The conventionality of geometry is merely incomplete

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Abstract

Empiricists following Poincaré have argued that spacetime geometry can be freely chosen by convention, while adjusting unobservable structure so as to maintain empirical adequacy. In this article, I first strengthen a no-go result of Weatherall and Manchak against the conventionality of geometry, and then argue that any remaining conventionality arises from scientific incompleteness. To illustrate, I discuss a new kind of conventionality that is available in the presence of higher spatial dimensions, and illustrate how the incompleteness in such models can be resolved by introducing new physical theories like Kaluza-Klein theory. Conventional choices of this kind may provide a fruitful starting point in the search for new science, but if successful would eliminate the conventionalist alternatives.

1. Introduction

If I ask you whether the edge of the page is straight, then you can surely check that it is. But, if I were to then hold up my wiggly ruler and ask you whether it is straight in that sense, then I suppose you would give a different answer. So, physical geometry depends to some extent on our conventions about straightness and distance, which Carnap (1922, §III) calls 'straightness and metrical stipulation'.

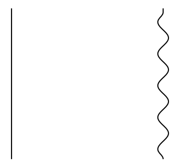

Figure 1: Two different conventions of straightness.

To say only this is not to say much.[1] It certainly does not follow that the structure of spacetime is a social constructivist free-for-all. For example, on

[1]This point has been argued by Eddington (1920, p.10), Grünbaum (1963, p.27), Putnam (1974, p.32), and especially Lewis (1969, p.1).

both straightness conventions of Figure 1, spatial geometry changes as one moves from an empty region towards a gravitating body. You can play the semantic game of stipulating a new referent for the word 'Euclidean', if that is the sort of thing you are into. But, this freedom is kind of trivial, in the sense that it is not unique to geometry alone. A more impressive observation about geometry is that, whatever metrical stipulations we make, spacetime geometry is related in a law-like way to the distribution of matter and energy in the universe. Thus physicists speak of "a geometrodynamical universe: a world whose properties are described by geometry, and a geometry whose curvature changes with time" (Wheeler 1962, p.361). That is a deep idea that is unique to geometry. The idea that we can trivially change the meaning of our words, called 'Trivial Semantic Conventionalism' (Grünbaum 1963, p.27), is not.

On the other hand, the language of spacetime geometry is quite different from most ordinary language, because it does not refer to anything directly observable.[2] The *empiricist response* to this is to deny that 'unobservable geometry' refers to anything at all. Then one can freely adopt the convention of choosing any spacetime geometry that is convenient, so long as it is compatible with the laws and provides accurate predictions. The *realist response* is to insist there is a true spacetime geometry, whether or not it is observable. The realist should then explain how the conventionalist's alternatives are inappropriate, for example because they are mathematically or physically impossible.

I will argue that the conventionality of geometry is not mathematically or physically impossible: it rather indicates the presence of incomplete science. Non-trivial conventionality can arise for the geometry of space, but it does so out of physical properties that are incompletely described, in the sense of being conceptually isolated from the rest of physics. Conventionality of this kind is rather hard to come by: to illustrate, I will prove two strengthenings of a theorem of Weatherall and Manchak (2014), which show that the there is little conventionality available through the introduction of hidden 'universal forces' in relativity theory. I will then consider an alternative form of conventionality of geometry that arises out of higher spatial dimensions, and argue that Kaluzi-Klein theory provides an indication of how this sort of conventionality actually amounts to incompleteness.

[2] As Riemann (1873, p.14) lamented, the darkness that shrouds physical geometry is "cleared up neither by mathematicians nor by such philosophers as concerned themselves with it."

2. Conventionality and its discontents

2.1. **The Poincaré disc.** A great awkwardness of spacetime is that it seems to have a geometry, which explains empirical measurements like spatial distance and temporal duration, but which on most textbook accounts is not directly observable. We can sure enough use measuring devices like rulers and clocks to access spacetime geometry indirectly. But, our conclusions are only as reliable as those rulers and clocks. If some hidden influence distorts our measuring devices, then our conclusions about spacetime geometry will be distorted too.

A classic illustration of this concern is the sphere of Poincaré (1905, p.65-68), commonly described in two-dimensions as a disc: consider a Euclidean surface of radius R that is hot at the centre and cold at the edges, with temperature at radius $r \in [0, R)$ proportional to $R^2 - r^2$. Suppose further that when rulers are placed on the surface, they expand and contract in proportion to the temperature so as to give the appearance of a non-Euclidean geometry. The rulers on this disc would not measure the shortest distance between two points p and q to be a straight Euclidean line, but rather a geodesic of hyperbolic 'Lobachevsky' geometry, illustrated in Figure 2.

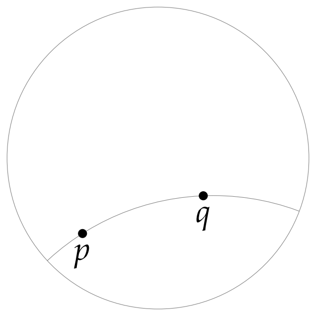

Figure 2: The shortest path between two points on Poincaré's disc is a hyperbolic geodesic, as measured by rulers distorted by the disc's heat gradient.

Of course, it would be hard to miss that the disc is heated if you had your wits about you. Better yet, by using a thermally insulated ruler one could avoid this distortion altogether. However, if unbeknownst to us there were some hidden 'universal' force defined so as to distort all rulers in the same way, then the true physical geometry of the disc would seem completely inaccessible. Reichenbach (1928, p.22) concludes from this that one can at most decide to "set the universal forces equal to zero by definition" as a matter of pure convention.

2.2. **Reactions and discontents.** The Poincaré's disc led many philosophers and scientists to interpret the spatial metric as a conventional choice, analogous to a choice of measurement units like metres or yards. Euclidean geometry, viewed for centuries following Newton as the science of space, was dramatically demoted, to a degree that some saw as "comparable in some respects to Kant's Copernican revolution" (Ben-Menahem 2006, p.5). In its place, a conventionalist philosophy of geometry was defended by empiricists[3] like Poincaré (1905), Schlick (1920, Chapter V), Carnap (1922), and Reichenbach (1928, §3), and developed in detail by Grünbaum (1962, 1963, 1969) as an "intrinsically metrically amorphous" interpretation of spacetime.[4] Even Einstein famously concluded:[5]

> Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G)+(P) is subject to experimental verification. Thus (G) may be chosen arbitrarily, and also parts of (P); all these laws are conventions. ... Envisaged in this way, axiomatic geometry and the part of natural law which has been given a conventional status appear as epistemologically equivalent." (Einstein 1921, p.236)

Despite these impressive early announcements, the conventionality of geometry soon fell into disrepute. Physics textbooks now generally agree that "the geometry of space is a new physical entity, with degrees of freedom and a dynamics of its own" (Misner, Thorne, and Wheeler 1973, p.ix)—and that it is given, for example, by the Minkowski metric in weak gravitational regimes, and by the Schwarzschild metric near a static and spherically symmetric black hole.

