Possibility in Physics

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Abstract

Physics not only describes past, present, and future events but also accounts for unrealized possibilities. These possibilities are represented through the solution spaces given by theories. These spaces are typically classified into two categories: kinematical and dynamical. The distinction raises important questions about the nature of physical possibility. How should we interpret the difference between kinematical and dynamical models? Do dynamical solutions represent genuine possibilities in the physical world? Should kinematical possibilities be viewed as mere logical or linguistic constructs, devoid of a deeper connection to the structure of physical reality? This chapter addresses these questions by analyzing some of the most significant theories in physics: classical mechanics, general relativity and quantum mechanics, with a final mention to quantum gravity. We argue that only dynamical models correspond to genuine physical possibilities.

1. Introduction

What is it about physical reality that allows us to walk from our flat to our favorite library in the city center and choose a book to read in the park, but prevents us from getting there instantaneously or through an admittedly thrilling flight path? Although we have shared intuitions about what is physically possible in everyday situations, it is generally accepted that science plays a key role in exploring and ultimately defining the domain of physical possibilities—possibilities that align with the way our universe is structured and functions.

A central starting point for the philosophical investigation of possibility is the observation that we can distinguish different notions. Are perpetual motion machines possible? In one clear sense, they are not: they would violate the first law of thermodynamics. At the same time, the concept of a perpetual motion machine does not appear self-contradictory: we can imagine a world with different physical laws in which perpetual motion machines exist. Perpetual motion machines thus seem impossible in one sense, and possible in another. For this reason, it is customary to distinguish between different kinds of possibility that can be ordered by strength: logical possibility stands as the broadest notion, while physical possibility—i.e., the set of possibilities compatible with the physical laws of our world—satisfies more restrictive requirements. [1] Perpetual motion machines could thus be construed as being logically possible, yet physically impossible. Another notion of possibility, which cannot as easily be cast in the inclusion ordering of 'objective' possibility spaces, is that of epistemic possibility. What is epistemically possible for a person or group is what is compatible with their knowledge. For instance, before the development of modern theories of thermodynamics, perpetual motion machines represented an epistemic possibility.

This chapter focuses on physical possibility, and on a feature of physical frameworks that might complicate the way we understand it: physical theories rely on two kinds of solution spaces, respectively associated with the kinematical and dynamical models of the theory. In general terms, the kinematical models are those consistent with the theoretical framework under consideration before adding dynamical equations, while the dynamical models are those that further obey the latter constraint.

The kinematical–dynamical distinction invites a number of questions. Are kinematical and dynamical models really representing distinct spaces of possibilities, or are kinematical solutions better thought of as a mere mathematical device? Are there two kinds of physical possibility, or is one of the two sorts of possibility to be identified with logical or another type of possibility?

To address these questions, the chapter analyzes how the distinction between kinematics and dynamics is carried out in different physical theories. We show how the mechanism through which dynamical models are generated from kinematical ones varies in different theoretical settings. In classical and general relativity, dynamical solutions correspond to a subset of the kinematical space, contrary to non-relativistic quantum mechanics where dynamical models are built out of weighted sums of kinematical solutions. The lack of universality for the subset approach suggests that kinematical possibilities should be construed as resulting from the application of a preliminary layer of constraints, on the way to even more constrained dynamical models. These dynamical models ultimately define the genuine space of physical possibilities. We thus conclude that kinematical solutions are not genuine physical possibilities, and should be seen as epistemic, logical or linguistic possibilities (we set aside the question of their exact nature).

[1] Another notion, popular in contemporary metaphysics but less so in philosophy of physics, is that of *metaphysical possibility*, born from the work of Kripke (1980). The notion has become popular in a priori approaches to modality, and its status remains a subject of ongoing debates (see, e.g., Divers 2018, Norton 2022). We will discuss it in the following.

Note also that here we will focus on a limited number of theoretical contexts.[2] We do so because they represent the foundational milestones to a proper understanding of the distinction between kinematical and dynamical solutions. In addition, general relativity and non-relativistic quantum mechanics embody the two major challenges for the tenability of the distinction between kinematics and dynamics: the former calls into question the very idea of temporal evolution dynamical solutions typically incorporate, while a certain understanding of the latter questions the ordering relations between kinematical and dynamical possibilities.

The chapter is structured as follows. We first articulate how dynamical models are selected from kinematical ones in the Lagrangian and Hamiltonian formulation of classical mechanics (Section 2). Based on our findings, we then discuss different possible interpretations of the kinematical–dynamical distinction (Section 3). We suggest that kinematical possibilities should be understood as epistemic, logical, or linguistic preconditions for the formulation of a theory's dynamics, which then defines the space of genuine physical possibilities. Section 4 discusses how the selection mechanism of dynamical solutions from kinematical ones should be generalized in the context of general relativity. We turn to non-relativistic quantum mechanics in Section 5, examining the nature of the kinematical–dynamical distinction there, and find that kinematical models appear to be highly idealized, not representing genuine physical possibilities. We conclude by discussing potential implications from speculative approaches to quantum gravity for the nature of physical possibilities and the distinction between kinematical and dynamical models (Section 6).

