation by adding stochastic, nonlinear terms. A number of such proposals have been made over the past few decades. A recently proposed equation based on the assumption that wave function collapse is induced by a sequence of correlating interactions of the kind that constitute measurements has been shown to maintain strict adherence to conservation laws in individual instances, and has also eliminated the need to introduce any new, ad hoc physical constants.

In this work it is shown that the stochastic modification to the Schrödinger equation is Lorentz invariant. It is further argued that the additional spacetime structure that it requires provides a way to implement the assumption that spacelike-separated operators (and measurements) commute, and that this assumption of local commutativity should be regarded as a third postulate of relativity.

I. INTRODUCTION

If quantum theory is regarded as an objective description of the physical world then it should be possible, at least in principle, to explain how individual measurement outcomes are generated from fundamental processes. Because quantum states change in a probabilistic and nonlocal manner during measurements it is reasonable to suppose that these features will play key roles in constructing such a fundamental explanation. One major approach to this issue takes these features into account by adding stochastic, nonlinear terms to the Schrödinger equation. These additional terms are designed to induce the wave function to collapse to one of its several branches. The nonlinearity is necessary in order to generate collapse, and the stochasticity is required in order to prevent superluminal signaling, as shown in a work by Gisin[1].

Gisin's work was one of a number of proposed stochastic modifications of the Schrödinger equation aimed at resolving the measurement problem[2–15]. Most of these proposals are designed to collapse the state vector to either an approximate position state or to an energy eigenstate. These attempted solutions have often been met with skepticism because they introduce new, ad hoc physical constants and imply small violations of conservation laws. A recent work has shown how to eliminate these problematic features[16]. It is based on the idea that wave function collapse is induced by the elementary interactions that establish correlations between physical systems, and was motivated by the fact that these correlating interactions play a central role both in the measurement of physical quantities and in the instantiation and transmission of physical information. In this work it will be shown that the proposed modification to the equation is Lorentz invariant.[1]

Since the proposed modification is Lorentz invariant it is reasonable to ask whether it can be considered "genuinely relativistic". In order to answer this question it is necessary to deal with the nonlocal nature of wave function collapse.

Concerns about nonlocality were first raised almost immediately after the Schrödinger equation was proposed (by Einstein at the 1927 Solvay conference; see also [17].) However, they were really sharpened by Bell when he showed that the correlations between entangled systems that are separated by spacelike intervals could not be explained by any account in which all physical processes are restricted to propagate only within the light cone[18]. At least in some sense, measurements do affect spacelike-separated systems.

The nonlocal nature of these effects has been, by far, the biggest challenge to developing a fully satisfactory account of quantum measurement because of the apparent conflict with relativity. This challenge has often seemed insurmountable because of the extreme reluctance to consider the possibility that the nonlocal correlations implied by quantum theory might require that we modify or supplement the metric structure of relativistic spacetime (which is based on classical physics).

The reason that the nonlocal effects do not generate any manifest conflicts with relativity is that they are fundamentally probabilistic. More specifically they obey the Born probability rule[19]. The rule was discovered empirically and was simply tacked on to quantum theory in

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[1] Of course, the nonrelativistic Schrödiger equation is not, itself, invariant. What will be shown here is that the stochastic modification to the equation *is* Lorentz invariant.

an ad hoc manner. There was no serious effort to integrate it into the mathematical structure of the theory; nor was it immediately associated with relativity.

Relativistic quantum field theory deals with the nonlocal correlations in a somewhat more formal manner. It assumes that spacelike-separated operators commute (or anticommute), and this assumption implies the Born rule, thus preserving the Lorentz invariance of the theory. In his text on quantum field theory Weinberg is quite explicit about the fact that this is an *additional* assumption that is necessary to maintain the relativistic character of the theory when he states that the assumption of the commutativity of spacelike-separated operators is made in order to preserve the Lorentz invariance of the scattering matrix[20]. He specifically states that he is not linking local commutativity with the notion of causality. It appears that his motivation for emphasizing this point is that 'causality' is used *both* as a synonym for the no-superluminal-signaling principle, and a shorthand for the idea that no physical processes can propagate faster than light. He seems to rightly regard the conflation of these two concepts as a mistake.

The critical point is that the assumption of local commutativity functions effectively as a third postulate for relativity. Nevertheless, very little consideration has been given to the possible implications this postulate has for spacetime structure. The reasons for the extreme reluctance are fairly obvious. The conventional picture of relativistic spacetime is very elegant, and it beautifully captures our intuitive notion that causal processes propagate through space in a continuous manner. Historically, both special and general relativity preceded the full development of quantum theory, and ideas about spacetime had become pretty firmly fixed by the time the Heisenberg and Schrödinger equations were published[21, 22].

The problem is that this reluctance has left the logical structure of contemporary physics in a very muddled state. The rules governing individual measurement outcomes imply a type of change at odds with the unitary evolution described by the Schrödinger equation, and there is no clear definition of the range of applicability of the two distinct types of change.