Philosophers have also given a variety of challenges to the conventionality of geometry. For example, Earman (1970) points out that each observer in a relativistic spacetime will define a unique induced spatial metric on the surface orthogonal to that observer's worldline, apparently eliminating the possibility of alternative spatial geometries for Poincaré's disc. Glymour (1977) argues that a

[3]Ben-Menahem (2006, §2.II) has given a chapter-length analysis of Poincaré's own conventionalism, although Worrall (1989) and Ivanova (2015a,b) have argued it is rather a kind of structural realism. See for an analysis and critique of Einstein's view.

[4]A discussion and critique of Grünbaum's view was given by Sklar (1972, 1974) among others, although Sklar remained convinced that nevertheless, Poincaré-style "'conventionalist alternatives' will arise" (Sklar 1974, p.112).

[5]See Bacelar Valente (2017) for an analysis and critique.

Bayesian perspective on theory confirmation should lead us to accept the standard metric over Poincaré-style alternatives. Friedman (1983, Chapter VII) argues that Poincaré-style conventionality can be dismissed by a principle of parsimony that was an important part of the development of relativity theory.[6] Thus, Friedman writes, "[t]here is no sense in which this metric is determined by arbitrary choice or convention" (Friedman 1983, p.26).

However, a particularly influential critique of conventionality began with a series of papers by Putnam (1959, 1963, 1974). Putnam's basic thesis is that spacetime geometry is built into scientific theories in such a way that if we were to replace a given geometry with a conventionally-chosen alternative, it would make an irreparable mess of those other parts of the theory. He concludes that although such conventionalist alternatives may be logically consistent, they fail to provide a 'usable' model of reality:

> "as far as we know, the choice of any non-standard space-time metric would lead to infinite complications in the form of the laws of nature and to an unusable concept of space-time distance. Thus, as far as we know, the metric of space-time is not relative to anything. There is no interesting sense in which we can speak of a conventional 'choice' of a metric for space-time in a general or special relativistic universe" (Putnam 1974, pp.34-35).

According to Putnam, the only sense in which we might say there are alternative geometries is by completely redefining other concepts in our theory. But this, he claims, collapses into Grünbaum's 'trivial semantic conventionalism', akin to my wiggly-ruler for measuring straightness, and rendering the conventionality of geometry no different from standard conventions of ordinary language.

One of Putnam's central examples is how Poincaré-style alternative geometries require us to redefine what a 'force' means in physics. Taking the case of Hooke's law, according to which the restoring force of a spring is proportional to its spatial distance out of equilibrium, Putnam writes,

> "If we decide by 'distance' to mean distance according to some other metric, then in stating Hooke's law we shall have to say that force

[6] Putnam (cf. 1974, p.33) makes a similar critique, although it is not clear that physics is beholden to such virtues as simplicity or parsimony (Norton 2021, Chapter 5). DiSalle (2002) responds to Friedman that some themes of conventionalism are compatible with relativity theory, but still finds that spacetime geometry is fixed by a process of conceptual analysis.

depends not on length but on some quite complicated function of length; but that quite complicated function of length would be just what we ordinarily mean by 'length."' (Putnam 1963, p.220)

There may be room to disagree that this complicated function of length really is best interpreted as 'just what we ordinarily mean' by length. However, I think that Putnam's conclusion here is still correct, because there is a stronger argument against such redefined forces in relativity theory. In particular, I will argue that a theorem due to Weatherall and Manchak (2014) shows the conventionalist does not have complete freedom to choose any spacetime metric whatsoever because the universal forces this would require cannot be defined, except in a semantically trivial sense.

3. Semantic triviality of universal forces

3.1. **On the meaning of 'force'.** What is a force? Various authors[7] have proposed that a force must at least be proportional to acceleration, as in Newton's second law, $F = ma$. However, since our concern is relativity theory, let me first motivate the relativistic version of Newton's second law. For, it is often suggested that Newtonian mechanics was falsified by Einstein's theories. That is not quite right: Newton's second law is carried over directly from Newtonian mechanics into general relativity, albeit in a slightly different language.

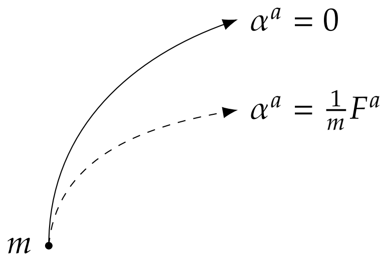

Figure 3: A massive test particle follows a geodesic ($\alpha^a = 0$) unless deflected by a force, in which case it accelerates according to $F^a = m\alpha^a$ (dashed line).

Given the usual definition[8] of a relativistic spacetime (M, g_{ab}), the metric

[7]Cf. Friedman (1983, p.258), Torretti (1983, pp.237-8), and Weatherall and Manchak (2014, p.236).

[8]A *relativistic spacetime* is a connected four-dimensional C^∞ manifold M without boundary, with a Lorentz-signature metric g_{ab}. The Levi-Civita connection ∇_a is the unique torsion-free connection satisfying *compatibility*, that a vector field is constant with respect to ∇_a if and only if it is constant with respect to g_{ab} (Malament 2012, §1.9).

g_{ab} uniquely determines an affine connection (or 'covariant derivative operator') ∇_a that allows one to define what it means to accelerate: if ξ^a is the velocity vector field tangent to the worldline of a test particle, then the acceleration of the particle is given by $\alpha^a := \xi^b \nabla_b \xi^a$. What it means to be the 'force' F^a on that curve can then be given by Newton's second law, in that if the test particle has rest-mass m, then,

$$F^a = m\alpha^a. \tag{1}$$

Newton's first law carries over as well, as the statement that in the absence of forces ($F^a = 0$), a test particle will follow a curve of zero acceleration ($\alpha^a = 0$) called a *geodesic* (Figure 3). These are the generalisation of straight lines to geometries with curvature. Thus, Newton's laws did not go extinct with relativity theory, but live on in loftier form, like those dinosaurs that evolved into birds.

