On Reissner's hypothesis:

A review of 20th Century relational models for the unification of gravity and inertia

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Abstract

This paper revisits a largely overlooked line of thought originating with Reissner's 1915 proposal: that gravity may be a necessary consequence of the relativity of inertia. We survey a range of historical models developed throughout the 20th Century that attempt to unify gravity and inertia by deriving the two in unison. While the unification of gravity and inertia is primarily recognized within the framework of Einstein's general relativity, we show that several lesser-known and largely classical models go further by attempting to explain this equivalence, thereby aiming at a deeper unification. Our analysis distinguishes four classes of models and argues that those incorporating internal particle motion—especially Cook's quantum-mechanical approach—offer the most comprehensive account of this unification. Cook's model, which derives gravitational attraction from the quantum zitterbewegung of elementary particles, suggests a direct link between gravity, inertia, special relativity, and quantum mechanics. By systematically comparing these approaches, we revive a promising and under-explored avenue toward a dynamical explanation of gravity and inertia, combining the physics of the largest and smallest scales in nature: of cosmology and the quantum properties of elementary matter.

Keywords: Relational inertia, Gravitational constant, Zitterbewegung, Quantum gravity

Contents

> If I am successful, gravity would be understood as a direct and necessary consequence of the relativity of acceleration.
>
> Hans Reissner, 1915

1. Introduction and context

1.1. Opening

While the historical relevance of relational principles to the foundations of modern gravitational physics has already been discussed at length in secondary literature (Barbour and Pfister, 1995; Assis, 2014; Renn, 2008; Hoefer, 1994; Norton, 1995; DiSalle, 1990, 2002), in the present paper we are concerned with what we regard as perhaps the most intriguing possibility that the relational-inertia research program has to offer, which in our view has not received its due attention. This idea, which we have named "Reissner's hypothesis", proposes that the existence of the force of gravity may be *explained* as a direct consequence of the relativity of inertia[1].

The aim of this paper is twofold: (1) to draw attention to Reissner's hypothesis (hypothesis 3) as a distinctive proposal which is logically stronger than the hypothesis proposed by Mach (hypothesis 1); and (2) to examine a range of classical models of gravity and inertia that, in different ways and to varying degrees, give expression to Reissner's hypothesis. Our goal is to clarify the distinctions between these models, highlight their respective strengths, and identify the problems they face. Although these models were often developed decades apart and by authors unaware of one another's work, they nonetheless converged repeatedly on many of the same core ideas.

The paper draws heavily from original historical resources, such as the papers of Reissner, Schrödinger, Treder, Barbour and Cook that have received very little attention, especially from philosophers of physics. The analysis of Sciama's 'toy model' of 1953 is informed by close reading of his doctoral thesis (Sciama, 1953a) which has not previously been cited and was recovered by one of the present authors from the University of Cambridge archives. Beyond Reissner's hypothesis, the models we explore here contain a wealth of fertile ideas, including links between gravito-inertial theory, special relativity and quantum mechanics, which we believe are ripe for further analysis. These various connections will be unpacked and discussed throughout the paper.

1.2. Reissner's hypothesis

Although the hypothesis of a connection between relative inertia and gravity is often attributed to Mach, it does not in fact originate with him, as Assis (2014, p.250) has remarked. Rather, Mach was responsible for a more general hypothesis:

Hypothesis 1. ***Mach's hypothesis:*** *"The motion of bodies observed as rectilinear and uniform with reference to the fixed stars, referred to in classical mechanics as 'inert', might be caused directly by the action of these distant stars upon those bodies."* (Fay, 2024)

1. Friedlaender. The hypothesis that will be discussed in this work involves the combination of this speculative idea with the notion that inertia and gravitation ought to be unified into a single law. This idea first appeared in print in 1896 in a short book "Absolute or relative motion?" written jointly by Benedict and Immanuel Friedlaender in which the idea of relativising inertia is discussed. In the section written by Immanuel a remark is made concerning the connection of this program to the unification of gravity and inertia[2]:

> However, it seems to me that the correct form of the law of inertia will only then have been found when relative inertia as an effect of masses on each other and gravitation, which is also an effect of masses on each other, have been derived on the basis of a unified law.

