Is the universe in a mixed state?

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Abstract

Quantum mechanics with a fundamental density matrix has been proposed and discussed recently. Moreover, it has been conjectured that the universe is not in a pure state but in a mixed state in this theory. In this paper, I argue that this mixed state conjecture has two main problems: the redundancy problem and the underdetermination problem, which are lacking in quantum mechanics with a definite initial wave function of the universe.

1 Introduction

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently (Dürr et al, 2005; Chen, 2019, 2021). It replaces the wave function in quantum mechanics (QM) with the density matrix and correspondingly the Schrödinger equation with the von Neumann equation. Since quantum dynamics can be formulated directly in terms of the density matrix, it seems reasonable to assume that the ontic state of the universe is represented not by a wave function in QM but by a density matrix in W-QM, which may be a mixed state. In other words, the universe may be not in a pure state but in a mixed state in W-QM (Chen, 2021). In this paper, I will present a new analysis of this interesting conjecture.

The rest of this paper is organized as follows. In Section 2, I discuss the empirical equivalence between W-QM and QM and argue that the mixed state conjecture has a redundancy problem. According to this conjecture, the universal density matrix in W-QM contains all possible universal wave functions in QM, while each of them is enough for explaining the empirical data. In Section 3, I analyze Chen's main argument against QM that there is no natural choice for an initial quantum state of the universe in QM. It is argued that W-QM has the same issue when the relevant Hilbert space is infinite-dimensional. In Section 4, I argue that the mixed state conjecture also has an underdetermination problem. Since W-QM and QM give the same empirical predictions, no experiments can determine whether the universe is in a mixed state or in a pure state. Moreover, I argue that QM with a definite initial universal wave function does not have these issues. Conclusions are given in the last section.

2 The redundancy problem

According to Dürr et al (2005) and Chen (2019), W-QM and QM are empirically equivalent when assuming that in QM a random wave function is assigned to the universe such that the associated statistical density matrix equals the fundamental density matrix assigned to the universe by W-QM. Suppose the fundamental density matrix of the universe at an initial instant t_0 is

$$W_0 = \sum_{i=1}^{N} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{1}$$

where N is the dimension of the Hilbert space of the universe, $p_i \in [0, 1]$ satisfies the nomalization relation $\sum_i p_i = 1$, and $|\psi_i\rangle$ is a set of orthogonal states in the Hilbert space. The equivalence between W-QM and QM means that one can assign a random wave function $|\psi_i\rangle$ with probability p_i or a mixed state W_0 to the universe and use either QM or W-QM for the same empirical predictions. In particular, when a subsystem of the universe is prepared such that QM assigns a wave function to it, W-QM must also assign the same pure state to it.[1] Otherwise, W-QM and QM will be not empirically equivalent; when measuring whether this subsystem is in the state represented by the wave function, QM will predict a definite positive result, while W-QM will not do. This means that one can do the same physics for subsystems of the universe in W-QM as in QM.

When one realizes that in W-QM the universal density matrix such as (1) contains all possible universal wave functions in QM, one will be not so surprising about the result that W-QM can be empirically equivalent to QM. However, this also introduces a serious problem for W-QM (with the mixed state conjecture), namely the redundancy problem. Once the random wave function of the universe is choosen at an initial instant in QM, all empirical data can be explained by this wave function and its time evolution. By contrast, W-QM includes all possible random wave functions of the universe, while each of them is enough for explaining the empirical data. The redundancy exists at both the mathematical and the physical

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[1] I thank Rodi Tumulka for helpful discussion here.

ontological levels. The degree of redundancy can be represented by N, the dimension of the Hilbert space of the universe. In a sense, the W-QM world is composed of N QM worlds. If N is a very large number for the initial quantum state of the universe, the degree of redundancy for W-QM will be very great.