This is not the only constraint that physical theory imposes on the meaning of forces. For example, if energy is bounded from below—as it is observed to be in Nature, and as it must be if matter is to avoid catastrophic collapse—then forces depend only on position and velocity, and not on any higher derivatives.[9] As a result, one can generally view a force as arising from a map $F^a{}_b$ that takes a test particle's velocity ξ^a at each point to a force vector, $\xi^a \mapsto F^a := F^a{}_b \xi^b$. A short calculation then shows that Newton's second law constrains this map to be antisymmetric.[10]

These constraints are not arbitrary. They summarise a large and interconnected body of ideas and practices in physics, which all agree to use the word 'force' to refer to whatever phenomenon is responsible for acceleration in space and time. Of course, nothing prevents you from choosing semantic conventions for these words, either by changing the meaning of 'force' to something arbitrary, or by changing what it means to be 'responsible for acceleration'. That is just the trivial semantic conventionalism that pervades all of ordinary language. But, in order to understand whether there is any separate sense in which physical geometry is conventional, one must hold those meanings fixed.

[9] Forces with higher derivatives lead to the unstable collapse of matter due to Ostrogradski instability; see Swanson (2019), in response to an argument for this property due to Easwaran (2014) on the basis of a causal reductionist account of change.

[10] Acceleration is always orthogonal to velocity, in that $\alpha_a \xi^a = 0$ (see Malament 2012, p.142). So, if $F^a = m\alpha^a$, then $0 = m\alpha^a \xi_a = F^a{}_b \xi^b \xi_a = F_{ab} \xi^b \xi^a = F_{ab} \xi^a \xi^b$, and so the symmetric part of F_{ab} vanishes. Therefore, F_{ab} is antisymmetric.

3.2. **Weatherall and Manchak's no-go theorem.** Weatherall and Manchak (2014) point out this accepted meaning of the word 'force' has immediate consequences for the conventionality of geometry. For instance, given a spacetime metric g_{ab} and a 'conventionally chosen alternative' metric $\tilde{g}_{ab}$, there is a tradition in the literature following Reichenbach (1928, §8) and Grünbaum (1963, Chapter 3A) to define the universal forces F_{ab} by the relation $F_{ab} := \tilde{g}_{ab} - g_{ab}$. But, this is impossible for the kind of forces we have just described, because g_{ab} and $\tilde{g}_{ab}$ are symmetric while F_{ab} is antisymmetric.

So, if one wants to give a recipe for determining the universal force that will make any conventional choice of metric $\tilde{g}_{ab}$ equivalent to the apparent metric g_{ab}, an alternative recipe is needed. Remarkably, Weatherall and Manchak show that no such general recipe exists. In particular, there are choices of an alternative metric $\tilde{g}_{ab}$ for which no universal force will satisfy Newton's law, whenever $\tilde{g}_{ab}$ is given by a non-constant 'rescaling' in the sense of a conformal transformation. Their theorem may be informally summarised:[11]

> *If a conventionally-chosen alternative metric is related to the original metric by a non-constant conformal transformation, then there is no force that satisfies $F = ma$ in the alternative geometry on exactly the curves that are (zero force) geodesics in the original geometry.*

This result lends some much-needed precision to a worry of Nagel (1961, p.265): "[i]t is by no means self-evident, however, that physical theories can in fact always be devised that have built-in provisions for such universal forces." The conventionalist might have hoped to enjoy the complete freedom to choose any arbitrary replacement for the spacetime metric, by conjecturing that there is a general prescription for devising a universal force that produces the same description of motion. Weatherall and Manchak (2014) have torpedoed that hope, by proving that this conjecture is false.

As Weatherall and Manchak themselves point out, this result does not necessarily refute all forms of conventionalism, so long as one changes what it means to 'freely choose a geometry by convention'. As an example, they consider

[11] The formal statement is: for a given spacetime (M, g_{ab}), if $\tilde{g}_{ab} = \Omega^2 g_{ab}$ with Ω nonconstant, and if ∇ and $\tilde{\nabla}$ are the respective Levi-Civita connections, then there is no tensor field $F^a{}_b$ such that a curve γ is a ∇-geodesic if and only if its acceleration with respect to $\tilde{\nabla}$ satisfies Newton's law, $\tilde{F}^a = \tilde{F}^a{}_b \tilde{\xi}^b = m\tilde{\xi}^b \tilde{\nabla}_b \tilde{\xi}^a = m\tilde{\alpha}^a$, where $\tilde{\xi}^b$ is the $\tilde{g}_{ab}$-unit tangent velocity field to γ (Weatherall and Manchak 2014, Proposition 2).

a conventionalist who associates their arbitrary geometry $\tilde{g}_{ab}$ with a different kind of 'force' map, which defines the force vector F^a using *two* vectors at a point rather than one, $\xi^b, \chi^c \mapsto F^a := F^a{}_{bc}\xi^b\chi^c$. Then one can always formally reproduce the geodesic motion of the original geometry as motion that follows from Newton's second law.[12] This proposal was recently defended by Dürr and Read (2024, §5.2.2). However, if those further vectors are in any way determined by the motion of the particle, then it would have to depend on higher derivatives in a way that is not possible due to the considerations above.

Duerr and Ben-Menahem and Dürr and Read (2024) defend a related response: for any given metric g_{ab} suppose we freely choose any alternative metric g'_{ab} as our preferred geometry, and then define $G_{ab} := g_{ab} - g'_{ab}$. In their view, "nothing *compels* us to interpret G_{ab} as a force: prima facie, we find nothing inherently absurd... in interpreting G_{ab} as a field, mediating a universal interaction" (Duerr and Ben-Menahem 2022, p.161). This proposal amounts to what is effectively the same thing, redefining what I have called 'the phenomenon that determines the acceleration of a test particle' as an effect determined by two vectors at a point rather than one. However, conventionalists in search of conventional freedom that goes beyond the semantically trivial would not be satisfied: this kind of redefinition is not unique to the nature of geometry. Of course one is also free to define one's terms. But, it is no more novel than my ability to label my wiggly ruler as 'straight'.

