Quantum Theory from Probability Conservation

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We present a first-principles derivation of the standard formalism of quantum theory through an information-theoretic analysis of single-variable physical systems –systems characterized by only one independent variable. These systems, with a minimal information capacity of exactly one bit, exhibit inherently indeterministic behavior under independent measurements while permitting probabilistic descriptions for dependent measurements. By enforcing probability conservation in the transformations of outcome probabilities across different measurement scenarios, we derive the core components of standard quantum theory, including the Born rule, the Hilbert space structure, and the Schrödinger equation. Furthermore, we demonstrate that the experimental requirements for observing quantum phenomena –specifically, preparing physical systems in coherent states under strict conditions, such as ultralow temperatures or high fields– effectively constrain the number of independent variables to one, thereby enforcing single-variable behavior. This first-principles, information-theoretic derivation establishes that quantum theory fundamentally describes the physics of single-variable systems and provides a concrete realization of Wheeler's "it from bit" idea.

I. INTRODUCTION.

Quantum mechanics constitutes a cornerstone of modern physics, providing fundamental insights into the behavior of particles at atomic and subatomic scales. It has been remarkably successful in explaining phenomena such as atomic structure, quantum tunneling, and electron behavior in solids. Despite its overwhelming empirical success, the foundational principles of quantum theory –particularly the derivation of its core equations from first physical principles– remain a subject of ongoing investigation. Traditional approaches to quantum theory often rely on abstract, axiomatic postulates [1-8], leaving the physical basis for its mathematical formalism unresolved.

In this paper, building on information-theoretic interpretations of quantum phenomena [9-14], we address this gap by deriving quantum theory through the study of physical systems with exactly one independent variable. Due to their minimal information capacity, such systems inherently exhibit indeterministic behavior upon measurements, necessitating a probabilistic description. By enforcing the principle of probability conservation in the transformations of outcome probabilities across different measurement scenarios, we derive the complete mathematical framework of standard quantum theory, including the Born rule, Hilbert space structure, and unitary dynamics. Furthermore, we demonstrate that the coherence requirements for conducting quantum experiments impose constraints on physical systems, effectively reducing their number of independent variables and confining their dynamics to a single variable in practice.

By completing this first-principles, information-theoretic derivation of quantum theory, we conclude that quantum mechanics fundamentally describes the physics of systems with a single independent variable. We subsequently apply this theoretical framework to re-examine foundational quantum concepts, offering novel insights into the meaning of coherence, the quantum-classical boundary, the epistemic nature of the state function, and the information-theoretic structure of entanglement. Beyond its theoretical significance, this work has the potential to drive future advancements in both quantum technologies and fundamental physics.

II. METHODS

A. Informational description of physical systems

Translating concepts from physics into the language of information theory is straightforward. A physical measurement M^K with N possible outcomes $\{m_1^K, \dots, m_N^K\}$ and their corresponding probabilities $\{p_1^K, \dots, p_N^K\}$ (where $\sum_{i=1}^{N} p_i^K = 1$) can be modeled as an informational query Q^{K}, asking, "What is the outcome of measurement M^K?", with answers $\{q_1^K, \dots, q_N^K\}$ obeying the same probabilities. The relationships between dependent and independent measurements mirror those between correlated and uncorrelated questions. These parallels enable the application of tools from information theory –such as Shannon entropy, information content, and information gain– to analyze physical systems.

Physical systems inherently contain and convey information. Each independent variable within a physical system can be viewed as a discrete communication channel carrying a single piece of information (message).

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Accordingly, the number of independent messages a physical system can convey is equal to the number of its independent variables. The length of each message is determined by the number of possible configurations of the variable. For instance, a variable with eight possible states can encode a three-bit message.

B. Single-variable physical systems

Microscopic systems often involve only a small number of independent variables. We define single-variable systems as those characterized by exactly one independent variable, constraining their information capacity to a single message. This property distinguishes them from classical systems, which can simultaneously convey multiple independent messages.