The redundancy problem can be illustrated with an example in Bohmian mechanics with a fundamental density matrix (W-BM). Suppose in Bohmian mechanics (BM) the universal wave function at a given instant t is a random wave function $|\psi_i\rangle$ with probability p_i, where $|\psi_i\rangle$ is a set of orthogonal states in the Hilbert space of the universe, and $p_i \in (0,1)$ satisfies the nomalization relation $\sum_i p_i = 1$. Then, in W-BM the universal density matrix can be written as

$$W_t(x,x') = \sum_{i=1}^{N} p_i \psi_i(x) \psi_i^*(x'), \tag{2}$$

where x, x' are the generic variables for the configuration of all particles in the universe, and N is the dimension of the Hilbert space of the universe. Then, the guiding equation for the world point Q in the configuration space is:

$$v(Q) = \frac{\sum_{i=1}^{N} p_i \rho_i(Q) v_i(Q)}{\sum_{i=1}^{N} p_i \rho_i(Q)}, \tag{3}$$

where $\rho_i(Q) = \psi_i(Q)\psi_i^*(Q)$, and $v_i(Q)$ is the velocity of the world point Q corresponding to the i-th random wave function $\psi_i(x)$.[2]

When the wave functions $\psi_i(x)$ overlap in the configuration space, $v(Q)$ is not equal to any $v_i(Q)$ in general. This means that the trajectory of the world point Q in W-QM will be different from the trajectory of the world point Q in QM with each random wave function $\psi_i(x)$. However, the trajectory of the world point Q corresponding to each random wave function $\psi_i(x)$ in QM is enough for explaining the empirical data. In this sense, W-QM is redundant, since it includes all these random wave functions.

When the wave functions $\psi_i(x)$ do not overlap in the configuration space, the redundancy of W-QM is more obvious. In this case, the world point Q will reside in one of these wave functions such as $\psi_n(x)$, and the above guiding equation will be $v(Q) = v_n(Q)$. This means that the trajectory of the world point Q in W-QM will be the same as the trajectory of the world point Q in QM with the random wave function $\psi_n(x)$, and thus other wave functions in the universal density matrix in W-BM will be redundant for empirical predictions.

Note that this redundancy problem does not plague the many-worlds interpretation of quantum mechanics (MWI). MWI provides a way to solve the

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[2] I thank Shelly Goldstein for helpful discussion here.

measurement problem without revising (unitary) QM or adding additional variables and their dynamics to the theory. If one removes the many-worlds ontology, one must change the mathematical formalism of QM in order to solve the measurement problem in a realist way. By contrast, one can remove all but one of the random wave functions in W-QM and use QM, a reduced form of W-QM for pure states, to explain all empirical data.

3 There is a natural choice?

Then, why not QM but W-QM? According to Chen (2019, 2021), in QM there is no natural choice for an initial quantum state of the universe, but in W-QM there is a natural choice for the initial quantum state of the universe, and it is the maximum mixed state.

Chen's argument is as follows. Given the past hypothesis (Albert, 2000), the initial wave function of the universe is randomly chosen from the past hypothesis subspace in QM. Suppose the low-entropy initial condition arises from a compact region in the configuration space such that any wave function in the subspace has compact support in that region and zero elsewhere. On the one hand, the most natural function defined in this compact region will be uniform in the entire region. This function will be discontinuous at the boundary, which means that it contains arbitrarily high momentum eigenstates and thus it is unphysical. On the other hand, if this function is made continuous and differentiable, it will be not unique since there are many choices for such a function, and none of them is more natural than the others. Thus, Chen concludes that there is no natural choice for the initial wave function of the universe even in this simplest case.

I agree with Chen that in QM there is no natural choice for a physical wave function when requiring that the wave function is nonzero in a compact region and zero elsewhere in the configuration space. But in this case the Hilbert space is infinite-dimensional, and W-QM has a more serious non-normalizable problem. According to Chen's (2021) initial projection hypothesis, the natural choice for the initial quantum state of the universe in W-QM is the projection onto this infinite-dimensional space, but the projection is non-normalizable and unphysical (Chen, 2019). This means that in W-QM there is no natural choice for the initial quantum state of the universe either when the Hilbert space is infinite-dimensional. Thus W-QM is not better than QM in this case.