4. Stronger limitations on universal forces

Another line of response to Weatherall and Manchak's proposal has been to restrict one's conventional freedom a little bit, but not entirely.[13] These proposals each point out that the conventionality of geometry sensitively depends on what it means to say we may 'freely choose an alternative geometry'. Tasdan and Thébault (2024) focus on empirical underdetermination:

"What is essential within our family of generalized notions of space-

[12] Namely, if we write $F^a{}_{bc} := (1/m)C^a{}_{bc}$, where $C^a{}_{bc}$ is the 'Christoffel' tensor defined by the difference $\tilde{\nabla} - \nabla$ (Malament 2012, Proposition 1.7.3), and if we also write $F^a := F^a{}_{bc}\xi^b\xi^c$, then $\xi^b\nabla_b\xi^a - \xi^b\tilde{\nabla}_b\xi^a = -\xi^b\xi^c C^a{}_{bc} = (1/m)F^a$, and thus $\alpha^a = \xi^b\nabla_b\xi^a = 0$ if and only if $F^a = F^a{}_{bc}\xi^b\xi^c = m\xi^b\tilde{\nabla}_b\xi^a = m\tilde{\alpha}^a$.

[13] See especially Duerr and Ben-Menahem (2022), Tasdan and Thébault (2024), Dürr and Read (2024), Mulder (2024), and Mulder and Read (2024).

time conventionalism is that in each and every case it is required that a basic structure of a spacetime theory is empirically underdetermined, and this underdetermination leads to the possibility for physical differences to arise between conventions regarding how to break the underdetermination." (Tasdan and Thébault 2024, p.490)

In this section, I will argue that if these generalised notions of conventionalism involve forces of any kind, then they do little to improve the case for conventionality.

4.1. **A strengthened no-go result.** The Weatherall and Manchak (2014) no-go theorem by itself only establishes that the conventionalist cannot choose an alternative metric that is conformally related to the original.[14] Duerr and Ben-Menahem (2022, p.158) argue that this "is too tight a constraint: it doesn't give conventionalism as our authors themselves understand it, a proper chance." This led both Duerr and Ben-Menahem (2022) and Tasdan and Thébault (2024) to propose that one might still replace the spacetime metric g_{ab} with a conventionally-chosen alternative, so long as they are not related by a conformal rescaling. Tasdan and Thébault (2024, p.492) call this "Spacetime Conventionality 1", although they do not endorse it.

As it turns out, such a conventionalist cannot be helped even with this 'proper chance', in that dropping the assumption of a conformal rescaling still does not allow for an empirically adequate universal force. A formal statement of this fact is the following, which we prove in the Appendix.

Theorem 1. *Let (M, g_{ab}) and $(M, \tilde{g}_{ab})$ be spacetimes, with respective Levi-Civita connections ∇ and $\tilde{\nabla}$. Suppose there is some tensor field $F^a{}_b$ such that, whenever ξ^a is a timelike ∇-geodesic, $F^a := F^a{}_b \xi^b$ satisfies Newton's equation with respect to $\tilde{\nabla}$:*

$$F^a = m\tilde{\alpha}^a \tag{2}$$

with $\tilde{\alpha}^a = \xi^b \tilde{\nabla}_b \xi^a$ and $m > 0$. Then $\nabla = \tilde{\nabla}$ and $F^a{}_b = 0$.

In other words, for a given spacetime metric, there is no alternative metric whatsoever that describes motion as arising from non-zero forces satisfying

[14] As Malament (1985) and Weatherall and Manchak (2014) interpret Reichenbach, this assumption is basically required by Reichenbach's causal theory of time, which seems to commit him to the view that conformal structure is not conventional. However, dropping this requirement should not bother conventionalists who do not follow this aspect of Reichenbach's philosophy.

Newton's law in a way that matches the non-accelerated motion for the original metric. This considerably strengthens the Weatherall and Manchak theorem: we not only drop their assumption of a conformal rescaling, but also drop one of their technical assumptions about the velocity vector field ξ^a being unit, which was similarly challenged by Duerr and Ben-Menahem (2022). In my view, this leaves little hope for 'Spacetime Conventionalism 1'.

4.2. **Tidal forces and geodesic deviation.** An alternative approach to a more restrictive kind of conventionality replaces 'forces' with the deviation of curves. It is commonly noted that one does not measure the curvature of spacetime by observing a single geodesic. However, by measuring the extent to which two nearby geodesics emerging from a perpendicular deviate from Euclidean behaviour, one can measure the curvature of spacetime. For example, two geodesics emerging perpendicular to the equator on the surface of a sphere will reveal its positive curvature, through their accelerated approach to one another instead of remaining equal distances apart (Figure 4).

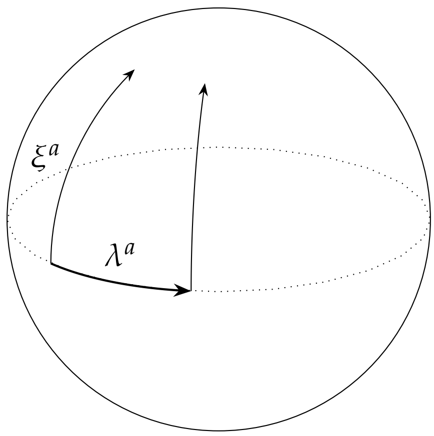

Figure 4: Deviation of a geodesic ξ^a with respect to a nearby geodesic determined by the orthogonal vector λ^a.

By measuring the deviation associated with two nearby curves, is it possible to determine the curvature of spacetime? If one is able to tell when those particles are following geodesics, then the answer is clearly yes: all the possible motions of geodesics are known to uniquely determine the spacetime curvature. However, if there is some hidden contribution to acceleration, whether it is a force or a more general 'effect' of the kind that Duerr and Ben-Menahem (2022) propose, then the deviation is described by a different equation, which includes the contribution of acceleration.

More formally, if we compare the motion of a test particle with tangent field ξ^a to that of a nearby curve determined by an orthogonal vector λ^a, the *deviation* of this deflection is defined by $\Delta^a := \xi^n \nabla_n(\xi^m \nabla_m \lambda^a)$. This quantity is provably related to the Riemann curvature $R^a{}_{bcd}$ determined by the metric, by what I will call the *deviation equation*,

$$\Delta^a = \xi^n \lambda^m \xi^b \underbrace{R^a{}_{bmn}}_{\text{curvature}} + \lambda^m \nabla_m (\underbrace{\xi^n \nabla_n \xi^a}_{\text{acceleration}}). \tag{3}$$

The deviation equation shows that if a test particle follows a geodesic, and thus has vanishing acceleration, then its deviation from a nearby geodesic is given entirely by curvature. In contrast, if one cannot determine whether a test particle is accelerating, then the deviation is only determined by curvature up to the additional contribution of that acceleration.

