A double-halfer embarrassment: Response to Pust

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Abstract

Titelbaum (2012) introduced a variant of the Sleeping Beauty problem in which a coin is tossed on both Monday and Tuesday, with the Tuesday toss not affecting Beauty's condition. Titelbaum argues that double halfers are committed to the embarrassing position that Beauty's credence that today's coin toss lands heads is greater than 1/2. Pust (2023) agrees with the result, but argues that it is not a distinctive embarrassment for halfers. I argue that thirders need not be embarrassed. Double halfers, on the other hand, must hold that Beauty's evidence is admissible for direct inference with respect to Monday's coin toss, but not with respect to today's coin toss. This is embarrassing because (1) a plausible argument exists for the opposite position, and (2) the position conflicts with the central motivation guiding double halfism.

1. Introduction

Sleeping Beauty, the renowned Bayesian reasoner, has enrolled in an experiment at the Experimental Philosophy Lab. On Sunday evening, she is put to sleep. On Monday, the experimenters awaken her. After a short chat, the experimenters tell her that it is Monday. She is then put to sleep again, and her memories of everything that happened on Monday are erased. The experimenters then toss a coin. If and only if the coin lands tails, the experimenters awaken her again on Tuesday. Beauty is told all this on Sunday. When she awakens on Monday – unsure of what day it is – what should her credence be that the coin toss on Monday lands heads?

The *thirders* argue that it should be 1/3 (Dorr, 2002; Elga, 2000; Horgan, 2004, 2007; Kim, 2022a, 2022b; Milano, 2022; Titelbaum, 2008), but some mischievous philosophers claim that it is 1/2. These *halfers* come in two kinds. *Lewisian halfers* further argue that Beauty's credence that the coin lands heads increases to 2/3 after she is told it is Monday (Lewis, 2001). *Double halfers* argue that her credence is 1/2 both before and after she is told it is Monday (Bostrom, 2007; Briggs, 2010; Meacham, 2008; Pust, 2012). This requires a denial that Beauty uses Bayesian conditionalization when she receives the evidence that it is Monday.

Titelbaum (2012) adds another detail to the experiment. The experimenters will also toss a coin on Tuesday, regardless of whether Beauty is awakened that day. When Beauty awakens on Monday, what should her credence be that *today's coin toss* lands heads?

Titelbaum's variant case is crafted to embarrass double halfers. The central motivation of the double-halfer position (according to Titelbaum) is that Beauty has no evidence that would justify a credence other than 1/2, both before and after she is told it is Monday. However, as Titelbaum shows, if Beauty's credence is 1/2 that the Monday coin toss lands heads (before being told it is Monday), then her credence that today's coin toss lands heads must be greater than 1/2. Besides being obviously wrong, such a credence conflicts with the central motivation of double halfism.

Pust (2023) agrees with Titelbaum that Beauty's credence in today's coin landing heads must be greater than 1/2 for halfers. However, he disagrees that this is a *distinctive* embarrassment for double halfers. Pust argues that two conflicting ways of using direct inference, both applying a direct inference principle akin to the *Principal Principle*, lead Beauty respectively to a credence of 1/2 and a credence greater than 1/2 that today's coin lands heads. Hence, according to Pust, both thirders and halfers have a problem. Neither the thirder nor the halfer has a straightforward undefeated direct inference leading to the credence that their position requires.

This is only a partial response. Titelbaum claims that only the double halfer needs some sort of direct inference, since that is essential for motivating double halfism (see footnote 2 in Titelbaum, 2012, p. 149), a claim not disputed in Pust (2023).[1] The thirder – who has other motivations available – need not be embarrassed by the unavailability of a direct inference.

However, even the partial response fails. I show that Pust's second derivation, which concludes that Beauty's credence that today's coin lands heads is greater than 1/2, relies on a premise that is akin to the position of halfism itself: that Beauty assigns a credence of 1/2 to *Monday's* coin landing heads. There is a plausible and intuitive argument that Beauty has *inadmissible evidence* with respect to this proposition, which is evidence that precludes the use of direct inference. Hence, there is no embarrassment for thirders. The double halfer, on the other hand, is forced to accept Pust's second

[1] Pust does appear to reject motivations based on direct inference, instead favouring a diachronic motivation (Pust, 2011; private communication). Titelbaum (2012, p. 149) takes such motivations to be "decisively undermined".

direct inference while rejecting the first (more intuitive) direct inference. The lack of an argument for this position only deepens the state of embarrassment that double halfers ought to be in.