2. Classical Mechanics

The original formulation of classical mechanics is Newtonian mechanics, which determines the evolution of a system's position with respect to a temporal parameter through the system's *equation of motion*. This equation of motion—referred to as Newton's second law—is a second-order differential equation which fixes the relationship between the total force acting on a system and the associated acceleration of the system itself.

However, the exact range of solutions in Newtonian mechanics can be quite difficult to evaluate explicitly. For example, consider the simple case of the classical pendulum's equation of motion: solutions are exactly defined for small

[2] We will not examine special relativity and relativistic quantum mechanics. Although these theoretical frameworks would require an in-depth evaluation, we believe that our analysis will naturally extend to them. We leave this task for future research.

angular values only. This is why more powerful formalisms with a broader scope of application, *Lagrangian* and *Hamiltonian mechanics*, were introduced in the 18th and 19th centuries, respectively. They helped drastically ease the calculation in many contexts. This is why, before comparing how the distinction between kinematical and dynamical possibilities is carried out, a preliminary discussion on the Lagrangian and Hamiltonian frameworks is necessary. This will allow us, in the next section, to give a more careful characterization of the general distinction between kinematical and dynamical possibilities.

Central to Lagrangian mechanics is the Lagrangian L of a system:[3]

$$L = K - V$$

where K (often denoted T in the Lagrangian framework) refers to the kinetic energy, and V to the potential energy. The kinematical solutions of the system are evaluated via an action S, itself defined as the integral of the Lagrangian function between two points in the configuration space, at two successive instants t_1 and t_2:

$$S = \int_{t_1}^{t_2} L(q(t), \dot{q}(t), t)dt$$

where q is the position and $\dot{q}(t)$ the associated velocity. The formalism is thus defined in terms of configuration-like and velocity-like quantities. The Lagrangian can be used to calculate equilibrium states satisfying an extremization principle for the paths a system can undergo. This can be done by evaluating the so-called Euler-Lagrange equations, a system of second-order differential equations, whose solutions correspond to stationary points of the action S:

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

In other terms, the Lagrangian formalism shows that the solutions of the equation of motion for a system of interest are obtained by extremizing a quantity, in this case the Lagrangian, on the space of possible trajectories a system can undergo while moving through spacetime. This procedure is called the *principle of least*

[3] For a more detailed, philosophical presentation, see e.g. North (2009, pp. 68-70).

action.[4] For systems that can be cast in this form, the Euler-Lagrange equations are generally construed as equivalent to Newton's equation of motion. The difference with respect to the latter is that the focus of Lagrangian mechanics is on the space of solutions, which can generically be interpreted as a space of possible 'worlds', out of which the dynamical solutions get selected via the extremization principle. Contrary to the picture captured by Newton's vectorial mechanics, which fixes initial states and allows temporal evolution via the application of dynamical laws (as encoded in Newton's second law of motion), the Lagrangian formalism hence ranges over the *temporally extended paths* a system can undergo, or *constrained histories*.

Turning to Hamiltonian mechanics, we recover the Newtonian idea of describing the evolution of states. The formalism uses the Hamiltonian function associated to a system, which reads:

$$H = K + V$$

where, again, K refers to the kinetic energy, and V to the potential energy of the system. The involved variables are the position of the system and its associated momentum (namely, the product of the system's velocity with its mass m). The Hamiltonian is then used to move from Newton's to Hamilton's equation of motion.

These correspond to a pair of first-order differential equations, respectively encoding the time derivative of the position and the momentum of the system under analysis:

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

The Newtonian and Hamiltonian equations are generally regarded as equivalent.[5] The focus of Hamiltonian mechanics is on the space of initial conditions. The underlying assumption is that, provided initial data plus the

[4] The variational principle is typically referred to as the principle of least action as dynamical solutions often correspond not just to stationary points, but more specifically to minima of the action (for a more detailed discussion see, e.g., Arnold 1989, Chapter 3).

[5] Yet, on the equivalence between the various formulations of classical mechanics, see Curiel (2014).

function representing the energy of the system, past and future states will follow deterministically.

The upshot is that the Lagrangian and Hamiltonian formulations of classical mechanics differ in that the first deals with constrained *histories* while the latter describes the evolution of *states*. Let us unpack this difference more thoroughly.