In a footnote, Friedlaender adds that the relativity of inertia could perhaps be implemented by applying an analogue of Weber's law for electromagnetism to gravitation and inertia; a suggestion which has foreshadowed numerous attempts to model gravity and inertia in this way. However, the idea was not developed any further than this by the Friedlaender brothers.[3]

2. Einstein. Friedlaender's idea of a unified law underlying both inertia and gravitation would not have been conceivable were it not for the remarkable fact, observed since Galileo, that all bodies in free-fall regardless of their composition are subjected to the same acceleration in a gravitational field; a property which gravity shares with the inertial or "fictitious" forces of classical mechanics. In 1907, Einstein saw in this idea a vague possibility of the extension of the relativity principle to accelerated reference frames (Einstein, 1907). According to Einstein, the assumption of "the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system [...] extends the principle of relativity to the uniformly accelerated translational motion of the reference system." Although it is now well understood that Einstein was wrong on this, and that in fact this idea enables the relativization of the gravity-inertia split rather than the rela-

[1] We have named this idea "Reissner's hypothesis" since it was first expressed by Reissner (1915), however, as we shall see the same idea was rediscovered independently by Barbour (1975) and Cook (1976).

[2] Immanuel Friedlaender would go on to have a career as a Geologist whereas Benedict would become an influential Sexologist and Sociologist. The original text "Absolute oder relative Bewegung?" can be found in Friedlaender and Friedlaender (1896, p.17), see Friedlaender (1995) for a partial English translation, and Friedlaender and Friedlaender (2007) for a full English translation.

[3] Weber's law for electromagnetism is a relational antecedent to Maxwell's electrodynamics founded by Wilhelm Eduard Weber, a colleague of Gauss; see Assis (1994) for further context. See also Assis (2014, 1995) for a discussion of the various gravity models which implement an electromagnetic analogy similar to Weber's law, including for instance: Schrödinger (1925); Brown (1955); Edwards (1974); Assis (1989). The models of Schrödinger and Assis will be discussed in section 3 of this paper.

tivization of motion (Janssen, 2012), Einstein nonetheless placed great weight in this insight which eventually grew into the various forms of the "equivalence principle", which we will here be referring to as the *equivalence hypothesis* (since it arises as a speculative generalisation of an empirical result).

Hypothesis 2. ***The equivalence hypothesis:*** *The local equivalence of a gravitational field and an accelerated reference frame suggests that gravity and inertia ought to be derived together as twin aspects of a unified law.*[4]

In 1912, Einstein began to consider his equivalence hypothesis in conjunction with Mach's hypothesis (Einstein, 1912). In the previous year, Einstein had deduced that it is a consequence of his equivalence hypothesis that the speed of light must vary spatially in the presence of large masses (Einstein, 1911); He also mentions that the inertial mass of a body should slightly increase in a gravitational field, in Einstein and Grossmann (1913) this is given as[5]:

$$m = \frac{\mu}{c} \tag{1}$$

where $m = m(r)$ is the inertial mass, μ "denotes a constant that is characteristic of the mass point and independent of the gravitational potential", and $c = c(r)$ is the variable speed of light in a gravitational field, given in terms of an invariant speed of light c_0 by $c = c_0(1 + \Phi/c^2)$, where $\Phi = \Phi(r) = -\frac{MG}{r}$ is the Newtonian gravitational potential around a central mass M (Einstein, 1911). The result of all this is that in the presence of a gravitational field, the inertial mass m increases, therefore, Einstein speculates (Einstein, 1912):

> This suggests that the entire inertia of a mass point is an effect of the presence of all other masses, which is based on a kind of interaction with the latter. [...] This is exactly the same point of view that E. Mach advanced in his astute investigations on this subject.

In later publications Einstein continued to uphold the idea that his equivalence hypothesis, combined with Mach's hypothesis, was central to his speculations on gravity. Indeed, in a discussion following the lecture version of his 1913 paper, "On the Present State of the Problem of Gravitation", he is recorded as stating: (Einstein, 1996, p.228):

> the fact that the identity of the inertial and the gravitational mass has proved correct with such a remarkable accuracy seems to me to be one of the most important pointers for the development of the theory. The need to find a more profound explanation for that identity, and besides that also the view concerning the relativity of inertia advanced by Mach, was actually the motive that compelled me to devote myself to the problem of gravitation.

The above passage came as a response to a question by Hans Reissner. As we will see, shortly after this, Reissner would write a paper endeavoring to prove just this sort of "explanation" for the identity of gravitational and inertial mass.

