Parallel Convergences: Cassirer and Vienna Indeterminism

Marco Giovanelli

Università di Torino
Department of Philosophy and Educational Sciences
Via S. Ottavio, 20 10124 - Torino, Italy

`marco.giovanelli@unito.it`

Stöltzner coined the expression 'Vienna indeterminism' to describe a philosophical tradition centered on the Viennese physicist Exner, serving as the 'historical link' between Mach and Boltzmann, on the one hand, and von Mises and Frank, on the other. During the early 1930s debate on quantum mechanics, there was a 'rapprochement' between Vienna indeterminism and Schlick's work on causality. However, it was Cassirer's 1936 monograph *Determinismus und Indeterminismus* that shows a full 'convergence' with major tenets of Vienna indeterminism: the fundamentality of statistical laws, the frequency interpretation of probability, and the statistical interpretation of the uncertainty relations. Yet, Cassirer used these conceptual tools to pursue 'in parallel' different philosophical goals. While for the Viennese quantum mechanics represented a fatal blow to the already discredited notion of 'causality,' for Cassirer it challenged the classical notion of 'substantiality,' the ideas of 'particles' as individual substances endowed with properties. The paper concludes that this 'parallel convergence' is the most striking and overlooked aspect of *Determinismus und Indeterminismus*, serving as the keystone of its argumentative structure.

Keywords: Ernst Cassirer • Determinism/Indeterminism • Individuality in Physics • Franz S. Exner • Philipp Frank • Richard von Mises • Quantum Mechanics

Introduction

At the turn of the 2000s, Michael Stöltzner (1999) introduced the category 'Vienna indeterminism' to describe a coherent philosophical tradition centered around the Viennese physicist Franz Serafin Exner. Starting from his 1908 inaugural address as the new rector of the University of Vienna, Exner (1909) provided a philosophical synthesis of Ernst Mach's empiricism, Ludwig Boltzmann's atomism, and Gustav T. Fechner's relative frequency interpretation of probability (Stöltzner 2002, 268). Max Planck (1914) could still be confident that Boltzmann's statistical mechanics would ultimately be integrated into a deterministic worldview (Stöltzner 2002, 270). By adopting Mach's definition of causality in terms of functional dependencies, Exner ventured to claim the universal applicability of Boltzmann's probabilistic approach (282). Exner's (1919; 1922) lectures on the 'physical foundations of the sciences,' written during the war, articulated this view in detail, launching a frontal attack against the Kantian *a priori* validity of the principle of causality. Thus, Exner served as the 'historical link' between the older generation of Viennese philosopher-physicists, Mach and Boltzmann, and the younger generation, Erwin Schrödinger (1929b), Richard von Mises (1928, 1930), and Philipp Frank (1932), who developed and promoted Exner's synthesis to wider audiences (Stöltzner 2003b, 3).

Through Frank and von Mises, Exner's indeterminism percolated into the Vienna Circle, led by Planck's former student Moritz Schlick. According to Stöltzner (2008), in his 1931 article on causality in quantum mechanics, Schlick shifted from his previous quasi-Kantian stance (Schlick 1920b), showing a certain *rapprochement* with Vienna indeterminism (Stöltzner 2003a, 22). Nevertheless, the détente did not develop into a full alliance. Indeed, as Schrödinger complained in private correspondence, Schlick did not even bother to mention Exner (Schrödinger to Schlick, Feb. 25, 1931; Schrödinger to Schlick, Mar. 31, 1931, Schlick Nachlass). On the contrary, Stöltzner (2003a, 22) points out that a more substantial *convergence* took place between Ernst Cassirer and Vienna indeterminism, the last representative of what was once the influential Marburg school of neo-Kantianism. In fact, as this paper will show, in his 1936 *Determinismus und Indeterminismus*, Cassirer seems to enthusiastically embrace all the major tenets of Vienna indeterminism that Schlick rejected: (a) Exner's (1919) hypothesis that fundamental laws of nature might be only statistical, (b) von Mises's (1928) frequentist interpretation of probability, (c) Frank's (1932) and von Mises's (1930, 1934) statistical interpretation of Heisenberg's uncertainty relations.

Stöltzner refrains from further discussion of Cassirer's work, and the strange case of the convergence between Vienna indeterminism and late Marburg neo-Kantianism has, as far as I can see, remained unexplored. Literature on the relationship between Cassirer and the Vienna Circle (Krois 2000; Mormann 2012) usually focuses on the similarities with Schlick and Carnap's 'structuralism' (Richardson 1998; Gower 2000; Neuber 2013) rather than on Frank and von Mises's 'indeterminism.' The literature on Cassirer's interpretation of quantum mechanics does not escape this interpretative framework (Cei and French 2009; French 2014, sec. 4.8; Ryckman 2015, 2018, 2021). *Determinismus und Indeterminismus*, although with different nuances, is portrayed as the culmination of the same 'structuralist' approach to classical physics that Cassirer defended in his previous epistemological monographs (Cassirer 1910, 1921, 1929). This continuist narrative, although endorsed by Cassirer himself, leaves an important part of the story untold, in my view. Indeed, upon closer inspection, the over 60-year-old Cassirer engages with the new quantum mechanics against the background of a substantially *revised* image of classical physics. In particular, in *Determinismus und Indeterminismus*, Cassirer tackles issues like irreversibility and statistical mechanics, on which he had been surprisingly reticent in his earlier writings. By confronting these problems, Cassirer encountered the works of Exner and his circle (see Karlik and Schmid 1982), which had remained completely foreign not only to his prior research but to neo-Kantianism more broadly.

This paper contends that Cassirer's *Nostrifizierung* of Vienna indeterminism is arguably the most striking aspect of *Determinismus und Indeterminismus*. For this reason, it can be used as Ariadne's thread for finding a way out of the somewhat labyrinthine argumentative structure of the book. In particular, it will be shown how Cassirer appropriated most of the key elements of Vienna indeterminism in pursuit of a very different agenda. The Viennese discussion on quantum mechanics centered around the status of the notion of *causality* in physics. Cassirer exploited the conceptual tools that the Viennese had put forward to shift the discussion surrounding the problem of *substantiality* in physics, specifically the conception of particles as individual substances bearing properties (see French and Krause 2006, sec. 3.7). In this way, Cassirer could attempt to converge with Vienna indeterminism while simultaneously aiming to continue along the "historical tendency of 'neo-Kantianism' as envisaged by the founders of the Marburg School, Hermann Cohen and Paul Natorp" (Cassirer 1936, VIII; tr. xxiii). One should speak of a sort of *parallel convergence*[1] The paper concludes that this endeavor put a significant strain on the framework of Cassirer's neo-Kantianism (VIII; tr. xxiii). As Cassirer anticipated, many indeed expressed "their agreement" about the conclusions of the book but questioned whether a 'neo-Kantian' "was permitted to draw such conclusions" (VIII; tr. xxiii).

The rationale behind Cassirer's curious strategy of exploiting the Exner-Frank-Mises 'anticausality' in defense of a broadly defined concept of 'causality' lies in his motivations for writing *Determinismus und Indeterminismus*. Cassirer seems to have planned to write on the new quantum theory before leaving Germany in 1933 (Cassirer Bondy 1981, 189), during a time of progressive destabilization of political and cultural institutions in the late Weimar Republic. Cassirer resolutely dismissed the claim that the *emergence* of quantum mechanics as a physical theory was milieu-dependent,[2] conditioned by the Weimar intellectual environment, which was hostile to mechanical causality (Forman 1971; Kraft and Kroes 1984). However, Cassirer was clearly concerned that the *reception* of quantum theory among educated general audiences was indeed milieu-dependent, influenced by the prevailing irrationalist, *lebensphilosophisch*, cultural tendencies of 1930s Germany (Cassirer 1930, 1933).[3] Despite showcasing Cassirer's substantial effort to master the technicalities of the new quantum physics, *Determinismus und Indeterminismus* was written against the backdrop of a broader cultural struggle.

The new theory challenged the notions of 'causality' (*Kausalität*), pictoriability (*Anschaulichkeit*), and individuality (*Individualität*) in their classical form (Forman 1984). According to Cassirer, the anti-scientific and anti-intellectual tendencies in the Weimar Republic sought to exploit these modifications of the classical world-picture in the name of reactionary cultural, if not political, agendas, often permeated by antisemitic undertones.[4] The aim of Cassirer's book was to show that, upon closer inspection, in each of these three instances, quantum mechanics should be considered nothing but a continuation of trends already present in classical physics. If at all, it was not the abandonment of 'causality,' but that of 'individuality,' where quantum mechanics appears to have created a stronger 'epistemological rupture.' However, Cassirer, as usual, immediately mends the rupture: even in classical physics, the concept of the 'individual' material point was at most a useful conceptual 'postulate,' rather than an unavoidable empirical 'fact.' Quantum mechanics has only shown that this postulate can be dropped if experience forces us to do so. As Cassirer put it in the closing of

---

[1] This admittedly paradoxical expression 'parallel converges' was once used by politician Aldo Moro, referring to a possible alliance between Communists and Christian Democrats in 1970s Italy.

[2] The expression *milieubedingt* was introduced by Schrödinger (1932).

[3] It is interesting to note that the role attributed to 'Vienna indeterminism' is a key distinction between the so-called 'Forman thesis' and what one might call the 'Cassirer thesis.' Paul Forman (1971) downplays Vienna indeterminism as a "subterranean anticausality current" (Forman 1971, 67; see Stöltzner 2011). In contrast, Cassirer elevates it to a fundamental tendency within the history of classical physics. Thus, from Cassirer's standpoint, portraying quantum theory as an unexpected irruption of 'irrationalism' in the history of physics ultimately results from ignoring the pre-quantum causality debate. I thank an anonymous referee for bringing this point to my attention.

[4] A similar concern motivated Cassirer to write on Einstein's theory (Cassirer to Einstein, Aug. 28, 1920; Einstein 1987–, Vol. 10, Doc. 112).

the book: “atomic physics has not destroyed the bases on which physical knowledge rests; rather it has made them known more clearly than ever before” (Cassirer 1936, 245; tr. 196).

1 The Principle of Causality: Laplace vs. Helmholtz

As was common in the literature of that time (see, *e.g.*, Weyl 1932, 33f. Frank 1932, chap. 2; Hermann 1935, secc. 1 and 11), Cassirer opens *Determinismus und Indeterminismus* by paying lip service to the infamous Laplace ‘demon’ or ‘spirit’ (*Geist*) (see Strien 2014). In the introduction of his *Théorie analytique des probabilités* (Laplace 1814a, IX; also see Laplace 1814b, 2f.), Laplace imagined an intelligence that knows with arbitrary accuracy the *initial state* of the world as a whole, say, the position q_0 and momentum p_0 of every point particle in the universe at the time t_0.[5] Thus, on the basis of the *laws* of classical mechanics—that is, given the ‘Hamiltonian function’ $H(p, q)$ of the universe—the demon could, in principle, calculate the values of the state variables q_t and p_t, at any t with equally arbitrary accuracy: “The human mind may be seen as the copy, though weak, of such a spirit when one considers the completeness to which it has brought astronomy, but it will certainly never reach the perfection of its original. No matter how great the effort to approach it, human understanding will always remain infinitely far behind” (Cassirer 1936, 7; tr. 3).

‘Laplace demon’ was (and often still is) usually considered the paradigmatic formulation of the principle of causality in classical physics. However, Cassirer immediately warns his readers that he does not agree with this characterization: “I begin with this picture of the Laplacean spirit, not because I consider this introduction logically appropriate or particularly suitable psychologically, but for exactly the opposite reason” (7; tr. 3). For Laplace, the ‘demon’ was merely a clever parable illustrating the distinction between probability and certainty. According to Cassirer, Laplace did not intend to turn it into a representation of a general principle of causality; indeed, initially, the parable did not attract much attention (8f.; tr. 4f.). It was Emil Du Bois-Reymond, in his famous 1872 address *Über die Grenzen des Naturerkennens,* who first rescued Laplace’s image from oblivion. Since then, in all discussions on the general problem of causality sparked by recent developments in atomic physics, Laplace’s demon has played a crucial role: “The defenders as well as the attackers of the causality principle of classical physics seemed to agree at least in this respect, that this picture may be taken as an adequate expression of the problem, that one may use it without hesitation in order to clarify the nature of a strictly deterministic view of the world” (Cassirer 1936, 7; tr. 3).