How about a finite-dimensional Hilbert space such as a spin space? In this case, there is indeed a natural choice for the initial quantum state of the universe in W-QM, which is the normalized projection onto this finite-dimensional space. But in QM, the situation is not so bad either; even though there is no natural choice for a unique state, it seems that the initial wave function of the universe may be an arbitrary (normalized) state vector in this space, since all these state vectors have equal status. Moreover, it may be the case that there is a particular initial condition that uniquely determines the initial wave function of the universe in QM (see below).

4 The underdetermination problem

My final complaint about W-QM is that the empirical equivalence of W-QM with QM leads to underdetermination. Since W-QM and QM give the same empirical predictions, no experiments can determine whether the universe is in a mixed state or in a pure state. Certainly, one may exclude W-QM by Occam's razor, since it is redundant for empirical predictions. However, underdetermination in ontology is still there if only W-QM (with the mixed state conjecture) exists. Then, why is there such a theory? It is because the wave function of the universe at an initial instant is random in QM. It is this random wave function assumption of QM that leads to underdetermination. In the following, I will give a more detailed analysis of this assumption.

The key is to understand the origin of these random wave functions and their probabilities in QM. There are two possible origins.[3] One is the deterministic origin: the "random" wave function is determined by a certain unknown condition at the initial instant of the universe. In this case, the probabilities are subjective or epistemic, relating to our incomplete knowledge about the initial state of the universe. The other is the stochastic origin: the random wave function results from a stochastic process of the universe at the initial instant. In this case, the probabilities are objective or ontic, inherent to the initial state of the universe itself.

Here one may argue, as Chen (2021) did, that W-QM unifies two types of probabilities, the statistical probabilities and the quantum probabilities, into one type of probabilities, the quantum probabilities. For example, in W-BM there is only the probability of the initial particle configuration of the universe, and it is equal to the product of the probability of the initial random wave function of the universe and the probability of the initial particle configuration of the universe in QM. This is indeed true. But the question is: are the statistical probabilities really necessary for explaining the empirical data?

In the deterministic case where the statistical probabilities are epistemic, the initial wave function of the universe is completely determined by a certain initial condition of the universe, and its randomess is only apparent, resulting from our ignorance about the initial condition. This means that the laws of nature require that the (objective) probability distribution of the initial wave functions of the universe is a delta distribution, and thus

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[3] Dürr et al (2005) seem to treat the probabilities as epistemic. Chen (2019) suggests taking the probabilities to be those given by the past hypothesis and the statistical postulate, which may be epistemic or ontic depending on how those principles are implemented.

W-QM will reduce to QM. In this case, we will have a theory of QM, in which the initial wave function of the universe is uniquely determined by an additional law such as a definite boundary condition,[4] and the (subjective) statistical probabilities do not appear in this fundamental theory.[5]

In the stochastic case where the statistical probabilities are ontic, the initial wave function of the universe is essentially random, resulting from a stochastic process of the universe at the initial instant. In this case, the statistical probabilities are necessary for empirical predictions. However, the existence of such a stochastic process is doubtful. First, in a deterministic QM, there is neither an external system outside the universe nor an internal dynamical process which "chooses" the wave function of the universe randomly with a particular probability at the initial instant. For example, in BM, the dynamics for the wave function is continuous and deterministic, and there is no stochastic process to make the wave function of the universe random. Anyway, it would be strange to assume that the initial state of the universe must have a random feature in a deterministic theory. Next, even in a stochastic theory such as collapse theories of QM, the initial state of the universe cannot be randomly collapsed from an earlier state by definition. Moreover, it would be also strange to assume that at the initial instant of the universe there exists another stochastic process which is different from the dynamical collapse of the wave function.

It is regarded as an advantage of W-QM over QM that the initial universal quantum state in the theory is assumed to be a *definite* mixed state (Chen, 2021). By the same reasoning, QM with a definite initial universal wave function is better than QM with a random initial universal wave function. In addition, QM with a definite initial universal wave function does not lead to underdetermination, since W-QM reduces to QM in this case.