The Lagrangian formulation focuses on the space of four-dimensional solutions, namely possible histories or trajectories of the system at hand. Provided the space of kinematically possible trajectories, dynamical solutions follow from the application of the principle of least action. Kinematical solutions can be viewed as a space of functions between independent variables (typically, spatial, temporal, or spatiotemporal coordinates) and dependent variables (field values for a quantity of interest, including spatial coordinates). Dynamical solutions correspond to the space of functions between independent and dependent variables which respect the additional constraint set by the Euler-Lagrange equations. In the Lagrangian formalism, dynamical possibilities thus correspond to a subset of the broader class of kinematically possible solutions. To provide an intuitive example (Belot 2007, p. 154), take a particle in Euclidean space on which a position-dependent force $F\big(x(t)\big)$ is applied. The independent variable acts as the temporal parameter, while the dependent variables are associated to the possible positions of the particle. In this case, an arbitrary continuous function $x(t)$ of the kind $t \in \mathbb{R} \rightarrow x(t) \in \mathbb{R}^3$ sets a kinematically possible trajectory for the particle, while the space of dynamically possible trajectories is defined by the subset that satisfies the Newtonian equation $\ddot{x} = F\big(x(t)\big)$. Within the Lagrangian framework, the role of differential equations, or laws of motion, is thus to select dynamically possible solutions among the broader class of kinematically possible ones.

On the other hand, the Hamiltonian formulation defines the space of initial data, or instantaneous states, and evaluates the flow of this space via the application of Hamilton's equations. These equations fix, when plugging in initial conditions, the space of dynamically possible models of the theory. The dynamics is thus provided by an action over the space of initial data, which implements a temporal evolution. This means that the history of the system under investigation corresponds to a trajectory through the space of initial values. As the emphasis is on instantaneous states—namely, a specification of the value of the field at each point of space, together with the associated temporal rate of change—the Hamiltonian formulation requires a global decomposition of spacetime into space and time (see, e.g, Belot 2007, p. 165). This partitioning of the spacetime manifold presupposes a notion of absolute simultaneity, defined by a preferred family of observers. Such a $3 + 1$ splitting can be uniquely defined for Newtonian spacetime without causing any particular issue. However, as we will discuss later on, the need to partition spacetime will become a burdensome challenge in the context of relativistic physics.

3. Kinematical and Dynamical Possibilities

In the last section, we have seen that the Lagrangian and Hamiltonian approaches concur in drawing a distinction between two spaces of models: kinematical models construed in terms of their compatibility with the general setting of the theory; and dynamical models satisfying additional constraints given by the equation of motion. The different spaces of solutions are constructed differently in the two formalisms: kinematical solutions are instantaneous states in Hamiltonian mechanics, and four-dimensional histories in Lagrangian mechanics. Dynamical models are selected by the Euler-Lagrange equations for Lagrangian mechanics and by Hamilton's equations for Hamiltonian mechanics.

As explained in the introduction, the existence of two realms of physical possibilities, suggested by the existence of two kinds of models, seems to clash with the traditional understanding of physical possibility as confineable to a single 'layer' (Ruyant and Guay 2024, p. 2). That there could be two sorts of possibilities in physics where one would expect to find only one leads to a number of intriguing questions. Is the distinction between kinematical and dynamical models latching onto a real separation between two sorts of possibilities, or is it a mere artifact of the methodology used in physics? And if there really are two sorts of possibilities in physics, are they both physical possibilities in the traditional sense? And, in particular, are kinematical models genuine possibilities at all?

To get a first grip on the kinematical–dynamical distinction, let us assume that the set of dynamical possibilities is a subset of the set of kinematical possibilities. Call this: the *subset view*. From what we discussed in the previous section, it should appear clear that classical mechanics supports the view. Ruyant and Guay (2024) entertain three different options for how to conceptualize the modal status of kinematical and dynamical possibilities. On the first option, kinematical models are identified with *metaphysical possibilities*—a distinct sort of possibilities situated between logical and physical ones—whereas dynamical models are defined as physical possibilities.[6] The nature (and existence) of these metaphysical possibilities remains debated, but they are supposed to capture the idea of metaphysical laws going beyond physical ones, thus associated with the natures or essences of the entities constituting the world (Fine, 1994). On the second option, kinematical models are associated with a new sort of possibilities

[6] This kind of view is defended by Hirèche et al. (2021), who argue that the kinematical space fixes metaphysical necessity, while the dynamical space determines nomic necessity.

situated between metaphysical and physical ones, the latter being again identified with dynamical possibilities. Finally, the third alternative holds that kinematical and dynamical models each form a subspecies of physical possibility.