The stochastic nonlinear modifications of the Schrödinger equation mentioned above do explain how individual measurement outcomes are selected in accord with the Born rule, and thereby provide a more complete and unified mathematical framework for contemporary physics. Since these modifications require adding structure to relativistic spacetime, let us consider the reasons for such a move.

In the conventional view relativistic spacetime is a four-dimensional manifold with a Lorentzian metric that defines a light cone structure. This framework implements Einstein's two postulates for relativity by prohibiting the assignment of an absolute temporal order to events that are separated by a spacelike interval. Why, then, did the advent of quantum theory necessitate the introduction of a third postulate to maintain this prohi-

bition? Prior to the development of quantum theory the relativistic prohibition on temporal order was strongly associated with the presumption that no physical processes could propagate outside the light cone. But the nonlocal correlations implied by quantum theory strongly suggest that there are physical effects that propagate across spacelike intervals. To "explain" why these effects do not result in any manifest inconsistencies with relativity the new theory simply ruled by fiat that they had to respect the prohibition on temporal ordering. As noted, this was done by requiring that spacelike-separated operators commute. For the reasons mentioned a few paragraphs back there was no inclination to consider the possibility that spacetime possessed additional structure. In fact, the assumption of local commutativity was often conflated with the limitation on the speed of light. The following quote from Gell-Mann, Goldberger, and Thirring illustrates this point[23]:

> "The quantum mechanical formulation of the demand that waves do not propagate faster than the speed of light is, *as is well known*, the condition that the measurement of two observable physical quantities should not interfere if the points of measurement are spacelike to each other...the commutators of two Heisenberg operators... shall vanish if the operators are taken at space-like points." (italics added)

(This quotation was cited by Bell in his discussion of local commutativity[24]).

As suggested by Weinberg's very careful characterization of local commutativity described above, the kind of conflation demonstrated in the quotation is simply wrong. The assumption that spacelike-separated operators commute is made to insure that any effects on physical systems that *do* propagate across spacelike intervals do not transmit any *physical* information. It involves the (at least) implicit recognition that there are nonlocal effects, and that there is a need to regulate them. But, this kind of regulation ought to be explained, and not simply imposed by fiat.

As mentioned above Einstein's postulate about the invariance of the speed of light is implemented by attributing a light cone structure to spacetime. In other words, this postulate is *explained* as a consequence of the fundamental nature of spacetime. Should we not then also consider adding structure to spacetime to explain how nonlocal effects propagate and how they can be regulated to maintain Lorentz invariance? This is exactly what stochastic collapse equations do. By assuming a foliation of spacetime into spacelike surfaces, and invoking a stochastic process (or processes) they provide the desired explanation. By making wave function collapse and the Born rule follow from the fundamental equation of the theory, they provide a coherent logical structure for contemporary physics.

To assess the extent to which these proposals can be

judged to be genuinely relativistic we can start by stipulating that the primary criterion for a relativistic theory is that the laws of physics have the same form in all inertial frames. In particular, this means that they must be Lorentz invariant. Assertions that a proposed account can be judged as "genuinely relativistic" only if it does not add structure beyond the standard relativistic metric cannot be accepted as a priori truths. They face a burden of proof that must take into account the nonlocal aspects of quantum theory. In order to reconcile relativity and quantum theory one must consider the possibility that both these pillars of contemporary physics will have to be modified. The evaluation of any particular proposal will then turn on whether it is Lorentz invariant and whether the Lorentz invariance follows from fundamental physical principles in a natural manner. It will be argued here that the proposal in [16] meets these criteria.

The next section describes the basic assumptions employed by nonlinear, norm-preserving stochastic collapse equations and illustrates how they work. Section III reviews the proposal of [16], which eliminates the need to introduce new ad hoc physical constants and insures that conservation laws are respected *in individual experiments*. It then goes on to demonstrate that it meets a critical test of Lorentz invariance in a very natural way. Section IV addresses the question of what it means for a theory to be "genuinely relativistic."

II. STOCHASTIC COLLAPSE EQUATIONS

This section first provides an explanation of why stochastic collapse equations need to add structures to spacetime, specifically a preferred foliation associated with a stochastic process (or processes). This is followed by an illustration of how these dynamic equations generate collapse in accord with the Born rule. The literature cited earlier contains more general and formal demonstrations of how these equations work; a proof that the equation described in Section III entails collapse in conformity to the Born rule has been presented in [16]. The purpose here is just to provide an intuitive understanding based on a simplified case that captures the essential features. The idea is to show that it is *not* necessary to paste the measurement postulates onto quantum theory in an ad hoc manner, but rather that they follow in a very natural way from a relatively simple modification of the Schrödinger equation that takes into account the fundamentally probabilistic nature of quantum theory as we currently understand it.