A significant consequence arises when attempting to extract two independent pieces of information from a single-variable system: the second measurement inevitably yields zero information content. From an information-theoretic perspective, zero information corresponds to randomness [14, 15]. Translated into the physics domain, this implies that performing independent measurements on a single-variable system will always yield random outcomes. This observed randomness is a direct consequence of fundamental information capacity limitations rather than any underlying hidden variables.

While independent measurements on single-variable systems yield random outcomes, dependent measurements permit probabilistic analysis of their behavior. In dependent measurements, the probability of obtaining a specific outcome in one measurement correlates with the outcome of the other. These probabilistic correlations are quantified by conditional probabilities, where $p(m_j^L|m_i^K)$ denotes the probability of obtaining outcome m_j^L in measurement L given outcome m_i^K in measurement K. The law of total probability ensures that for a given outcome in the first measurement, the conditional probabilities sum to unity, formally expressed as

$$\sum_{j=1}^{N} p(m_j^L|m_i^K) = 1, \tag{1}$$

for any fixed i.

C. Probability calculations

Given the probability state of the system for measurement K, denoted as $\boldsymbol{P}^K = [p_1^K, \dots, p_N^K]^T$, the probabilities for the outcomes of measurement L can be calculated using their conditional probabilities as

$$p_j^L = \sum_{i=1}^{N} p(m_j^L|m_i^K) p_i^K. \tag{2}$$

The mapping $P_{ji}^{LK} = p(m_j^L|m_i^K)$ preserves total probability:

$$\sum_j p_j^L = \sum_j \sum_i P_{ji}^{LK} p_i^K = \sum_i p_i^K \sum_j P_{ji}^{LK} = \sum_i p_i^K \sum_j p(m_j^L|m_i^K) = \sum_i p_i^K = 1, \tag{3}$$

given the law of total probability (1). Conversely, mapping the probabilities from measurement L to measurement K involves $P_{ij}^{KL} = p(m_i^K|m_j^L)$, and the probabilities can be calculated as

$$p_j^K = \sum_i P_{ji}^{KL} p_i^L. \tag{4}$$

While these mappings conserve total probability, they are not bidirectionally reversible. Reversibility would require

$$p_j^K = \sum_i P_{ji}^{KL} p_i^L = \sum_i P_{ji}^{KL} \sum_n P_{in}^{LK} p_n^K = \sum_n \sum_i P_{ji}^{KL} P_{in}^{LK} p_n^K, \quad \forall j, n. \tag{5}$$

Since this equation must hold for all j and n, the linear independence of the probability vectors implies:

$$\sum_i P_{ji}^{KL} P_{in}^{LK} = \delta_{jn}, \tag{6}$$

where δ_{jn} is the Kronecker delta. This condition is generally not satisfied, because (6) requires that the product of strictly positive values sum to zero when $j \neq n$, which is mathematically impossible. The positivity constraint on the mapping elements fundamentally prevents bidirectional reversibility in direct probability mappings.

To resolve this limitation and establish consistent, reversible transformations between probability spaces, we introduce probability amplitudes σ_i^K related to probabilities through a function f such that $\boldsymbol{\sigma}^K = f(\boldsymbol{P}^K)$. These probability amplitudes transform according to

$$\sigma_j^L = \sum_{i=1}^{N} \rho_{ji}^{LK} \sigma_i^K, \tag{7}$$

(and likewise for K and L swapped), where the revised mappings ρ_{ji}^{LK} must permit non-positive elements. Consequently, we require a probability measure that can accommodate non-positive values while preserving fundamental properties of probability.

