Determinism and Asymmetry in General Relativity

JB Manchak, TW Barrett, HP Halvorson, JO Weatherall

Abstract

This paper concerns the question of which collections of general relativistic space-times are deterministic relative to which definitions. We begin by considering a series of three definitions of increasing strength due to Belot (1995). The strongest of these definitions is particularly interesting for spacetime theories because it involves an asymmetry condition called "rigidity" that has been studied previously in a different context (Geroch 1969; Halvorson and Manchak 2022; Dewar 2024). We go on to explore other (stronger) asymmetry conditions that give rise to other (stronger) forms of determinism. We introduce a number of definitions of this type and clarify the relationships between them and the three considered by Belot. We go on to show that there are collections of general relativistic spacetimes that satisfy much stronger forms of determinism than previously known. We also highlight a number of open questions.

1 Introduction

This paper concerns the question of which collections of general relativistic spacetimes are deterministic relative to which definitions. Consider an intuitive but imprecise idea due to Montague (1974), Lewis (1983), and Earman (1986): a collection of possible worlds is "MLE deterministic" if, for any worlds in the collection that agree on some initial segment, the worlds agree entirely. Belot (1995) considers three natural ways of making MLE determinism precise. Within the discussions of the hole argument (Earman and Norton 1987), the first two definitions have long played a central role (Butterfield 1989; Melia 1999; Pooley 2021). Belot's third definition is much stronger than the other two and has only recently become a focus of attention (Halvorson and Manchak 2022; Landsman 2023; Cudek 2023). For each definition, Belot provides a highly symmetric example that seems to be (i) deterministic according to the particular formulation but (ii) indeterministic by the lights of certain haecceitist positions. This leads him to conclude that "determinism cannot be a formal property of theories" (Belot 1995, p. 88).

In a recent paper, Halvorson et al. (2025) push back on this position. They hold that Belot's third definition is actually a promising precisification of MLE determinism that was unfortunately ignored for thirty years. They also argue that Belot's third example is ambiguous; until it is made precise, it is neither deterministic nor indeterministic according to his third definition. It is emphasized that whether or not a theory is deterministic depends crucially on the precise formulation of the theory and the precise formulation of determinism.

Here in this paper, we follow this line but focus our attention on (variants of) general relativity. Belot's third definition is particularly interesting for spacetime theories because it involves an asymmetry condition called "rigidity" that has been studied previously in a different context (Geroch 1969; Halvorson and Manchak 2022; Dewar 2024). We explore other (stronger) asymmetry conditions that give rise to other (stronger) forms of MLE determinism. We introduce a number of definitions of this type and clarify the relationships between them and the three considered by Belot. We go on to show that there are collections of general relativistic spacetimes that satisfy much stronger forms of determinism than previously known. We also highlight a number of open questions.

2 De Dicto and De Re Determinism

Let $\mathscr{U}$ be the collection of all general relativistic spacetimes (M, g). Here M is a smooth, connected, Hausdorff, n-dimensional (for $n \geq 2$) manifold and g is a smooth Lorentzian metric on M. In order to make sense of the notion of "initial segment" in the MLE definition of determinism, we first restrict attention to the collection $\mathscr{H} \subset \mathscr{U}$ of globally hyperbolic spacetimes with a chosen temporal orientation (see Wald 1984, Section 8.3). Any n-dimensional globally hyperbolic spacetime (M, g) admits an $(n-1)$-dimensional Cauchy surface $\Sigma \subset M$ representing all of space at a given time. One can show that M can be foliated by a collection of Cauchy surfaces such that the topology of M is $\Sigma \times \mathbb{R}$. For any spacetime $(M, g) \in \mathscr{H}$, we define an **initial segment** $U \subset M$ to be the timelike past $I^-[\Sigma]$ of any Cauchy surface Σ in (M, g). One can verify that any initial segment U of any globally hyperbolic spacetime (M, g) counts as a connected, open subset of M. So (U, g) is a spacetime in its own right. Let us now consider two forms of MLE determinism adapted from Belot (1995).

Definition 1. A collection $\mathscr{C} \subseteq \mathscr{H}$ is **de dicto deterministic** if, for any spacetimes $(M, g), (M', g') \in \mathscr{C}$ and any initial segments $U \subset M$ and $U' \subset M'$, if there is an isometry $\varphi : U \to U'$, then there is an isometry $\psi : M \to M'$.

Definition 2. A collection $\mathscr{C} \subseteq \mathscr{H}$ is **de re deterministic** if, for any spacetimes

$(M, g), (M', g') \in \mathscr{C}$ and any initial segments $U \subset M$ and $U' \subset M'$, if there is an isometry $\varphi : U \to U'$, then there is an isometry $\psi : M \to M'$ such that $\psi_{|U} = \varphi$.