3. Reissner. Hans Reissner (1874–1967) was a pioneering German aeronautical engineer and amateur physicist, known in part for having designed and built the world's first successful all-metal aircraft, the *Reissner Canard* while holding a professorship at the University of Aachen. To theoretical physicists, he is primarily known today for being the first person to have derived the *Reissner-Nordström metric* for a spherically symmetric, stationary, charged body in Einstein's theory of general relativity (Reissner, 1916). Before this however, Reissner had been attentively following Einstein's reasoning in the course of the development of the general theory, and in 1914 and 1915 he published a pair of little-known papers (Reissner, 1914, 1915) addressing the possibility of the fulfillment of Mach's hypothesis concerning the material origin of inertia.

Whereas, as we saw in section 1.2, Einstein believed he had discovered an extension of the principle of relativity in his equivalence hypothesis. In his first paper, Reissner correctly points out that these are "essentially different requirements" (Reissner, 1914). Concerning the relativity principle he writes[6]:

[4]The present formulation of the equivalence hypothesis is inspired by what Lehmkuhl (2021) calls the "Einstein equivalence principle". See Lehmkuhl (2014, 2021) for more context on the role of Einstein's various formulations of the equivalence principle and their role in motivating the unification of gravity and inertia.

[5]Notation has been adjusted for consistency with the rest of this article.

[6]It is worth noting, as Barbour has remarked, that Reissner's strategy in 1914 is very similar to a former work of Wenzel Hofmann from 1904 (Hofmann, 1904), however Hofmann did not develop his ideas mathematically very far. In his 1913 lecture "*On the present state of the problem of gravitation*", Einstein referred to Hofmann's booklet as a "witty little book" (Einstein, 1996, p.220) which independently advances the same view that Mach had developed previously. It is plausible that Reissner was aware of Hofmann's booklet since Reissner did attend this lecture by Einstein and participated in the subsequent discussion, however Reissner did not cite Hofmann's work.

> It is to be required that not only absolute speed but also any absolute motion whatever, in particular acceleration, must be undetectable. This requirement was already formulated by Mach, but it has not yet been carried out.

In his 1915 paper, however—which is titled "*On the possibility of deriving gravity as the direct consequence of the relativity of inertia*"—he returns to Einstein's hypothesis, having unexpectedly realized that it contained hidden fruits. Referring to his previous essay, Reissner writes (Reissner, 1915):

> In the essay in question, I stated and for the first time quantitatively formulated the idea that the relativity of acceleration can only be implemented if the centrifugal forces of a rotating body correspond to centripetal forces of all other masses [...] However, my knowledge of the equality of inertial and gravitational masses had not been included as necessary, since my approach involved separate kinetic and potential energy functions.

Reissner continues:

> Mr. Einstein's equivalence hypothesis which asserts the mechanical and optical identity of an acceleration field with a field of constant gravity seems to imply the deeper meaning that gravity is also a resistance to acceleration. [...] it is precisely this idea that makes it possible to fulfill Hertz's ideal requirement of representing gravity as an inertial force.[7]

In other words, despite having previously dismissed Einstein's equivalence hypothesis as an inappropriate approach to generalizing the relativity principle, to his surprise, Reissner now found that the equivalence hypothesis enables exactly this, though not in the way that Einstein had suggested. The equivalence hypothesis suggests that gravity and inertia should be unified as two aspects of a single law, in such a way that the two laws need not be postulated independently of one another:

> If I am successful, gravity would be understood as a direct and necessary consequence of the relativity of acceleration, the identity of the gravitational and inertial masses would be shown to be self-evident and the gravitational field would not only be equivalent to an accelerated space, as Einstein proposes, but gravity itself would be identified as a resistance to relative acceleration.

Reissner's hypothesis hearkens back to Friedlaender's comments from 1896, however, by exploring this idea more explicitly, Reissner raises the possibility that a relativized theory of inertia would not only unify inertia with gravity but *explain* gravity as a necessary consequence of relativized inertia.

The mechanism by means of which gravity would arise according to Reissner involves the internal motion of the constituent particles within massive bodies, and this particular idea would be rediscovered independently by both Barbour (1975) and Cook (1976) six decades later. However, in a broader sense, we will define *Reissner's hypothesis* independently of the particular mechanism which is being postulated:

Hypothesis 3. ***Reissner's hypothesis:*** *Gravity arises entirely as a necessary consequence of the relativity of inertia.*

1.3. Roadmap

In this paper we will be discussing various classical mechanical unified models of gravity and inertia proposed over the course of the 20th Century which to some degree embody Reissner's hypothesis as we have defined it above. These will include the models of Hans Reissner, Erwin Schrödinger, Dennis Sciama, Hans-Jürgen Treder, Richard Cook, Julian Barbour and André Assis. The aim of the paper is to categorise these various models according to their methods, assumptions and implications in order to bring out the differences between them, and in particular assess their respective strengths, weaknesses and the degree to which they genuinely embody Reissner's hypothesis.