This probability measure must satisfy several mathematical constraints: it must preserve probabilities at 0 and 1, i.e., $P(0) = 0$, and $P(1) = 1$; it must be a monotonic function with $P(x) \in [0,1]$ for all permissible inputs; and finally, it must be scale-invariant, i.e., $P(\lambda x) = \lambda^\alpha P(x)$. These criteria uniquely determine the form of the probability measure as

$$P(x) = |x|^\alpha, \text{where } \alpha \in \mathbb{R}^+, \tag{8}$$

which is defined by the α-th power of the absolute values of the probability amplitudes associated with events. This power function represents the general class of probability measures that satisfy these constraints while allowing non-positive elements in transformations. Thus, the revised mappings take the form

$$\sigma_j^L = \sum_i \rho_{ji}^{LK} \sigma_i^K, \tag{9}$$

where

$$\left|\rho_{ji}^{LK}\right|^{\alpha} = p\left(m_j^L \middle| m_i^K\right), \quad \text{and} \quad |\sigma_i^K|^{\alpha} = p_i^K. \tag{10}$$

III. RESULTS

A. The quadratic probability measure

We now determine the exponent α that enables bidirectional reversible mappings, satisfying the condition:

$$\sum_n \rho_{an}^{LK}\rho_{nb}^{KL} = \sum_i \rho_{ai}^{KL}\rho_{ib}^{LK} = \delta_{ab}, \tag{11}$$

while conserving total probability. From the law of total probability, we have $\sum_n P\left(m_n^{\mathrm{K}} \middle| m_i^{\mathrm{L}}\right) = 1$, which can be equivalently written as

$$\sum_n |\rho_{ni}^{KL}|^{\alpha} = 1. \tag{12}$$

Comparing this with the (i,i) element of $\boldsymbol{\rho}^{LK}\boldsymbol{\rho}^{KL}$ from (11) yields:

$$\sum_n |\rho_{ni}^{KL}|^{\alpha} = \sum_n \rho_{in}^{LK}\rho_{ni}^{KL}, \forall n, i. \tag{13}$$

Since this equation must hold for any n and i, it must hold term-by-term. Given that our formulation allows for complex-valued elements, we express

$$\rho_{ni}^{KL} = |\rho_{ni}^{KL}|e^{i\phi_{ni}}, \tag{14}$$

where $e^{i\phi_{ni}}$ is a phase factor satisfying $\left|e^{i\phi_{ni}}\right| = 1$. Substituting this into (13) results in the term-wise constraint:

$$\rho_{in}^{LK} = e^{-i\phi_{ni}}|\rho_{ni}^{KL}|^{\alpha-1}. \tag{15}$$

Combining (15) with the law of total probability for $\boldsymbol{\rho}^{LK}$, i.e.,

$$\sum_i |\rho_{in}^{LK}|^{\alpha} = \sum_i P\left(m_i^{\mathrm{L}} \middle| m_n^K\right) = 1, \tag{16}$$

leads to:

$$1 = \sum_i |\rho_{in}^{LK}|^{\alpha} = \sum_i \left|e^{-i\phi_{ni}}|\rho_{ni}^{KL}|^{\alpha-1}\right|^{\alpha} = \sum_i |\rho_{ni}^{KL}|^{\alpha^2-\alpha}. \tag{17}$$

Using (15) along with the (n,n) element of $\boldsymbol{\rho}^{KL}\boldsymbol{\rho}^{LK}$ from (11), we obtain

$$1 = \sum_i \rho_{ni}^{KL}\rho_{in}^{LK} = \sum_i \rho_{ni}^{KL}e^{-i\phi_{ni}}|\rho_{ni}^{KL}|^{\alpha-1} = \sum_i |\rho_{ni}^{KL}|^{\alpha}. \tag{18}$$

Comparing (17) and (18), we obtain $\alpha^2 - \alpha = \alpha$, with the unique nonzero solution:

$$\alpha = 2. \tag{19}$$

Substituting this value into (8), we arrive at the quadratic probability measure:

$$P(x) = |x|^2, \tag{20}$$

which is based on the square of probability amplitudes.

Unitarity of the mappings

Moreover, substituting (19) into (12) and (18) shows that

$$\sum_n |\rho_{ni}^{KL}|^2 = 1, \quad \text{and} \quad \sum_i |\rho_{ni}^{KL}|^2 = 1, \tag{21}$$

indicating that the sum of the squares of the mapping elements in each row and each column equals one. This property reveals that the transformation matrices, $\boldsymbol{\rho}^{LK}$ and $\boldsymbol{\rho}^{KL}$ are unitary.