A. The Need for Additional Spacetime Structure

Interactions play a critical role in stochastic collapse equations. In the proposal of [16] that will be described in detail in Section III they play the central role because it is assumed that it is interactions that actually induce the collapse. In other proposals they are crucial to establishing the large scale entanglement that allows nonlinear collapse to occur on macroscopic scales while leaving the (almost) linear quantum behavior of microscopic systems essentially undisturbed. Because these equations typically require an extremely large number of elementary systems to become entangled in order to generate collapse it is essential that the entanglement relations are well defined throughout the collapse process. To insure this there must be some means of sequencing the interactions that generate these relations. In typical measurement processes some of these interactions are spacelike-separated. The assumption of a preferred reference frame (or more generally, a foliation of spacetime) provides the necessary sequencing.

In addition to a foliation (or some similar structure) the other critical feature that must be introduced is a stochastic process (or processes). As mentioned earlier any nonlinear and nonlocal modification of the Schrödinger equation must be stochastic in order to prevent superluminal signaling. The stochastic process to be described here is based on the Wiener integral of a white noise Gaussian process. This can be thought of as the continuous time limit of an unbiased random walk with zero mean. As such, it scales with time as $\sqrt{t}$. It is designated as $\xi(t)$, and its differential, which plays a key role in the equations, is designated as $d\xi(t)$. In general, $\xi(t)$ can be complex. The process is governed by the rules of the Itô stochastic calculus[25]: $d\xi^* d\xi = dt, \quad dt d\xi = 0$.

More general stochastic processes can be considered, and a number of proposals employ a stochastic field which is a function of both space and time, $\xi(x,t)$, rather than just time. There is also another form of the stochastic calculus due to Stratonovich. See the references for details. A good general reference is the text by Gardiner.[26]

B. Nonlinear, Norm-Preserving Stochastic Collapse Equations

The overall aim of this work is to show how relativity and the nonlocal aspects of quantum theory can be encompassed in a unified mathematical structure describing the dynamics of physical systems without the need to introduce ad hoc rules that limit the applicability of the mathematics. However, the general form of the stochastic collapse equations outlined here (which includes the proposal described in Section III) is formulated in a nonrelativistic framework. The main reason for this is to avoid mathematical complexity. The assumption of a preferred reference frame that will be used here should be seen as just the simplest special case of a randomly evolving spacelike surface. The restriction to nonrelativistic quantum mechanics makes it possible to illustrate the essential ideas of the proposal in [16] while avoiding many of the complications involved in quantum field theory. It also makes it possible to build on the substantial body of work developed in [1–15].

Measurements of quantum systems can have a very large number of possible outcomes, but at the most basic level they come down to either a detection or a failure to detect. The essentially binary character of measurement processes means that at each stage the Hilbert space (of any number of dimensions) can be decomposed into two orthogonal subspaces, and the state vector of the total system can be represented as the sum of two components, one in each subspace. Although the particular decomposition can vary during the process (thus, allowing any number of possible outcomes) the fundamentally binary nature makes it possible to model measurement processes as a random walk between two alternatives.[2] This is essentially what stochastic collapse equations do.[3]

To see how these equations work we can begin with a simple theorem about the relative probabilities of a random walk ending at either of the two end points. If the walk continues for enough steps it will eventually finish at one or the other of the end points. Label the end points as 0 and 1, and suppose that the walk begins at a point, p, between them. It will be shown that the probability of ending at 1 is p and that the probability of ending at 0 is $1-p$. The step size is labeled δ. It is allowed to vary anywhere between 0 and the distance to the nearest end point: $0 \le \delta \le p, 0 \le \delta \le 1-p$. Label the probability of reaching 1 as $Pr(p)$. Because the walk is assumed to be unbiased we get: $Pr(p) = \frac{1}{2}Pr(p-\delta) + \frac{1}{2}Pr(p+\delta)$. Because this relationship holds for all values of p and δ it is linear. Therefore, the probability of reaching 1 is p, and the probability of reaching 0 is $1-p$. What follows is an attempt to present an intuitive explanation of how nonlinear, norm-preserving stochastic collapse equations map onto this simple, binary picture.

The binary character of measurement processes noted above allows us to illustrate the essential operation of stochastic collapse equations using a simple two state system. Consider a system with a wave function ψ and Hamiltonian, $\mathbf{H}$. The system evolves under the action of the Hamiltonian and of a stochastic collapse operator that is constructed from a self-adjoint operator, $\mathbf{O}$, with two eigenstates, $|x\rangle$ and $|y\rangle$, associated with eigenvalues $\mathbf{a}$ and $\mathbf{b}$. The eigenstates of the operator, $\mathbf{O}$, define the collapse basis. The nonlinear stochastic operator is defined as: $\mathcal{O} \equiv k[\mathbf{O} - \langle\psi|\mathbf{O}|\psi\rangle]$, where $\langle\psi|\mathbf{O}|\psi\rangle$ is the expectation value of the operator, $\mathbf{O}$, in the state, ψ, and k is a constant that helps to determine the strength and

scale of the collapse effects and also insures that $\mathcal{O}$ is dimensionless. For example, if $\mathcal{O}$ is based on the position operator k could determine the range of the collapse effect.