These two forms of determinism (or close variations) go by different names and have long played a central role in discussions of the hole argument. The first is called "Dm2" in (Butterfield 1989), "Definition 1" in (Belot 1995), "Lewis's analysis" in (Melia 1999), "Det2" in (Pooley 2021), and "D1" in (Halvorson et al. 2025). The second is called "Definition 2" in (Belot 1995), the "second resolution" in (Melia 1999), "Dm2+" in (Cudek 2023), and "D2" in (Halvorson et al. 2025). Here, we use the names "de dicto" and "de re" from (Dewar 2016, 2024). The two forms of determinism capture slightly different senses in which a collection of spacetimes might be considered deterministic. Clearly, de re determinism implies de dicto determinism. They differ in how one understands "agreement of worlds" in the MLE formulation of determinism. To better understand the definitions, consider the following example.

Example 1. Consider two-dimensional Minkowski spacetime in standard (t, x) coordinates and let (M, g) be the $t < 0$ region. The collection $\mathscr{C} = \{(M, g)\}$ is de dicto deterministic. This follows easily since there is only one spacetime in the collection. But $\mathscr{C}$ is not de re deterministic. To see this, consider the initial segments $U, U' \subset M$ given by the $t < -2$ and $t < -1$ regions respectively. Clearly there is a time translation isometry $\varphi : U \to U'$ defined by $\varphi(t, x) = (t + 1, x)$. But there can be no corresponding global isometry $\psi : M \to M$ such that $\psi_{|U} = \varphi$ because of the $t = 0$ "edge" of the spacetime (see Figure 1).
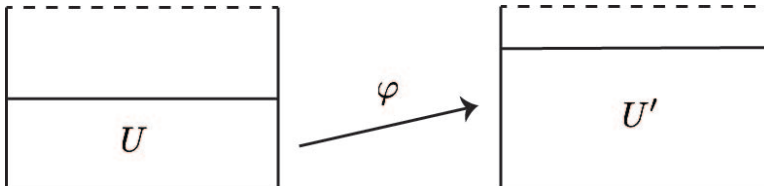

Figure 1: Two copies of the spacetime (M, g) from Example 1. The time translation isometry φ from the initial segment U to the initial segment U' does not extend to a global isometry because of the $t = 0$ "edge" of the spacetime (dotted line).

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Example 1 illustrates a curious state of affairs. The agreement of the initial segments U and U' given by φ does not extend to a global agreement. So one form of MLE determinism (de re) does not hold. And yet there is global agreement given by the identity map on M showing that another form of MLE determinism (de dicto) does hold. A similar situation

arises in the "collapsing columns" example due to Wilson (1993) often used to distinguish the two definitions. Here is Belot (1995, p. 191):

> "But now consider a world W, in which centrally loaded columns collapse by buckling, which contains nothing but: (i) a large, homogeneous, perfectly spherical planet; (ii) a relatively small perfectly cylindrical column resting on the planet so that its axis of symmetry is normal to the planet at the point of contact; (iii) and a cone, which is moving through space in such a way that at time $t = 0$ its apex will impact the column at the exact center of its top surface, with sufficient force to cause the column to buckle. We also assume that W has a Newtonian spacetime structure; that the laws concerning motion are completely deterministic; and that the law governing column collapse fully determines the shape that the column assumes upon collapse."

The classical analogue to de dicto determinism is satisfied by the singleton collection consisting of the collapsing columns world W. However, by the lights of certain haecceitist positions, there is indeterminism present in the example: because of the symmetries of the $t < 0$ initial segment, the columns "could have toppled in a different direction" (Melia 1999, p. 649). Accordingly, the de dicto form of MLE determinism is considered not strong enough. With respect to the classical analogue of de re definition however, the collapsing columns example fails to be de re deterministic. In this way, the example is similar to Example 1. Stepping back, we have the following.

Proposition 1. If a collection $\mathscr{C} \subseteq \mathscr{H}$ is de re deterministic, then it is de dicto deterministic; the other direction does not hold.

Which collections of $\mathscr{C} \subseteq \mathscr{H}$ satisfy de re determinism? Consider the collection $\mathscr{V}^+ \subset \mathscr{H}$ of four-dimensional, inextendible, globally hyperbolic, vacuum solutions to Einstein's equation. Let (M, g) and (M', g') be any spacetimes in $\mathscr{V}^+$ and let $U \subset M$ and $U' \subset M'$ be any initial segments. If there is an isometry $\varphi : U \to U'$, it follows from a theorem by Choquet-Bruhat and Geroch (1969) that there is an isometry $\psi : M \to M'$ such that $\psi_{|U} = \varphi$. Thus, we have the following which captures a sense in which large swaths of general relativistic spacetimes of physical interest are deterministic.