Mathematically, our goal was to identify a diffeomorphic transformation between probability spaces associated with different measurements**.** We found that probability conservation in bidirectional transformations uniquely leads to the quadratic probability measure as the only solution that ensures the preservation of total probability in both forward and reverse mappings.

This analysis reveals that the fundamental object undergoing transformation is the "probability amplitude state" of the system, defined by

$$\boldsymbol{\sigma}^K = [\sigma_1^K, \dots, \sigma_N^K]^T, \tag{22}$$

which satisfies:

$$|\sigma_i^K|^2 = p_i^K. \tag{23}$$

Rather than the "probability state" $\boldsymbol{P}^K$, it is the probability amplitude state that transforms consistently between measurement probability spaces. These transformations are governed by the unitary mappings:

$$\boldsymbol{\sigma}^L = \boldsymbol{\rho}^{LK}\boldsymbol{\sigma}^K, \tag{24}$$

where the mapping elements are related to the conditional probabilities of measurement outcomes through

$$\left|\rho_{ij}^{LK}\right|^2 = p\left(m_i^L \middle| m_j^K\right). \tag{25}$$

These unitary transformations ensure the preservation of total probability across different measurement scenarios by maintaining the norm of the probability amplitude vectors:

$$\sum_{i=1}^N |\sigma_i^L|^2 = \sum_{i=1}^N |\sigma_i^K|^2 = \sum_{i=1}^N p_i^K = 1. \tag{26}$$

B. The Hilbert space formalism

The algebraic structure derived above forms the foundation of the Hilbert space, where the systems are represented by N-

dimensional complex vector spaces associated with measurements. The unitary transformations and quadratic probability measure –rigorously derived from first principles in our framework– are naturally embedded within this Hilbert space structure. Specifically, the probability amplitude state $\boldsymbol{\sigma}^K$ resides in a linear vector space spanned by the independent outcomes of a specific measurement, ranging from m_1^K to m_N^K. These states transform between different probability spaces via the unitary mappings given in (24).

In this representation, the unitary transformations of the system's probability amplitude state correspond to rotations of its state vector in the Hilbert space. Using conventional bra-ket notation, the state of the system in the derived algebraic structure can be written as an N-dimensional vector:

$$|\sigma^K\rangle = \sum_{i=1}^{N} \sigma_i^K |m_i^K\rangle, \tag{27}$$

with the norm:

$$\langle\sigma^K|\sigma^K\rangle = \sum_{i=1}^{N} |\sigma_i^K|^2 = 1, \tag{28}$$

defined through the quadratic probability measure. The evolution of these states is described by unitary transformations that map them between probability spaces of different measurements, according to:

$$|\sigma^L\rangle = \boldsymbol{\rho}^{\mathrm{LK}} |\sigma^K\rangle. \tag{29}$$

It is mathematically significant that the fundamental principle of probability conservation in single-variable physical systems naturally gives rise to the complete Hilbert space structure of quantum theory, including its key features such as linearity, inner product preservation, and unitary evolution. This result demonstrates that mathematical formalism of quantum mechanics emerges directly from the information-theoretic constraints imposed on single-variable systems.

C. The Born probability rule

The Born rule for calculating probabilities is naturally embedded in our framework through the quadratic probability measure. The probability of a specific measurement outcome is given by the square of the inner product between the system's current state and the outcome state:

$$p_{ab}^{LK} = |\rho_{ab}^{LK}|^2 = |\langle\sigma_b^K|\sigma_a^L\rangle|^2. \tag{30}$$

Our derivation reveals that the Born rule is not merely an axiom but rather a conservation law, arising directly from probability conservation in bidirectional transformations between different measurements. This provides a principled explanation for one of the most fundamental postulates of quantum mechanics, grounding it in first principles rather than assuming it axiomatically.