The only view we take on this is that the third option is incorrect: purely kinematical models do not represent physical possibilities. Our position resonates well with Wallace (2022)'s distinction between the kinematics and dynamics of a theory, according to whom the former represents the ensemble of models with the proper mathematical features to represent a physically possible world, while the latter stands for the ensemble of models which actually represent a physically possible world. Put differently, the kinematics fixes the space of solutions or data which display suitable features (such as continuity, four-dimensionality, differentiability, and so forth) to properly represent a physical state of the universe, while the dynamics defines the space of solutions or data which actually represent a physical state of the universe.

On our view, we can think of the kinematical space as the space that defines the preconditions for the application of what are often described as *dynamical laws*. This means that the kinematical space somehow fixes the space of possibilities in a general sense, while the dynamical space adds dynamical laws as a second layer of constraints on the range of what is physically possible.

Before turning to physical settings other than classical mechanics, let us highlight another interesting problem about physical possibility that arises from the co-existence of multiple physical frameworks. Different physical theories—while all of them simultaneously accepted and in use by physicists, think for example about general relativity and quantum mechanics—yield different possibilities, which makes it far from clear how to delineate the border between the physically possible and the physically impossible. Theoretical physics comprises a collection of frameworks forming a complex web that provides different answers to these questions: special and general relativity, non-relativistic quantum mechanics, quantum field theories, thermodynamics, and statistical mechanics, to mention the most fundamental building blocks of our best empirically confirmed physics. What seems bewildering is that a certain phenomenon may be possible according to one framework, yet impossible according to another (for a discussion of this issue, see Baron et al. 2025). For example, signals propagating faster than the speed of light are forbidden by special and general relativity but appear to be possible, at least in the sense of not being clearly ruled out, based on non-relativistic quantum mechanics. What lessons need to be drawn from this? Despite being an important point, we will not focus on this aspect in the present context. Let us simply point out that one solution would be to accord a privileged status to some theories, and perhaps to one only—a final theory of everything—in discerning what is, and what is not, physically possible.

4. General Relativity

What happens to the kinematical–dynamical distinction in general relativity (GR), and how are physical possibilities represented within the theory? A defining feature of GR, which marks the difference with respect to other theoretical contexts, is that it follows the *principle of general covariance*. The latter states that the physical laws need to be written in an invariant form under arbitrary coordinate transformations. The terminology is thus somewhat confusing as the term of art, *co*variance, expresses a form of *in*variance of the laws under arbitrary coordinate transformations. As we will discuss, the notion of general covariance, which has sparked much interpretative debate (see, e.g., Belot and Earman 2001), calls for a generalization of the selection mechanism of dynamical solutions from kinematically allowed models. Despite this complication, we will show that in GR too kinematical solutions can be regarded as a valuable methodological tool, guiding the construction of genuine physical possibilities via dynamical models. To illustrate this, we will begin with a semi-technical overview.

GR is currently our best empirically established theory of spacetime. In GR, a spacetime can be described as a differentiable manifold M defined by an ensemble of points with specific topological properties (such as four-dimensionality and continuity), and a metric tensor field g (with a Lorentzian signature -+++). Matter and energy are represented by a stress-energy tensor field T. The Einstein Field Equation (EFE) connects the metric structure (especially, the local spacetime curvature) with the matter distribution. The formal expression reads:

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

where R_{ab} is the Ricci tensor (specifying how curvature infinitesimally varies in all directions with respect to every other direction), R the Ricci scalar (describing the local curvature), g_{ab} the metric field (encoding the lightcone structure of spacetime), Λ the cosmological constant (a coefficient providing a counterbalance effect to gravity), and T_{ab} the stress-energy tensor (where natural units are adopted, namely $c = G = 1$). A model of GR is then a triple $\langle M, g_{ab}, T_{ab} \rangle$ satisfying the EFE. The solution of this tensor equation describes how the metric evolves given a specific distribution of the matter fields. Note, however, that the stress-energy tensor does not simply operate as a source for the spacetime curvature, for its physical meaning is also contingent upon knowledge of the associated metric. As a result, the EFE requires solving concomitantly the spacetime metric and the matter distribution. This is why exact solutions are only available for specific, highly symmetric cases.

In the context of GR, it is then necessary to outline which specific class of solutions one is considering, as different sectors, i.e. classes of solutions of the theory, will have different implications for the availability of the Hamiltonian and

Lagrangian formalism. To this end, an important consideration is to evaluate whether the specific sector under examination allows to define a *globally hyperbolic manifold*, as this condition is required for the adoption of the Hamiltonian formulation: only globally hyperbolic solutions can be cast in the Hamiltonian form. Let us note in passing that, if one takes non-globally hyperbolic solutions seriously, this seems to provide a compelling argument for the superiority of the Lagrangian formalism over the Hamiltonian one, at least in GR.