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[4] Hartle and Hawking (1983)'s no-boundary proposal can be regarded as one example of such a theory. According to this proposal, the wave function of the universe, which obeys the Wheeler-DeWitt equation, is given by a path integral over all compact Euclidean four-dimensional geometries and matter fields that have the three-dimensional geometry and matter field configuration as its only boundary. Recently, Halliwell, Hartle and Hertog (2019) gave a formulation of the no-boundary wave function directly in terms of a collection of saddle points that satisfy a specific minimal set of criteria, and argued that it determines a unique wave function. Moreover, Feleppa, Licata and Corda (2019) showed that the no-boundary wave function can be obtained in a simple way starting from the Friedmann-Lemaitre-Robertson-Walker line element of cosmological equations for a de Sitter universe.

[5] Here there is a natural speculation on how to solve the puzzle of the arrow of time. It is usually thought that the past hypothesis and the statistical postulate are needed to solve this puzzle (see, e.g. Loewer, 2020). It is the statistical postulate that motivated the mixed state conjecture (Chen, 2021). Now it seems that QM with a definite initial universal wave function may solve the puzzle of the arrow of time in a simpler way. In this theory, the statistical postulate is dropped. Moreover, the past hypothesis can be naturally derived: since the universe has only one possible initial wave function, the initial entropy of the universe is zero, which means that the universe begins with the minimum entropy. Penrose's (1994) Weyl curvature hypothesis can be regarded as one example of such a theory.

If there were indeed two fundamental theories such as W-QM and QM that are completely empirical equivalent, then God would have another type of freedom to choose the theory for the universe (besides choosing the initial state of the universe), and we would be never able to find God's choice even if we obtain all empirical evidence. In this case, to paraphrase Einstein, God would be not only subtle, but also malicious.

5 Conclusion

Is the universe in a mixed state? Probably not. In this paper, I argue that the mixed state conjecture, which says that the universe is not in a pure state but in a mixed state in quantum mechanics with a fundamental density matrix (W-QM), has two main problems. One problem is redundancy: W-QM includes all possible random wave functions of the universe, while each of them is enough for explaining the empirical data in QM. The other problem is underdetermination: since W-QM and QM give the same empirical predictions, no experiments can determine whether the universe is in a mixed state or in a pure state. Moreover, I argue that QM with a definite initial universal wave function does not have these problems. However, how to construct this theory remains a great challenge.

References

[1] Albert, D. Z. (2000). Time and Chance. Harvard University Press.

[2] Chen, E. K. (2019). Quantum States of a Time-Asymmetric Universe: Wave Function, Density Matrix, and Empirical Equivalence. http://philsci-archive.pitt.edu/15658/.

[3] Chen, E. K. (2021). Quantum Mechanics in a Time-Asymmetric Universe: On the Nature of the Initial Quantum State. The British Journal for the Philosophy of Science, 72(4), 1155-1183.

[4] Dürr, D., Goldstein, S., Tumulka, R., and Zanghi, N. (2005). On the role of density matrices in Bohmian mechanics. Foundations of Physics, 35(3), 449-467.

[5] Feleppa, F., Licata, I. and Corda, C. (2019). Hartle-Hawking boundary conditions as Nucleation by de Sitter Vacuum. Phys. Dark Universe 26, 100381.

[6] Halliwell, J.J., Hartle, J.B. and Hertog, T. (2019). What is the no-boundary wave function of the Universe? Phys. Rev. D 99, 043526.

[7] Hartle, J. B. and Hawking, S. W. (1983). Wave function of the Universe. Phys. Rev. D 28, 2960.

[8] Loewer, B. (2020). The Mentaculus Vision. in: Allori, V. (ed.), Statistical Mechanics and Scientific Explanation: Determinism, Indeterminism and Laws of Nature, World Scientific. pp.3-30.

[9] Penrose, R. (1994). On the second law of thermodynamics. J. Stat. Phys., 77, 217.