D. The Schrödinger equation

Time evolution can be incorporated into our framework by associating the unitary transformations with a time parameter. Specifically, we introduce time dependence into the unitary mappings in (24) by defining them as time-dependent operators:

$$\boldsymbol{\sigma}^L = \boldsymbol{\rho}^{LK}(t)\boldsymbol{\sigma}^K. \tag{31}$$

Recall that these mappings are functions of the correlations between two measurements K and L. Since the correlations between the measurements are time-independent, the time evolution of $\boldsymbol{\rho}^{LK}(t)$ over an interval $\Delta t = t_2 - t_1$ is governed by a unitary transformation $\mathbf{U}(\Delta t)$ according to:

$$\boldsymbol{\rho}^{LK}(t_2) = \mathbf{U}^{-1}(\Delta\mathrm{t})\boldsymbol{\rho}^{LK}(t_1)\mathbf{U}(\Delta t). \tag{32}$$

Under the standard requirements for a time-translation operator –i.e., continuity ($\lim_{\Delta\mathrm{t}\to 0} \mathbf{U}(\Delta\mathrm{t}) = \mathbf{1}$) and the semigroup property ($\mathbf{U}(\mathrm{t}_1)\mathbf{U}(\mathrm{t}_2) = \mathbf{U}(\mathrm{t}_1 + \mathrm{t}_2)$)– this transformation takes the form:

$$\mathbf{U}(\Delta t) = e^{-i\boldsymbol{H}(\Delta t)}, \tag{33}$$

where $\boldsymbol{H}$ is a self-adjoint operator (the Hamiltonian) governing the system's dynamics. Shifting the focus to the time evolution of the state itself, $\boldsymbol{\sigma}$, rather than the transformation, $\boldsymbol{\rho}$ (i.e., switching from the Heisenberg picture to the Schrödinger picture), we derive the time evolution of the probability amplitude state $\boldsymbol{\sigma}^{\mathrm{K}}(t)$:

$$\frac{d}{dt}\boldsymbol{\sigma}^{\mathrm{K}}(t) = -i\boldsymbol{H}\boldsymbol{\sigma}^{\mathrm{K}}(t). \tag{34}$$

Equation (34) is the Schrödinger equation in our formalism, demonstrating that the time evolution of the state of single-variable systems is governed by unitary dynamics under the Hamiltonian $\boldsymbol{H}$.

IV. DISCUSSION

In the above, we have developed a mathematical framework to describe the behavior of single-variable physical systems with a finite number of states. This framework can be systematically extended to the infinite-dimensional case using standard techniques from functional analysis. Remarkably, our framework directly yields the three fundamental elements of standard quantum theory –namely, the Hilbert space structure with its operator algebra, the Born probability rule, and the Schrödinger equation– through an analysis of information-theoretic constraints imposed on single-variable systems.

By identifying the minimal physical principles required to establish the mathematical foundation of quantum theory, we reveal that the principle of probability conservation in single-variable physical systems serves as the cornerstone of

quantum mechanics. Although we have now derived the complete mathematical structure of quantum theory, we must still address how this formalism –derived specifically for single-variable systems– corresponds to and explains quantum phenomena observed in practice, and whether physical quantum systems indeed satisfy this single-variable criterion.

Quantum mechanics (QM) is traditionally regarded as the framework for describing microscopic systems and elementary particles. This perspective, however, has been contested, as the formalism of QM does not inherently impose a scale limit, and there appears to be no clear boundary between classical and quantum domains, often referred to as the quantum-classical boundary or the *Heisenberg cut*. While Schrödinger's cat thought experiment [16] was originally conceived to highlight the paradoxical consequences of applying QM to macroscopic objects, numerous experiments have since expanded the scale at which quantum effects can be observed. These include interference experiments with progressively larger macromolecules [17-21] and Bose-Einstein condensates comprising millions of atoms. Despite these advances, the question of whether QM can serve as a truly universal theory remains unresolved in foundational physics.