This observation lead Tasdan and Thébault (2024) to suggest a reformation of the conventionality of geometry, which they call 'Spacetime Conventionality 3'. I will formulate it as the claim that one is free to choose whatever metrical geometry one wants, so long as that geometry produces the same geodesic deviation. The result will always be empirically adequate, insofar as our empirical evidence is associated with geodesic deviation rather than the motion of individual particles.

As it turns out, this does not provide any more leeway for the conventionality of geometry either. Having the same geodesic deviation uniquely determines the spacetime geometry, in a sense given by the following statement, which is proved in the Appendix.

Theorem 2. *Let (M, g_{ab}) and (M, g'_{ab}) be relativistic spacetimes with Levi-Civita connections ∇ and $\tilde{\nabla}$. Suppose that for all timelike ξ^a they display equal deviation, $\Delta^a = \tilde{\Delta}^a$, for all λ^a such that $[\xi, \lambda] = 0$. Then $\nabla = \tilde{\nabla}$.*

Taken together, these two results show that there is no non-trivial conventionality of geometry arising from the acceleration of a particle by a hidden force, nor from the acceleration of geodesic deviation. I take this as an indication that there is little hope in seeking the conventionality of geometry through universal forces, either Newtonian or tidal.

5. Conventionality as merely incomplete

We have seen serious challenges to the claim that physical geometry is conventional. In this section, I would like to point out what I find to be an interesting alternative convention about physical geometry, which arises from the choice of how many spatial dimensions there are. After presenting this sense of conventionality, I will argue that it is neither mathematically nor physically impossible, but rather scientifically incomplete. I then discuss how such frameworks might be completed using the example of Kaluza-Klein theory, and find that they appear to succeed only insofar as they eliminate the conventionalist alternatives.

5.1. **Conventionality in higher dimensions.** A tiny and very flat being constrained to a two-dimensional spatial surface might struggle to perceive geometric facts originating in a third dimension. Similarly, human beings constrained to three dimensions of space might struggle to infer the existence of yet higher spatial dimensions. Mathematicians since Riemann (1873) have developed detailed studies of metrics in such higher dimensional spaces. These have now come to play a central role in modern string theory, which generally postulates at least nine dimensions of space and one of time. I would like to point out that they also introduce an interesting new sense in which physical geometry is conventional.

The Nash (1954) Embedding Theorem says that every Riemannian manifold, no matter how curved, can be smoothly embedded in a metric-preserving (isometric) way into an ordinary Euclidean manifold with some higher number of dimensions. To see why this is so surprising, consider the flat torus, defined by taking a unit square of the Euclidean plane and identifying its opposite sides. Thus, two lines that cross at the centre of the square are in fact a pair of intersecting 'circles' of the same length. But, these two lines have different lengths under the standard embedding of the torus into three-dimensional Euclidean space, since one of the lines will be mapped to a circle of larger radius than the other (Figure 5).

To achieve an isometric embedding, a much more creative map is needed, which includes 'ripples' flowing across the torus in such a way that the small meridian circles become larger in just the right way (Borrelli et al. 2012). This isometry in $\mathbb{R}^3$ also fails to be twice-differentiable, which means that its curva-

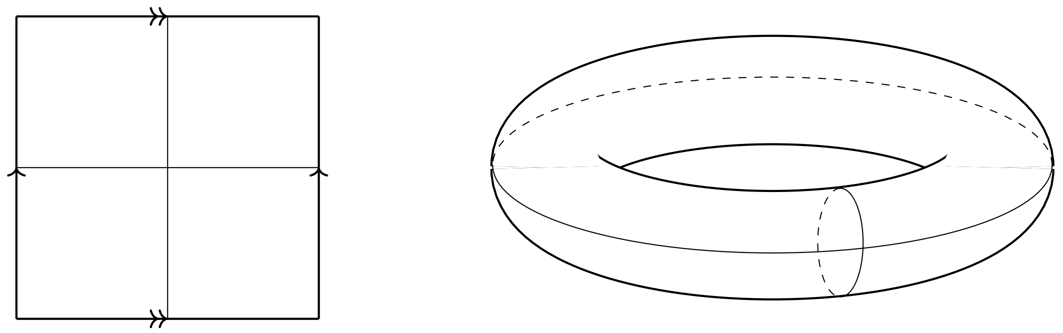

Figure 5: Crossing lines of equal length on a flat torus (left) no longer have equal length in the standard Euclidean embedding (right).

ture is not defined. However, Nash's theorem provides a general method for smoothing this isometry out through the use of higher dimensions of space.

Thus, whatever Riemannian metric we encounter in three dimensional space, we are free to adopt the convention of using the Euclidean metric in some higher dimensional space instead, and all the same geometrical facts will still be recovered on a submanifold given by the Nash embedding. Nash's same technique can be adapted to the Lorentzian metrics of general relativity as well: Greene (1970) and Clarke (1970) independently showed that a Lorentzian manifold can always be isometrically embedded into Minkowski spacetime with a sufficiently high number of dimensions. Of course, this by itself does not give one the complete freedom to choose any geometry that one wants. That would require an extension of Nash's theorem to embeddings into an arbitrary Riemannian manifold, which is not necessarily Euclidean. However, examination of Nash's proof suggests to me that such an extension may be possible:

> *Conjecture*. Every Riemannian manifold of dimension n can be isometrically embedded into every Riemannian manifold of dimension m for some integer $m = f(n)$.

A similar conjecture can be formulated for Lorentzian manifolds. However, at the moment, both of these statements appear to be open mathematical problems.

5.2. **Incompleteness and the fine tuning problem.** A conventionalist who describes the curvature of spacetime as arising from its embedding in higher dimensional Euclidean space has a lot of explaining to do. As a representation of physical space, at least three important features of this convention have been left unexplained:

- *The higher dimensions are hidden.* The observable world appears to consist in only three dimensions of space and one dimension of time. Why are the extra dimensions of space hidden from view?

- *A specific embedding is needed*. The observable world is recovered as a very special surface in this higher dimensional spacetime, which might in general be quite complicated to specify. Why is this particular embedding the relevant one?

- *The laws of nature are unspecified.* The observable laws of nature are regularities of four dimensional spacetime, which is only a partial (submanifold) description in the higher dimensional spacetime. What are the general laws of nature in higher dimensional spacetime, and how are they motivated?