Proposition 2. Any collection $\mathscr{C} \subseteq \mathscr{V}^+$ is de re (and thus de dicto) deterministic.

3 De Re + Uniqueness

Given a collection of $\mathscr{C} \subseteq \mathscr{H}$, both the de dicto and de re forms of MLE determinism require that for any spacetimes $(M, g), (M', g') \in \mathscr{C}$ and any initial segments $U \subset M$ and $U' \subset M'$, if there is an isometry $\varphi : U \to U'$, then a certain type of isometry $\psi : M \to M'$ must exist. In the natural way, one can strengthen each definition by adding a uniqueness clause to ψ. Indeed, the third form of MLE determinism considered by Belot (1995) is de re determinism with uniqueness: worlds must agree in only one way.

Definition 3. A collection $\mathscr{C} \subseteq \mathscr{H}$ is **de re* deterministic** if, for any spacetimes $(M, g), (M', g') \in \mathscr{C}$ and any initial segments $U \subset M$ and $U' \subset M'$, if there is an isometry $\varphi : U \to U'$, then there is a unique isometry $\psi : M \to M'$ such that $\psi_{|U} = \varphi$.

Curiously, this definition has received relatively little attention in the hole argument literature until just recently. De re* determinism is called "Definition 3" in (Belot 1995) and "Dm2++" in (Cudek 2023). It is endorsed by Halvorson et al. (2025) who call it "D3" determinism. The definition is also briefly considered by Halvorson and Manchak (2022) and Landsman (2023). Why might one be interested in de re* determinism? It has been argued that de re determinism is not strong enough to count all instances of indeterminism as such. Belot (1995) introduces his own symmetric example which leads to the formulation of his Definition 3 (de re* determinism). We will return to a similar example in due course. For now let us turn to another case: classical Leibnizian spacetime (Earman 1989). One can verify that the spacetime (considered as a singleton collection) counts as deterministic under the classical analogue of the de re formulation. And yet there are reasons to think that such a spacetime ought to be considered indeterministic. Here is Earman (1977, p. 96):

> "If the history of a particle is represented by a timelike world line (i.e. a world line which is everywhere oblique to the planes of simultaneity) on [Leibnizian spacetime], then determinism cannot hold. For among the automorphisms [of Leibnizian space-time] are those which are the identity on the portion of [Leibnizian spacetime] on or below some given time slice but which differ from the identity above; such a mapping leaves fixed the entire past history of the particles while changing their future behavior. Since the automorphisms of the space-time should be symmetries of the dynamical laws (whatever they are), there will be two solutions which describe the same particle histories for all past times but which describe different future behaviors."

Whether or not Leibnizian spacetime (or theories of matter set therein) ought to count as deterministic in some sense is an interesting question (see Stein 1977; Weatherall 2020). We do not enter the debate here. At this stage, we emphasize that we are not interested in capturing, once and for all, a single notion of "determinism" within within spacetime theories. There are many interesting forms of determinism to explore (Doboszewski 2019; Smeenk and Wüthrich 2021). In a pluralistic way, we are simply interested in mapping out which collections of general relativistic spacetimes are deterministic relative to which definitions. Here, we draw attention to the fact that there are definitions of determinism – in particular de re* – which are strong enough to capture Earman's sense in which Leibnizian spacetime is indeterministic.

Let us now return to general relativity. The isometry uniqueness clause which is added to de re determinism to obtain de re* determinism has the effect of requiring a certain type of spacetime asymmetry known as "rigidity" (Geroch 1969) which has recently become a focus of attention in the hole argument literature (Halvorson and Manchak 2022; Menon and Read 2023; Dewar 2024). Consider the following definition.

Definition 4. A collection $\mathscr{C} \subseteq \mathscr{U}$ is *rigid* if, for any $(M, g), (M', g') \in \mathscr{C}$ and any open set $O \subset M$, if there are isometries $\varphi, \psi : M \to M'$ such that $\varphi_{|O} = \psi_{|O}$, then $\varphi = \psi$.

If a collection of spacetimes is rigid, then for any pair of spacetimes in the collection, if there are isometries from one to the other that agree on an open set, they must agree entirely. It is easily verified that any collection $\mathscr{C} \subseteq \mathscr{H}$ that is both rigid and de re deterministic must be de re* deterministic. Indeed, one can think of the rigidity condition as a type of "dual" to de re determinism: the latter requires the existence of a certain isometry $\psi : M \to M'$ while the former requires (because initial segments are open sets) the uniqueness of such an isometry (Dewar 2024). One can use a general rigidity result due to Geroch (1969, Theorem A1) to show the following (Halvorson and Manchak 2022, Theorem 1).

Proposition 3. Any collection $\mathscr{C} \subseteq \mathscr{U}$ is rigid.