Firstly, in section 2 we will discuss the approaches of Reissner and Barbour (Reissner, 1914, 1915; Barbour, 1974, 1975). In their respective second papers, both Reissner and Barbour derive gravity from a single relative kinetic energy expression, along with the supplementary hypothesis that the irregularly distributed "zitterbewegung" of the particles constitutive of massive bodies is responsible for providing the relative motion necessary for gravitational effects. These models face the difficulty that the resultant inertial mass is

[7] Reissner's reference to "Hertz's ideal requirement of representing gravity as an inertial force" here refers to Heinrich Hertz's posthumously published *Principles of mechanics presented in a new form* (Hertz, 1899), in which Hertz endorses the unification of all mechanical phenomena under a single law. Interestingly, Julian Barbour also appeals to Hertz's ideas in a paper in which he independently rediscovers Reissner's hypothesis 60 years after Reissner's paper (Barbour, 1975). See section 2.2 for further discussion.

anisotropic, which became grounds for Julian Barbour to abandon this approach.[8]

In section 3, we first examine the models developed by Schrödinger and Assis (Schrödinger, 1925; Assis, 1989) which do not rely on the hypothesis of internal motion, but leverage an analogy with Weberian electromagnetism in order to derive gravitational and inertial effects in unison, thus directly fulfilling Immanuel Friedlaender's suggestion that we encountered in section 1.2. Whereas the models of Schrödinger and Assis also face the problem of mass anisotropy, the model of Treder which we will examined in section 3.3 overcomes this by using the Riemann potential rather than the Weber potential, however this undermines the desired invariance of the model under rotations of the universe. In all of these models moreover, Reissner's hypothesis is not perfectly fulfilled, since the ratio δ between the inertial and gravitational coupling strengths is not strictly predicted but rather fitted to experimental measurements of Mercury's perihelion shift.

Finally, in section 4, we examine the model of Dennis Sciama, which differs quite substantially on the methodological side from previous models, and moreover suffers from some serious flaws, mostly on account of the fact that it was only constructed for illustrative purposes and not intended as a consistent theory. Nonetheless, we find that Sciama's results are highly suggestive since this is the only one of the various models which makes contact with results from special relativity. In section 4.2, we explore the paper of Cook (1976) which adopts Sciama's treatment of linear accelerations and attempts to derive gravity through the same mechanism involving internal motion that Barbour and Reissner had independently discovered before him. Cook's model, however, is novel in that he is able to determine that a gravitational force of the appropriate strength is not produced in a classical model but is predicted when the vibrating particles are modelled using quantum mechanics as spin-$\frac{1}{2}$ fermions. Therefore, just as Sciama's model suggests a connection between gravity, inertia and special relativity, Cook's model suggests an intimate connection between gravity, inertia and quantum mechanics.

In various places throughout the paper, we have chosen to modify the notation used in the original papers so as to ensure it is standardised across models. In general, we use γ to denote the Newtonian gravitational constant and δ for the inertial coupling strength. The invariant masses are given by m_i, whereas the inertial mass is often given as m_i^*. The distance vector between two masses m_i and m_j is given by $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_i$, whereas $r_{ij} = |\mathbf{r}_i - \mathbf{r}_i|$ is the magnitude of this distance. $\dot{r}_{ij}$ denotes the rate of change of r_{ij} whereas $\mathbf{v}_{ij} = \dot{\mathbf{r}}_{ij}$ is the relative velocity of a pair of particles in a given frame of reference, and $v_{ij} = |\mathbf{v}_{ij}|$ is its magnitude. In general we use $\Phi = \int_U \rho / r d^3r = \frac{3M_U}{2R_U}$ to denote the total Newtonian gravitational potential of the universe (unnormalised by γ) at a given point, where the universe U is assumed to be homogeneous of density ρ, and M_U and R_U are the universe's total mass and radius respectively. In general summations $\sum_i$ run over all particles i in the universe.

The models discussed in sections 2 and 3 all follow straightforwardly from a postulated Lagrange function, therefore we will focus on comparing the different Lagrangians across models. In section 4 we adopt somewhat different methods since these models were not presented using the Lagrangian formalism.