However, according to Cassirer, by turning Laplace’s image into the formulation of the general principle of causality, Du Bois-Reymond fundamentally distorted the very formulation of the problem: “For how shall we think the condition fulfilled on which the foresight of the Laplacean spirit depends? How should he have obtained the complete knowledge of the initial positions and velocities of all particles? Does he attain to this knowledge in a human or in a ‘superhuman’ fashion, in an empirical or in a ‘transcendent’ manner?” (15; tr. 9; slightly modified). In the first case, even for him, the conditions that apply to our empirical knowledge would not be lifted. Measurements would have to be conducted, and all measurements have finite precision; they yield

---

[5] A system with n particles is described by $2n$ variables (q_i, p_i) where $i = 1, \ldots, n$; henceforth, the subscript index is dropped for simplicity.

only a decimal number with finitely many digits. Speaking of a precise knowledge of the initial conditions q_0 or p_0 at t_0 does not have any absolute meaning and depends on the problem at stake.

This difficulty can be circumvented only "if we ascribe to the Laplacean intelligence not merely a mediate but an immediate, an 'intuitive,' knowledge of the initial conditions" (Cassirer 1936, 15; tr. 9). However, in this way, the very distinction between laws and initial conditions would collapse. This intelligence would not need to integrate Hamilton's equations to know the trajectory of all particles but would 'see' them by direct intuition: "For an intelligence equipped with such intuitive knowledge would be spared all the pains of mediate inference and calculation. It would not need to 'conclude' from the present to the past or future. It would possess, in one single, undivided act, a complete comprehension, an immediate intuition of the whole time series and of its infinite extent" (15; tr. 9). From this point of view, a chaotic world that the demon knows with absolute precision would be just as 'causally determined' as a law-like world (see Schlick 1920a, 465).

Cassirer can then conclude that in the idea of the Laplace demon, "two heterogeneous and mutually incompatible tendencies" coexist (Cassirer 1936, 15f.; tr. 9). In Kantian terms, one can say that the demon is at the same time a 'discursive' and an 'intuitive' understanding, an *intellectus archetypus* and the *intellectus ectypus*, an understanding that is bound to the form of *mediate* comprehension and an understanding that has access to an intuitive *immediate* knowledge that can dispense with all calculation. By abolishing the distinction between laws and initial conditions, the image of the Laplace demon abolishes the distinction between 'necessary' and 'contingent' that characterizes modern philosophy and modern physics (124–128; tr. 99–103s). *Fatalism* is confused with *determinism*, destiny with causality, blurring the opposition "between the mythical worldview and the empirical and theoretical worldview" (123; tr. 99; slightly modified). The mythical world is dominated by a sense of *inevitability*, by the idea that nothing can change what happens. The scientific world-image is based on *control*; it implies that by freely changing the initial conditions, the outcome can be changed (see Cassirer 1946, chap. XVIII).

This confusion dissolves if the principle of causality is properly understood not as a statement about the nature of reality (metaphysical determinism) but as a statement about the structure of scientific experience (critical determinism) (Cassirer 1936, 17-34; tr. 11–25). For the neo-Kantian Cassirer, it was, of course, Kant's merit to have transformed 'causality' from a metaphysical into a methodological principle. This key insight, in Cassirer's view, can still be maintained in the face of the advances of contemporary physics. In part 2 of the book, Cassirer famously argues that the structure of physical theories (mechanics, electrodynamics, etc.) is articulated on different levels of statements, each of which constrains without determining the lower ones: statements of measurement, statements of law, statements of principle (see Ryckman 2015, sec. 4). The principle of causality is a statement of still a higher order, a statement *a priori*, that lies beyond the scope of the three classes of statements:

> What is the significance of the causal principle and what new insight does it add to what we have already learned from the foregoing epistemological analysis?[. . .] I would like to give an answer to this question, which at first sight will perhaps seem paradoxical. *There is in fact nothing left over* [. . .] For us the causal principle belongs to a new type of physical statement, insofar as it is a statement about measurements, laws, and principles. It says that all these can be so related and combined with one another that from this combination there results a system of physical knowledge and not a mere aggregate of

> isolated observations [...] But the search after ever more general laws is a basic feature, a regulative principle of our thought. It is precisely this regulative principle, and nothing else, that we call the causal law. In this sense it is given *a priori*, it is a transcendental law: for a proof of it from experience is not possible. It is true on the other hand, however, that we have no other warrant for its applicability than its success. (Cassirer 1936, 75ff.; tr. 60ff.)

Cassirer can still maintain the Kantian characterization of the principle of causality as a principle *a priori*; however, he clearly modifies the meaning of *a priori* not only with respect to Kant but also to the Marburg school of neo-Kantianism, of which he still considers himself a proud member (VIII; tr. xxiii). Cassirer's principle of causality is a *regulative*, rather than a *constitutive* principle. It does not impose any specific constraint on the *structure* of the laws of nature; it only demands that we never abandon the *search* for laws of increasing generality, whatever they may be.[6]

Modern physics forces us to question Du Bois-Reymond's (1872) conception of causality as the requirement of complete *prediction* about the future states of a physical system.[7] This conception is personified by Laplace's demon (Cassirer 1936, 79; tr. 63). However, one should by no means simply give up the principle of causality; rather, one should appeal to a different and more adequate epistemological form of the principle (see Cassirer 1939; 1944, 236). One does not have to look far (see Ryckman 2015, sec. 3). At around the same time, none other than Hermann von Helmholtz, in his 1878 address *Die Tatsachen in der Wahrnehmung,* provided an alternative definition. Helmholtz presented the principle of causality as the requirement of complete *lawlikeness* (*Gesetzmäßigkeit*) between the successive states of a system. Unfortunately, "in modern discussions of the causal problem the name of Laplace is met with almost constantly, the name of du Bois-Reymond very often, but the name of Helmholtz seldom or never" (Cassirer 1936, 77; tr. 61).

In Helmholtz's (1879) mature formulation, the principle of causality was a maxim, the imperative that we never abandon the search for increasingly more general and comprehensive 'laws' (Cassirer 1936, 78; tr. 62). One cannot expect a proof of the principle of causality in the usual sense, whether it be logical, which would necessarily be empty, or empirical, which would lead to an inescapable circle. Helmholtz's piece of advice is simply: 'Trust and act!' (*Vertraue und handle!*) (Helmholtz 1879, 42). The *search* for the laws of nature would be meaningless without the *belief* in the lawlikeness of nature: "What Helmholtz demands and what he regards as the necessary and sufficient condition for the validity of the principle is precisely the gradation of knowledge that we attempted to present in detail: the procedure from experimental

---

[6] The claim that the causality principle is a 'regulative principle' was also defended at about the same time by Schlick (1931, 154) and the young Popper (1935, sec. 78), albeit with varying shades of meaning. For Schlick, the principle of causality is not a 'statement' that can be true or false, but a 'demand' to search for *predictive* laws (see also Schlick 1936). However, it is not a 'postulate,' that is, a rule to which we must *always* adhere. Indeed, experience, as in the case of quantum mechanics, can show us that the principle has become useless in certain domains. On the contrary, for Popper, the principle of causality is a methodological postulate to never abandon the search for *dynamical* laws (even after quantum mechanics). As we shall see, for Cassirer, the principle of causality is a postulate to never abandon the search for laws *in general*, be they dynamical or statistical.

[7] Cassirer explicitly rejects Schlick's (1931) identification between 'causality' and 'prediction' (Cassirer 1936, 79; tr. 63). He adds a passing reference to Grete Hermann's (1935) criticism of 'predictive causality' (Cassirer 1936, 82fn1; tr. 64fn13). However, he does not show interest in Hermann's 'rectrodictive causality' thesis. For a collection of essays on Hermann's work and English translations of the latter, see Crull and Bacciagaluppi 2016.

findings and their exact formulation to ever stricter statements of laws and ever more general statements of principles" (Cassirer 1936, 79; tr. 63).

2 Cassirer's Appropriation of Vienna Indeterminism

In the first two parts of *Determinismus und Indeterminismus*, Cassirer seems to be mainly concerned with guiding his readers in transitioning from the Laplacian to the Helmholtzian understanding of the principle of causality: "The constitutive, essential characteristic of causality consists in the general requirement of order according to law, not in instructions as to how this order can be discovered and followed through in detail" (Cassirer 1936, 203; tr. 163). The principle states that we must search for the laws of nature, but it does not specify which kind of laws we have to search for. With this more flexible version of the causality principle in hand, Cassirer could devote the third part of the book to reframing the issue of the relationship between determinism and indeterminism in classical physics. This part includes Cassirer's direct confrontation with Vienna indeterminism. It is, in my view, one of the most remarkable sections in the book, as it presents an image of classical physics that has no counterpart in young Cassirer's epistemological monographs.

2.1 Exner and the Fundamentality of Statistical Laws

In *Determinismus und indeterminismus*, Cassirer, as one might expect, gives great emphasis to the fundamental shift from dynamical to statistical laws that took place in the history of 19th-century physics—an issue that he had surprisingly not addressed in his previous writings. Following roughly Planck's (1910a) historical account, Cassirer points out that by the end of the 19th century, Helmholtz was able to show that the laws of all *reversible* processes—mechanical, electromagnetic, etc.—are governed by dynamical laws that could all be derived from the principle of least action. However, Rudolf Clausius's principle of entropy regulating *irreversible* processes introduced a "foreign and intrusive element" (Cassirer 1936, 95; tr. 76) into the system of classical mechanics and electrodynamics. The gap between reversible and irreversible processes was bridged by Boltzmann's definition of entropy as probability, as a statistical law pertaining to the mixture of large numbers of particles. The dualism between 'reversible' and 'irreversible' processes was reframed into the dualism of 'dynamical' and 'statistical' laws:

> Boltzmann gave it exact form in his law that entropy is proportional to the logarithm of the probability ($S = k \log W$) This approach, however, did not solve the riddle epistemologically but only reiterated it more emphatically. For Boltzmann's solution was successful only by introducing a new kind of physical conformity to law and by giving it equal rank with 'dynamic' laws. The probability laws on which he based the kinetic theory of gases, however, do not have the same epistemological quality and 'dignity' that had previously been ascribed to the laws of nature. [...] The statistical procedure was applied solely to the formulation of initial conditions, whereas the further course of events was regarded as dominated completely by strict dynamic laws, the laws of conservation of energy and of momentum during molecular collisions. (96; tr. 77f.)

Boltzmann's H-theorem, which sought to connect mechanics with the second law of thermodynamics, faced serious objections (*e.g.*, Loschmidt's 'reversibility objection' and Zermelo's 'recurrence objection') (98; tr. 79). Indeed, Boltzmann conceded that the 'one-sidedness' of reversible processes expressed by the second law of thermodynamics could not be derived solely from the *laws* of mechanics. It required an assumption about the *initial conditions*, namely, that the universe started in an improbable initial state (Cassirer 1936, 98; tr. 79). However, this assumption cannot be logically or empirically proved: "The special statistical innovations [*Ansätze*] from which Boltzmann set out in the construction of his theory thus retained a precarious and not strictly demonstrable character" (98; tr. 79).

Boltzmann's revolution was an 'unfinished revolution.' As Planck (1914) insisted in his famous 1914 address *Dynamische und statistische Gesetzmäßigkeit,* Boltzmann's result was not incompatible with a belief in the fundamentality of dynamical laws. Physicists use statistical methods due to uncertainty in a system's initial conditions, but probability calculus still assumes that the latter, albeit unknown, is well defined. As Cassirer points out, it was Exner (1919) who, in direct polemic against Planck's *a priori* assumption, outlined the opposite program. The fact that macroscopic phenomena appear to be regulated by dynamical laws is the consequence of the 'law of large numbers'; nothing excludes that the laws governing elementary processes are only statistical (712f.). The advent of quantum mechanics has transformed Exner's program from a speculative hypothesis into a concrete possibility: "the attempt made by Franz Exner in his *Vorlesungen über die physikalischen Grundlagen der Naturwissenschaften* (1919) [...] is of special significance in the development of recent quantum mechanics, inasmuch as Schrödinger in his inaugural address in Zürich in 1922 formulated his own basic view with reference to it" (Cassirer 1936, 99; tr. 79). Indeed, as a student of Exner, Schrödinger (1929b, 1929a) was the first to promote Exner's program outside of Vienna and to insist that it anticipated the turn that physics has taken regarding the question of causality (Schrödinger 1932; see Stöltzner 2012).