The modified stochastic Schrödinger equation is defined as:

$$d\psi = \frac{-i}{\hbar}\mathbf{H}\,\psi\,dt + \mathcal{O}\,\psi\,\sqrt{\gamma}\,d\xi(t) - \frac{1}{2}\mathcal{O}^2\,\psi\,\gamma\,dt. \quad (1)$$

The first term on the right represents the standard Schrödinger evolution. The primary stochastic action is described by the middle term. The parameter, γ, determines the rate at which the stochastic operator acts. The square root operator is applied to the rate parameter, γ, because it works in conjunction with the stochastic differential, $d\xi$, which scales as $\sqrt{dt}$ (as described above). In the definition of the stochastic operator, $\mathcal{O}$, the subtraction of the expectation value, $\langle\psi|\mathbf{O}|\psi\rangle$, acts, as in the Gram-Schmidt procedure, to insure that the stochastic modification of ψ is orthogonal to the existing wave function. The small orthogonal addition to the wave function slightly alters the norm. This alteration is compensated for by the third term on the right which involves $\mathcal{O}^2$.

The action of the stochastic term is quite small in comparison to that of the Hamiltonian. So, in order for it to be effective in generating collapse it needs to act in a manner that is essentially independent of the Hamiltonian. This can be achieved in several ways, through appropriate choices for the rate parameter, the operator, $\mathbf{O}$, and possibly other parameters.

The stochastic term is designed to drive the system to one of the eigenstates of the operator, $\mathbf{O}$. In this simple example the wave function is represented as: $\psi = \alpha|x\rangle + \beta|y\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. To simplify the example these amplitudes can be taken as real and positive with no loss of generality. The action of the self-adjoint operator, $\mathbf{O}$, on the wave function is: $\mathbf{O}\psi = \mathbf{a}\alpha|x\rangle + \mathbf{b}\beta|y\rangle$, and its expectation value is $\langle\psi|\mathbf{O}|\psi\rangle = \mathbf{a}\alpha^2 + \mathbf{b}\beta^2$. The action of the stochastic operator on the wave function can be expanded as:

$$\begin{aligned} \mathcal{O}\,\psi &= k\{\mathbf{a}\alpha|x\rangle + \mathbf{b}\beta|y\rangle - (\mathbf{a}\alpha^2 + \mathbf{b}\beta^2)\big[\alpha|x\rangle + \beta|y\rangle\big]\} \\ &= k\{\,\alpha\,[\mathbf{a}(1-\alpha^2) - \mathbf{b}\beta^2]|x\rangle + \beta\,[\mathbf{b}(1-\beta^2) - \mathbf{a}\alpha^2]|y\rangle\} \\ &= k\{\alpha\,\beta\,(\mathbf{a}-\mathbf{b})\big[\beta|x\rangle - \alpha|y\rangle\big]\}. \end{aligned}$$
$$(2)$$

So the middle term of 1 can be written as:

$$k\{\alpha\,\beta\,\big[\beta|x\rangle - \alpha|y\rangle\big]\,(\mathbf{a}-\mathbf{b})\sqrt{\gamma}\,d\xi(t)\}. \quad (3)$$

The expression in square brackets can be recognized as a normalized vector that is orthogonal to ψ. As long as the rate parameter, γ, is independent of α and β the only dependence on the amplitudes (aside from the orthonormal state vector) is the term, $\alpha\beta$.

In this form and with the simplifying assumptions described above it is possible to trace the evolution of the wave function through Hilbert space under the influence of the stochastic operator. Since α and β are assumed to be real and positive the evolution can be modeled as

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[2] The claim that measurement processes are essentially binary might seem implausible when one considers many common-place measurements that we make such as length, height, weight, etc. However, that is because we typically describe these kinds of measurements by reference to extremely complex human actions. Imagine eliminating any reference to humans and try to design a robot to carry out the measurement. Eventually one must get back to describing the fundamental physical processes involved.

[3] Wave function collapse can occur either inside or outside a laboratory, but for ease of explanation the discussion here will focus primarily on typical measurement situations.

a random walk along the arc joining x and y axes (corresponding to the eigenstates $|x\rangle$ and $|y\rangle$).[4] The state, $\psi = \alpha|x\rangle + \beta|y\rangle$, lies on the arc; the orthogonal state, $\beta|x\rangle - \alpha|y\rangle$, is tangent to the arc and it drives the state to one or the other of the eigenstates in infinitesimal steps. The magnitude and direction of the steps are determined by the coefficient of the tangent state vector, $\alpha \beta k(\mathbf{a} - \mathbf{b})\sqrt{\gamma}\, d\xi(t)$.