2. Internal motion models

Before examining Reissner's model, let us briefly recall the form of the Lagrangian for a system of particles subject to the law of inertia and the law of gravity in classical mechanics. It is given by the difference between the kinetic ($T_i = \frac{1}{2}m_i v_i^2$) and potential ($U_i = -\sum_{j\neq i} \gamma \frac{m_i m_j}{r_{ij}}$) energies:

$$L = \sum_i \frac{1}{2} m_i v_i^2 + \gamma \sum_{i<j} \frac{m_i m_j}{r_{ij}}, \tag{2}$$

where γ is the Newtonian gravitational constant. Whereas the gravitational potential is defined relationally between particles, the kinetic energy term, which is responsible for the inertia of matter in classical mechanics, is defined in terms of the particles's absolute velocities, i.e. their velocity with respect to absolute space and absolute time.

2.1. Reissner's models

1. Reissner (1914). In order to carry out the generalisation of the relativity principle, in accordance with Mach's ideas, Reissner replaces the classical kinetic energy term ($T_i = \frac{1}{2}m_i v_i^2$) with one defined entirely in terms of the changes of relative distance between masses:

$$T_i = \sum_{j\neq i} \delta m_i m_j \dot{r}_{ij}^2 f(r_{ij}). \tag{3}$$

Reissner is able to determine that $f(r)$ must tend to a constant asymptotically and so sees no reason not to regard it as a constant. By applying the condition that

[8] See sections 2.2 and 5 for discussions the anisotropy issue.

Newton's equations of motion should hold with good accuracy for a large number of masses, Reissner derives the following condition on his inertial coupling constant:

$$\delta = \frac{3}{\sum_i m_i} \tag{4}$$

So that his Lagrangian for the system effectively becomes:

$$L = \frac{3}{\sum_i m_i} \sum_{i<j} m_i m_j \dot{r}_{ij}^2 + \gamma \sum_{i<j} \frac{m_i m_j}{r_{ij}}, \tag{5}$$

As we can see however, although Reissner has implemented Mach's hypothesis in this first paper, inertia and gravity are treated independently of one-another, so that there is no unification of inertia and gravity in this model.

2. Reissner (1915). The main difference in starting point of Reissner's second paper from that of the first is that he eliminates the need to consider an additional gravitational potential U, and instead derives both inertia and gravitation from a single relative kinetic energy function T with a different choice of $f(r)$; referring to his previous paper, Reissner (1915) writes:

> at that time I saw no reason not to equate $f(r)$ to a constant and assumed a separate force function for gravitation. Here, however, the function $f(r)$ of the mutual distance should be used in such a way that no additional force function is required for gravitation.

Reissner introduces two postulates for the origin of inertial and gravitational forces respectively:

Postulate 1. *The inertial force of mechanics can be represented as the resistance to translational accelerations of a mass relative to all other masses in space.*

Postulate 2. *Weight, or Newtonian gravitation, can be represented as an inertial force arising from the rotation of mass particles relative to each other.*

To achieve this, Reissner then proposes:

$$T = \frac{1}{2} \sum_{i>j} \frac{m_i m_j}{r_{ij}} \dot{r}_{ij}^2 \tag{6}$$

for the total relative kinetic energy of a system of many particles. Reissner's Lagrangian $L = T$ now only has a single term, so there is no need for either an inertial or gravitational coupling constant, instead, the gravitational coupling is determined by identifying the predicted mass-mass attraction of the model with Newtonian gravity.

Since gravity arises in Reissner's model due to motion internal to a body, he is forced to assume that:

> The structure of gravitating matter may be imagined such that, in every volume element, rotating particles are distributed without preference for any particular axis of rotation.

Given this assumption, Reissner works out that his model produces a attraction between masses given by:

$$K_r = \frac{1}{3} m_i m_j \frac{k_i^2 \omega_i^2 + k_j^2 \omega_j^2}{r_{ij}^2}. \tag{7}$$

where k_i is the radius of gyration and ω_i the angular velocity of the body i. Gravity will then have been *explained* as a phenomenon of relative inertia provided that K_r can be identified with the Newtonian gravitational force $\gamma \frac{m_i^* m_j^*}{r^2}$ where m_i^* and m_j^* are the *inertial* masses. In both of Reissner's models, the inertial masses are tensorial and anisotropic, and can be given as:

$$m_i^* = m_i \sum_{j \neq i} \frac{m_j}{r_{ij}^3} (\mathbf{r}_{ij} \otimes \mathbf{r}_{ij}), \tag{8}$$

where $\mathbf{r}_{ij} \otimes \mathbf{r}_{ij}$ is the tensor product of the distance vector between i and j composed with itself. However, if the potential function $\sum \frac{m}{r}$ is assumed to change gradually enough, the inertial mass can be approximated as:

$$m_i^* = \frac{1}{3} \phi_i m_i, \qquad \phi_i = \sum_{j \neq i} \frac{m_j}{r_{ij}}, \tag{9}$$