Cassirer immediately clarifies that "[t]he validity of strictly universal scientific laws was not denied by Exner, but he declared them problematical" (Cassirer 1936, 101; tr. 81). It is generally agreed that the laws of kinetic gas theory do not have absolute or exact validity, but only statistical validity. However, it is not obvious that, say, Galileo's law of parabolic motion of projectiles is an exact law. Classical physics makes the *assumption* that if the conditions of the experiment could be fixed precisely, say, q_0 and p_0, then the trajectory of the projectile $q(t)$ and $p(t)$ could be predicted exactly once one knows $H(q, p)$. Exner, however, argued that there is no contradiction in making the opposite *assumption*: that even with continuous improvements in experimental methods, it would still be impossible to reduce the variability of final results to any arbitrary degree by simply controlling the initial conditions: "It is possible that although we formerly believed we were dealing with natural laws that had absolute validity, actually we are dealing only with laws of averages which lose their validity in sufficiently minute ranges of time and space. Dogmatic assertion, at least, is here no more justified than dogmatic negation" (102; tr. 81f.)

Planck (1914)'s 'Berlin determinism' was based on the assumption that, if the initial conditions of the experiment could be fixed with precision and the statistical 'dispersion' of the measurement results could be eliminated, it would lead to the formulation of dynamical laws valid in the ideal case. Exner's (1919) 'Vienna indeterminism' argues that the opposite assumption is just as legitimate: that dispersion can never be reduced below a defined limit that could be fixed by a natural constant[8]:

---

[8] The argument cannot be found in Exner's work in this form. Cassirer's source seems to be von Mises (1930) or Frank (1932, 162), who wrote after quantum mechanics was established. Reichenbach (1930) made a similar point independently. See Stöltzner 2009 for more detail.

> On the basis of this distinction, Exner's attempt to formulate anew the concept of natural law contains a speculative or, better, a purely methodological characteristic. Exner did not lean on new empirical facts; and considering the prevailing state of research at the time, he could hardly have found an adequate justification in the merely factual state of scientific knowledge. What moved Exner was, above all, his interest in reason, which caused him to object to the indissoluble dualism of dynamic and statistical laws. A way out of this dualism did not seem to be available so long as one held to the current view, so long as one regarded dynamic laws as the proper and indispensable foundation of all genuine scientific knowledge. For it seemed impossible to carry through the idea that statistical laws can be viewed as merely provisional ones, replaceable later by laws of dynamics. [. . .] Therefore if unity was again to be secured, it could be so only by reversing the procedure: statistical law must be regarded as the comprehensive genus, a concept of *higher order* [*überordnet*] than that of dynamic laws and including them as a special case. This was the thesis advocated and defended by Exner in a new and original way. (Cassirer 1936, 101; tr. 81)

Exner's point, Cassirer emphasizes, was not historical but purely systematic. Exner did not deny that classical physics achieved its greatest successes through dynamical laws, as in the case of celestial mechanics. Exner, one might say, was 'just asking questions': "But must we forever adhere to this particular method of research? Is it not rather advisable to remember that other ways are also possible and practicable and that the time may come in which physics will see itself definitely forced to take the step from dynamic to statistical laws?" (103; tr. 84). Exner did not dispute that, as a matter of fact, physics is based on the assumption that dynamical laws are fundamental; what he disputed is the alleged *necessity* of this assumption. It is true that "[t]he factum of classical physics cannot be removed or upset by the presentation of mere possibilities" (106; tr. 85). However, the very existence of these mere possibilities reveals that the 'fact of science' from which critical philosophy is supposed to take its cue is fundamentally *contingent*, and one cannot exclude that it might change (124–129; tr. 100–103).

2.2 Von Mises and the Frequency Interpretation of Probability

At first sight, Cassirer's appeal to Exner's empiricist 'openness' to statistical laws against Planck's *a priori* 'restriction' to dynamical laws seems a strange philosophical move for a 'neo-Kantian.' One would have expected a Marburg-Berlin rather than a Marburg-Vienna convergence. However, the arc of Cassirer's argumentative strategy was broader than that of the less sophisticated defenders of the 'apriority' of the causality principle (see Bergmann 1929). Cassirer needed to first recognize statistical and dynamical laws as two different, equally legitimate types of laws. In this way, he could argue that the opposition between indeterminism and determinism was ill-posed. As we have seen, for Cassirer, the principle of causality is nothing but the imperative never to abandon the *search* for progressively more encompassing laws; the principle does not impose any preference for dynamical laws over statistical laws or vice versa (cf. Popper 1935, sec. 78).

The apparent conflict between the Berlin and Vienna research programs in the name of the principle of causality arises from the implicit reliance on the 'Laplacean' concept of causality rather than the 'Helmholtzian' one. Planck and Exner agreed on the existence of two "different basic types of law" (Cassirer 1936, 111; tr. 89). However, they considered that "dynamic and statistical laws were not regarded as two complementary methods and directions, as two different modes of description; they were instead opposed as the 'determined' and the 'undetermined'" (111; tr. 89). In this way, the debate was framed in terms of the opposition between a 'deterministic' and an ‘indeterministic’ metaphysics that, in Cassirer’s view, “gives rise to the most dangerous equivocations” (Cassirer 1936, 112; tr. 89).

According to Cassirer, the problem of the relations between statistical and dynamical laws can only be resolved by clarifying the true nature of the problem that physics associates with the concept of ‘probability.’ Cassirer briefly presents the different interpretations of the concept of probability that were considered at that time: Laplace’s (1812) classical interpretation of probability as the fraction of the total number of possible cases; Keynes’s (1921) subjective interpretation as ‘degree’ of belief; and von Mises’s (1928) objective interpretation in terms of observed frequencies. Cassirer examines the various pros and cons and swiftly concludes that a clear winner emerges: “It seems to me that of the modern theorists of probability, von Mises has offered the simplest and most consistent solution” (Cassirer 1936, 117; tr. 94). Once again, Cassirer’s choice is counter to the usual philosophical classifications. The neo-Kantian Cassirer embraced von Mises’s (1919) frequentist *a posteriori* interpretation of probability, which was empiricist-positivist in its inspiration; by contrast, the empiricist Schlick defended von Kries’s (1919) logical-objective *a priori* ‘range theory of probability’ stemming from the Kantian tradition (Schlick 1931; Waismann 1930–31; see Heidelberger 2001).

As is well known, von Mises (1919, 1928) bases his theory on the concept of ‘collective’ (*Kollektiv*). A collective is a sequence of events or occurrences that can, in principle, be continued indefinitely. Each of these events has a certain property, P. The relative frequency of P is the number of times P occurs up to the n-th element. One can then take the sequence of the relative frequencies of P. Probability is defined as the *limit* of relative frequencies as the number of trials tends to infinity. However, not all series of events form a ‘collective.’ To be treated as such, they must satisfy specific axiomatic requirements: the limit axiom and the randomness axiom. It was not Cassirer’s style to engage in a technical discussion of the status of these axioms, as attempted, for example, by Reichenbach (1932) and Popper (1935) around that time (Cassirer 1936, 120; tr. 96).[9] In order to ‘nostrify’ the notion of ‘collective,’ Cassirer needed only to follow von Mises in presenting statistical collectives as instances of ‘idealizations’ commonly used in physics, like the ‘perfect gas’ or ‘massless spring’:

> When von Mises explains a collective as being a mass phenomenon, or a repetitive process, an extended sequence of individual observations appearing to justify the assumption that the relative frequency of the occurrence of each particular observed feature tends toward a definite limiting value, he is emphasizing that such a collective is not an empirical object but an idealized conception similar to that of the sphere in geometry or of the rigid body in mechanics [. . .] A collective as such contains no inaccuracy within itself; rather, it consists of a series of intrinsically exact observations. In any case, statistics can only begin where univocal and precise observations are available. It is true that concrete statistics never lead to numerical series appearing immediately and exactly as collectives in the sense of the ideal concept here indicated; this, however, does not matter. What does matter is solely the discovery of those applications of probability theory based on this concept that are possible in the realm of empirical events. (117f.; tr. 94)

Von Mises argues that a finite empirical collective is represented mathematically by an infinite collective. Von Mises’s critics in Vienna questioned the legitimacy of this representation of the large finite by the infinite (Waismann 1930–31, 230f.). However, according to Cassirer, this objection is without merit. The epistemological justification

---

[9] Reichenbach rejected the idea of random sequences, while Popper tried to eliminate the limit axiom. A discussion of this point goes beyond the scope of this paper.

for the abstract notion of ‘collective’ is not dissimilar to that of any idealized theoretical entites used in physics. A collective is as little found directly in experience as a ‘body left to itself’ that moves indefinitely at constant velocity can be found in experience: “The objects of statistical statements are mass phenomena or recurring events, but it does not follow that the statement as such consists of a direct description of what is observed in these phenomena” (Cassirer 1936, 119; tr. 95f.). The increasing stability of statistical frequencies can be *observed*; however, the convergence of sequences to a limiting distribution must be *postulated*—as even empiricists like Reichenbach (1916, 1930) could not avoid conceding (Cassirer 1936, 120–122; tr. 96–98).

Contrary to Reichenbach (1930), Cassirer shows no interest in extending the frequency interpretation of the ‘probability of events’ to the ‘probability of hypotheses.’ The abandonment of classical logic in favor of three-valued logic appeared to Cassirer as unnecessarily far-fetched (Cassirer 1936, 116f.; tr. 93). Cassirer’s interest in the frequency interpretation lies in the fact that it demonstrates the distinction between deterministic and statistical laws as, at most, a gradual one. At first sight, Cassirer points out, the very idea of a statistical law seems to be a paradox (see Schlick 1931, sec. 10), what the Germans like to call a ‘wooden iron’ (*hölzernes Eisen*) (Cassirer 1936, 123; tr. 98). The frequency interpretation dissolves this paradox. Each physical measurement, such as that of our q and p values, is a repetitive event that can be seen as part of a collective. A collective is described by a ‘distribution’—the probabilities attached to different q- and p-values—which, in turn, determines the ‘mean’ value, and the ‘variance’ or ‘dispersion’ from the latter (Mises 1928, 113). Dynamical laws can be considered a special case of statistical laws in which individual values of initial conditions q_0, p_0 form a dispersion-free collective, meaning they coincide with each other and, consequently, with the mean value. In this case, the ‘mean value’ is traditionally identified with the ‘true value’ of the initial conditions. However, from a frequentist point of view, the latter concept is meaningless without reference to the dispersion-free collective to which it belongs.

The philosophical question arises: can any unknown magnitude be measured with arbitrarily small, or even zero, dispersion? Planck and most physicists did not question this possibility, leading to the idea that the ‘mean value’ approximates the ‘true value’ (Mises 1930, 151; 1936, 255–261). Exner and his Viennese acolytes contested this unanalyzed prejudice, arguing that dispersion-free collectives might not, in fact, exist (see also Reichenbach 1930, 179). Rather surprisingly, Cassirer sides with Exner over Planck. However, he turns Exner’s conjecture against the philosophical goal it was designed to support. For Cassirer, in contrast to the Viennese, *both* dynamical and statistical laws are compatible with the principle of causality, if correctly understood: “In classical physics, causality refers essentially to the knowledge of the course of the event, and probability to the knowledge of its initial conditions” (Cassirer 1936, 130; tr. 104). Statistical and dynamical laws are both ‘laws’; their difference hinges on whether the values of the ‘initial conditions’ can be measured collectively without dispersion.