One way to understand the significance of the proposition is as follows: within the context of general relativity, there can be no collection of spacetimes that is de re deterministic but not de re* deterministic (cf. Cudek 2023, footnote 46). To distinguish between these two notions of determinism, one has to look to certain classical spacetime theories (eg. the Leibnizian case mentioned above) or non-standard general relativity such as its non-Hausdorff generalization (Manchak and Barrett 2023). From Proposition 3, we have the following.

Corollary 1. A collection $\mathscr{C} \subseteq \mathscr{H}$ is de re deterministic if and only if it is de re* deterministic.

Recall that $\mathscr{V}^+ \subset \mathscr{H}$ is the collection of four-dimensional, inextendible, globally hyperbolic, vacuum solutions of Einstein's equation. From Corollary 1 and Proposition 2, we have the following result which shows that large swaths of general relativistic spacetimes of physical interest satisfy a much stronger form of determinism than previously known (cf. Landsman 2022, footnote 23).

Corollary 2. Any collection $\mathscr{C} \subseteq \mathscr{V}^+$ is de re* deterministic.

4   De Dicto + Uniqueness

As we have seen, Belot (1995) naturally strengthens de re determinism (his Definition 2) by introducing a uniqueness clause to obtain de re* determinism (his Definition 3). In the analogous way, he could have also considered a natural strengthening of the de dicto form of determinism (his Definition 1). Consider the following.

Definition 5. A collection $\mathscr{C} \subseteq \mathscr{H}$ is **de dicto* deterministic** if, for any spacetimes $(M, g), (M', g') \in \mathscr{C}$ and any initial segments $U \subset M$ and $U' \subset M'$, if there is an isometry $\varphi : U \to U'$, then there is a unique isometry $\psi : M \to M'$.

We have seen that within the context of (standard) general relativity, de re determinism and de re* determinism are equivalent. This is not the case for the de dicto and de dicto* types of determinism; we will soon show that the latter condition is strictly stronger than the former. First, we establish that the de re* and de dicto* forms of MLE determinism are independent conditions in the sense that neither implies the other. Consider the following examples.

Example 2. Let (M, g) be four-dimensional Minkowski spacetime. The collection $\mathscr{C} = \{(M, g)\}$ is de re* deterministic as a consequence of Corollary 2. So the collection is de re deterministic from Corollary 1 and hence de dicto deterministic from Proposition 1. But $\mathscr{C}$ cannot be de dicto* deterministic since there are non-trivial isometries $\psi : M \to M$ of Minkowski spacetime to itself (e.g. rotations, translations).

Example 3. Let (M, g) be two-dimensional Minkowski spacetime in standard (t, x) coordinates. Let M_1 be the $t < 0$ region of M and let M_2 be the $t < x$ region of M. Consider the spacetime (M', g) where $M' = M_1 \cup M_2$ (see Figure 2). One can verify that (M', g) is a globally hyperbolic spacetime and, due to the "missing" region, the only isometry $\psi : M' \to M'$ is the identity map. It follows that the collection $\mathscr{C} = \{(M', g)\}$ must de dicto* deterministic. But the collection fails to be de re deterministic. To see this, consider the initial segment $U \subset M'$ given by the $t < -1$ region. Clearly there is a reflection isometry $\varphi : U \to U$ defined by $\varphi(t, x) = (t, -x)$. But since the only isometry $\psi : M' \to M'$ is the identity map, we find that $\psi_{|U} \neq \varphi$. Since de re determinism is not satisfied, de re* determinism is also not satisfied.
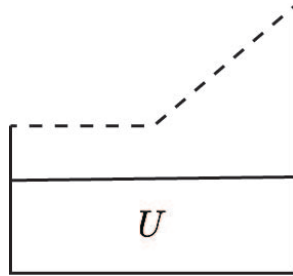

Figure 2: The spacetime (M', g) from Example 3. Because of the "missing" region (above the dotted line), the only global isometry is the identity map. But there is a non-trivial reflection isometry from the initial segment U to itself.

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Examples 2 and 3 show the unusual character of de dicto* determinism. Example 2 shows a sense in which the definition is quite strong since it will count as indeterministic some collections of spacetimes that might seem intuitively deterministic. Strong as it is, Example 3 shows a sense in which the definition is also quite weak since it does not imply either of the de re forms of MLE determinism. We see that the agreement of the initial segment U with itself given by φ does not extend to a global agreement. So the example does not satisfy the de re forms of MLE determinism. Of course, the agreement of the initial segment U with itself given by $\psi_{|U}$ does extend to a global agreement via ψ. Moreover, this global agreement is unique which ensures the example satisfies the de dicto* form of MLE determinism. Stepping back, we see that the curious state of affairs obtains because there are non-trivial symmetries of the initial segment U but no such global symmetries. An analogous situation obtains in the collapsing columns example of Wilson (1993). At least, that is the case if the columns collapse in a sufficiently asymmetric way (see Dewar 2024, footnote 28). This suggests that the de dicto* form of MLE is not strong enough by the

lights of certain haecceitist positions. In the next section, we will explore the possibility of stronger MLE definitions of determinism that have the effect of ruling out non-trivial local symmetries like φ. Here, we continue to explore the de dicto* form and its relationship to other determinism definitions.