Of course, not every structure in science requires explanation: one might well hit 'bedrock' and arrive at fundamental concepts for which no further explanation is possible or needed. However, the unexplained concepts in a conventionalist philosophy of geometry are not of this kind: I will argue that they represent an incompleteness in the model of physics. That incompleteness may yet be useful for the purposes of exploring new theories of physics. But, it is quite different from the conventionality of geometry as it was originally envisaged by empiricists.

One way to spot incompleteness in science is through the presence of a fine-tuning problem.[15] For example, in the conventional choice of a flat geometry described above, one must make a number of finely-tuned choices: the higher dimensional space, the embedded surface, and the higher dimensional laws of nature must all be chosen in exactly the right way, or else the description of four-dimensional spacetime will not match observations, and the theory will be empirically inadequate. This is a generic problem for the conventionality of geometry: whether it arises through universal forces, a connection with torsion, or an embedding into a higher dimensional space, conventionality requires introducing a remarkable coincidence, that our world just happens to arise in just the right kind of way to produce the appearance of the standard curved geometry.

[15] The phrase 'fine-tuning problem' is used by physicists to criticise unexplained free parameters in scientific modelling, originally associated with the cosmological constant (Weinberg 1989), and later in contexts like inflationary cosmology (Earman 1995, §5.12) and the constants of nature (Rees 1999).

Fine tuning is a different kind of problem than the charge of trivial semantic conventionality discussed in previous sections. The problem is not the introduction of trivial new definitions for words that already have meaning, like 'force' or 'phenomenon that determines acceleration'. Rather, fine tuning requires the introduction of new concepts that can be adjusted and changed in complete isolation from the other properties of a physical system.

By introducing an abstract higher dimensional spacetime, or universal forces, or it seems any other structure that makes geometry conventional, it appears that one must keep certain concepts isolated and independent from any other physical quantities. By its very nature, the freedom to choose one's conventions about a concept requires isolating that concept from the rest of physical theory. That isolation that indicates a theory is incomplete. In contrast, more complete physical theories satisfy a much greater degree of semantic holism: they are highly structured objects, with models that interlink a variety of concepts in a coherent fashion, and few free parameters.[16] Free parameters are not in general a problem, and are inevitable to some extent in physics. But, they are usually a sign that a physical theory is incomplete, because it is otherwise too easy to use free parameters to invent spurious but empirically adequate models. As John von Neumann is rumoured[17] to have remarked: "with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

Let me give a simple little example of this, before turning to a more interesting one. Consider the harmonic oscillator, one of the most ubiquitous forces physics. It is defined abstractly as a force proportional to distance from a preferred point in space, $F = -kx$. At this level of description, we can conventionally choose any value that we want for the constant k, even after a choice of units, because multiplying both k and m by the same factor we get the same solutions to Newton's equation, $F = ma$.

However, that conventional freedom is eliminated when the description is 'completed' with a more detailed account of the origin of the force, which generally determines values for both k and m. The former may be given by the elasticity of a rubber-band, or by the length of a pendulum in a gravity

[16]Semantic holism in twentieth century empiricism is commonly associated with Carnap (1934) and Quine (1951), but has more recently become an important aspect of the 'categorical turn' in interpreting scientific theories, developed by Halvorson (2012, 2019), Barrett (2015, 2020) Dewar (2016, 2022), Rosenstock (2016), and Weatherall (2016, 2021).

[17]According to Enrico Fermi, as reported by Dyson (2014).

field, or by the wave properties of a sound wave in water, among countless other things. The abstract harmonic oscillator is ubiquitous in physics precisely because it approximates *every* locally-defined force field.[18] So, it is no mystery that it can be associated with many different conventional choices of constants: the abstract harmonic oscillator contains great conventional freedom because it has been left radically incomplete. Once the description is completed, those conventionalist alternatives are eliminated. In the next section, I will argue that the conventionality associated with higher spatial dimensions has exactly the same character.

5.3. **The Kaluza-Klein miracle.** What would it mean for the conventionalist to provide a completion of a framework for alternative geometries like the one I have described above? It is not so easy to answer this kind of question, since it invariable requires some insight into new laws of nature. But, the example of Kaluza-Klein theory helps to illustrate the kind of thing that would be needed.

To introduce Kaluza-Klein theory, let me first recall Reichenbach's alternative proposal on how to 'geometrise' electromagnetic forces, through a conventional choice of metric not unlike the ones discussed above, and which he communicated in a letter to Einstein.[19] I think it is fair to say that Einstein did not like the idea, writing:

> "So, you have come among theoretical physicists, and chosen a bad area, at that. ... [Your] theory is not a connection between electricity and gravitation insofar as there is no mathematically unified field equation that simultaneously provides the field law of gravitation and that of electromagnetism; it does not provide a connection between electricity and gravitation either in the sense that it would tell us from which electromagnetic quantities the gravitational field arises.— I would not publish this; otherwise the same will happen to you as to me, who must disown his own children." (Einstein 1926, p.274).

As Einstein points out, Reichenbach's model—like the conventional geometry arising from an arbitrary higher dimensional embedding—does not provide a

[18] By 'locally-defined' I mean one described by an analytic function of space, which has a Taylor expansion $F = k_0 + k_1 x + k_2 x^2 + \cdots$. Thus, its first-order approximation is an abstract harmonic oscillator. For a philosophical discussion see Roberts (2022, p.103).

[19] See Giovanelli (2016) for an analysis.

law of nature explaining how gravitation and electromagnetism depend on one another. That is, there is a sense in which Reichenbach's proposal is dramatically incomplete as a physical model, because the choice of geometry and forces are chosen without describing any law characterising their dependence on one another or on other fields.

Now, compare this to Einstein's reaction to Kaluza's geometrisation of electromagnetism just a few years earlier. In his own letter to Einstein, Kaluza proposed what might be considered a kind of conventionality of geometry, which arises by viewing spacetime as arising as a four dimensional submanifold embedded in a five dimensional spacetime. This construction is an example of the very higher dimensional embedding that I have described above. Einstein responded:

> "I see that you have also thought about this matter quite thoroughly. I have great respect for the beauty and boldness of your idea" (Einstein 1919)

After encouraging Kaluza to develop the idea further, Einstein communicated Kaluza's revised theory to the Prussian Academy of Sciences himself, and it captivated Einstein for the remainder of his career.[20] So, it is worth examining what made this form of conventionality more acceptable to Einstein. The key difference, I claim, is the complete description of the laws of electromagnetism that Kaluza proposed.