By adopting von Mises’s (1919) frequentist interpretation of probability, Cassirer can attribute both dynamical and statistical laws the same dignity of ‘strict laws’ that the principle of causality compels us to pursue. This approach allows him to challenge von Kries’s (1919) distinction between nomological and ontological regularities—the exact laws of nature and the lawless initial conditions—, a distinction defended by Schlick (1931, sec. 10):

> If one wishes, as von Kries did, to differentiate the two types of laws which are thus obtained as ‘nomological’ and ‘ontological’ laws, it becomes clear that the two nowhere contradict each other, that no factual or methodological conflict exists between them. The only requirement which restricts the probability approach, but which already follows from the general determination of scientific knowledge and hence needs no special formulation, is that this approach must be nomologically permissible—that is, not contrary to a known law of nature. The characteristic difference between probability laws and dynamic laws nevertheless persists, but on the other hand it becomes clear how the two interweave and how only in this way the universal form of ‘lawlikeness’ in general [*Gesetzlichkeit überhaupt*] arises. (Cassirer 1936, 131; tr. 105; translation modified)

Again, the conventional philosophical taxonomies are put into question. The ‘empiricist’ Schlick, the leader of the Vienna Circle, sided with von Kries, whose aim was to reconcile probability with the Kantian idea of a deterministic universe governed by strict laws of nature. Indeed, for Schlick (1931, sec. 10), statistical laws are not laws; there can only be strict dynamical laws and total lawlessness in the initial conditions. On the contrary, the ‘Kantian’ Cassirer fully embraced the Viennese indeterminist tradition in considering statistical laws not only as laws in a proper sense but possibly as the most comprehensive genus of laws. Dynamical laws can be considered as idealized limiting cases of statistical laws where the probability can approach 1 without limit (see also Reichenbach 1930).

3 Beyond Vienna Indeterminism: Cassirer and Quantum Mechanics

This unexpected ‘convergence’ between Cassirer and the von Mises-Frank faction of the Vienna Circle does not preclude the ‘parallelism’ of their philosophical paths. Contrary to Schlick (1931, 1936), Cassirer agreed with the Vienna indeterminists regarding the status of statistical laws; however, needless to say, he did not embrace their empiricist epistemology. By replacing Laplacean causality with Helmholtzian causality, Cassirer could retain the validity of the principle of causality, as *a priori*, although only as a regulative principle, as the postulate *never* to abandon in the search for laws, be they dynamical or statistical. The question of which kind of laws is more fundamental is irrelevant to the status of the causality principle. Once Cassirer established this result for classical physics, he needed only to extend it to the new quantum mechanics in the fourth part of *Determinismus und Indeterminismus*. Indeed, from the Viennese perspective, quantum mechanics, and in particular Heisenberg’s famous uncertainty relations, could be considered a sort of scientific confirmation of Exner’s speculative suggestion that it might be impossible *in principle* to prepare dispersion-free collectives.

3.1 The Statistical Interpretation of the Uncertainty Relations

Cassirer dedicates the long chapter IV.1 of *Determinismus und Indeterminismus* to the uncertainty relations. Most of Cassirer’s somewhat untidy presentation seems to be oriented toward refuting Heisenberg’s *‘disturbance interpretation’* of the uncertainty relations as limitations in measurement precision. Heisenberg’s (1927) famous γ-ray microscope thought experiment seems to lead to the impression that the uncertainty relations refer to errors in simultaneous measurements of q and p on an *individual* system: one of these measurements might cause an error in the other; for example, when localizing a particle using light of a specified wavelength, a change in the momentum of the particle occurs (Cassirer 1936, 152; tr. 122). Cassirer, like others, complained that Heisenberg’s parlance of ‘uncontrollable disturbance’ and ‘inexactness’ in our measurements of a particle's position or momentum is misleading, as it suggests that the uncertainty relations somehow arise from peculiarities of our measuring apparatuses rather than from the formalism of the theory (see, *e.g.*, Jammer 1974, 79ff.). It is this interpretation that leads to the misunderstanding of the uncertainty relations as a challenge to the causality principle.

Cassirer seems to consider it sufficiently established that the philosophically sound interpretation of the uncertainty relations is the so-called *statistical interpretation*, which was defended, among others, by Frank (1932, 176–191) and more explicitly by von Mises (1930, 153; 1934, 149; 1936, 254–262): the uncertainty relations refer to statistical dispersions in measurements on an *collective* of identically prepared systems, say, electrons (see also Reichenbach 1930, 180f. Popper 1935, chap. IX). The uncertainty relations claim that the product of the dispersions $\Delta q \Delta p$ cannot both be reduced below a fixed limit, Planck's h divided by 4π. In classical physics, starting from a dispersive collective of electrons with various q and p, one can always filter dispersion-free sub-collectives with both definite q and p. In quantum mechanics, however, this selection is declared impossible. For example, selecting a homogeneous collective in which all particles have the same sharp momentum $\Delta p = 0$ (using a velocity filter) results in maximal dispersion of positions $\Delta q = \infty$—*i.e.*, position measurements provide maximally different results for the elements of this collective. The impossibility of homogeneous, dispersion-free collectives can be derived from the quantum formalism alone, without invoking 'inaccurate' measurements from instrument interaction. Inaccuracy in measurement can only be discussed if the 'true' value the measured quantity is known. However, even in classical physics, we have empirical access only to the 'mean' or 'expected' value over a 'collective' of repeated measurements (Mises 1934; 1936, 255–261; see also Margenau 1931, 1937).

By adopting von Mises's statistical interpretation of the uncertainty relations, Cassirer could easily transfer his considerations on the role of statistical laws in physics and the nature of probability statements from classical to quantum mechanics. In doing so, Cassirer believed he could challenge the widespread rhetoric of a 'demise' of the principle of causality. After quantum mechanics, it was argued that physics should abandon the search for 'strict' laws (Heisenberg 1930, 62) and be content with 'sloppy' (*schlampig*) laws of nature, as Sommerfeld once put it (Sommerfeld to Schlick, Dec. 18, 1932). However, Cassirer counters that the laws of quantum mechanics are 'strict laws' just like the laws of classical mechanics; only they govern the behavior of collectives rather than individual particles:

> One can refer to a basic lack of precision in the statements of quantum theory only so long as one presupposes that statistical statements are necessarily 'inexact' statements. In reality they are strict statements referring, however, not to an individual thing or event but to definite collectives. [...][E.g. predictions about the time of decay of atoms] are all extremely precise conclusions, even though they say nothing about the fate of the individual atom and the precise instant of its decomposition. There is here no thought of relinquishing causality, for causality in principle has nothing to do with 'fate' but simply and solely with law. Thus in quantum theory also this problem should be understood exclusively in this, its solely 'critical' sense. [...] But a real indeterminism, truly worthy of the name, can only be arrived at if we insist in going a step farther, when the attack, instead of being leveled at the determinateness of the individual event, is leveled at the determinateness of the laws which we consider as governing the event. [...] The definiteness, the logical determination of these concepts and principles would not be nullified if it should become evident that an event within atomic physics can no longer be represented by dynamic laws of the classical type but only through statistical laws.

> (Cassirer 1936, 148; tr. 118f.)

The interpretation of the uncertainty relations as a statistical statement has the advantage of discouraging those "far-reaching 'metaphysical' or 'worldview-related' [*weltanschaulichen*] consequences" (142; tr. 115; translation modified) that were often derived in some Weimar intellectual circles.[10]

As Cassirer remarks, Schrödinger (1935, 39f.) once distinguished between a 'conservative' and a 'revolutionary' viewpoint within quantum theory. According to the 'conservatives,' dynamical laws are fundamental, and chance is only apparent. According to the 'revolutionaries,' statistical laws are fundamental, and chance cannot be further explained, as there is no 'true' value for physical quantities. Even so, one should not speak of 'abandoning causality.' This renunciation would apply only to Laplacean causality, which demands absolute precision in initial conditions. From Helmholtz's methodological point of view, focused on the requirement of legality, the rhetoric of 'abandoning causality' becomes less compelling, even for the revolutionary physicist: "For none of these revolutionaries wanted to dispense altogether in their actual physical procedure with conformity to law [*Gesetzlichkeit*] of events; rather they ask how they may express and establish this conformity unobjectionably under the conditions of our observation of nature" (Cassirer 1936, 143f.; tr. 115).

It is true that many quantum theoreticians, like Heisenberg (1927) and Born (1927b, 240), rushed to claim that causality was meaningless because of the impossibility of reproducing identically the conditions of an experiment: "But this deduction is promptly supplemented by Born's declaration that statistical statements are thoroughly strict statements; the probabilities themselves, as he emphasizes, are by no means indefinite; they are strictly determined by the formalism of the quantum theory" (Cassirer 1936, 144; tr. 116). Indeed, in quantum mechanics, identical initial conditions do lead to the prediction of identical probability distributions according to a strict law. Quantum mechanics does not imply that there are no laws, which would be a proper violation of the principle of causality; it only necessitates treating a particular type of law, statistical law, as fundamental.

Following Eddington (1935), Cassirer compares the change in modern physics to the economic shift from 'commodity currency,' in which money is backed by a commodity, like gold, to 'fiat currency,' which is based on the trust that it will be accepted as a means of payment.[11] From the traditional point of view, statistical laws were considered 'paper money,' and dynamical laws the 'gold standard.' Most physicists are beginning to concede, as Exner surmised, that there is only *paper money* and accept the new monetary system. It is true that more conservative physicists, like Einstein, fear inflation and insist on a return to the *gold standard*, searching for additional hypothetical variables determining dispersion-free states. However, Cassirer was skeptical about the conservative approach[12]:

> The hope of ever successfully reducing all physical statements to the one type of classical dynamic law seems to dwindle more and more. Instead of looking for such a reduction,

---

[10] A similar concern was also expressed by Frank, who was worried about the cultural and even political misuse of Bohr's complementarity. See the letter by Frank to Bohr, likely from 1936, cited by Beller and Fine (1993, 31; fn. 11).

[11] Britain abandoned the gold standard in 1931, followed by the USA in 1933.

[12] Cassirer does not mention von Neumann's (1932) famous proof of the impossibility of dispersion-free states criticized by Hermann (1935, §§7-8). See, Seevinck 2016. However, I suppose he would have likely embraced Neumann's result.

> we must be ready to acknowledge statistical laws as a particular and fundamental type of physical statement. They must be evaluated as nonderivable, equally basic and equally valid elements of knowledge. The backing for our physical judgments has thus been shifted; but it is in no sense abolished. It now consists in that which proves invariant in the face of this shift. There must be some Archimedean point, some secure basis, immune to all uncertainty, if the construction of modern physics is to succeed. [...] Another essential precision and constancy is obtained by maintaining the invariability of certain fundamental quantities and by presupposing this invariability in all theoretical descriptions of natural events. [...] It is in these determinations—to continue Eddington's metaphor—that the real 'gold reserves' of modern physics are to be found. (Cassirer 1936, 150f.; tr. 120f.)

It is not about abandoning any 'backing' for physical laws; the challenge lies in determining where to seek it. In the traditional monetary system, gold backs the value of money, whereas in the new monetary system, it is backed by the trustworthiness of the central bank. Likewise, in physics, the value of a theory that was previously grounded in the determinateness of state 'variables' now hinges on the invariance of certain universal 'constants'[13]: "and the security and firmness of this frame alone ought to be sufficient to protect the indeterminism of the theory against those speculative interpretations to which it was exposed in the transition from physics to general conclusions concerning man's *Weltanschauung*" (151; tr. 122).

By relinquishing the ideal of a dispersion-free state of sharp q and p, physics has acquired a new fundamental constant, h, which relates their spreads, $\Delta q \Delta p = h/4\pi$. The scheme of causal connection is not abandoned: 'if q_0, p_0, then q_t, p_t' on the basis of some function $q_t, p_t = f(q_0, p_0)$. It is merely asserted that the values for the variables q, p must be "'permissible' values, in order to give the causal relation a definite unambiguous meaning" (155; tr. 125). Classical mechanics *requires* the attribution of sharp values to both q and p, while the uncertainty relations *forbid* such attribution. In a 'pure,' homogeneous collective, at most half of such a set of variables are *permitted* to have 'sharp' values: "Only through this limiting condition does the 'causal principle' attain a physically comprehensible significance—and its legitimate application remains confined to this condition" (155; tr. 125).