Global hyperbolicity encodes a particular condition on the causal structure of the spacetime manifold. A fruitful way to identify a globally hyperbolic manifold is via the existence of Cauchy surfaces. Given (M, g), a set of points $\Sigma \subset M$ is a Cauchy surface of (M, g) if every inextendible timelike curve in (M, g) intersects Σ just once. A manifold M is globally hyperbolic if it can be foliated into Cauchy surfaces, namely decomposed in the form of $\Sigma \times \mathbb{R}$, where Σ stands for a stack of $3d$ (for a 4−dimensional manifold) space, ordered via a universal time parametrized by $\mathbb{R}$. Within this class of solutions, both the Lagrangian and the Hamiltonian formulations are available. [7] Intuitively, the Hamiltonian approach is applicable within this sector of the theory for a Cauchy surface can be described as a spatial state at a certain time, while successive Cauchy surfaces can be linked together to form a history.

On the one hand, the Lagrangian formulation of GR is defined via the application of the corresponding extremization principle to the so-called Einstein-Hilbert action. The peculiarity of the formalism in this context comes from the fact that, as already discussed, GR has to respect general covariance. This implies that, in this context, certain transformations do not change the physical content. They represent different ways of describing the same physical situation by smoothly shifting the spacetime coordinates without altering the underlying geometry. This is why solutions can be grouped into so-called *gauge orbits*. Two metrics, call them g and g^1, belong to the same gauge orbit in case there exists a diffeomorphism $d: M \to M$ such that $g^1 = d^* g$. The idea, roughly, is that if you can transform one metric into the other by smoothly changing the coordinates, then the two metrics belong to the same gauge orbit. We then define the reduced space of solutions as the one that divides the solutions in gauge orbits. And this is the starting point for the application of the Lagrangian machinery to GR.

On the other hand, the Hamiltonian formulation operates in analogy to the classical case, where a $3 + 1$ slicing of the four-dimensional manifold M in terms of $\Sigma \times \mathbb{R}$ for some 3-dimensional manifold Σ is realized to define the space of

[7] For a detailed discussion of the Hamiltonian and Lagrangian formulation of GR, and the more specific conditions required for the Hamiltonian approach to be applicable (e.g., orientability), see Wald (1984, Appendix E).

initial data and successively evaluate Hamilton's equations. In this context, the main issue to apply the procedure is that GR does not allow a solution-independent decomposition of the manifold M. Even if one provides a procedure to deal with this additional complication (see Belot 2007, p. 201), the more dramatic aspect of the Hamiltonian formalism in GR is that it directly leads to a frozen formalism. This is the so-called *problem of time in GR*. When we apply the standard dynamical framework to analyze how the relevant variables evolve, it turns out that the dynamical space is made up of those paths in the space of initial data that remain within the same gauge orbit. In other words, the evolution we track only occurs along gauge-related curves (such as diffeomorphisms), typically interpreted as encoding unphysical information (yet, on this, see Belot and Earman 2001). The general result is that, within this sector of the theory, no genuine notion of time, evolution, or change seems available, for the physical variables remain confined within the gauge orbit, where the physical content, by definition, remains unchanged.[8]

Clearly, the broader set of solutions, in which some of the so-far assumptions get abandoned, renders the problem even more dramatic. If we drop the global-hyperbolicity condition, the manifold M cannot be sliced into Cauchy hypersurfaces, and the Hamiltonian formalism is no longer applicable. For this sector, only the Lagrangian formalism is available. Let us then focus on the latter with the intent of spelling out how the construction of the kinematical and the dynamical spaces is realized, and what implications should be drawn concerning the status of physical possibilities.

Starting from the Lagrangian,[9] the solution space is evaluated from the action S, which in classical mechanics was defined as the integral of the Lagrangian between two points in the configuration space at two successive instants t_1 and t_2. More specifically, it was shown that the actual physical path is the one that minimizes the action S, and that the minimization procedure is obtained by evaluating the Euler-Lagrange equations.

Analogously, in the context of GR, the Lagrangian formalism can provide the starting basis for the definition of an action principle which allows evaluating dynamical solutions. The main issue arising in the context of GR is how to provide

[8] While *prima facie* a possible way out of the conundrum would be to focus on the Lagrangian formalism, which does not require the $3+1$ splitting, it can be shown that, under the so-far introduced assumptions concerning the kinematical space, an isomorphism between the reduced space of (Lagrangian) solutions and the corresponding reduced space of (Hamiltonian) initial data should be expected. This means that the problem of time figures in this sector of GR, no matter what formalism one aims to adopt (see Belot 2007, pp. 202-203; 209-211).