In this way, Reissner derives the following expression for the gravitational constant:

$$\gamma = 3 \frac{k_i^2 \omega_i^2 + k_j^2 \omega_j^2}{\phi_i \phi_j}, \tag{10}$$

where the term $\phi_i \phi_j$ has units of mass/distance, since the units of m_i^* is mass, however the invariant quantity m_i has different units.

2.2. Barbour's models

In one of the most striking coincidences in the history of relational theories, the two models of Reissner would be reinvented exactly 60 years after their initial publication in a pair of papers by Julian Barbour (Barbour, 1974, 1975) which correspond almost exactly to the two papers by Reissner of 1914 and 1915 respectively, despite Barbour not having come across Reissner's papers at the time.

1. Barbour (1974). One of Barbour's more significant contributions in his first paper was to introduce the concept of the *relative configuration space* (RCS) of the universe, which—for a universe consisting of a system of N particles in Euclidean space—consists of "all the distinct relative configurations of these N particles". The generalised concept of the RCS would go on to be an important feature in all of Barbour's later works and in the field of *shape dynamics* in general.

The Lagrangian Barbour uses in his 1974 model differs slightly from Reissner's as it is given as a product rather than a sum of two terms:

$$L = \Psi\Gamma, \tag{11}$$

with the two potentials given by:

$$\Psi = \sum_{i>j} m_i m_j / r_{ij} \qquad \Gamma = \left(\sum_{i>j} m_i m_j \dot{r}_{ij}^2 \right)^{1/2} \tag{12}$$

Barbour justifies his use of a product as a means of ensuring that "$Ld\lambda$ is independent of the time parameter" since "a sum like $(\Psi + \Gamma)d\lambda$ would not be λ-independent)".

Although Barbour's lagrangian in his 1914 paper uses two separate potentials, they cannot have independent couping strengths since the Lagrangian is written as a product rather than a sum. This means that despite using two potentials, Barbour is able to derive an expression for the gravitational constant:

$$\gamma = \frac{1}{M\Psi}, \tag{13}$$

where $M = \sum_i m_i$. This is achieved of course by considering a universe with an "environment broadly similar to ours (very many stars distributed uniformly over a large region)", and identifying the resultant force on a particle—in a frame fitted to classical mechanics—with Newtonian gravity.

2. Barbour (1975). In his second paper, Barbour makes essentially the same move as Reissner made in his second paper: he modifies his velocity dependent potential $\Gamma \to \tilde{\Gamma}$ by introducing a $1/r_{ij}$ dependence:

$$\tilde{\Gamma} = \left(\sum_{i>j} \frac{m_i m_j}{r_{ij}} \dot{r}_{ij}^2 \right)^{1/2} \tag{14}$$

and proposes the same hypothesis as Reissner, namely, that whereas inertia arises "due to the relative motion of masses as a whole", gravity arises "due to the additional relative motion generated by internal motion". Which corresponds exactly to the two postulates of Reissner (1915) respectively. Although Barbour's Lagrangian is the square root of that of Reissner's second paper, the theory should be identical.

Barbour obtains his results by considering a "highly idealised model" in two dimensions where matter is modelled as being constituted of rings of mass rotating with a velocity of internal motion given as c. An attractive force between pairs of particles is derived which is identified with Newtonian gravity, giving the expression:

$$\gamma \sim \frac{Rc^2}{M} \tag{15}$$

for the gravitational constant. Where R is the radius of the universe and M its total mass. Barbour notes that this expression resembles one of the famous "cosmic coincidences"—that had been brought to attention by Dirac (1938) and was discussed in some detail by Bondi (1961)—provided that the velocity of rotation c is the speed of light.This had not been considered in Reissner's paper of 1915, in which the velocity of inernal rotation was left as $k\omega$. The possibility of deriving gravity as a necessary consequence of relative inertia in Barbour's theory then hinges on this intriguing hypothesis about the nature of matter, that:

> the rest-mass energy of the particles is associated with an incessant internal motion at velocity c (all energy then being kinetic in origin) or alternatively with the zitterbewegung of elementary particles, which occurs at c.