3.2 The Problem of the 'Material Point'

The Marburg-Vienna convergence seems to be reaching a point of intersection. Cassirer enthusiastically acknowledges that Exner's speculation turned out to be remarkably prophetic (Schrödinger 1932). In classical mechanics, it is often *in practice* impossible to prepare dispersion-free collectives of point particles, that is, collectives all members of which have the same sharply defined values of position q and momentum p. However, quantum mechanics shows that it is indeed achievable to construct a physical theory in which the selection of non-dispersive collectives is *in principle* impossible (see Neumann 1932, 160). However, the process of convergence appears to stall abruptly from here onward. From the non-existence of dispersion-free states, it was natural for the Viennese to consider quantum mechanics as the final blow against the classical notion of '*causality*' in physics, the requirement that all phenomena obey 'strict' laws. This rhetoric was also adopted by physicists like Born (1927a) and Heisenberg (1931, 1934). However, for Cassirer, this attitude was the consequence of a rather misleading conception of 'causality.' The uncertainty relations do not imply the rejection of 'strict

---

[13] As Cassirer is certainly aware (see Cassirer 1921, 42), this is an old argument by Planck (1910b). However, Cassirer does not explicitly refer to Planck here.

laws'; they impose a constraint on the form that such 'strict laws' are allowed to take (Cassirer 1936, 151; tr. 122).

Cassirer's strategy was to steer the discussion along a more favorable direction: it was not the notion of 'causality' but that of '*substantiality*' that is at stake (see French 2014, 95–99). The impossibility of dispersion-free collectives of q- and p-values means that the notion of 'trajectory' has become meaningless, and with it, the possibility of tracking the trans-temporal identity of particles as the constant substrate of their properties. As suggested by physicists like Max von Laue (1932, 1933, 1934), Schrödinger (1934), and even more explicitly by Paul Langevin (1931, 1934),[14] quantum mechanics represents a crisis of 'corpuscularism' rather than 'determinism' (Cassirer 1936, 205f.; fn. 1; tr. 165; fn. 17).[15] Cassirer, true to his style, embeds this line of argument in a historical-critical reconstruction of the notion of 'individuality' in physics, aiming to show that this seemingly radical step of abandoning the notion of 'individual particle' is not so radical upon closer inspection[16]; rather, it merely brings to the fore a tendency that was already present in classical physics.

The 'particle'-picture of matter emerged to assure the identifiability of the parts of a homogeneous medium over time (220f.; tr. 177f.). Impenetrability, extension, and rigidity of 'atoms' were ultimately conceptual presuppositions of atoms that prevent us from losing track of their identity (182f.; tr. 145f.). However, it progressively became apparent that the constancy of the subject of motion does not require the "immutability and indestructibility of mass particles" (221; tr. 185); identity over time is already assumed in the very *definition* of what counts as the subject of motion. Heinrich Hertz[17] ultimately suggested that one should *define* the 'material point' by this possibility of identification, the univocal correlation between a point in space at a particular time and a given point in space at any other time (221; tr. 185). However, Cassirer pointed out that even this minimal requirement cannot be satisfied as soon as we make the transition to a 'field' theory. The electromagnetic field is not an aggregate of material points. We may, and must, indeed, speak of parts of the field; but these parts have no separate existence. There would be "no longer any meaning in speaking of one and the same part at different times" (Cassirer 1929, 552; tr. 1957, 473). In a hypothetical 'field'-picture of matter, an electron would be conceived as a changing field and not "as being 'identical with itself' throughout the course of time" (Cassirer 1936, 222; tr. 178).

The renunciation of particle individuality, Cassirer writes, "could not be retracted even in the quantum theory for which the problem presented itself in a new and more general sense" (222; tr. 178). Cassirer attempts to show that the question of particle individuality runs through the history of quantum mechanics. In his reconstruction, Heisenberg-Born-Jordan matrix mechanics (Born and Jordan 1925; Born, Heisenberg, and Jordan 1926) was initially a theory of particles. The relations between the matrices Q_{nm} and P_{nm} were defined to retain the Hamiltonian formalism and the analogy with position q and momentum p in classical mechanics. However, matrix theorists viewed electron orbits in Bohr's model as fundamentally unobservable, arguing that a proper physical theory should focus only on observable quantities associated with those orbits, such as the frequency, intensity, and polarization of emitted radiation. While the

---

[14] See Ullmo 1934.

[15] See also Planck 1932.

[16] One might contrast Cassirer's stance with that of Émile Meyerson (1933), who considered the very idea of a 'non-individual real' to be the most fundamental departure from the classical worldview.

[17] Hertz 1894, 54.

concept of point-like electrons was not entirely abandoned, “any statements about individual distinguishable particles” (Cassirer 1936, 229; tr. 184) and their orbits were carefully avoided. However, this compromise was epistemologically unsatisfying: “For what *are* these electrons whose path we can no longer follow?” (222; tr. 178). Why do we persist in referring to them as ‘particles’ even though their orbits remain, in principle, inaccessible?

In contrast, de De Broglie’s (1924) wave theory of matter and Schrödinger’s (1928) wave mechanics rejected the idea of electrons and protons as ‘material points’ from the outset, in favor of a field picture of matter (Cassirer 1936, 225; tr. 180). As is well known, Schrödinger initially conceived the ‘wave function’ $\psi(q)$ as a physical field analogous to the electromagnetic radiation field; it described a real wave in three-dimensional space. What appears to be an ‘electron,’ he claimed, is actually a ‘wave packet,’ the peak intensity of a cluster of waves confined within particle-like dimensions (230; tr. 184). In this interpretation, $e|\psi(q)|^2$ represents the charge density of the electron at the point q (e is the total charge of the electron): “The charge of an electron is no longer linked to a definite location but is distributed throughout a ‘charge cloud.’ The corpuscular character of the electron is abandoned” (225; tr. 181). However, the limits of such approach immediately began to emerge. The particle, as a superposition of de Broglie waves, turned out to be unstable. Furthermore, the state function ψ is complex, and in the case of multiple particles, it extends in the configuration space of many dimensions and cannot be interpreted as a real wave field in ordinary space.

For Göttingen theorists, the shortcomings of Schrödinger’s original approach confirmed the view that quantum mechanics was inextricably connected to the corpuscular representation of matter—after all, protons and electrons do behave like particles, with fixed state-independent properties like mass and charge (229f.; tr. 182f.). Born’s (1926a, 1926b) famous probabilistic interpretation of the ψ-field could be seen as support for the particle interpretation. The square of the probability amplitude $|\psi(q)|^2$ is proportional to the probability (or relative frequency) that, say, an electron will be found at a given place at a given time in a collective of subsequent measurements (Cassirer 1936, 230; tr. 184). As Cassirer points out, “[f]rom this, Born concluded that we can continue to represent matter as before as a picture of moving, point-like particles (electrons or protons), but he immediately adds that in many cases these corpuscles cannot be ‘identified as individuals at all’[18]” (231; tr. 184), since their trajectories can be determined only with a restricted accuracy. However, Cassirer retorts that it is hard to fathom what a non-individual corpuscle is supposed to be. After all, classical physics, as Hertz pointed out, virtually defines the ‘material point’ by this possibility of identification (230; tr. 184).

Cassirer recognizes that, with the probability interpretation of ψ-function, the dialectic between the two classical ‘models’ or ‘pictures’ (*Bilder*), the discreteness of the ‘particle’ and the continuity of the ‘field,’ entered a new phase. Cassirer mentions with approval Bohr’s notion of ‘complementarity’ (186; tr. 212). However, in Cassirer’s rather loose interpretation, Bohr’s doctrine simply brings to full display what was already implicit in classical physics: wave and particle are at most useful ‘symbols’ and cannot be treated literally as ‘images’ of real entities (Cassirer 1932, 128f.; tr. 1956, 114).[19] Those ‘symbols’ might turn out to be inadequate. Schrödinger’s attempt at a realistic wave interpretation of matter failed. The statistical interpretation seems

---

[18] Born 1927b, 240.

[19] On the role of the ‘symbolic method’ in quantum mechanics, see Ryckman 2018, 2021.

initially to bring water to the mill of the realistic particle interpretation of matter. Nonetheless, this alternative was also met with considerable difficulties. The discrete nature of particles, endowed with permanent properties (mass, charge, spin), could still be maintained, but it was not possible to treat them as 'individuals' whose identity could be tracked through time:

> And the statistical approach points with particular emphasis to the fact that where descriptions of microcosmic phenomena are concerned we can no longer maintain and fulfill the demand for *individualization and identification* in the same way that appears possible for macroscopic objects. For statistical statements are, in themselves, strict statements, which apply, however, to collectives and not to individual cases, and which result in a determination solely for these collectives and not for a particular member picked from them. If quantum mechanics demonstrates that the possibility of determination does not reach beyond these collectives, then every means is lacking for going further and postulating the existence of an isolated particle. Therefore the statistical character of the premises of quantum theory must not be viewed in an exclusively negative way. It does not state that we are uncertain about location and momentum, *about the path and the entire 'fate' of the individual electron* [...][. It] intends to assert that it now considers as its ultimate goal not the determination of individual events but the determination of whole *systems* of events. (Cassirer 1936, 230f.; tr. 185; my emphasis)

For Cassirer, in truly Viennese fashion, all measurements presuppose collectives. The assumption that sharp q- and p-values can be determined is identical to the assumption that dispersion-free collectives of possible q- and p-values can be selected. In quantum mechanics, no physical method exists to obtain such homogeneous, dispersion-free collectives. As a consequence, the notion of 'position-cum-momentum' or well-defined 'trajectory' of particles loses its "empirical reality" (223; tr. 179).[20] However, without a trajectory, qualitatively identical particles cannot be treated as material points endowed with individuality. If wave packets of two identical particles $\psi(q_1)$ and $\psi(q_2)$ overlap in a region, one can predict the probability *that* a particle will be found there, but it becomes impossible to identify *which* one. In the case of systems containing multiple non-interacting identical particles, non-classical statistics replace the old Maxwell-Boltzmann statistics.[21] Particles do not even act as separate and distinct parts of the system as a whole; they cannot even be 'labeled' as particle 1 and particle 2.[22]

Not only are all quantum particles of the same kind qualitatively indistinguishable (all electrons have the same state-independent properties—charge, mass, etc.), but they cannot even considered as as different individuals in the ordinary sense of word since they cannot be re-identified over time (259; tr. 208).[23] Cassirer acknowledges that physics finds it challenging to relinquish the "'individuality' of the physical object" and "the conception that photons and electrons exist as objective individual things"

---

[20] Cassirer seems implicitly to disagree with Popper (1934; 1935, ch. IX) that the uncertainty relations prohibit the simultaneous 'preparation' of dispersion-free collectives but allow for simultaneous 'measurement' of q and p in a single particle.

[21] The move from uncertainty relations to new statistics to challenge individuality is also adopted by Langevin (1931, 1934). See also Ullmo 1934.

[22] On the whole-part relationship, Cassirer refers to Hermann Weyl (1931, 88; 1932, 55). See French and Krause 2006, sec. 3.7. In private correspondence, Einstein presented Cassirer with the case of *interacting* particles that had been separated but lack independent states (Cassirer to Einstein, Mar. 16, 1937; Cassirer 1995–2022, Vol. 18, Doc. 158). Cassirer's reply is not extant, but I guess he would have seen 'entanglement' as another step toward a 'non-individuals' physics.

[23] See Weyl 1928, 188; Jordan 1933, 86ff. According to Jammer (1966, 344), this point was initially seldom emphasized by physicists. Nor was it by philosophers. Logical empiricists never mention the issue. Hermann (1935, 90–93[23–25]) mentions it, but does not consider it central.