2. Pust's second derivation

I summarize Pust's (2023) argument using his own formalism (slightly adapted). Let *ch* be the objective probability function[2] and let P be Beauty's credence function after she awakens on Monday in Titelbaum's variant of the problem. Pust uses the following direct inference principle based on Wallmann and Hawthorne (2020).

> **The Generic Direct Inference Principle (G-DIP).** Let E be evidence that is consistent with and *admissible* with respect to the proposition Ta and the objective probability evidence $ch(Tx \mid Rx) = r \,\&\, Ra$. If one's total evidence is given by $ch(Tx \mid Rx) = r \,\&\, Ra \,\&\, E$, then it is rationally required to have $P(Ta) = r$.[3]

It is controversial what it means for evidence to be *admissible*, but there are some straightforward cases of inadmissibility, such as non-certain disjunctions with the outcome, to which I will turn below (Wallmann & Hawthorne, 2020).

Let $\mathrm{TSB}(s)$ be the proposition that s is a Two-Toss Sleeping Beauty experiment (that is, Titelbaum's variant). Let $\mathrm{Toss}(x, s)$ be the proposition that x is a toss occurring on a day that Beauty is awake (an awakening day) in s. Let $\mathrm{H}(x)$ be the proposition that x lands heads. After Beauty has just awakened, she uses τ to refer to today's coin toss and σ to refer to the current experiment. Hence, she knows that $\mathrm{TSB}(\sigma)$ and $\mathrm{Toss}(\tau, \sigma)$.

The first derivation (endorsed by myself and implicitly by Titelbaum) uses only one step of direct inference based on G-DIP. Beauty knows that

$$ch(\mathrm{H}(x) \mid \mathrm{Toss}(x, s) \,\&\, \mathrm{TSB}(s)) = 1/2. \quad (1)$$

Supposing she has no inadmissible evidence, G-DIP requires her to set

$$P(\mathrm{H}(\tau)) = 1/2. \quad (2)$$

Pust's second derivation works by considering three possibilities that are mutually exclusive and exhaustive. The probability of the three scenarios is obtained using

[2] This can be a chance function or objective probability in another sense.

[3] This is based on the "Generic Direct Inference Principle" from Wallmann and Hawthorne (2020, p. 958).

direct inference. He then gives some plausible probabilities for today's coin landing heads conditional on each possibility. These probabilities are combined using the law of total probability to obtain Beauty's credence that today's coin lands heads. The three scenarios are as follows.

1. The experiment has only a single toss occurring on an awakening day, which lands heads. This possibility has an objective probability of 1/2, since it happens just in case the Monday coin lands heads. Formally:

$$\begin{aligned} \mathrm{S}_1(s) &:= \exists! x(\mathrm{Toss}(x,s) \mathbin{\&} \mathrm{H}(x) \mathbin{\&} \neg\exists y(y \neq x \mathbin{\&} \mathrm{Toss}(y,s))) \\ ch(\mathrm{S}_1(s) \mid \mathrm{TSB}(s)) &= 1/2. \end{aligned} \tag{3}$$

2. The experiment has one awakening day toss landing heads and another awakening day toss landing tails. This possibility has an objective probability of 1/4, since the first toss must land tails and the second heads. Formally:

$$\begin{aligned} \mathrm{S}_2(s) &:= \exists! x(\mathrm{Toss}(x,s) \mathbin{\&} \mathrm{H}(x)) \mathbin{\&} \exists! y(\mathrm{Toss}(y,s) \mathbin{\&} \neg\, \mathrm{H}(y)) \\ ch(\mathrm{S}_2(s) \mid \mathrm{TSB}(s)) &= 1/4. \end{aligned} \tag{4}$$

3. There is no awakening day toss that lands heads. This possibility has an objective probability of 1/4 since it requires that the toss lands tails on both days. Formally:

$$\begin{aligned} \mathrm{S}_3(s) &:= \neg\exists x(\mathrm{Toss}(x,s) \mathbin{\&} \mathrm{H}(x)) \\ ch(\mathrm{S}_3(s) \mid \mathrm{TSB}(s)) &= 1/4 \end{aligned} \tag{5}$$