The dependence on α and β means that the step size varies as the wave function, ψ, traverses the arc under the influence of the stochastic operator. Therefore, the calculation of the distance to the end points, $|x\rangle$ and $|y\rangle$, must take this variation into account. This can be done by introducing a parameter, θ, with $\alpha = \cos\theta$ and $\beta = \sin\theta$, and integrating the coefficient of the tangent state vector along the arc:

$$k(\mathbf{a} - \mathbf{b})\sqrt{\gamma}\, d\xi(t) \int \cos\theta \, \sin\theta \, d\theta \;=\; A\sin^2 = A\beta^2.$$
(4)

where A is some term independent of α, β and θ. If we associate the two eigenstates with the end points of a random walk as described earlier, with $|x\rangle$ corresponding to 0 and $|y\rangle$ corresponding to 1, then the position of ψ along the arc can be parameterized as β^2. As shown earlier, this is the probability that the random walk ends at $|y\rangle$, and $\alpha^2 = 1 - \beta^2$ is the probability that it ends at $|x\rangle$. So the Born rule follows from the basic structure of the collapse equation in a straightforward manner. It is also worth noting that the random walk terminates because the term, $\alpha\beta = \cos\theta \sin\theta$ goes to 0 as ψ approaches one of the end points.[5]

I have assumed here that the terms, k and γ, are constant, and, hence, independent of the amplitudes, α and β. This independence from the amplitudes is crucial for the derivation of the Born rule. It is this feature that must be maintained when k and γ are replaced by parameters that vary in a manner that is dependent on the interaction strength. These parameters are discussed in the next section.

As stated at the beginning of this section what has been shown here is that, if we are willing to countenance some additional fundamental structure for relativistic spacetime and incorporate the probabilistic character of quantum theory at the fundamental level it is possible to modify the Schrödinger equation in a fairly simple way so that it yields the measurement postulates as dynamic consequences. In this way contemporary physical theory is rendered much more coherent.

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[4] In this simplified picture with α and β real and positive the process can be pictured on a standard Cartesian graph with x and y axes. It is very straightforward to transfer the analysis to the Bloch sphere afterward.

[5] The fact that $\alpha\beta$ approaches 0 at the end points also creates a problem (the "tails problem") in that the walk does not end in a finite number of steps. This problem will not be dealt with in detail here, but I will offer a speculative solution later.

With this background we can now review the proposed equation described in [16], and see how the stochastic modification maintains Lorentz invariance.

III.   INTERACTION-INDUCED WAVE FUNCTION COLLAPSE

Measurements consist of interactions that establish correlations between physical systems. Correlations are established through the exchange of conserved quantities. Given the probabilistic nature of quantum theory, the generation of stable information and its transmission depend on these correlating interactions. These considerations are what motivated and guided the construction of the equation described below. Roughly speaking, the idea is that the magnitude of the collapse effect associated with an interaction is proportional to the amount of correlation that is generated.

The degree of correlation between two systems that is generated during an interaction depends on the extent to which the interaction changes the individual state of each system. This, in turn, depends on the strength of the interaction and the resistance of each of the systems to a change of state. The strength is measured by the interaction potential energy. This is assumed to depend on the separation between the two systems and to decrease as the separation, $\mathbf{r}$, increases. For systems j and k it will be indicated as $\mathbf{V_{jk}} = \mathbf{V_{jk}(r_{jk})}$. The resistance to change depends on the mass of the systems, m_j and m_k. The effective mass of elementary systems such as electrons in bound states is altered by the binding interactions. So atoms, molecules, and other complex structures are treated as single systems with a total mass and a net charge (or electric multipole moment).

So the collapse operator is based on the interaction potential energies, $\mathbf{V_{jk}}$, and is proportional to the ratio, $\mathbf{V_{jk}}/(m_j + m_k)$. As mentioned in Section II the stochastic operator must be dimensionless. To convert the denominator to an energy it is multiplied by the square of the speed of light, c^2. This is the only nonarbitrary speed, it is crucial for maintaining Lorentz invariance, and it eliminates the need to introduce an arbitrary constant, k. It is also necessary, as in Section II, to subtract the expectation value, $\langle\psi|\mathbf{V_{jk}}|\psi\rangle$. So the component of the collapse operator associated with the interaction between systems j and k is:

$$\mathcal{V}_{jk} \;\equiv\; \frac{\mathbf{V_{jk}} - \langle\psi|\mathbf{V_{jk}}|\psi\rangle}{(m_j + m_k)c^2}.$$
(5)

In most proposed collapse equations the rate or frequency parameter, γ, is a constant. It is chosen in a rather ad hoc manner to minimize deviations from linearity at an elementary level while insuring collapse on a macroscopic scale. In contrast, since it is assumed here that collapse effects are induced by the physical processes that establish correlations between systems it is possible to define this parameter in terms of the rate at which the

correlations are generated. Since correlations are established through the exchange of conserved quantities, and since these exchanges are associated with variations in the interaction potentials, $\mathbf{V_{jk}}$, we can define a rate parameter, γ_{jk}, associated with each interaction in terms of the rate at which the interaction proceeds. As described in Section II the term, $\sqrt{\gamma_{jk}}$, is multiplied by the stochastic differential, $d\xi$.