(Cassirer 1936, 231; tr. 186). However, his historical-critical analysis revealed that even when classical physics "spoke of 'individual' material points, the impression was still to be taken with a grain of salt" (231; tr. 186). Classical physics did not simply assume this 'individuality' from experience; individuality was, so to speak, 'put into' experience as a conceptual condition. In a multiple-particle system, like a gas, what counts as 'one' depends on the assumption regarding which case can be considered 'equiprobable' in the equilibrium state. In classical Maxwell-Boltzmann statistics, a permutation of qualitatively identical particles is counted as giving a different arrangement. However, this is not the case in both quantum statistics, where two particles in different states are counted as one: "the determination of the individual, of what should truly count as 'one,' is not the *terminus a quo* but always only the *terminus ad quem*, *i.e.*, a result of the theory that cannot be dogmatically presupposed in advance, almost as if it were based on an immediate intuition" (234; tr. 187f.).

After a long detour, Cassirer could then finally bring the discussion back to his famous opposition between substance-concepts and function-concepts (Cassirer 1910). Quantum mechanics does not so much question the categories of 'cause and effect' but rather those of 'substance and accident' (Cassirer 1936, 235; tr. 188). Indeed, if causality is nothing more than the requirement of a *functional* relation between the subsequent 'states' of a system, quantum mechanics is still a causal theory in this sense if one properly redefines the notion of the 'state' of a system: the ψ-state refers not to 'individuals' but to 'collectives.' For Cassirer, the impossibility of dispersionless 'states' is not a failure of the cause-effect relation (cf. Neumann 1932, 160). Indeed, quantum mechanics provides a 'strict law' for the evolution of such ψ-states, based, like in classical mechanics, on a Hamiltonian function $H(q, p)$ characteristic of the system.[24] The inevitable 'dispersiveness' of pure, homogeneous collectives in quantum mechanics is a challenge to the substance-accident relation.[25] As we have, in principle, no empirical *access* to finer-grained states in which both q and p assume sharp values, the notion of position-cum-momentum, *i.e.*, the path of an individual 'particle' as a property-transcending substance cannot be considered as legitimate empirical *object*[26]:

> In this respect the uncertainty relations have no skeptical but solely a critical meaning. The necessary correction of the physical object concept of the older theory could only then appear as a skeptical abandonment, when this concept was not yet overcome in principle, when the viewpoint was once more assumed which one desired to abandon and which, with the establishment of the uncertainty relations, had already been left

---

[24] Of course, this remark misses the point: upon measurement the system is no longer in the same ψ-state as before. This transition to a different ψ'-state is not law-like and can indeed be described as 'acausal' (see, *e.g.*, Margenau 1937). See also fn. 25.

[25] According to Cassirer, classical physics implicitly assumes the principle of *omnimoda determinatio*: the 'state' of a particle at a given t is perfectly determined in terms of *definite* numerical values of q, p, E (Cassirer 1936, 234f.; tr. 189). In quantum mechanics, however, particles do not 'possess' definite q- and p-values as 'properties.' If the same q-measurement is conducted on a collective of particles in the *same* of sharp p-state, it will yield *different* result each time (dispersion). Following Dirac (1930), Cassirer understood that this paradoxical situation is captured by the notion of *superposition* (Cassirer 1936, 237f.; tr. 190f.). The question that Cassirer does not address, however, is why we measure a *single* position each time. See Ryckman 2015, 80f. for more detail.

[26] The 'conditions of accessibility' established by the uncertainty relations, as Cassirer put it, paraphrasing Kant, are 'conditions of the objects of experience' (Cassirer 1936, 222; tr. 178). French (2014, 59) calls the latter 'Cassirer's condition' (see also French and Ladyman 2003, sec. 3; Ryckman 2015, 80). For Cassirer, the situation is not dissimilar to, say, 'absolute position' in classical mechanics, which is, in principle, 'inaccessible' given the homogeneity of Euclidean space. In this context, he evokes Leibniz's 'principle of observability' (Cassirer 1936, 155f.; tr. 122f.)

> behind. We must face squarely the new problems thus created. *There seems to be no return to the lost paradise of classical concepts*; physics has to undertake the construction of a new methodological path. I do not wish to claim at all that the end of this path is already clearly in sight. But the direction in which the solution is to be found seems to me clearly recognizable. Physics and epistemology cannot continue to posit a being with the full realization that it contradicts the conditions of physical knowledge. If it appears that certain concepts, such as those of position, of velocity, or of the mass of an individual electron can no longer be filled with a definite empirical content, we have to exclude them from the theoretical system of physics, important and fruitful though their function may have been. (Cassirer 1936, 242f.; tr. 194f.; my emphasis)

As one can infer from this passage, Cassirer, more or less consciously, departed strongly from what would become the 'Göttingen-Copenhagen orthodoxy' (see Howard 2004). For Bohr, Heisenberg, and Born, no one will expel us from the 'paradise of classical concepts.' The latter are *indispensable* for describing the results of experiments involving quantum phenomena, despite the impossibility of their simultaneous applicability: either waves or particles. On the contrary, Cassirer seems to align with the minority position of physicists like Laue and Schrödinger, according to whom quantum mechanics has forced us out of the classical Eden to which there is no return. Classical concepts, the pictorial models of older physics, might turn out to be 'dispensable' and should possibly be replaced by something more adequate: neither waves nor particles.[27] The probabilistic interpretation of wave function ruled out Schrödinger's early wave theory of matter. However, without trajectories, the alternative Göttingen attempt to uphold a particle theory of matter was also unsuccessful. 'Models,' Cassirer insists, are not 'pictures' that depict entities but 'symbols' that are implicitly defined by the structure of the theory as a whole (Cassirer 1936, 243; tr. 196). As often happens in the history of physics, if the theory fails to 'save the phenomena,' previously successful physical models are required to reorient themselves: "[t]he concept of the mass point seems to face such a necessity of reorientation" (245; tr. 196).

Conclusion

Stripping away the details, one might conclude that Cassirer appropriated one central tenet of the so-called 'Vienna indeterminism': 'collectives' can be regarded as ideal objects that can figure in 'strict laws' (Stöltzner 2003a, 297). Since only the 'long-term' average or mean over repeated measurements is empirically accessible, *both* classical and quantum mechanics can be regarded as, ultimately, theories about 'collectives.' The crux of the matter is that, in quantum mechanics, homogeneous collectives are *not* dispersion-free as in classical mechanics. In this perspective, the question of whether each individual member of the quantum collective 'possesses' sharp, though unknown, 'true' q and p values is empty.[28] The notion of 'true value' is meaningful only when the variance from the 'mean value' is permitted to be zero, which is not the case in quantum mechanics.

As argued throughout this paper, Cassirer and the Vienna indeterminists broadly converged on this 'interpretation' of quantum mechanics.[29] However, they used it as a

---

[27] Cassirer shows no interest in a 'Kantian' reading of Bohr's 'complementarity,' which was embraced, *e.g.*, by Hermann (1935). See Soler (2016). Hermann (1935, 115f.[47f.]) explicitly criticizes Laue's and Schrödinger's position that Cassirer endorses. See also Weizsäcker's (1937, 861) criticism of Cassirer mentioned in fn. 33.

[28] In contrast to, say, Popper (1935, ch. IX).

[29] The Vienna/Marburg 'interpretation' of quantum mechanics is, of course, a variant of the 'ensemble

starting point to pursue parallel philosophical goals. For the Viennese, the impossibility of fixing the initial conditions with arbitrary precision forces us to reject the notion of *causality* that dominated classical physics. According to Cassirer, this takeaway is the result of the Laplacian identification of 'causality' with 'predictability,' rather than 'lawlikeness.' In his view, quantum mechanics challenges the classical notion of *substantiality*. By denying the possibility of well-defined trajectories, quantum mechanics forces us to abandon the idea of the trans-temporal individuality of particles independent of all possessed 'properties'.[30]

Determinismus und Indeterminismus first appeared in November 1936 in the rather obscure Swedish journal *Göteborgs Högskolas Årsskrift*. Cassirer sent copies of the 1937 *separatum* to several leading physicists of his time: Einstein, von Laue, Born, and others.[31] Of course, Cassirer also hoped to receive a reaction from philosophers, especially from the Vienna Circle, with which he was intensively engaged during the emigration period (Mormann 2012). In particular, Cassirer hoped for a review in *Erkenntnis*, the house journal of logical empiricism (Cassirer to Reichenbach, Sep. 1, 1936; Cassirer 1995–2022, Vol. 18, Doc. 122). Reichenbach, to whom Cassirer was particularly close, offered to find a reviewer, but he remarked: "I would rather not choose one of the gentlemen from the Vienna Circle, as they are too distant from your ideas" (Reichenbach to Cassirer, Mar. 20, 1937; Vol. 18, Doc. 130). However, the opposite turned out to be the case. In private correspondence, von Mises recognized the "agreement on essential points" (Mises to Cassirer, Mar. 3, 1937; Vol. 18, Doc. 1131), and so did Frank (Frank to Cassirer, undated[32]; Vol. 18, Doc. 137), who also wrote a lengthy review of *Determinismus und Indeterminismus* in the Swedish journal *Theoria* (Frank 1938). The review, at least on a cursory reading, was glowing.

By reducing causality to legality, Cassirer comes close to full agreement with the "purely positivistic conception of science" (73; tr. 175). In particular, Cassirer senses that the validity of the principle of causality depends on the variables used to describe the state of a system (73; tr. 175f.). Since a law can always be found to connect an arbitrary number of variables, at first the causality principle amounts to a mere *convention*, a definition of what counts as the system's 'state' (73; tr. 176). In Frank's reading, to avoid this conclusion, Cassirer tacitly imposes the additional but rather ambiguous requirement that such laws must be *simple*, meaning they connect a 'relatively small' number of state variables (see Frank 1932, sec. IX.8). Thus, as Frank argues in his 1932 book, the causality principle can ultimately be treated as a *non-empty* statement, only if it is rendered rather *vague* (Frank 1938, 73; tr. 176): "Exactly this character, however, is possessed by Cassirer's conception of causality, which I have therefore designated approvingly as 'disintegrating' [*zersetzend*]" (73; tr. 175). Frank's parlance of 'disintegration' or, better, 'decomposition' (*Zersetzung*) betrays the backhanded nature of his praise. He used Cassirer's example to show that any attempt at integrating

---

interpretation' (Ballentine 1970), as opposed to a 'single-particle' interpretation. However, it does not assume that individual particles possess definite but unknown q and p values at all times (see fn. 25). Yet, no explanation is provided for the scattering of those values in a homogeneous ensemble. See also fn. 30.

[30] In search of a label, one might argue that Cassirer proposed a sort of 'non-individuals interpretation of quantum mechanics ' (Krause, Arenhart, and Bueno 2022). Cassirer's stance may come close to that of the late Schrödinger (1950, 1955), although Cassirer does not seem to endorse a wave ontology (see Bitbol 1996, secs. 4.1, 4.2).

[31] See Heijden 2014 for more details.

[32] After February 1937.

modern physics into traditional philosophy inevitably erodes its foundations from within. As Frank did not refrain from pointing out in private correspondence (Frank to Cassirer, undated; Cassirer 1995–2022, Vol. 18, Doc. 137), Cassirer's philosophy represents the most prominent example of a "decomposition process [*Zersetzungsprozess*] of school philosophy" (Frank 1938, 71; tr. 174; slightly modified).

Frank concedes that this process of 'decomposition' (Frank 1932, V) was not complete; there was still a "dark background" in Cassirer's book (Frank 1938, 77; tr. 184). Interestingly, the main source of disagreement was Cassirer's stance towards the notion of *material point*. Frank (1937) relied on a talk he had just given at the Copenhagen congress for the unity of science—to which Cassirer had also been invited to but could not attend (Neurath to Cassirer, Jan. 15, 1938; Cassirer 1995–2022, Vol. 18, Doc. 139). Frank distinguished two possible misinterpretations of the notion of 'material point' in quantum mechanics: (a) there are material points, but their q and p at t are immeasurable in quantum mechanics; (b) there are material points, but their q and p at t are indeterminate. Cassirer vacillates between (a) and (b), which is, nonetheless, no less metaphysical than (a). According to Frank, the proper *positivistic* version of complementarity was presented by Bohr (1937) at the Copenhagen congress: quantum mechanics is not concerned with position and momenta of particles at all, but with the *experimental arrangements* measuring the 'position of a particle' and 'momentum of a particle' (Frank 1938, 76; tr. 179). Complementarity claims that, in quantum mechanics, those experimental arrangements are mutually exclusive. Cassirer's *metaphysical* urge to replace the classical concept of the 'material point' with a more fitting non-classical concept was merely a remnant of 'school philosophy' (77; tr. 179f.).