In the preceding section, we derived quantum theory as the fundamental framework for physical systems with a single independent variable. Our findings not only establish quantum theory from first principles but also offer a fundamentally new perspective on the nature of quantum systems. Specifically, we propose that quantum systems can be fundamentally characterized as physical systems with no more than one independent variable. This perspective suggests that the Heisenberg cut is not dictated by physical scale but rather by the number of independent variables in a system. In the following sections, we demonstrate how this interpretation aligns with the physical realization of quantum experiments and precisely delineates the scope of quantum theory's applicability.

A. Coherence

As experimental physicists recognize, the success of quantum experiments relies on the ability to prepare and maintain systems in coherent states, typically achieved under stringent laboratory conditions, such as extremely low temperatures, strong fields, or high vacuum environments. In these conditions, many of the system's variables are effectively suppressed, reducing the number of independent variables. We assert that strong coherence –essential for observing quantum phenomena– enforces a single independent variable in the system, which becomes the sole degree of freedom accessible to measurement.

To illustrate this, consider electron guns and lasers, two essential tools in quantum experiments for generating coherent electron and photon beams, respectively. An electron gun (Fig.1) extracts electrons from a heated filament via thermionic emission. These electrons are accelerated by an anode and focused through a collimator, directing them into a uniform beam. This design ensures that the electrons possess nearly identical energy and momentum distributions, leaving spin as their only effectively independent variable. Similarly, a coherent beam of light consists of photons with well-defined energy and propagation direction, leaving polarization as the sole remaining independent variable. While laser pointers have made the demonstration of interference phenomena widely accessible, it is crucial to note that ordinary incoherent light sources fail to produce interference patterns.

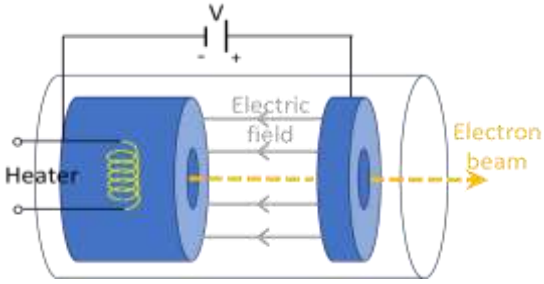

FIG 1. **Schematic of an electron gun structure, a device that produces single-variable systems (not to scale):** *Electrons are produced through thermionic emission from a heated filament at the cathode. Attracted to the anode, they travel in parallel trajectories through a collimator. The applied voltage accelerates the electrons to high velocities. The resulting electron beam consists of electrons with nearly identical energy and momentum distributions, leaving spin as their only independent variable. This device effectively generates a stream of single-variable particles suitable for quantum experiments.*

Achieving coherence involves constraining the values of a system's variables, a task that becomes increasingly difficult as the size of the system –and concurrently, the number of variables– increases. Figure 2 illustrates the patterns observed in interference experiments with particle beams exhibiting varying degrees of coherence. The appearance of interference fringes is directly linked to the presence of coherence within the particle ensemble. While coherence can be easily achieved in microscopic systems, macroscopic systems, such as golf balls, cannot be prepared in coherent states, and their patterns exhibit classical distributions, lacking the characteristic undulations in intensity that signify quantum behavior.

At intermediate scales, full coherence remains elusive, although partial coherence is achievable. This results in hybrid interference patterns (Fig.2, bottom), where interference fringes are superimposed on a classical pattern, indicating the simultaneous presence of quantum effects alongside the classical expectations. Unlike in fully coherent cases, interference fringe minima in partially coherent cases do not reach zero, as observed in experiments with macromolecules. The results, however, can be rescaled or framed (see, for example, Refs. [21-23]) to closely resemble

full coherence, potentially creating the impression of pure quantum behavior even in macroscopic systems.