The rate can be defined so that it integrates to a value of approximately, 1, over the course of the interaction, and goes to zero when the interacting systems either settle into a stationary state or separate to a distance at which the interaction effectively ends. This insures that the stochastic collapse equation reduces to the ordinary Schrödinger equation in these situations. This can be done by taking the ratio of the rate of change of potential energy to the maximum magnitude of potential energy that occurs during the interaction:
(*rate of change of P.E. / max P.E*).

Both the rate of change and the maximum value are affected by the action of the Hamiltonian and the stochastic operator. However, because the stochastic action is unbiased and because the changes that it induces in the numerator and denominator are correlated its effects tend to cancel out. Because collapse equations involve stochastic differentials, $d\xi$, time derivatives are not well defined. But, it is possible to use the Hamiltonian to express its contribution to the rate of change of potential energy in terms of spatial derivatives:

$$\left|\left| (i\hbar) \int \mathbf{V_{jk}} \left(\frac{\psi^* \nabla_{\mathbf{j}}^2 \psi - \psi \nabla_{\mathbf{j}}^2 \psi^*}{2m_j} + \frac{\psi^* \nabla_{\mathbf{k}}^2 \psi - \psi \nabla_{\mathbf{k}}^2 \psi^*}{2m_k} \right) \right|\right|. \tag{6}$$

The norm is taken to insure a positive rate.

To establish that the expression for the denominator in γ_{jk}, $||\langle \psi | \mathbf{V_{jk}} | \psi \rangle||_{max}$, is well defined note that the collapse equation can be integrated both backward and forward in time. Because the Hamiltonian is independent of time it is clear that the associated Scrödinger equation is integrable. The fact that the stochastic operator is unbiased implies that averaging over all of the unravellings of the collapse equation will reproduce the value obtained by the Scrödinger equation integral. Observe also that the stochastic changes in the denominator are matched by those in the numerator; so their overall effect on γ_{jk} is to leave it essentially unchanged. It is, of course, not necessary to actually carry out the integration since γ_{jk} is designed to integrate to a value of order, 1, over the course of the interaction. With this understanding the variable rate parameter is defined as follows:

$$\gamma_{jk} \equiv \frac{||(i\hbar) \int \mathbf{V_{jk}} \left(\frac{\psi^* \nabla_{\mathbf{j}}^2 \psi - \psi \nabla_{\mathbf{j}}^2 \psi^*}{2m_j} + \frac{\psi^* \nabla_{\mathbf{k}}^2 \psi - \psi \nabla_{\mathbf{k}}^2 \psi^*}{2m_k} \right) ||}{||\langle \psi | \mathbf{V_{jk}} | \psi \rangle||_{max}}. \tag{7}$$

An important point to note about 7 is that both numerator and denominator pick out *only the interacting components of the the wave function*. This insures that γ_{jk} is dependent only on the portion of the wave function that is involved in the interaction. This insures that the rate parameter is independent of the amplitudes of the interacting and noninteracting components. This feature is necessary to insure compliance with the Born probability rule as illustrated in the discussion in Section II.

The period over which γ_{jk} is significantly different from zero depends on the initial conditions of the interaction. However, the time during which the vast majority of the momentum and energy are exchanged (and the amplitude is transferred) depends essentially on the maximum interaction energy:

$$dt_{int} \approx \frac{\hbar}{\mathbf{V_{jk-max}}}, \tag{8}$$

For two electrons with a maximum interaction energy equal to the potential at the Bohr radius the duration would be about $2.5 * 10^{-17}$ seconds. Rates and durations for other interactions can be scaled from this estimate, taking into account the mass and charge of the systems involved.

As promised a few paragraphs back this formulation for γ_{jk} implies that when multiplied by dt it integrates to a value of order 1 over the course of the interaction. This is also true for the expression $\sqrt{\gamma_{jk}} d\xi$. This fact allows one to treat each interaction as a discrete event, and makes it possible to estimate the scale on which collapse occurs and the duration of the collapse process. This kind of analysis shows how the scale and duration depend on the average strength of the interactions involved. The estimates of the average interaction rate and duration were given above.

Finally, the full collapse operator is obtained by multiplying the operators by the square root of the rate parameters and summing over the terms for each (j, k) pair:

$$\mathcal{V} \equiv \sum_{j<k} \mathcal{V}_{jk} \sqrt{\gamma_{jk}}. \tag{9}$$

The proposed collapse equation takes the form:

$$d\psi = (-i/\hbar)\mathbf{H}\psi\, dt + \sum_{j<k} \mathcal{V}_{jk}\, \psi \sqrt{\gamma_{jk}} d\xi(t) - \frac{1}{2}\left(\sum_{j<k} \mathcal{V}_{jk}\right)^2 \psi\, \gamma_{jk} dt. \tag{10}$$

A detailed proof that 10 results in collapse with the correct probabilities is given in [16], along with estimates of the scale and duration of collapse processes. As indicated these depend on the average strength of the interactions involved. Such processes can involve anywhere from about 10^8 to 10^{16} elementary interactions. Since many of the interactions can be occurring in parallel the durations are typically very small fractions of a second as can be seen from the estimates of the durations associated with individual interactions given above.