Such a hypothesis is immediately suggestive of a possible link with quantum mechanics and special relativity:

> The ideal would therefore be a theory based on a generalization of (14) in which special relativity and quantum mechanics play an integral part, providing a natural framework for the necessary internal motion. In such a theory, the four universals of Nature—gravity, inertia, special relativity and quantum mechanics—would clearly be linked in an intimate and inseparable manner.

What is all the more remarkable about Barbour's remark here is that the very next year, Richard Cook, a physicist working for a US air force weapons laboratory, would rediscover the same idea entirely independently and explicitly develop the suggested connection with quantum mechanics by deriving gravity as an effect of the quantum "zitterbewegung" of elementary particles. Cook's model (Cook, 1976) will be discussed in section 4.2.

3. Mass anisotropy. Just as in Reissner's model, the inertial mass in Barbour's model turns out to be anisotropic due to the dependence of the lagrangian on the derivative of relative positions $\dot{r}_{ij}$ rather than relative velocity $\mathbf{v}_{ij}$.

This phenomenon of mass anisotropy has been a source of continued controversy for theories of this kind ever since. As Barbour has remarked in the notes to his translation of Reissner's first paper (Reissner, 1995), it is "in clear disagreement with both experiment and general relativity".

Some important experiments concerning this question were carried out by Hughes et al. (1960); Drever (1961), and showed a high precision null-result for the anisotropy of inertial mass. However, in defence of these Machian theories, Dicke (1961) has argued that "because of the universal character of the inertial anisotropy, being present in the same way for all particles (or fields), the spatial anisotropy is unobservable locally".[9] In fact, the anisotropy tensor f_{ij}, which gives the tensorial inertial mass as

$$m^*_{ij} = f_{ij} m, \tag{16}$$

effectively functions as a metric tensor in these theories. Dicke then argued that[10]:

> a coordinate system can always be chosen in such a way as to cause f_{ij} to be locally Minkowskian with vanishing first derivatives.

Nonetheless, drawing in particular from the experiments of James E. Faller, Barbour himself considered that the anisotropy problem had experimentally disproven this class of Machian models and subsequently directed his attention towards his collaborative work with Bruno Bertotti (Barbour and Bertotti, 1982), and later on to the development of his theory of *shape dynamics* (Mercati, 2018), whose main connection to the pair of papers discussed here is the continuation of the (generalised) concept of the *relative configuration space*. The issue of mass anisotropy was also a motivation for Treder's model which we will discuss in section 3.3.[11]

[9] A similar idea was expressed in Reissner's original paper: "the fact that classical mechanics, with mass as a constant scalar quantity, serves so well must be seen as an indication that we find ourselves in a region of space with a sufficiently symmetrical mass distribution—unless it should turn out that the tensorial and variable nature of inertial mass appears to us as scalar and unchanging due to the variable properties of our measuring instruments" (Reissner, 1915). However Reissner himself was not decided one way or the other. In correspondence, Richard Cook (see sec. 4.2) has indicated that he endorses Dicke's view.

[10] This claim has been disputed by Treder (1973).

[11] In a little-known paper from 1973, Treder has also argued that general relativity itself should also predict mass anisotropy (Treder, 1973).

3. The Weber and Riemann potentials

3.1. Schrödinger's model

Whereas Barbour had rediscovered Reissner's results independently six decades after their original publication, Schrödinger's contribution to the literature on relational physics models was submitted for publication to *Annalen der Physik* a single decade after the publication of Reissner's papers, in the summer of 1925 (just a few months before he would derive the equation that bears his name). Schrödinger's paper is in many ways similar to Reissner's, indeed, in a note written following its original publication, Schrödinger profusely apologises to Reissner for having unconsciously plagiarised his 1914 paper, which he would have read at the time, and calls both of Reissner's papers "very interesting".[12] Nonetheless, Schrödinger's approach does offer some novelties.