Recall that Example 1 is the collection $\mathscr{C} = \{(M, g)\}$ where (M, g) is the $t < 0$ portion of Minkowski spacetime in standard (t, x) coordinates. Because $\mathscr{C}$ is a singleton collection, it is immediate that it is de dicto deterministic. But it is not de dicto* deterministic since there is a non-trivial global reflection isometry $\psi : M \to M$ given by $\psi(t, x) = (t, -x)$. We now see that Examples 1, 2, and 3 give us the following.

Proposition 4. If a collection $\mathscr{C} \subseteq \mathscr{H}$ is de dicto* deterministic, then it is de dicto deterministic; the other direction does not hold. Moreover, neither de dicto* determinism nor de re* determinism implies the other condition.

The entire situation so far is captured in the diagram below. Arrows correspond to implication relations. If two conditions in the diagram are not connected by an arrow (or series of arrows), then the corresponding implication relation does not hold. We emphasize that these implication relations hold within (standard) general relativity but not generally (e.g. Leibnizian spacetime is de re but not de re* deterministic).

$$
\begin{array}{ccc}
\text{(de re*)} & & \text{(de dicto*)} \\
\Updownarrow & & \Downarrow \\
\text{(de re)} & \Rightarrow & \text{(de dicto)}
\end{array}
$$

Why might one be interested in de dicto* determinism? Belot (1995) has argued the de dicto, de re, and de re* forms of MLE determinism all fail to be strong enough to count all instances of indeterminism as such. Near the end of the paper, an example is presented. Belot claims the example is deterministic according to de re* determinism (and hence the de re and de dicto forms as well) and yet it is indeterministic by the lights of certain haecceitist positions (for related examples, see Melia 1999; Pooley 2021). Here is Belot (1995, p. 193):

> In this example, W is a world with spacetime points and Newtonian spacetime structure. It initially contains a single α particle. The laws of nature decree that thirteen years later, at $t = 0$, the α particle decays into continuum many β particles; arranged so that at time t, the β particles form a spherical shell of radius t; with each β particle moving away from the center of the sphere along its radius.

A global rotational symmetry of W takes the worldline of one particle (β_1) into the worldline of another (β_2). The presence of such a symmetry supposedly allows one to make sense of the idea that "β_1 *could have been* β_2" which signals the presence of a type of indeterminism (Belot 1995, p. 194).

One certainly can object to this line of reasoning (Brighouse 1997). Indeed, it has been argued that the example is ambiguous as it stands (Halvorson et al. 2025): one natural way of making it precise renders it de re* deterministic while another way – a less natural approach using names for the β particles – renders it de re* indeterministic. We suspect that there will be some philosophers who will nonetheless want to hold that the example is de re* deterministic (as Belot claimed) and yet will still be tempted to argue that "β_1 *could have been* β_2." This is nonsense: the example is de re* deterministic only if the β particles are not named and "β_1 *could have been* β_2" makes sense only if β particles are named (Halvorson et al. 2025).

Here, we wish to highlight that even if one went along with the nonsensical line that both (i) the example is de re* deterministic and also (ii) "β_1 *could have been* β_2" is a meaningful statement, then there exist other versions of MLE determinism – e.g. de dicto* – that have the effect of ruling out this type of symmetric example within the context of general relativity. These other definitions are of independent interest since they will allow us to show that general relativity is deterministic in ways that have not been previously appreciated.

Consider any collection $\mathscr{C} \subseteq \mathscr{H}$ which counts as de dicto* deterministic. It follows that for any $(M,g),(M',g') \in \mathscr{C}$, if there is an isometry $\psi : M \to M'$, it is unique. So every isometry from a spacetime (M,g) to itself must be the identity map; all non-trivial (e.g. rotational) symmetries that take one world line into another are ruled out. This illustrates the way in which de dicto* determinism differs from de re* determinism (which permits non-trivial symmetries).

We close this section with a question that concerns a connection between de dicto* determinism and the theorem of Choquet-Bruhat and Geroch (1969). Note that because of Example 2 (the singleton collection of Minkowski spacetime), there is no analogue to Corollary 2 for de dicto* determinism: some collections $\mathscr{C} \subseteq \mathscr{V}^+$ of four-dimensional, inextendible, globally hyperbolic, vacuum solutions to Einstein's equation fail to be de dicto* deterministic. Which sub-collections of $\mathscr{V}^+$ do count as de dicto* deterministic? Consider the following asymmetry condition (cf. Barrett et al. 2023).