To see this, it will be helpful to review the Kaluza (1921) proposal in a little more detail. The idea is that our universe, viewed as a four dimensional curved spacetime filled electromagnetic fields, can be viewed as arising from a five dimensional flat spacetime that is devoid of matter-energy at every point. Kaluza also introduced a law of nature for this higher dimensional spacetime, which is nothing more than the Einstein equation of general relativity, together with a symmetry known to hold of electromagnetism ($U(1)$ gauge symmetry). A short calculation[21] then shows that the empty five-dimensional spacetime with these laws has a four-dimensional submanifold living inside it, subject to the Einstein equations for a *non*-empty universe filled with electromagnetic fields. Writing G_{AB} with capital-letter indices for the five-dimensional Einstein tensor,

[20]For a brief history see Van Dongen (2002).

[21]See Wesson (1999, §1.5) for an introduction, and Gomes and Gryb (2021) for a recent philosophical application.

and G_{ab} for the four-dimensional one on a subspace, this is to say that by assuming $G_{AB} = 0$ together with a symmetry condition, we recover the vacuum Maxwell equation $\nabla_a F^{ab} = 0$, together with the Einstein equation $G_{ab} = \kappa T_{ab}$ for gravitation, where T_{ab} is the ordinary energy-momentum tensor for electromagnetic fields. One of the developers of modern gauge physics Abdus Salam (1979) referred to this surprising result as "the Kaluza-Klein Miracle".[22] As Klein (1926) later pointed out, the invisibility of the extra spatial dimension can be explained in this framework by viewing it as rolled-up or 'compactified' into a tiny tube.

Thus, all three unexplained features of higher dimensional conventionality of geometry identified in Section 5.2 are explained in Kaluza-Klein theory. There are simple laws of nature for the five-dimensional spacetime. Indeed, they are the very same laws that we observe in our four dimensional experience of the world: Einstein's equation for gravity, together with the symmetries of electromagnetism. The 'hidden' nature of the extra dimension of space is explained by Klein (1926) compactification. And, the specific four-dimensional embedding describing our experience of the world arises from our ignorance of this small extra dimension. This is not to say that Kaluza-Klein theory does not have its own challenges as a physical theory—of course it does, and a number of open problems remain (cf. Wesson 1999). But, developments in Kaluza-Klein theory in the last forty years have also shown that it provides a fruitful gauge theory in its own right, especially as a mechanism for recovering the emergence of the observable world in the low-energy limit of supergravity (Duff, Nilsson, and Pope 1986, 2025).

Sexl (1970, p.177) suggested Kaluza-Klein is a replacement for the "standard convention" of using measuring devices that are not sensitive enough to capture information about matter and energy in the compactified fifth dimension.[23] However, as a philosophy of geometry, this is a conventionality of a completely different kind. Kaluza-Klein theory is not just an arbitrary conventional choice of metric, but a proposal for how to construct new physical theories that unify gravity and gauge physics. For the moment, some might view this as

[22]Not all physicists were sympathetic, as when Weinberg (1972, p.vii) famously wrote, "now the passage of time had taught us not to expect that the strong weak and electromagnetic interactions can be understood in geometrical terms, and too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics." But, a little more passage of time soon led to an explosion of geometric approaches to gauge physics and a renewed interest in Kaluza-Klein theory (Wesson 1999, Ch.1).

[23]This view of unified field theory as a kind of conventionalism was also defended by Pitowsky (1984).

a conventional choice that provides a productive framework for the endeavour seeking better theories.[24] But, insofar as that endeavour succeeds, the result is not an alternative convention for the geometry of general relativity, but a replacement for it.

Other creative proposals for conventionalist philosophies seem to confront the same issue. For example, Glymour (1977, p.241) pointed out that flat Newtonian law of gravity in flat Newtonian spacetime might be viewed as a conventional choice, as compared to a curved spacetime formulation of Newtonian gravity with no forces.[25] There is an analogue of this curious alternative in general relativity as well, through the so-called teleparallel gravity formulation of relativity theory on flat spacetime. In this theory, gravitational phenomena is obtained through the use of a connection ∇ that 'twists' in the sense of admitting torsion. Philosophers have recently argued that this choice too is an example of the conventionality of geometry.[26] However, what makes this kind of conventionality fruitful is that they may provide a framework for seeking alternative theories that might eventually succeed general relativity as the appropriate description of reality. I take this to be what Dürr and Read (2024) have in mind when they suggest that teleparallel gravity is a "conventionalist alternative" to general relativity that provides a fruitful framework for exploring new physics:

> "empirically equivalent theories can differ in terms of their heuristic power: they needn't exhibit the same fertility and potential to suggest novel applications and natural extensions (which, if empirically borne out, might advance gravitational research)." (Dürr and Read 2024, p.37).

I would only add that, insofar as these advances succeed, they would eliminate the choice of general relativity as an alternative convention.

[24]See von Achen (2023, Chapter 2) for a general perspective on conventionality of this kind.

[25]See Malament (2012, Chapter 4) for a detailed study of Newtonian gravitation in flat and curved contexts.

[26]Conventionality interpretations of Teleparallel Gravity have been defended by Dewar, Linnemann, and Read (2022), Dürr and Read (2024), and Mulder and Read (2024). Notably, Knox (2011) argued that these formulations effectively say the same thing, whereas Weatherall and Meskhidze (2024) argue that there are structural differences that distinguish between them.

6. Conclusion

Norton (1994) has given a colourful appraisal of Poincaré-style conventionalism, but which does not instill one with optimism:

> "[W]e should just ignore [universal forces] and for exactly the sorts of reasons that motivated the logical positivists in introducing verificationism. Universal forces seem to me exactly like the fairies at the bottom of my garden. We can never see these fairies when we look for them because they always hide on the other side of the tree. I do not take them seriously exactly because their properties so conveniently conspire to make the fairies undetectable in principle. Similarly I cannot take the genuine physical existence of universal forces seriously. Thus to say that the values of the universal force field must be set by definition has about as much relevance to geometry as saying the colors of the wings of these fairies must be set by definition has to the ecology of my garden." (Norton 1994, p.165)

I agree, although my perspective is somewhat more optimistic.

I do not think one needs to ascend to such austere principles as the verifiability criterion of meaning to dismiss universal forces. That would prove awkward for any interpreter of spacetime who treats geometry as unobservable. There is a more elementary reason to be unconvinced by fairies at the bottom of Norton's garden, which is that they are effectively forbidden by any reasonable definition of a 'force' that avoids trivial semantic conventionalism. And, when alternative structures can be used to describe conventionalist alternative geometry, the very features that secure the conventionalist alternatives appear to also make them incomplete as physical theories. It is in this sense that I find conventionalist philosophies unconvincing: without an account of how they depend on other physical properties, the conventionalist alternatives are radically incomplete; and, insofar as they are successfully completed, they eliminate the other conventionalist alternatives.