[9] Note that, though the Lagrangian formalism is also available for more complex sectors of GR, here we will mention, for a matter of simplicity, the manageable scenario of vacuum solutions, whereby the only available field is the metric field.

a generalization of the concept of temporal integration typical of the Lagrangian formalism in classical and quantum mechanics. This issue can be framed in terms of establishing how to define a volume element, call it $d\mu_g$. A possibility would be to justify its definition on the basis that the integral

$$\int_M f\, d\mu_g$$

for a function f defined on the manifold M and a measure $d\mu_g$ which may be dependent upon the metric, is independent of a choice of coordinates (as prescribed by general covariance). This requires the integral to be invariant under the smooth coordinate change $\{x^i\} \rightarrow \{\hat{x}^i\}$. . Without entering too technical details, it can be shown that this implies imposing $d\mu_g = \sqrt{|g(x)|}d^4x$ (where $g(x)$ stands for the determinant of the metric tensor). The volume element $d\mu_g$ is typically called the measure corresponding to the metric g.

What this last condition teaches us is that the Lagrangian formalism of GR represents a generalization of the one employed in the context of classical mechanics, whereby no unique notion of temporal evolution is assumed, let alone adopted. For the Lorentzian metric one typically employs $\sqrt{-g}$ instead of $\sqrt{|g(x)|}$. As the volume element is contingent upon the metric, it should figure in the variation as long as the metric represents the field variable the variational principle applies to. From this, one arrives at the definition of the Einstein-Hilbert action:

$$S_{EH} = \int R d\mu_g = \int \sqrt{-g} R d^4x$$

Where R is the Ricci scalar, and the integral is performed over a compact region D with smooth boundary conditions. From the subsequent application of the variational principle, it is then possible to find out the solutions that minimize the Einstein-Hilbert action, namely the ones corresponding to dynamical solutions.

From the above discussion, three aspects emerge that are of particular interest for the kinematics–dynamics distinction. First, as GR allows no solution-independent decomposition of the manifold M in a global space Σ and a global time $t \in \mathbb{R}$, the Lagrangian formalism is more apt to track the *solution space* of the theory. Second, GR calls for a redefinition of the notion of evolution, whereby the volume element for the integration of the Lagrangian needs to account for the principle of general covariance. This means one should allow for a generalization of the selection mechanism of dynamical solutions with respect to the space of kinematically allowed models. Third, the subset view still applies as long as the just mentioned generalization of the Lagrangian formalism remains available: the Einstein-Hilbert action encodes kinematical possibilities, while the successive application of the variational principle generates the dynamical space, in our interpretation the one which captures genuine physical possibilities.

5. Non-Relativistic Quantum Mechanics

Is the characterization of the distinction between kinematical and dynamical possibilities from above also suited for non-relativistic quantum mechanics? The situation is quite complex as quantum mechanics can be formulated in different ways.[10] Historically, the three major are the *matrix formulation*, the *wavefunction formulation* and the *path integral approach*, respectively advanced by Heisenberg, Schrödinger and Feynman.

Here, we restrict our attention to the path-integral approach, for a number of reasons. First, it is a natural and fruitful method to convey the main insights of quantum mechanics. Second, the classical limit is relatively straightforward in the approach (MacKenzie 2000, p. 2). Third, while it might be more complicated than the matrix and the wavefunction formulations, it becomes extremely useful in various contexts, such as classical and quantum field theory, statistical mechanics, and condensed matter physics (MacKenzie 2000, p. 1, Styer et al. 2002, p. 290), so that the conceptual results here endorsed can be straightforwardly generalized to those settings. Fourth, many approaches to quantum gravity embody the spirit of the path integral formalism.

The path integral formulation deals with transition probabilities between initial and final states. The idea is analogous to what happens in the Lagrangian formulation of classical mechanics. Suppose we have a particle located at x_1 at time t_1, and assume we want to calculate the probability of finding it at x_2 at time t_2. This is what the path integral approach prescribes:

- Evaluate all the possible classical paths from x_1 to x_2.
- Determine the action for each classical path (recall that the action is provided by the temporal integral of the associated Lagrangian).
- Ascribe to each classical path a transition amplitude, which can be shown to be proportional to $e^{\frac{iS}{\hbar}}$.

[10] See, e.g., Styer et al. (2002), which provides no less than nine formulations.

- Sum the amplitude over all possible classical paths by using the summing rule provided by quantum mechanics.
- The sum, which, assuming the possible paths vary continuously, corresponds to a path integral, provides the transition amplitude from x_1 to x_2, and its square magnitude corresponds to the transition probability.

The total probability that a certain particle, located at x_1 at t_1, gets to x_2 at t_2, is calculated as a weighted sum of all the possible paths the particle can follow from the initial to the final state. If one assumes, for the sake of the present argument, that such an approach to quantum mechanics is not a mere instrumental way of expressing transition probabilities directly measurable in laboratories, the picture of reality we obtain is quite peculiar. Indeed, the lesson appears to be that quantum particles take all viable paths from initial to final states, and not only one of them.