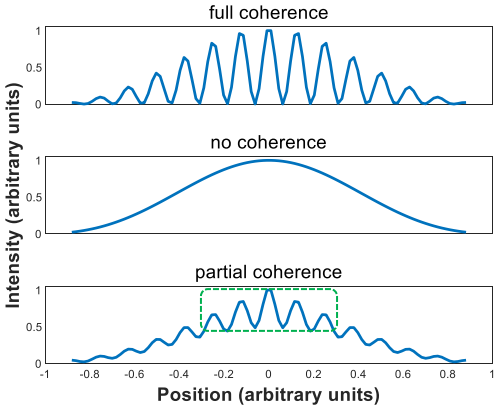

FIG 2. **Effect of coherence on the intensity of interference fringes:** *This illustration shows the resulting patterns observed in interference experiments performed with particle beams of different degrees of coherence. Top: full coherence (quantum behavior). Middle: no coherence (classical behavior). Bottom: partial coherence (hybrid behavior) where interference fringes remain visible but less pronounced. Notably, in partially coherent cases, the fringe minima do not reach zero intensity, as seen in interference experiments with macromolecules. However, selective data framing (dashed line inset) can make these patterns resemble those of fully coherent systems.*

B. The Heisenberg cut

Our results offer new insights into the Heisenberg cut, suggesting that the boundary where quantum theory ceases to apply is not determined by the size of a system, but by the number of its independent variables. Systems with many independent variables, such as cats or measuring devices, cannot be characterized as single-variable systems and thus fall outside the formalism of quantum theory. On the other hand, quantum theory can be applied on large scales in degenerate stars [24, 25], which exist in coherent states under extreme gravitational fields. Systems with partial coherence exhibit a blend of quantum and classical behavior, with quantum effects manifesting according to the degree of coherence.

This perspective resolves the traditional difficulty in locating the quantum-classical boundary and the apparent arbitrariness of the Heisenberg cut in conventional interpretations –what Bell called the "shifty split" [26]. Rather than representing a fundamental size-based division in nature that distinguishes 'microscopic' from 'macroscopic', or a subjective choice made by the observer [27], the quantum-classical boundary emerges from the informational structure of physical systems. As our framework indicates, measuring devices are not single-variable systems, and coupling a single-variable system to them effectively transitions the latter into the classical realm. Furthermore, in experiments involving ensembles of identically prepared systems, as the number of independent variables increases, the ability to induce and maintain efficient coherence becomes progressively more difficult, leading to a transition toward predominantly classical behavior. This perspective implies that in such experiments, the quantum-to-classical transition occurs gradually as the ensemble evolves from predominantly single-variable behavior to multi-variable dynamics, rather than at an arbitrary division point. This explains the traditional difficulty in pinpointing this boundary [28]: coherence exists on a continuum rather than as a binary property.

C. The state function

Our derivation also sheds light on the nature of the quantum state function, which we identify as the probability amplitudes of single-variable systems corresponding to different measurement outcomes. Essentially, the state function represents the system's propensities for specific measurement outcomes. These propensities are inherently relational, depending on both the system's current state and the type of measurement being performed. While they are objective, the state function does not correspond to any intrinsic physical attribute of the system itself but represents knowledge about correlations between measurements. Borrowing from philosophical terminology [29], quantum state functions are best characterized as epistemic realities –representations of knowledge about measurement correlations– rather than as ontological objective realities.

That being said, one must still address where in the physical world these propensities reside. Our derivation suggests that these propensities are mathematically embedded in the transformations $\boldsymbol{\rho}^{II,I}$ that map the state of the system to the planned measurement. These transformations, in turn, reflect the correlations between two measurements –one that defines the state of the system and the other that constitutes the planned measurement. As our derivation demonstrates, the transformations that map the state function between different measurements involve conditional probability amplitudes of the measurement outcomes. Therefore, the propensities are not contained within the quantum system itself but are encoded in the correlations between measurements.