Conditional on S_1, since there is only one toss, today's toss must be it. Given S_1, this only toss lands heads, so it has a credence of 1. Conditional on S_2, today's coin lands heads if and only if it is Tuesday. This is clearly possible, so this probability must be greater than 0. Conditional on S_3, the probability that today's toss lands heads is 0. Formally, we have these conditional credences:

$$P(\mathrm{H}(\tau) \mid \mathrm{S}_1(\sigma)) = 1 \tag{6}$$

$$P(\mathrm{H}(\tau) \mid \mathrm{S}_2(\sigma)) > 0 \tag{7}$$

$$P(\mathrm{H}(\tau) \mid \mathrm{S}_3(\sigma)) = 0. \tag{8}$$

Applying G-DIP to each scenario, we get:

$$P(\mathrm{S}_1(\sigma)) = 1/2 \tag{9}$$

$$P(\mathrm{S}_2(\sigma)) = 1/4 \tag{10}$$

$$P(\mathrm{S}_3(\sigma)) = 1/4. \tag{11}$$

Finally, applying the law of total probability to (6)-(11), we get:

$$P(\mathrm{H}(\tau)) = 1/2 + 1/4 \cdot P(\mathrm{H}(\tau) \mid \mathrm{S}_2(\sigma)) > 1/2. \tag{12}$$

Pust takes the conflict between the two derivations to show that there is a failure of admissibility in *both* derivations. This conclusion is clearly unwarranted. Admissibility of evidence is defined with respect to a proposition and its objective probability (Wallmann & Hawthorne, 2020), and the above direct inferences involve different propositions and chances. It is thus possible that there is only a single failure of admissibility. Since the applications of G-DIP yielding (10) and (11) are not needed for the conclusion (see below), either (2) or (9) must involve inadmissible evidence.

Nevertheless, Pust's final conclusion is correct: lacking an argument that there is a failure of admissibility in one case but not the other, the above poses a problem for thirders and halfers alike. However, there *is* such an argument.

3. The inadmissible evidence

The first scenario, $\mathrm{S}_1(\sigma)$, occurs if and only if the Monday coin in σ lands heads. Hence, $\mathrm{S}_1(\sigma)$ is the analogue of the proposition of interest in the original Sleeping Beauty problem, to which thirders assign a probability of 1/3. I argue that Beauty has inadmissible evidence with respect to this proposition and the objective probability evidence (3).

Upon waking up, Beauty knows that the experimenters have awakened her today. As thirders have often argued, this is evidence that affects her credences. The fact that she is awakened today entails that the coin landed tails or it is Monday (Mon). Hence, Beauty knows the logical disjunction $\neg\,\mathrm{S}_1(\sigma) \vee \mathrm{Mon}$. This evidence is relevant for the proposition that the Monday coin landed tails, and therefore it is relevant for heads. Since it is relevant, it is inadmissible. This is clear intuitively, but there is also a plausible argument to that effect. (The next section will consider an objection against this argument.)

The admissibility of logical disjunctions involving the outcome has been proven by Wallmann and Hawthorne (2020, p. 963). The following is a simplified version of their theorem 3.

Inadmissible disjunctions. Let P_0 be an initial credence function.[4] Let F be any proposition, and let E be evidence that is consistent with and admissible with respect to the proposition Ta and the objective probability evidence $ch(Tx \mid Rx) = r \,\&\, Ra$, with $0 < r < 1$. Hence, by G-DIP, we have $P_0[Ta \mid ch(Tx \mid Rx) = r \,\&\, Ra \,\&\, E] = r$. Suppose also that $P_0[Ta \vee F \mid ch(Tx \mid Rx) = r \,\&\, Ra \,\&\, E] < 1$. Then we have

$$P_0[Ta \mid ch(Tx \mid Rx) = r \,\&\, Ra \,\&\, E \,\&\, (Ta \vee F)] > r.$$

To apply *Inadmissible disjunctions*, we assume that Beauty's credence function $P(X)$ after being woken up equals a suitable conditional probability function $P_0(X \mid Y)$, where Y is the conjunction of all of Beauty's evidence after she is woken up.

For Beauty to find out whether she has inadmissible evidence after being woken up, she would need to imagine that she doesn't know $\neg\, \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$, i.e., that she does not know that she has been woken up. There has been some controversy over whether such a credal state is possible, since it may be argued that a logically possible rational agent must always be sure that she is conscious (Pust, 2008). However, this is not a problem for the present case, since an agent who is asleep but dreaming is logically possible and conscious.