What is the relation between the classical and quantum versions of Lagrangian mechanics? As we have seen, a transition amplitude is associated to each of the possible paths the particle can follow, and the total transition amplitude is given by summing them. How are we then to combine this with the fact that, classically, objects appear to follow clearly defined, and arguably unique, trajectories? By evaluating interferences between adjacent paths, it can be shown that the closer the paths are to the classical one, the more they interfere constructively. The net effect is that, on average, paths close to the classical one will interfere constructively, while arbitrary paths will interfere destructively (MacKenzie 2000, p. 13).

From classical mechanics, we know that stationary states are the ones that correspond to dynamical solutions. In that context, we showed that the latter can thus be regarded as a subset of the kinematical space. Correspondingly, one may wonder whether what we dubbed the subset view is still available in the context of non-relativistic quantum mechanics. At first glance, the path integral approach to quantum mechanics does not seem to differ from the classical case in this respect: among the various possible paths the particle can follow there is a subset that maximizes, in the classical limit, constructive interference. This subset is what should be accorded the status of dynamical solutions.

However, Ruyant and Guay (2024) challenge this view. They argue that quantum mechanics raises an issue for the metaphysics of modality. Their argument goes as follows: classical mechanics and quantum mechanics frame the relationship between kinematical and dynamical solutions differently. While in classical physics dynamical possibilities get selected from kinematical ones (which thus turn out to be both kinematical and dynamical possibilities), in quantum mechanics kinematical solutions are *not* dynamical solutions, suggesting that dynamical possibilities are not kinematical possibilities. This is because the kinematical space is modeled as continuous, while the transition amplitudes (which stand for the quantum analogue of classical dynamical possibilities) correspond to discontinuous jumps. As a consequence, kinematical possibilities never get realized in nature: we need them to construct dynamical possibilities, but they are 'impossible histories' (Ruyant and Guay 2024, p. 11).

We agree with Ruyant and Guay that the subset approach no longer works in the quantum context, or at the very least requires substantive adjustments. Still, we believe that a broader approach to the relation between kinematical and dynamical solutions applies equally to the classical and quantum contexts. This broader view is that dynamical models are built from kinematical ones, and that we should not take the latter to indicate genuine physical possibilities, even in the quantum context.

An immediate challenge against this deflationist attitude towards kinematical possibilities in quantum mechanics arises from the fact that dynamical models appear to be constructed from weighted sums of kinematical models, making the latter seem physically possible. How could kinematical models fail to represent genuine physical possibilities if they are, in some important sense, constitutive of dynamical possibilities? To address this, it is helpful to distinguish between two opposing interpretations of the path integral: one that treats it as a mere mathematical tool, and another that regards it as ontologically significant.

The first approach suggests that the path integral should not be taken too seriously, ontologically. This perspective may involve adopting an alternative formulation of quantum mechanics and viewing the path integral as a mere mathematical representation, useful for empirical predictions but without ontological bearing. Under this view, dynamical possibilities would not literally be constituted by kinematical possibilities. Instead, the kinematical-dynamical structure would be an artifact of the mathematical framework employed.

The second approach adopts the view that quantum particles do not follow *one* path, but *all* possible paths from initial to final states. This perspective aligns naturally with Everettian quantum mechanics (EQM), the many-worlds interpretation of quantum mechanics (Wallace, 2012). According to EQM, the many worlds or branches are grounded in the fundamental universal quantum state. This implies that all trajectories are realized in nature. Kinematical models, therefore, can be regarded as describing the branches of an extraordinarily vast multiverse. If one considers the branches to represent physical possibilities, rather than proper parts of such a multiverse, it follows that kinematical models do represent physical possibilities, after all.[11] But note that even if one does not endorse the modal interpretation of EQM, the kinematical models still represent

[11] The modal interpretation of EQM is endorsed by Wilson (2020).

physical possibilities, since the particles actually follow them. Indeed, what is actually happening in our world should also, a fortiori, be physically possible.

Does this mean that EQM implies that branches are kinematical possibilities, thereby undermining our claim that kinematical models do not represent genuine physical possibilities? Not quite.

First, it is important to note that in an important sense a particle cannot, in this interpretation, follow one trajectory only. It must also follow other trajectories—or, alternatively, other particles must do so, depending on how one individuates entities across the quantum multiverse. For consider this: If EQM is the correct interpretation of quantum mechanics, the fundamentality of the universal quantum state rules out the possibility that each physically possible world coincides exactly with one classical world only. This is because the branches are emergent, i.e., less fundamental than the universal quantum state. According to EQM, physical branches are a package deal—they always come in large numbers, never individually. So, if the branches correspond to physical possibilities, those are of a quite peculiar sort, requiring the existence of other co-existing, entrenched possibilities. In our view, this speaks against taking the branches as genuine classical possibilities, and thus against the view that purely kinematical models can be interpreted as genuine physical possibilities in EQM.