In private correspondence, Frank praises Cassirer for having put the "logical constitution to the forefront more than any other" (Frank to Cassirer, undated; Cassirer 1995–2022, Vol. 18, Doc. 137). Physics is understood as a system of symbols rather than a representation of a 'true world.' For this reason, Cassirer's philosophical path should have fully converged with the scientific *Weltauffassung*—that is, ultimately Carnap's doctrine of the "arbitrariness in logical constitution" (Frank to Cassirer, undated; Vol. 18, Doc. 137). However, as Frank argues, Cassirer concurrently upheld the ideal of a scientific *Weltbild*, a unified and objective description of the 'real world' as a limit-concept. The latter, however, "does not actually seem to fit with [Cassirer's] overall line of thought but rather appears to be merely a remnant of metaphysical idealism" (Frank to Cassirer, undated; Vol. 18, Doc. 137).

Frank's remark is somewhat simplistic; yet he perceptively sensed the tension that what we have called 'parallel convergence' imposes on Cassirer's neo-Kantianism. According to Frank, following the internal logic of his book, Cassirer should have concluded that the causality is merely a *definition* of what counts as the state of a system. However, *Determinismus und Indeterminismus* retains a residual, and ultimately extraneous, element of 'Kantianism'.[33] Cassirer holds fast the causality principle as the *a priori maxim* never to abandon the search for the final unified system of laws of nature. However, the very idea of a 'limit' toward which the series of scientific theories converge lies, by definition, beyond the domain of science (see Frank 1932, secs. X.18 and 22). It is a metaphysical slag that must be shed. In my view, however, Frank misses the point here. This last remnant of *a priori* was not due to Cassirer's

---

[33] Weizsäcker (1937) accuses Cassirer of having conceded too much to his opponents and prefers Hermann (1935)'s form of 'Kantianism.' On the contrary Frank prefers Cassirer to Hermann (Frank 1938, 74; tr. 177).

stubborn attachment to a declining philosophical tradition. For Cassirer, the Vienna Circle's reduction of science to a system of symbols for mastering observations, might adequately account for its logical structure; however, amid growing skepticism toward the scientific endeavor, it ultimately falls short of explaining why science remains worth pursuing.[34] The assumption of the lawlikeness of nature appeared to Cassirer as the last glimmering beacon of scientific rationality, piercing the storm of anti-intellectualism that engulfed Europe in the 1930s[35]: "the scientist does not give us a logical or empirical proof of this fundamental assumption. The only proof that he gives us is his work" (Cassirer 1944, 219).

References

Ballentine, Leslie E. 1970. "The Statistical Interpretation of Quantum Mechanics." *Reviews of Modern Physics* 42:358–381.

Beller, Mara, and Arthur Fine. 1993. "Bohr's Response to EPR." In *Niels Bohr and Contemporary Philosophy,* edited by Jan Faye and Henry J. Folse, 1–31.

Bergmann, Hugo. 1929. *Der Kampf um das Kausalgesetz in der jüngsten Physik.* Braunschweig: Vieweg.

Bitbol, Michel. 1996. *Schrödinger's Philosophy of Quantum Mechanics.* Dordrecht: Springer.

Bohr, Niels. 1937. "Kausalität und Komplementarität." *Erkenntnis* 6:293–303.

Born, Max. 1926a. "Zur Quantenmechanik der Stoßvorgänge." *Zeitschrift für Physik* 37:863–867.

———. 1926b. "Zur Quantenmechanik der Stoßvorgänge." *Zeitschrift für Physik* 38:803–827.

———. 1927a. "Physical Aspects of Quantum Mechanics." *Nature* 119:354–357.

———. 1927b. "Quantenmechanik und Statistik." *Die Naturwissenschaften* 15:238–242.

Born, Max, and Pascual Jordan. 1925. "Zur Quantenmechanik." *Zeitschrift für Physik* 34:858–888.

Born, Max, et al. 1926. "Zur Quantenmechanik, II." *Zeitschrift für Physik* 35:557–615.

Cassirer, Ernst. 1910. *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik.* Berlin: B. Cassirer. Repr. in Cassirer 1998–2009, Vol. 6.

———. 1921. *Zur Einstein'schen Relativitätstheorie: Erkenntnistheoretische Betrachtungen.* Berlin: B. Cassirer. Repr. in Cassirer 1998–2009, Vol. 10.

———. 1929. *Philosophie der symbolischen Formen.* Dritter Teil: Phänomenologie der Erkenntnis. Berlin: B. Cassirer. Repr. in Cassirer 1998–2009, Vol. 13.

———. 1930. "Geist und 'Leben' in der Philosophie der Gegenwart." *Die neue Rundschau* XLI:244–264. Repr. in Cassirer 1998–2009, Vol. 17, 185–205.

———. 1932. *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit.* Vol. 4: Von Hegels Tod bis zur Gegenwart(1832–1932). Berlin: B. Cassirer. German manuscript 1940.

———. 1933. "Henri Bergsons Ethik und Religionsphilosophie." *Der Morgen* 9:20–29, 138–151. Repr. in Cassirer 1998–2009, Vol. 18, 229–253.

———. 1936. "Determinismus und Indeterminismus in der modernen Physik: Historische und systematische Studien zum Kausalproblem." *Göteborgs Högskolas Årsskrift* 42. Repr. in Cassirer 1998–2009, Vol. 19.

———. 1937. *Determinismus und Indeterminismus in der Modernen Physik.* Göteborg: Wettergren & Kerbers Förlag. Repr. in Cassirer 1998–2009, Vol. 19.

———. 1939. *Naturalistische und humanistische Begründung der Kulturphilosophie.* Göteborg: Wettergren & Kerber. Repr. in Cassirer 1998–2009, Vol. 22, 140–166.

———. 1944. *An Essay on Man: An Introduction to a Philosophy of Human Culture.* New Haven: Yale University Press. Repr. in Cassirer 1998–2009, Vol. 23.

———. 1946. *The Myth of State.* New Haven: Yale University Press. Repr. in Cassirer 1998–2009, Vol. 25.

———. 1956. *Determinism and Indeterminism in Modern physics: Historical and Systematic Studies of the Problem of Causality.* Translated by Theodor Benfey. With a foreword by Henry Margenau. New Haven: Yale University Press.

---

[34] Giovanelli (2022) characterizes Cassirer's late 'Kantianism' as 'motivational': it does not search for the conditions of the possibility of science, but for the conditions of its pursue worthiness.

[35] See Cassirer 1946, chap. XVIII.

Cassirer, Ernst. 1957. *The Philosophy of Symbolic Forms: Volume Three: The Phenomenology of Knowledge.* With an introduction by Charles W. Hendel. New York: Yale University Press.

———. 1995–2022. *Nachgelassene Manuskripte und Texte.* Edited by John Michael Krois. 19 vols. Hamburg: Meiner.

———. 1998–2009. *Gesammelte Werke: Hamburger Ausgabe.* Edited by Birgit Recki. 26 vols. Hamburg: Meiner.

Cassirer Bondy, Toni. 1981. *Mein Leben mit Ernst Cassirer.* Hildesheim: Gerstenberg.

Cei, Angelo, and Steven French. 2009. "On the Transposition of the Substantial into the Functional: Bringing Cassirer's Philosophy of Quantum Mechanics into the Twenty-first Century." In *Constituting Objectivity: Transcendental Perspectives on Modern Physics,* edited by Michel Bitbol et al., 95–115. Dordrecht: Springer.

Crull, Elise, and Guido Bacciagaluppi, eds. 2016. *Grete Hermann: Between Physics and Philosophy.* Dordrecht: Springer.

De Broglie, Louis. 1924. "Recherches sur la théorie des quanta." PhD diss., Masson.

Dirac, Paul Adrien Maurice. 1930. *The Principles of Quantum Mechanics.* Oxford: Oxford University Press.

Du Bois-Reymond, Emil Heinrich. 1872. *Über die Grenzen des Naturerkennens.* Leipzig: Veit.

Eddington, Arthur Stanley. 1935. *New Pathways in Science: Messenger Lecture 1934.* Cambridge: Cambridge University Press.

Exner, Franz S. 1909. *Über Gesetze in Naturwissenschaft und Humanistik.* Wien/Leipzig: Hölder.

———. 1919. *Vorlesungen über die physikalischen Grundlagen der Naturwissenschaften.* Leipzig/Wien: Franz Deuticke.

———. 1922. *Vorlesungen über die physikalischen Grundlagen der Naturwissenschaften.* 2nd ed. Leipzig/Wien: Franz Deuticke.

Forman, Paul. 1971. "Weimar culture, causality, and quantum theory: adaptation by German physicists and mathematicians to a hostile intellectual environment." *Historical Studies in the Physical Sciences* 3:1–115.

———. 1984. "*Kausalität*, *Anschaulichkeit* and *Individualität*, or How Cultural Values Prescribed the Character and the Lessons Ascribed to Quantum Mechanics." In *Society and Knowledge: Contemporary Perspectives in the Sociology of Knowledge and Science,* edited by Nico Stehr and Volker Meja. New Brunswick: Transaction.

Frank, Philipp. 1932. *Das Kausalgesetz und seine Grenzen.* Wien: Springer.

———. 1937. "Philosophische Deutungen und Mißdeutungen der Quantentheorie." *Erkenntnis* 6:303–317.

———. 1938. "Discussion of E. Cassirer: *Determinismus und Indeterminismus in der modernen Physik*." *Theoria* 4:70–80.

———. 1950. *Modern Science and its Philosophy.* Cambridge: Harvard University Press.

French, Steven. 2014. *The Structure of the World: Metaphysics and Representation.* Oxford: Oxford University Press.

French, Steven, and Decio Krause. 2006. *Identity in Physics: A Historical, Philosophical, and Formal Analysis.* Oxford: Oxford University Press.

French, Steven, and James Ladyman. 2003. "Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure." *Synthese* 136:31–56.

Giovanelli, Marco. 2022. "Motivational Kantianism: Cassirer's Late Shift Towards a Regulative Conception of the a Priori." *Studies in History and Philosophy of Science* 95:118–125.

Gower, Barry. 2000. "Cassirer, Schlick and 'Structural' Realism: The Philosophy of the Exact Sciences in the Background to Early Logical Empiricism." *British Journal for the History of Philosophy* 8:71–106.

Heidelberger, Michael. 2001. "Origins of the Logical Theory of Probability: von Kries, Wittgenstein, Waismann." *International Studies in the Philosophy of Science* 15:177–188.

Heijden, Pim van der. 2014. "From the Frying Pan into the Fire: Cassirer's Conception of Causality and Determinism and the Responses of Contemporary Physicists." Thesis, University of Utrecht, December 22, 2014.

Heisenberg, Werner. 1927. "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik." *Zeitschrift für Physik* (Leipzig) 43:172–198.

———. 1930. *Die physikalischen Prinzipien der Quantentheorie.* Leipzig.

———. 1931. "Kausalgesetz und Quantenmechanik." *Erkenntnis* (Leipzig) 2:172–182.

Heisenberg, Werner. 1934. “Wandlungen der Grundlagen der exakten Naturwissenschaft in jüngster Zeit.” *Die Naturwissenschaften* 22:669–675.

Helmholtz, Hermann von. 1879. *Die Tatsachen in der Wahrnehmung.* Rede gehalten zur Stiftungsfeier der Friedrich-Wilhelms-Universität zu Berlin am 3. August 1878, überarbeitet und mit Zusätzen versehen. Darmstadt: Wiss. Buchges.

Hermann, Grete. 1935. “Die naturphilosophischen Grundlagen der Quantenmechanik.” *Abhandlungen der Fries’schen Schule* 6:69–152.

Hertz, Heinrich. 1894. *Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt: Mit einem Vorworte von H. von Helmholtz.* In *Gesammelte Werke,* vol. 3. Leipzig: Barth.

Howard, Don. 2004. “Who Invented the ‘Copenhagen Interpretation’? A Study in Mythology.” *Philosophy of Science* 71:669–682.