Hence, Beauty could imagine a rational agent who is dreaming, but sufficiently lucid to have rational credences. While asleep, Beauty's state is unaffected by the outcome of the coin toss. Hence, her sleeping situation is just like a normal situation in which one assigns a credence to the uncertain outcome of an unbiased coin toss. Therefore, her evidence while asleep is admissible with respect to the proposition $\mathrm{S}_1(\sigma)$ and the objective probability $ch(\mathrm{S}_1(s) \mid \mathrm{TSB}(s)) = 1/2$. By G-DIP, her imagined dreaming probability is given by $P_0[Hx \mid ch(\mathrm{S}_1(s) \mid \mathrm{TSB}(s)) = 1/2 \,\&\, \mathrm{TSB}(\sigma) \,\&\, E] = 1/2$, where E is her remaining evidence. (For a more extensive argument about Beauty's sleeping credences and admissibility of her evidence, see Ackermans, 2024.)

While dreaming, it is clear that the probability of $\neg\, \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$ is lower than 1.[5] Hence, the conditions of *Inadmissible disjunctions* are satisfied. Adding the evidence $\neg\, \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$ would increase the probability in $\neg\, \mathrm{S}_1(\sigma)$, and so it would decrease the probability in $\mathrm{S}_1(\sigma)$. This means that $\neg\, \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$ is a *defeater*, and on any plausible

[4] An initial credence function $P_0(X \mid Y)$ describes a rational agent's credence in X when her evidence is Y. This assumes that her credences in different evidential situations can be described by a single conditional probability function P_0.

[5] In the dream, Beauty does not know whether it is Monday or Tuesday. She also doesn't know the outcome of the Monday coin. Hence, it is possible that it is Tuesday and that the Monday coin landed heads. It follows that $\neg\, \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$ has a probability less than 1.

account of admissibility, evidence containing a defeater is inadmissible (Wallmann & Hawthorne, 2020, p. 959).

Hence, there is a strong argument available for the inadmissibility of Beauty's evidence with respect to the direct inference in (9). At the same time, the thirder has good reason to accept the admissibility of Beauty's evidence with respect to the proposition that today's coin lands heads, which she can therefore set to the chance (equation 1). This is simply overwhelmingly plausibly. As Titelbaum puts it:

> Imagine the experimenters put the coin in Beauty's hand and say, "This is the coin we're going to flip in ten minutes. It's fair – it has a 1/2 objective probability of coming up heads. And however the flip comes out, its outcome has no influence on your present condition. Heck, if you like you can be the one to flip it." Standing there with the coin in her hands, Beauty is supposed to be more than fifty percent confident that it'll come up heads? (Titelbaum, 2012, p. 149)

In summary, Pust's derivations do not show that thirders have any reason to doubt the admissibility of Beauty's evidence with respect to the proposition that today's coin lands heads, while they do have strong reasons to reject the admissibility of her evidence with respect to the proposition that Monday's coin lands heads.

4. An objection

The thirder's inadmissibility argument assumes that Beauty can bracket a part of her evidence (that she is woken up today) in order to determine its admissibility. Arguments offered by Pust (2012, 2014) can be used to doubt that this is possible.[6]

The objection is as follows.

1. For $\neg S_1(\sigma) \vee \text{Mon}$ to be a defeater after Beauty woke up, there must be some scenario (imagined or real) in which a rational agent's credence in Hx goes from its chance (1/2) to a different value as a result of learning $\neg S_1(\sigma) \vee \text{Mon}$.

2. The only possible scenario of this type consistent with the thirder's inadmissibility argument is one in which a rational agent uses Bayesian conditionalization on $\neg S_1(\sigma) \vee \text{Mon}$ with respect to her credence function while dreaming.

3. Bayesian conditionalization on $\neg S_1(\sigma) \vee \text{Mon}$ is impossible.

4. Hence, the thirder's inadmissibility argument does not establish that $\neg S_1(\sigma) \vee \text{Mon}$ is a defeater.

[6] Thanks to Joel Pust for pointing out this type of objection.