D. Quantum entanglement

An intriguing implication of our findings relates to quantum entanglement. Single-variable physical systems can, in principle, be multipartite and physically extended, with parts separated by large distances. This is precisely the case for entangled quantum systems, where the entire system behaves as a single-variable entity despite its multipartite nature. Consider, for example, the generalized Greenberger-Horne-Zeilinger (GHZ) state [30], which is a superposition of n subsystems, all in the state $|\uparrow\rangle$ with all in the state $|\downarrow\rangle$, represented as

$$|\mathrm{GHZ}>=\frac{1}{\sqrt{2}}\left(|\uparrow\rangle^{\otimes n}+|\downarrow\rangle^{\otimes n}\right). \qquad (35)$$

Although a collection of n single-variable subsystems might initially suggest n independent variables, the $n-1$ correlations between the subsystems in the GHZ state (i.e., $|s_1\rangle = |s_2\rangle$, $|s_2\rangle = |s_3\rangle$, …, and $|s_{n-1}\rangle = |s_n\rangle$, where $|s_i\rangle$ represents the state of a subsystem) reduce the number of independent variables in the system to just one. Thus, the entire n-component entangled system effectively comprises a single variable.

The unified description of quantum systems –single-variable systems constrained to convey only one piece of information– offers a novel interpretation of quantum entanglement. Rather than viewing entanglement as "spooky action at a distance," the fundamental information-theoretic constraint manifests in entangled systems by restricting them to convey only one piece of information, regardless of their physical separation. This confinement of shared information capacity enforces perfect correlations between subsystems as the entire system can convey only one piece of information.

The GHZ state exemplifies this principle perfectly, as it represents a maximally entangled state where the measurement of any single qubit immediately determines the state of all others. Viewed through our single-variable framework, this behavior is not mysterious but rather a natural consequence of the fundamental limitation on the system's information capacity. This perspective provides a conceptually clear resolution to the presumed 'nonlocality' of quantum mechanics by recognizing that entangled systems –despite being spatially distributed– are fundamentally single-variable systems whose subsystems exhibit strong informational correlations.

V. CONCLUSIONS

In this work, we have presented a rigorous, step-by-step derivation of the mathematical formalism of quantum theory by analyzing the properties of physical systems with minimal information capacity –specifically, those with only one independent variable. The constraints imposed by probability conservation in these systems, which are fundamentally probabilistic in nature, necessarily lead to the quadratic probability measure, which in turn yields the complete formalism of quantum theory. Furthermore, we have demonstrated how the practical requirements for performing quantum experiments –specifically, preparing physical systems in coherent states– effectively transform these systems into single-variable systems. Thus, we conclude that quantum theory fundamentally describes the physics of single-variable systems.

This approach represents a significant departure from traditional axiomatic formulations, offering instead a first-principles derivation grounded in information-theoretic principles applied to physical systems. Rather than postulating the mathematical structure of quantum mechanics, we demonstrate how it emerges naturally from the analysis of systems with inherently limited information capacity. By deriving quantum theory from first principles and clearly specifying its domain of applicability, we provide new insights into key concepts such as coherence, the quantum-classical boundary, and entanglement. Our framework reveals that the Heisenberg cut is determined not by a system's size but by the number of independent variables it possesses, while quantum entanglement arises as a direct consequence of the informational constraints imposed on spatially extended single-variable systems. These findings support Wheeler's "it from bit" idea, suggesting that physical reality emerges from information-theoretic constraints rather than the reverse. This perspective also offers a unified lens through which other well-established quantum concepts can be reinterpreted.

The implications of our derivation extend beyond theoretical insights. By establishing that quantum behavior is fundamentally linked to information capacity rather than physical scale, we provide clear criteria for identifying systems that are expected to exhibit quantum effects. This perspective resolves longstanding paradoxes in quantum foundations while suggesting practical guidelines for experimental design and technological implementation. It not only clarifies existing quantum phenomena such as the mechanisms underlying coherence and entanglement but also suggests novel approaches to quantum technology development, particularly in identifying candidate systems for quantum information processing and communication –with a focus on the informational capacity of physical systems. Ultimately, the cohesive and consistent framework for understanding quantum physics presented here has the potential to reshape and enrich our comprehension of the fundamental physical principles that govern the universe.

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