In the case of higher dimensional spacetime theories, it is a matter of open scientific investigation whether these conventionalist alternatives can be completed in a physically plausible way. Kaluza-Klein theory does suggest a sense in which the geometry of spacetime may be replaced with a conventionalist alternative in higher dimensions. But, if it also turns out to be a successful and complete physical theory, it would replace the alternative conventions entirely.

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Appendix

Theorem 1. *Let (M, g_{ab}) and $(M, \tilde{g}_{ab})$ be spacetimes, with respective Levi-Civita connections ∇ and $\tilde{\nabla}$. Suppose there is some tensor field $F^a{}_b$ such that, whenever ξ^a is a timelike ∇-geodesic, $F^a := F^a{}_b \xi^b$ satisfies Newton's equation with respect to $\tilde{\nabla}$:*

$$F^a = m\tilde{\alpha}^a \tag{2}$$

with $\tilde{\alpha}^a = \xi^b \tilde{\nabla}_b \xi^a$ and $m > 0$. Then $\nabla = \tilde{\nabla}$ and $F^a{}_b = 0$.

Proof. For any such pair of connections ∇ and $\tilde{\nabla}$ there is a smooth tensor field $C^a{}_{bc}$, symmetric in its lower indices, such that for any ξ^a,

$$\xi^b \tilde{\nabla}_b \xi^a = \xi^b \nabla_b \xi^a + C^a{}_{bc} \xi^b \xi^c \tag{4}$$

(Malament 2012, Proposition 1.7.3). So, for any ∇-geodesic ξ^a, our assumptions imply,

$$F^a{}_b \xi^b = mC^a{}_{bc} \xi^b \xi^c. \tag{5}$$

Now, consider three timelike ∇-geodesic vector fields given by ξ^a, ψ^a, and $\xi^a + \psi^a \neq 0$ at p. Since each of them will satisfy Equation (5) at p,

$$\begin{aligned} mC^a{}_{bc}\xi^b\xi^c + mC^a{}_{bc}\psi^b\psi^c &= F^a{}_b\xi^b + F^a{}_b\psi^b = F^a{}_b(\xi^b + \psi^b) \\ &= mC^a{}_{bc}(\xi^b + \psi^b)(\xi^c + \psi^c) \\ &= m\left(C^a{}_{bc}\xi^b\xi^c + C^a{}_{bc}\xi^b\psi^c + C^a{}_{bc}\psi^b\xi^c + C^a{}_{bc}\psi^b\psi^c\right). \end{aligned} \tag{6}$$

Collecting terms, this implies that $C^a{}_{bc}\xi^b\psi^c + C^a{}_{bc}\psi^b\xi^c = 0$. But, since $C^a{}_{bc}$ is symmetric in the lower indices and ξ^b and ψ^c were arbitrary timelike vectors, it follows that $C^a{}_{bc} = 0$. From the definition of $C^a{}_{bc}$ it immediately follows that $\nabla = \tilde{\nabla}$ and $F_{ab} = 0$. □

Theorem 2. *Let (M, g_{ab}) and (M, g'_{ab}) be relativistic spacetimes with Levi-Civita connections ∇ and $\tilde{\nabla}$. Suppose that for all timelike ξ^a they display equal deviation, $\Delta^a = \tilde{\Delta}^a$, for all λ^a such that $[\xi, \lambda] = 0$. Then $\nabla = \tilde{\nabla}$.*

Proof. Define $C^a{}_{bc}$ as above. Then by its definition we have $\xi^n \tilde{\nabla}_n \lambda^a = \xi^n \nabla_n \lambda^a - \xi^n \lambda^b C^a{}_{bn}$, and a second application implies,

$$\underbrace{\xi^m \tilde{\nabla}_m (\xi^n \tilde{\nabla}_n \lambda^a)}_{\tilde{\Delta}^a} = \underbrace{\xi^m \nabla_m (\xi^n \nabla_n \lambda^a)}_{\Delta^a} - \xi^m \xi^n (\nabla_n \lambda^b) C^a{}_{bm} \\ - \xi^m \nabla_m (\xi^n \lambda^b C^a{}_{bn}) + \xi^m \xi^n \lambda^b C^c{}_{bn} C^a{}_{cm}. \tag{7}$$

Thus, equal deviation $\Delta = \tilde{\Delta}$ holds if and only if, for all ξ^a and λ^a such that $[\xi, \lambda] = 0$,

$$\xi^m \xi^n (\nabla_n \lambda^b) C^a{}_{bm} = \xi^m \xi^n \lambda^b C^c{}_{bn} C^a{}_{cm} - \xi^m \nabla_m (\xi^n \lambda^b C^a{}_{bn}). \tag{8}$$

Consider the special case that ξ^a is a ∇-geodesic. Applying the Leibniz rule to get $\xi^m \nabla_m (\xi^n \lambda^b C^a{}_{bn}) = \xi^m \xi^n (\nabla_m \lambda^b) C^a{}_{bn} + \xi^m \xi^n \lambda^b \nabla_m C^a{}_{bn}$, Equation (8) may be rewritten,

$$\xi^m \xi^n \left(\lambda^b \nabla_m C^a{}_{bn} - \lambda^b C^c{}_{bn} C^a{}_{cm} + (\nabla_n \lambda^b) C^a{}_{bm} + (\nabla_m \lambda^b) C^a{}_{bn} \right) = 0. \tag{9}$$

Now, choose a vector field λ^a with $\lambda^a = 0$ and $\nabla_a \lambda^b = \delta_a{}^b$ at some point p. The former implies that the first two terms vanish, while the latter implies that the last two terms reduce to,

$$(\nabla_n \lambda^b) C^a{}_{bm} + (\nabla_m \lambda^b) C^a{}_{bn} = C^a{}_{nm} + C^a{}_{mn} = 2C^a{}_{nm}, \tag{10}$$

where the last equality uses the fact that $C^a{}_{bc}$ is symmetric in the lower indices. Thus, with this choice of λ^b, Equation (9) reduces to $2\xi^m \xi^n C^a{}_{nm} = 0$. Since ξ^m was an arbitrary timelike geodesic, this implies that $C^a{}_{nm} = 0$. Therefore, $\nabla = \tilde{\nabla}$. □

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