The first premise (to which I object below) can be seen as a constraint on when a piece of evidence can be considered a *defeater*. Let P_1 be the credences of a rational agent at time 1 whose evidence is E, let X be the proposition of interest, and suppose E includes the chance evidence $ch(X) = r$. It is suggested that some piece of evidence Y is a defeater for X given the evidence E & Y only if E is not a defeater and every rational agent's credence goes from $P_1(X) = r$ to $P_2(X) \neq r$ after learning Y. Hence, this conception of a defeater requires that there exists a scenario in which a rational agent's credences change diachronically in the way described here.

The second premise is plausible. Let P_1 be the credence function of a rational agent whose evidence is that of Beauty after waking up except for $F := \neg \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$. (This rational agent could be Beauty while asleep but dreaming.) The inadmissibility argument shows that $P_1(Hx) = r$ and $P_1(Hx \mid F) > r$. To establish that F is a defeater as understood by the first premise, the rational agent's credence in heads after learning F, $P_2(Hx)$, must be equal to her conditional credence $P_1(Hx \mid F)$. This is precisely what the principle of Bayesian conditionalization would require.

The third premise, stating that this kind of conditionalization is impossible, is defended by Pust (2012), who argues that Bayesian conditionalization on evidence that is essentially temporally indexical is impossible given the three main accounts of what such beliefs are. An essentially temporally indexical belief is a belief whose truth or meaning is dependent on a temporal indexical such as "Today". The belief $\neg \mathrm{S}_1(\sigma) \vee \mathrm{Mon}$ is essentially temporally indexical. Hence, the third premise is true if the conclusion of Pust (2012) is true.

I argue that both premise 1 and premise 3 should be rejected. Note that my argument interprets $P_0(X \mid Y)$ as giving the rational degree of belief in X of every rational agent who has evidence Y. In this case, premise 1 amounts to an assumption that for the relevant evidence to be a defeater, there must a possible world in which a rational agent's credences at two points in time are described by the following two probabilities, in which the second probability is derived from the first using Bayesian conditionalization:

$$P_0[Hx \mid ch(\mathrm{S}_1(s) \mid \mathrm{TSB}(s)) = 1/2 \,\&\, \mathrm{TSB}(\sigma) \,\&\, E] = 1/2, \tag{13}$$

$$P_0[Hx \mid ch(\mathrm{S}_1(s) \mid \mathrm{TSB}(s)) = 1/2 \,\&\, \mathrm{TSB}(\sigma) \,\&\, E \,\&\, \neg \mathrm{S}_1(\sigma) \vee \mathrm{Mon}] > 1/2. \tag{14}$$

The problem with this assumption is that P_0 can be interpreted to describe rational credences independently of the way in which an agent derives them. They may not even be held in the same possible world: for example, the first may be held by Beauty in a world in which she has a dream (during her dream), while the second is held by Beauty in another world in which she does not have a dream. This gives a plausible interpretation of what it means for some piece of evidence Y to be a defeater for X

relative to admissible evidence E: that every agent assigns a greater credence to X in worlds in which her evidence is E & Y than in worlds in which her evidence is E.

When P_0 is understood in this way, the argument might still be said to use some form of Bayesian conditionalization, since the second probability is the first with the additional evidence conditioned one. However, it is not the type of diachronic conditionalization referred to in premise 3, defended by Pust (2012). Hence, if the argument is understood such as to be valid (requiring premise 1 and 3 to refer to the same type of conditionalization), and we use the above interpretation of P_0, then premise 1 is false.

Another way in which we might be able to reject premise 1 is to understand the second probability as a conditional probability. That is, let P_1 describe a rational agent's credence who has evidence $ch(S_1(s) \mid TSB(s)) = 1/2 \,\&\, TSB(\sigma) \,\&\, E$. Then equations (13) and (14) can be interpreted as follows.

$$P_1(Hx) = 1/2 \tag{15}$$

$$P_1(Hx \mid S_1(\sigma) \vee Mon) > 1/2. \tag{16}$$

On a standard definition of evidential relevance, a piece of evidence Y is relevant for a proposition X if and only if every rational agent's conditional probability in X given Y is different from her unconditional probability in X. We might further say that Y is a defeater if every rational agent's evidence without Y is admissible and Y is relevant to X in the above sense. According to this notion of a defeater (13) and (14) establish that $\neg S_1(\sigma) \vee Mon$ is a defeater. If this notion is correct, no type of Bayesian conditionalization is required for the argument.