Definition 6. A collection $\mathscr{C} \subseteq \mathscr{U}$ is *giraffe* if, for any $(M,g),(M',g') \in \mathscr{C}$, if there are isometries $\varphi,\psi : M \to M'$, then $\varphi = \psi$.

One can verify that any collection $\mathscr{C} \subseteq \mathscr{U}$ that is giraffe must also be rigid. The other direction does not hold since the collection $\mathscr{U}$ is rigid from Proposition 3 but fails to be giraffe since it contains Minkowski spacetime. Moreover, it follows easily that any collection $\mathscr{C} \subseteq \mathscr{H}$ is de dicto* deterministic if and only if it is both de dicto deterministic and giraffe. The isometry uniqueness clause which is added to de dicto determinism to obtain de dicto* determinism has the effect of requiring collections of spacetimes be giraffe. In the same way that the rigidity condition is a type of dual to de re determinism, we see that the giraffe condition is a type of dual to de dicto determinism: the latter condition requires the existence of an isometry $\psi : M \to M'$ while the former requires its uniqueness. Because any collection $\mathscr{C} \subseteq \mathscr{H}$ that is both giraffe and de dicto deterministic must be de dicto* deterministic, Proposition 2 implies the following.

Corollary 3. Any giraffe collection $\mathscr{C} \subseteq \mathscr{V}^+$ is de dicto* deterministic.

One naturally wonders: is there a non-empty giraffe collection $\mathscr{C} \subseteq \mathscr{V}^+$? Presumably there is. The collection of all spacetimes with trivial global symmetries is a giraffe collection and there is reason to think that its spacetimes are, in some sense, "generic" among the collection $\mathscr{U}$ of all spacetimes (see Mounoud 2015, Theorem 1).

5   De Dicto/De Re + Local Uniqueness

There is a sense in which de dicto* and de re* forms of MLE determinism, even as strong as they are, could still be considered too weak by the lights of certain haecceitist positions. The de dicto* form is considered not strong enough since the classical analogue counts the collapsing columns example as deterministic. That example is counted as indeterministic under the classical analogue of the de re* form. But by the lights of certain (in our view, misguided) haecceitist positions, the de re* definition has its own problems. Belot has argued that it is also not strong enough in the sense that it wrongly counts the particle decay example as deterministic.

In both examples, symmetries are present; local symmetries are present in the initial segment of the collapsing columns example and global symmetries are present in the particle decay example. This naturally suggests a way to strengthen each definition: rule out all symmetries. We shall do this by adding a "local uniqueness" clause to both the de dicto and de re forms of MLE determinism: even parts of worlds (including initial segments) must agree in only one way.

For any spacetimes $(M, g), (M', g') \in \mathscr{U}$, let us say that a map $\psi : M \to M'$ is a (not

necessarily proper) **isometric embedding** if ψ is an isometry from M to $\psi[M]$. Consider the following local asymmetry condition (cf. Manchak and Barrett 2023).

Definition 7. A collection $\mathscr{C} \subseteq \mathscr{U}$ is **Heraclitus** if, for any $(M, g), (M', g') \in \mathscr{C}$ and any open set $O \subseteq M$, if there are isometric embeddings $\varphi, \psi : O \to M'$, then $\varphi = \psi$.

We see that the Heraclitus condition ensures that even local isometries between spacetimes must be unique. Let $\mathscr{C} \subseteq \mathscr{U}$ be any Heraclitus collection and consider any $(M, g), (M', g') \in \mathscr{C}$. For any initial segment $U \subset M$, we find there is at most one initial segment $U' \subset M'$ isometric to U. Moreover, if there is an isometry relating U and U', it is unique. One can verify that any collection that is Heraclitus must also be giraffe (and thus rigid). The other direction does not hold since the collection $\mathscr{C} = \{(M', g)\}$ from Example 3 is giraffe but fails to be Heraclitus. Because the Heraclitus asymmetry condition is the natural strengthening of the giraffe asymmetry condition, we immediately see the natural way to strengthen the de dicto* and de re* forms of determinism – at least within the context of general relativity. We have the following.

Definition 8. A collection $\mathscr{C} \subseteq \mathscr{H}$ is **de dicto** determinism** if it is de dicto and Heraclitus.

Definition 9. A collection $\mathscr{C} \subseteq \mathscr{H}$ is is **de re** determinism** if it is de re and Heraclitus.

Because the Heraclitus condition in each of the definitions ensures that even local isometries between spacetimes must be unique, it serves to rule out even the local version of a "β_1 could have been β_2" argument. Because of Examples 2 and 3 and the way we have set things up, we have the following.