Jammer, Max. 1966. *The Conceptual Development of Quantum Mechanics.* New York: McGraw-Hill.

———. 1974. *The Philosophy of Quantum Mechanics: The Interpretations of QM in Historical Perspective.* Wiley.

Jordan, Pascual. 1933. *Statistische Mechanik auf quantentheoretischer Grundlage.* Braunschweig.

Karlik, Berta, and Erich Schmid. 1982. *Franz Serafin Exner Und Sein Kreis: Ein Beitrag Zur Geschichte Der Physik in Österreich.* Wien.

Keynes, John Maynard. 1921. *A Treatise on Probability.* London: Macmillan.

Kraft, Peter, and Peter Kroes. 1984. “Adaptation of Scientific Knowledge to an Intellectual Environment: Paul Forman’s “Weimar Culture, Causality, and Quantum Theory, 1918–1927”: Analysis and Criticism.” *Centaurus* 27:76–99.

Krause, Décio, et al. 2022. “The Non-Individuals Interpretation of Quantum Mechanics.” In *The Oxford Handbook of the History of Quantum Interpretations,* edited by Olival Freire.

Kries, Johannes von. 1919. “Über Wahrscheinlichkeitsrechnung und ihre Anwendung in der Physik.” *Die Naturwissenschaften* 7:2–7 & 17–23.

Krois, John Michael. 2000. “Ernst Cassirer und der Wiener Kreis.” In *Elemente moderner Wissenschaftstheorie,* edited by Friedrich Stadler, 105–122. Vienna.

Langevin, Paul. 1931. “Y a-t-il une crise du déterminisme?” *La revue du Mois* 2:273–275.

———. 1934. *La notion de corpuscules et d’atomes.* Paris: Hermann. Conference faite le 16 octobre 1933 a la séance d’ouverture de la Reunion Internationale de Chimie Physique.

Laplace, Marquis de, Pierre Simon. 1812. *Théorie analytique des probabilités.* Paris: Courcier.

———. 1814a. *Théorie analytique des probabilités.* 2nd ed. Paris: Courcier.

Laplace, Pierre Simon de. 1814b. *Essai philosophique sur les probabilités.* Paris: Courcier.

Laue, Max von. 1932. “Zu den Erörterungen über Kausalität.” *Die Naturwissenschaften* 20:915–916.

———. 1933. “Materie und Raumerfüllung.” *Scientia* 27:402–412.

———. 1934. “Über Heisenbergs Ungenauigkeitsbeziehungen und ihre erkenntnistheoretische Bedeutung.” *Die Naturwissenschaften* 22:439.

Margenau, Henry. 1931. “Causality and Modern Physics.” *The Monist* 41:1–36.

———. 1937. “Critical Points in Modern Physical Theory.” *Philosophy of Science* 4:337–370.

Meyerson, Émile. 1933. *Réel et déterminisme dans la physique quantique.* With a foreword by Louis de Broglie. Paris: Hermann.

Mises, Richard von. 1919. “Grundlagen der Wahrscheinlichkeitsrechnung.” *Mathematische Zeitschrift* 5:52–99.

———. 1928. *Wahrscheinlichkeit Statistik und Wahrheit.* Berlin/Heidelberg: Springer.

———. 1930. “Über kausale und statistische Gesetzmäßigkeit in der Physik.” *Erkenntnis* 1:189–210.

———. 1934. “Über Heisenbergs Ungenauigkeitsbeziehungen und ihre erkenntnistheoretische Bedeutung.” *Naturwissenschaften* 22:822.

———. 1936. *Wahrscheinlichkeit Statistik und Wahrheit.* 2nd ed. Berlin/Heidelberg: Springer.

Mormann, Thomas. 2012. “A Virtual debate in Exil: Cassirer and the Vienna Circle after 1933.” In *Rudolf Carnap and the Legacy of Logical Empiricism,* edited by Richard Creath, 149–167. Dordrecht et al.: Springer.

Neuber, Matthias. 2013. “Trefpunkt Struktur—Cassirer, Schlick und Carnap.” *Archiv für Geschichte der Philosophie* 95:206–233.

Neumann, Johann von. 1932. *Mathematische Grundlagen der Quantenmechanik.* Berlin: Springer.

Planck, Max. 1910a. *Acht Vorlesungen über theoretische Physik gehalten an der Columbia University in the City of New York im Frühjahr 1909.* Leipzig: Hirzel.

———. 1910b. *Die Stellung der neueren Physik zur mechanischen Naturanschauung.* Vortrag gehalten am 23. September 1910 auf der 82. Versammlung deutscher Naturforscher und Ärzte in Königsberg.

Leibzig: Hirzel. Repr. in Planck 1958, Vol. 3, Doc. 124. *Physikalische Rundblicke.* Leipzig: Hirzel, 1922.

———. 1914. *Dynamische und statistische Gesetzmäßigkeit.* Bericht über die Feier zum Gedächtnis des Stifters der Berliner Universität König Friedrich Wilhelms III am Jahrestage seiner Geburt dem 3. August 1914. Leipzig: Barth.

———. 1932. *Der Kausalbegriff in der Physik.* Leipzig: Barth.

———. 1958. *Physikalische Abhandlungen und Vorträge.* Edited by Verband Deutscher Physikalischer Gesellschaften and Max-Planck-Gesellschaft zur Förderung der Wissenschaften. 3 vols. Braunschweig: Vieweg.

Popper, Karl R. 1934. "Zur Kritik der Ungenauigkeitsrelationen." *Naturwissenschaften* 22:807–808. With a reply by Carl Friedrich von Weizsäcker.

———. 1935. *Logik der Forschung: Zur Erkenntnistheorie der modernen Naturwissenschaft.* Berlin: Springer.

Reichenbach, Hans. 1916. *Der Begriff der Wahrscheinlichkeit für die mathematische Darstellung der Wirklichkeit.* Leipzig: Barth.

———. 1930. "Kausalität und Wahrscheinlichkeit." *Erkenntnis* 2–4:158–188.

———. 1932. "Die logischen Grundlagen des Wahrfcheinlichkeitsbegriffs." *Erkenntnis* 3:401–425.

Richardson, Alan W. 1998. *Carnap's Construction of the World: The 'Aufbau' and the Emergence of Logical Empiricism.* Cambridge, New York/Melbourne: Cambridge Univerity Press.

Ryckman, Thomas. 2015. "A Retrospective View of Determinism and Indeterminism in Modern Physics." In *The Philosophy of Ernst Cassirer: A Novel Assessment,* edited by J. Tyler Friedman and Sebastian Luft, 65–102. Berlin/Boston: De Gruyter.

———. 2018. "Cassirer and Dirac on the Symbolic Method in Quantum Mechanics: A Confluence of Opposites." *Journal for the History of Analytical Philosophy* 6:213–243.

———. 2021. "Quantum Mechanics As the Ultimate Mode of Symbol Formation." In *Interpreting Cassirer: Critical Essays,* edited by Simon Truwant, 89–108. Cambrdige: Cambridge University Press.

Schlick, Moritz. 1920a. "Naturphilosophische Betrachtungen über das Kausalprinzip." *Die Naturwissenschaften* 8:461–474.

———. 1920b. *Space and Time in Contemporary Physics: An Introduction to the Theory of Relativity and Gravitation.* Edited by Henry L. Brose. Oxford/New York: Oxford University Press.

———. 1931. "Die Kausalität in der gegenwärtigen Physik." *Die Naturwissenschaften* 19:145–162.

———. 1936. "Quantentheorie und Erkennbarkeit der Natur." *Erkenntnis* 6:317–326.

Schrödinger, Erwin. 1928. *Collected Papers on Wave Mechanics.* Translated by James Fleming Shearer and Winifred M. Deans. London: Blackie & Son Limited.

———. "Antrittsrede des Herrn Schrödinger." *Sitzungsberichte der Preußischen Akademie der Wissenschaften,* C–CII.

———. 1929b. "Was ist ein Naturgesetz?" *Die Naturwissenschaften* 17:9–11.

———. 1932. *Über Indeterminismus in der Physik: Ist die Naturwissenschaft milieubedingt? Zwei Vorträge zur Kritik der naturwissenschaftlichen Erkenntnis.* Leipzig: Barth.

———. 1934. "Über die Unanwendbarkeit der Geometrie im Kleinen." *Die Naturwissenschaften* 22:518–520.

———. 1935. "The Law of Chance: The Problem of Causation in Modern Science." In *Science and the Human Temperament,* 33–42. London: George Allen & Unwin, Ltd.

———. 1950. "What is an Elementary Particle?" *Annual Report of the Board of Regents of The Smithsonian Institution,* 183–196.

———. 1955. *The Interpretation of Quantum Mechanics: Dublin Seminars (1949-1955) and Other Unpublished Essays.* Edited by Michel Bitbol. Woodbridge, Connecticut: Oxbow Press.

Seevinck, Michiel. 2016. "Challenging the Gospel: Grete Hermann on von Neumann's No-Hidden-Variables Proof." In Crull and Bacciagaluppi 2016, 107–117.

Soler, Léna. 2016. "The Convergence of Transcendental Philosophy and Quantum Physics: Grete Henry-Hermann's 1935 Pioneering Proposal." In Crull and Bacciagaluppi 2016, 55–69.

Stöltzner, Michael. 1999. "Mach, Boltzmann, Exner." *Synthese* 119 (Ludwig Boltzmann 'Troubled Genius as Philosopher'): 85–111.

———. 2002. "Franz Serafin Exner's Indeterminist Theory of Culture." *Physics in Perspective* (Berlin) 4:267–319.

———. 2003a. "Causality, Realism and the Two Strands of Boltzmann's Legacy (1896-1936)." PhD diss., Universität Bielefeld.

Stöltzner, Michael. 2003b. “Vienna Indeterminism II: From Exner to Frank and von Mises.” In *Logical Empiricism: Historical and Contemporary Perspectives,* edited by Paolo Parrini et al., 194. Pittsburgh: Pittsburgh University Press.

———. 2008. “Can meaning criteria account for indeterminism? Moritz Schlick on causality and verificationism in quantum mechanics.” In *Moritz Schlick Leben, Werk und Wirkung,* edited by Fynn Ole Engler and Matthias Iven, 215–245. Berlin, Parerga.

———. 2009. “The Logical Empiricists.” In *The Oxford Handbook of Causation,* edited by Helen Beebee et al. New York: Oxford University Press.

———. 2011. “The Causality Debates of the Interwar Years and Their Preconditions: Revisiting the Forman Thesis from a Broader Perspective.” In *Weimar Culture and Quantum Mechanics: Selected Papers by Paul Forman and Contemporary Perspectives on the Forman Thesis,* edited by Cathryn Carson and Alexei Kojevnikov, 505–522. London/Singapore: Imperial College Press / World Scientific Publishing.

———. 2012. “Erwin Schrödinger, Vienna Indeterminist.” In *Probabilities, Laws, and Structures. The Philosophy of Science in a European Perspective,* edited by Dennis Dieks et al., 3:481–495. Dordrecht: Springer.

Strien, Marij van. 2014. “On the origins and foundations of Laplacian determinism.” *Studies in History and Philosophy of Science Part A* 45:24–31.

Ullmo, Jean. 1934. “L’évolution de la notion de corpuscule d’après M. Langevin.” *Scientia* 28:103–117.

Waismann, Friedrich. 1930–31. “Logische Analyse des Wahrscheinlichkeitsbegriffs.” *Erkenntnis (1930-1938)* 1:228–248.

Weizsäcker, Carl Friedrich von. 1937. “Rev. of Cassirer 1936.” *Physikalische Zeitschrift* 38:860–861.

Weyl, Hermann. 1928. *Gruppentheorie und Quantenmechanik.* Leipzig: Hirzel.

———. 1931. *Gruppentheorie und Quantenmechanik.* 2nd ed. Leipzig: Hirzel.

———. 1932. *The Open World: Three Lectures on the Metaphysical Implications of Science.* New Haven: Yale University Press. Repr. in Weyl 2009, Ch. 4.

———. 2009. *Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics.* Edited by Peter Pesic. Princeton: Princeton Univerity Press.