Second, and relatedly, it is sometimes underappreciated that the quantum multiverse, as described by EQM, is emergent—at least in the standard case of decoherence-based EQM. This implies that there are infinitely many ways to define branches in the quantum multiverse. Consequently, either the branching involves some degree of indeterminacy, or the branches should be excluded from the ontology of EQM, regarded instead as mere heuristics to interpret the universal quantum state (Glick and Le Bihan 2024). This suggests that, in this context, kinematical models should not be viewed as representing classical possibilities rather than quasi-classical ones. They appear classical only if one neglects the subtle residual entanglement with degrees of freedom localized in other branches. Viewing them as truly classical possibilities is thus an idealization. Hence, dynamical models are constructed from classical ones, which in turn are understood as approximations of quasi-classical models. Quantum states, although they come after classical states in the order of theory construction, are more fundamental than classical states. Thus, even assuming EQM, or more generally any form of realism about the paths involved in the path integral, we should not take purely kinematical possibilities to represent genuine physical possibilities.

6. Concluding Remarks

As we have seen, the distinction between kinematical and dynamical models reflects a specific methodology used by physicists: the kinematics provides the stage setting from which dynamical models, corresponding to physical possibilities, are constructed. A central theme of our investigation has been the coexistence of two distinct mathematical frameworks in theoretical physics: the Lagrangian/path integral and Hamiltonian approaches.

Interestingly, general relativity appears to suggest that the Lagrangian formulation is more fundamental, as certain solutions within the theory cannot be accommodated by the Hamiltonian framework. Furthermore, quantum mechanics suggests that dynamical models are more fundamental than kinematical ones. A natural conclusion, then, is that physical possibilities are best represented by dynamical solutions within the Lagrangian or path-integral framework. Kinematical models, by contrast, do not represent genuine physical possibilities, but merely serve as preliminary approximations or rough characterizations of potential candidates for physical possibilities.

Looking ahead to the future of physics, what insights on physical possibility might a theory of *quantum gravity*, going beyond relativistic and quantum physics, offer? While a comprehensive study is necessary, we can already highlight one key point. As extensively discussed in the literature, virtually all approaches to quantum gravity suggest that spacetime is not, in a way to be further specified, fundamental (see, e.g., Oriti 2009, Crowther 2018, Vistarini 2019, Huggett and Wüthrich, forthcoming). Rather, it would emerge from a non-spatiotemporal structure.

The possible emergence of spacetime leads to a number of distinct issues, both conceptual and empirical (Le Bihan, 2021). That the Lagrangian approach is less temporally-flavored than the Hamiltonian one is suggestive of its potential capacity to describe non-spatiotemporal physical systems and thus address at least some of those issues. By deconstructing space and time more deeply, spacetime emergence could thereby further underscore the metaphysical superiority of the Lagrangian approach over the Hamiltonian framework. [12] A Lagrangian formulation would thereby offer our best chance of identifying physical possibilities within such a non-spatiotemporal theory.

However, the task will not be easy: the Lagrangian formulation is traditionally framed in a spatiotemporal setting, as it involves integrals over paths typically parameterized by spatiotemporal coordinates. In classical physics, the action is a

[12] Note that this is consistent with the fact that, at the moment, many quantum gravity approaches fruitfully put the Hamiltonian formulation to work. For the approach might be highly useful as a way to develop a more fundamental, perspicuous formulation.

function of positions, velocities, and time, which are intrinsically spatial and temporal notions. In quantum mechanics, the path integral formulation extends this idea by summing over all possible trajectories in spacetime. Thus, in both cases, spacetime plays an essential role in defining the possibilities and dynamics of the system. Our Lagrangian understanding of a non-spatiotemporal physics will thus have to be clearly articulated to get a sense of the nature of non-spatiotemporal, physical possibilities.

What about the kinematical–dynamical distinction itself, in non-spatiotemporal contexts? If the distinction does not hinge on a temporally-flavored picture of dynamical laws acting on otherwise non-dynamical systems, but rather echoes the particular way we get to physical possibilities via a methodological process, then the challenge posed by spacetime emergence on our understanding of physical possibilities might become conceptually easier to address. Moreover, a reduced emphasis on the modal significance of kinematical models—viewed as mere tools to guide the search for true physical possibilities—suggests that purely kinematical models should not be overvalued when attempting to understand the nature of reality through research programs in quantum gravity. Overall, the lesson seems to be that, when seeking genuine physical possibilities in theoretical physics, the primary focus should be on the maximally constrained solution space of the theory.

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