Proposition 5. If a collection $\mathscr{C} \subseteq \mathscr{H}$ is de dicto** deterministic, then it is de dicto* deterministic (and thus de dicto deterministic); the other direction does not hold. If a collection $\mathscr{C} \subseteq \mathscr{H}$ is de re** deterministic, then it is de re* deterministic (and thus de re deterministic); the other direction does not hold.

Curiously, we find that de dicto** determinism and de re** determinism are equivalent. Consider the following.

Proposition 6. A collection $\mathscr{C} \subseteq \mathscr{H}$ is de dicto** deterministic if and only if it is de re**

deterministic.

Proof. One direction is trivial. Suppose a collection $\mathscr{C} \subseteq \mathscr{H}$ is de dicto** deterministic. So $\mathscr{C}$ is Heraclitus. We need only show that it is also de re deterministic. Let (M, g) and (M', g') be any spacetimes in $\mathscr{C}$ and let $U \subset M$ and $U' \subset M'$ be any initial segments. Suppose there is an isometry $\varphi : U \to U'$. Since $\mathscr{C}$ is de dicto** deterministic, it is de dicto deterministic. So there is an isometry $\psi : M \to M'$. We find that $\psi_{|U}$ and φ are both isometric embeddings of the open set U into M'. Because $\mathscr{C}$ is Heraclitus, we have $\psi_{|U} = \varphi$. Thus $\mathscr{C}$ is de re deterministic and we are done. $\qquad\square$

The entire situation is captured in the diagram below. As before, arrows correspond to implication relations. If two conditions in the diagram are not connected by an arrow (or series of arrows), then the corresponding implication relation does not hold. We emphasize that, although these implication relations hold within general relativity, there are open questions here about what the situation looks outside of this context (e.g. various classical spacetime theories).

$$
\begin{array}{ccc}
(\text{de re**}) & \Leftrightarrow & (\text{de dicto**}) \\
\Downarrow & & \Downarrow \\
(\text{de re*}) & & (\text{de dicto*}) \\
\Updownarrow & & \Downarrow \\
(\text{de re}) & \Rightarrow & (\text{de dicto})
\end{array}
$$

Despite the strength of de dicto**/de re** determinism, these definitions are still weak enough to be satisfied by non-empty collections of spacetimes. Consider the following.

Example 4. Let (M, g) be the two-dimensional spacetime constructed in Manchak and Barrett (2023). This spacetime has the "Heraclitus" property of spacetime which requires that, for any open regions $U, U' \subseteq M$, if there is an isometry $\psi : U \to U'$, then $U = U'$ and ψ is the identity map. Let (M', g) be any globally hyperbolic region of (M, g). (We know that arbitrarily small such regions exist around any point in any spacetime.) It follows easily that (M', g) inherits the Heraclitus property of spacetime from (M, g). So $\mathscr{C} = \{(M', g)\}$ must be a Heraclitus collection . But $\mathscr{C}$ is also de dicto deterministic. This follows easily since there is only one spacetime in the collection. Since $\mathscr{C}$ is both Heraclitus and de dicto deterministic, it is de dicto** deterministic. Thus, by Proposition 6, it is de re** deterministic as well.

We now consider a question analogous to one posed at the end of the previous section. It concerns a connection between de dicto**/de re** determinism and the theorem of Choquet-Bruhat and Geroch (1969). Recall that $\mathscr{V}^+$ is the collection of four-dimensional, inextendible, vacuum, globally hyperbolic spacetimes. Which sub-collections of $\mathscr{V}^+$ do count as de dicto**/de re** deterministic? Because any collection $\mathscr{C} \subseteq \mathscr{H}$ that is both Heraclitus and de dicto deterministic must be de dicto** deterministic, Propositions 2 and 6 imply the following.

Corollary 4. Any Heraclitus collection $\mathscr{C} \subseteq \mathscr{V}^+$ is de dicto** deterministic and thus de re** deterministic.

As before, one naturally wonders: is there a non-empty Heraclitus collection $\mathscr{C} \subseteq \mathscr{V}^+$? As with the giraffe case, presumably there is. The collection of all spacetimes with trivial local symmetries is a Heraclitus collection and there is reason to think that its spacetimes are, in some sense, "generic" among the collection $\mathscr{U}$ of all spacetimes (see Sunada 1985, Proposition 1).

We close by drawing attention to a strong connection between de dicto**/ de re** determinism and another (more metaphysical) way of capturing MLE determinism which requires an "intrinsic" agreement of possible worlds. This form of determinism goes by many names. It is unfortunately called "de re" by Hawthorne (2006). It is also called "Det1" by Pooley (2021) and "full" by Teitel (2019) and Halvorson et al. (2025). To avoid confusion, let us follow the latter usage: a collection of possible worlds satisfies "full determinism" if, for any worlds in the collection that "intrinsically" agree on some initial segment, the worlds "intrinsically" agree in their entirety.