The claim that conservation laws hold *exactly* in individual instances of collapse obviously runs counter to the prevailing presumption that conservation laws hold only on average in quantum theory. However, this presumption is based on an artificial division of the world into classical and quantum systems, and also on an overly

idealized concept of elementary physical systems being in strictly factorizable states. In order to properly assess the status of conservation laws in quantum theory it is necessary to treat all systems, both macroscopic and microscopic, as quantum systems, and also to recognize that all these systems have a history of interaction with other systems (which include preparation apparatuses). As emphasized in [27, 28] interaction generates entanglement. Therefore, the idealization of elementary systems as being in purely factorizable states is never fully realized in practice. This is pointed out by the authors of [27] where they say:

> "Thus it is, strictly speaking, unjustified to describe a particle in a box, which is part of an interacting quantum system, by a wavefunction.

In other words, the common textbook example of a particle in a box ignores the small amount of entanglement that results from the interaction between the particle, the box and whatever apparatus was used to prepare the system. The interactions involved in both the preparation and measurement of quantum systems are conservative interactions. When these interactions induce branching of the wave function they lead to a different distribution of conserved quantities among the interacting systems in the various branches, but they do not alter the total amount (up to normalization of the branches). Based on these kinds of observations a number of articles in recent years have demonstrated that conservation laws hold exactly in individual instances, and not just when applied to ensembles of identical measurement situations. In addition to [16] these include [29–35].

In discussing the status of conservation laws with reference to stochastic collapse equations there are two issues that must be addressed. First, some of the collapse proposals imply small violations of conservation laws even with regard to statistical averages. For example, consider the most widely known collapse proposal, the continuous spontaneous localization (CSL) model[36, 37]. It implies small violations of energy conservation as described in [38]. One of the main reasons for these violations is that this proposal uses a stochastic *field*, $\xi(x,t)$, which is a function of both position and time, as opposed to a single, global stochastic process, $\xi(t)$, which is a function only of time. The stochastic field induces spatial variations in the wave function - in effect, pumping energy into it. Other proposals, such as one of the variants discussed in [1] use a single, global stochastic process, and base the stochastic operator on the Hamiltonian. In this way it maintains energy conservation on average, but still violates it in individual instances.

Because the proposed equation in [16] uses a single global stochastic process it avoids the kinds of statistical violations of conservation laws just described. The reason that it is able to maintain strict conservation in individual measurement situations is that it is based on the assumption that the amplitude shifts between branches that bring about collapse are generated by conservative interactions. So these interactions are responsible both for the splitting of the wave function into branches and for the redistribution of amplitude among the branches. As stated above subsequent to the splitting under ordinary Schrödinger evolution conserved quantities are identical in each branch (up to normalization of the branch). Thus, the amplitude shifts between branches do not alter any of these quantities, and the eventual selection (or elimination) of a particular branch leaves the surviving wave function with the same values that it had prior to the chain of interactions (including preparation) that led to the collapse.

It is shown in [16] that the proposed equation conserves momentum and angular momentum exactly. Because the proposal is formulated in a nonrelativistic framework it is only able to conserve energy within the accuracy allowed by the limited forms of energy describable in nonrelativistic theory.

Experimental consequences of the proposal are also discussed in the earlier work. These deal with very small discrepancies in the correlations between entangled systems predicted by conventional quantum theory and the equation described above. They are a result of the nonlinearity of the equation.

This review of the proposal in [16] is intended as background for the demonstration that the action of the stochastic operator in 10 is Lorentz invariant. The general question of Lorentz invariance hangs over any proposal that deals with the measurement problem. But there is a particular concern that arises in connection with collapse equations of the type discussed here because they are formulated in a preferred reference frame. There is essentially zero probability that any reference frame in which one chooses to analyze a collapse process will coincide with the preferred frame. The critical question is whether the magnitude of the collapse effects is independent of the frame in which they are considered.

The sort of discrepancies in predictions of correlations mentioned two paragraphs back occur in any nonlinear collapse equation. (Other experimental deviations from conventional theory may also occur.) If the predicted magnitude of the collapse effects differs from one frame to the next there will be, in principle, observable differences from the preferred frame and the proposal will be in explicit conflict with relativity. So the task here is to show that the magnitude of the collapse effects predicted by equation 10 is independent of the reference frame in which they are viewed.