Turning to premise 3, there are now many accounts in the literature that make sense of Bayesian conditionalization on temporally indexical evidence (e.g., Bradley, 2011; Kim, 2009; Schulz, 2010; Schwarz, 2012; Titelbaum, 2013). Most of these accounts provide an amended version of the traditional principle of Bayesian conditionalization and show how this leads to very plausible results. While amended, these accounts are all consistent with Bayesian conditionalization for cases such as the present case.[7] That is, these accounts all entail that after learning $\neg S_1(\sigma) \vee Mon$, we have $P_2(Hx) = P_1(Hx \mid \neg S_1(\sigma) \vee Mon)$.

If the argument in Pust (2012) is intended to show that Bayesian conditionalization is impossible in both its original form and all of its amendments, then the argument

[7] The present case involves what Bradley (2011) calls *belief discovery* as opposed to *belief mutation*. These involve the learning of self-locating information that has not changed in truth value between the prior and posterior times.

should be interpreted as a modus tollens. The authors cited above have made it plausible that a version of Bayesian conditionalization is a requirement of rationality. Hence, if Pust (2012) is correct, then the three main accounts of temporally indexical belief are wrong. A different account should then be sought that is compatible with Bayesian conditionalization on essentially temporally indexical evidence.

In conclusion, the above argument seems to fail because both premise 1 and premise 3 are implausible. Perhaps this is not a knock-down argument. However, even without a knock-down argument, double halfers have a problem in the Two-Toss Sleeping Beauty problem. Thirders can use my argument, and the above defence against objections, to reasonably claim that their direct inference is undefeated. Double halfers, on the other hand, are missing such a positive argument in favour of their own direct inference.

5. What the embarrassment consists of

Double halfers must accept that Sleeping Beauty can use direct inference to set her probability in the Monday coin landing heads to $1/2$. Pust's second derivation can be simplified to use only this direct inference, that is, the inference yielding (9). (This is just another way of making Titelbaum's argument to which Pust responded.[8])

Consider that $S_2(\sigma)$, the scenario that the current experiment has one awakening day toss landing heads and another awakening day toss landing tails, is possible. Hence, we can conclude $P(S_2(\sigma)) > 0$ without using direct inference.

Consider also that Beauty's credence in $S_3(\sigma)$ is irrelevant for the final step (12), since we have $P(H(\tau) \mid S_3(\sigma)) = 0$.

Hence, just using (6)-(9), $P(S_2(\sigma)) > 0$, and the law of total probability, we get:

$$P(H(\tau)) = 1/2 + P(H(\tau) \mid S_2(\sigma)) \cdot P(S_2(\sigma)) > 1/2. \tag{17}$$

The double halfer has to accept the above derivation, since the only direct inference used is the direct inference that is central to her own position. At the same time, the double halfer must reject the intuitively plausible direct inference based on (1). The double halfer must hold that Beauty has inadmissible evidence with respect to at least one of these two direct inferences. Since the double halfer accepts admissibility in the former case, she must deny it in the latter, despite the overwhelming plausibility.

[8] Titelbaum's argument also starts out from the assumption (9), leading to the conclusion (17), using just the probability axioms.

Let's take a look at the dialectic again. Titelbaum (2012) took it to be embarrassing that the double halfer has to assign a credence greater than 1/2 to today's coin landing heads, because the central motivation for double halfism is "that all this sleeping/waking/day-of-the-week stuff doesn't make it okay for Beauty to assign credence other than 1/2 to the outcome of an objectively fair coin flip" (Titelbaum, 2012, pp. 147–148). In Pust's interpretation of Titelbaum's argument, the issue is that the double halfer must assign a credence greater than 1/2 to an event for which a plausible direct inference mandates a credence of exactly 1/2. According to Pust, however, there is an equally serious problem for thirders.

This response does not address Titelbaum's argument directly, since Titelbaum maintains that only double halfers require a direct inference to motivate their position, whereas thirders and Lewisian halfers have other motivations available. (Pust appears to disagree, but does not attack this position in his response; see footnote 1.) The lack of such a direct inference would thus affect double halfers differently than thirders.

My argument adds another layer to the embarrassment. As I showed, thirders have a strong argument available to defend the inadmissibility that their position requires in the Two-Toss Sleeping Beauty problem. Double halfers, so far, do not.

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