It would seem that in order to make sense of "intrinsic" agreement, one must restrict attention to theories in which objects are somehow named, e.g. the β particles in Belot's particle decay example (see Halvorson et al. 2025). Due to the possibility of symmetries, collections of general relativistic spacetimes will not have such names in general. But we emphasize here that collections of general relativistic spacetimes that satisfies de dicto**/de re** determinism are necessarily Heraclitus. This means that each point in each spacetime is effectively named since isometries (even local isometries) among Heraclitus spacetimes are unique. In Example 4, one can verify that each spacetime point can be distinguished from any other via its unique invariant curvature properties. So we see that a collection of spacetimes that is sufficiently asymmetric will have sufficiently many names to make sense of the "intrinsic" agreement of possible worlds. It follows that a collection of general relativistic spacetimes that satisfies de dicto**/de re** determinism will also satisfy full

determinism (Halvorson et al. 2025). Since we know that there are collections of general relativistic spacetimes that satisfy de dicto**/de re** determinism (e.g. the one given in Example 4), we know that there are collections of general relativistic spacetimes that satisfy full determinism.

6   Conclusion

We have considered several precise formulations of MLE determinism idea: if worlds agree on some initial segment, then the worlds agree entirely. We emphasize that we have not tried to capture, once and for all, a single notion of "determinism" within general relativity. In a pluralistic way, we have simply mapped out which collections of general relativistic spacetimes are deterministic relative to which definitions. We have also clarified the relationships between the definitions and posed some open questions. Here is a summary of the situation.

The de dicto and de re formulations of MLE determinism have long played a central role in discussions of the hole argument (Butterfield 1989; Melia 1999; Pooley 2021). Within that context, de re* determinism was first introduced by Belot (1995). This definition introduces a uniqueness clause into the consequent of the de re form of MLE determinism: worlds must agree in only one way. Within this context, this uniqueness clause amounts to the asymmetry condition of rigidity (Geroch 1969) which has recently become a focus of attention in the hole argument literature (Halvorson and Manchak 2022; Menon and Read 2023; Dewar 2024). Unlike the de dicto and de re forms of determinism, the de re* form is strong enough to capture the intuition of Earman (1977) that Leibnizian spacetime is indeterministic. Even so, by the lights of certain haecceitist positions, de re* determinism is still too weak in the sense that Belot's particle decay example is wrongly counted as deterministic under this form.

One can certainly object to the significance of the particle decay example (Brighouse 1997). Indeed, it has been argued that one cannot hold both of the following: (i) the example counts as de re* deterministic and (ii) it makes sense to say that "β_1 *could have been β_2*" (Halvorson et al. 2025). But in this paper, we have explored an altogether different type of response. We have looked for even stronger versions of MLE determinism to rule out such examples. We first considered de dicto* determinism which is the analogue to re de* determinism. This definition introduces a uniqueness clause which amounts to the giraffe asymmetry condition (Barrett et al. 2023). We found that de dicto* determinism is neither stronger nor weaker than de re* determinism but that it is strong enough to rule out the globally symmetric problem cases (e.g. the particle decay example) central to the hole argument literature. Even so, it still may not be strong enough by the lights of certain

haecceitist positions: some locally symmetric examples that seem to be indeterministic (e.g. the collapsing columns) are not counted as such under the de dicto* form.

We then introduced a local uniqueness clause into both the de dicto and de re forms of determinism: even parts of worlds (including initial segments) must agree in only one way. This local uniqueness clause amounts to the Heraclitus asymmetry condition (Manchak and Barrett 2023). The resulting definitions of de dicto** and de re** determinism turn out to be equivalent and also seem to be sufficiently strong to rule out, not only the globally symmetric problem cases, but locally symmetric variants as well. We have shown that the satisfaction of de dicto**/re re** determinism by a collection of general relativistic spacetimes implies that each point in each spacetime is effectively named since isometries (even local isometries) among Heraclitus spacetimes are unique. This allows one to make sense of "intrinsic" agreement of possible worlds which is required by "full" determinism (Hawthorne 2006; Teitel 2019; Pooley 2021). It follows that any collection of general relativistic spacetimes that satisfies de dicto**/de re** determinism will also satisfy full determinism. We have shown that de dicto**/de re** determinism, strong as it is, is weak enough to be satisfied by some non-empty collections of general relativistic spacetimes. It is an open question whether there are large swaths of general relativistic spacetimes of physical interest that are de dicto**/de re** deterministic. Perhaps there are given that the Heraclitus condition seems to be generically satisfied (see Sunada 1985, Proposition 1).

Acknowledgments

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