According to equation 10 during the brief time in which an interaction generates a correlation between two systems it also transfers amplitude between the interacting and noninteracting branches of the wave function. The magnitude of the transfer is:

$$\mathcal{V}_{jk}\sqrt{\gamma_{jk}}d\xi(t)\,\psi \;\equiv\; \frac{\mathbf{V_{jk}} - \langle\psi|\mathbf{V_{jk}}|\psi\rangle}{(m_j + m_k)c^2}\sqrt{\gamma_{jk}}d\xi(t)\,\psi.$$

(11)

We want to show that this magnitude is the same whether it is calculated in the rest frame of the laboratory in which the interaction takes place or in the preferred frame in which the collapse equation is formulated.

Consider, first, the term, $\sqrt{\gamma_{jk}}d\xi(t)$, that governs the rate at which the transfer takes place. As argued above, this term integrates to a value of order, 1, over the course of the interaction. As viewed from the preferred frame the duration of the interaction is increased by the relativistic time dilation factor, and the rate parameter, γ_{jk}, is reduced by the same proportion. But, from the way in which the rate parameter is constructed it is clear that it will still integrate to the same value over the course of the interaction. So we need only show that the expression, $\frac{\mathbf{V_{jk}}-\langle\psi|\mathbf{V_{jk}}|\psi\rangle}{(m_j+m_k)c^2}$ is invariant.

To demonstrate this it is necessary to show that the relativistic transformation of the numerator is the same as that of the denominator. The numerator is based on the interaction potential energy. The elementary interactions that occur in nonrelativistic quantum theory are electromagnetic. Therefore, the potential energy is an electromagnetic potential. From a relativistic point of view this potential is the timelike component of an electromagnetic four-potential, and transforms accordingly. The denominator consists of the sum of the relativistic energies of the interacting systems. The relativistic energy is the timelike component of the energy-momentum four vector. So the denominator transforms in the same way as the numerator, and, hence, the expression for the collapse term is Lorentz invariant.

In Section II it was shown that stochastic collapse equations provide a very natural extension of the mathematical formalism of quantum theory and that they entail the measurement postulates as straightforward consequences of the fundamental equation. This approach eliminates the need to insert them into the logical structure of the theory at some very vaguely defined point. In this section it was shown that it is possible to formulate such an equation that respects conservation laws in individual measurement processes without introducing any new, ad hoc physical constants, and is also Lorentz invariant. The next section will consider whether this general approach can be considered to respect "serious" Lorentz invariance.

IV. RECONCILING RELATIVITY AND QUANTUM NONLOCALITY

One common attitude is that no proposal that requires adding structure to spacetime beyond a Lorentzian metric can be considered to be genuinely relativistic. It implies that any new physics must work within the limited framework that was erected in a classical context prior to the development of quantum theory.

To evaluate this viewpoint it is necessary to take into account the fundamentally probabilistic and nonlocal nature of quantum theory as we currently understand it.

As argued in Section I these characteristics necessitated the introduction of an additional postulate for theoretical physics - local commutativity. Because the new theory was such a large break from the past, and because many of its consequences only became apparent over time the central role that this additional postulate plays in contemporary physics has not been fully appreciated.

The Lorentzian metric and light cone structure that we use to characterize relativistic spacetime were adopted in order to provide a fundamental physical explanation for Einstein's two original postulates for relativity. The assumption of local commutativity deals with the novel aspects of quantum theory by forbidding the transmission of stable physical information across spacelike intervals. It is intended to prevent any manifest violations of relativity and should be regarded as a third postulate. But, just as the earlier structures were designed to *explain* how the original postulates are enforced we should be willing to entertain the possibility of adding structure to explain how this third postulate is enforced. That is exactly what a spacetime foliation associated with a stochastic process does.

The adoption of this sort of additional stochastic structure transfers the probabilistic nature of quantum theory from the macroscopic to the elementary level. This turns what were ad hoc postulates at the macro level into straightforward consequences of the mathematical description of elementary processes. The probabilistic nature of elementary correlating interactions also explains why a preferred reference frame (or spacetime foliation) remains hidden, and explains why the description of spacetime at the macroscopic level is limited to the standard relativistic account involving just a Lorentzian metric. This is why accounts involving these sorts of additional stochastic structures ought to be regarded as genuinely relativistic.

Because the proposal described here is formulated in a nonrelativistic framework it must be considered as incomplete. But since it can, arguably, be regarded as Lorentz invariant there do not appear to be any serious conceptual obstacles to a full relativistic account. Such an account would encompass quantum field theory, and might well offer solutions to some of the key problems with this more limited proposal. As argued in [16] there is reason to believe that such an extension could resolve the small discrepancies with energy conservation that are implied by equation 10. One might also hope that a relativistic account might provide a solution to the tails problem (mentioned in a footnote in section II). With the transition from distinguishable to indistinguishable particles the suppression of the very small amplitudes in the wave function tail below the level of vacuum fluctuations might point the way to a resolution. But these are just speculations that must be addressed in future work.

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