Schrödinger derives inertia and gravity—like Reissner (1914)—from two independent terms:[13]

$$W_{ij} = \delta\gamma \frac{m_i m_j}{r_{ij} c^2} \dot{r}_{ij}^2 - \gamma \frac{m_i m_j}{r_{ij}}, \tag{17}$$

where δ is an inertial constant to be determined. The first term in Schrödinger's potential, however, differs from Reissner's 1914 paper, in fact it is identical to the expression in Reissner (1915) which includes the $1/r_{ij}$ dependence. However Schrödinger does not obtain his results from the *Reissner-Barbour-Cook hypothesis* of internal rotation of masses, instead, gravity is derived from the second term, which is just the Newtonian gravitational potential. For this reason, unlike the theories of Reissner (1915) and Barbour (1975), Schrödinger's theory does not exactly fulfill Reissner's hypothesis; i.e. gravity is not shown to arise *necessarily* out of relative inertia. Instead, it makes more sense to read Schrödinger's model the other way around: *an extension of the gravitational interaction to include a dependence on relative velocity as well as relative position leads to a relational derivation of inertial effects.*

[12] See Barbour and Pfister (1995, p.156) and Assis (2014, p.483) for discussions of Schrödinger's apology to Reissner.

[13] In Schrödinger's original paper the gravitational constant γ is left out and Schrödinger uses γ to denote the inertial constant which we denote with δ. Like Reissner, Schrödinger also uses μ to denote the invariant masses, however we have replaced there with m. Schrödinger's Lagrangian is given in his paper as $W_{ij} = \delta\frac{\mu\mu'}{r}\dot{r}^2 - \frac{\mu\mu'}{r}$, however for the sake of clarity and consistency with our discussion of other models we have chosen to write it as equation 17.

Although Schrödinger did not mention this fact, his potential function is formally identical to Weber's potential for electromagnetism, so that Schrödinger's model can be seen as the first real fulfillment of Immanuel Friedlaender's comment from 1896 in which it was suggested that the application of Weber's law to gravity may enable the unification of gravity and inertia (Friedlaender and Friedlaender, 1896).

One might think that an expression for the gravitational constant γ may be derived by simply fitting the velocity-dependent term in Schrödinger's Lagrangian to the classical term for inertia. However it is not so simple, since the dependence on $\dot{r}_{ij}$ rather than v_{ij} produces anisotropic effects. It was therefore necessary for Schrödinger to integrate the tensorial inertial mass (8) over a homogeneous and isotropic universe in order to calculate γ. The result calculated comes out as:

$$\gamma = \frac{3c^2}{2\delta\Phi} \tag{18}$$

Where Φ is the total gravitational potential of the universe calculated at the location of our test-mass.

On the basis of an analogy with Weberian electromagnetism, or equivalently, on the basis of the assumption that both terms in the potential are due to a common "gravitational" interaction, it is reasonable to assume that δ is of order 1, and thereby an approximate expression for the gravitational constant would be calculable. However, Schrödinger obtains a more precise value for δ by fitting the model to the observed results for the perihelion shift of Mercury, which gives $\delta = 3$. So that his final expression for the gravitational constant is:

$$\gamma = \frac{c^2}{2\Phi} \tag{19}$$

Following from this, Schrödinger raises the implications that this result has for cosmology. Since the gravitational potential of our solar system and indeed our galaxy appears to be vanishingly small in comparison to the total $\sum \frac{m}{r}$ needed for the correct prediction of Mercury's perihelion, it follows that (Schrödinger, 1995):

> only an entirely vanishing fraction of the inertial effects observed on the earth and in the planetary system arises from the interaction with the masses of our Milky Way system.

This is a remarkable and bold result given that at this time very little was known experimentally about the true size of the universe. In fact, it was only the year before, in 1924, that Hubble demonstrated that there were galaxies other than our own milky way; before this many would have identified the universe with our galaxy.

3.2. Assis's model

In André Assis's original article "On Mach's principle" of 1989, Schrödinger's Weberian model is independently reinvented, 64 years after Schrödinger's original publication, however, in Assis's case the analogy with Weber's law is explicitly recognised (Assis, 1989). Indeed Assis would go on to advocate in favour of the application of Weber's law to both gravity and electromagnetism on the basis of it's relational character, writing several books on the history of Weberian electromagnetism (Assis, 2021a,b,c,d, 2024). Assis's results are essentially exactly the same as those of Schrödinger since the same Lagrangian is used, although since Assis is explicitly applying the analogy with electromagnetism he uses Weber's original unit conventions.[14] Just as with Schrödinger's theory, Assis's inertial coupling constant $\xi = 6$ is derived by fitting the predicted perihelion advance to that obtained by using general relativity, although the analogy with electromagnetism suggests that it should be of order 1.

In addition to rediscovering Schrödinger's results, Assis also shows that the model produces classical inertial forces. By considering an isotropic universe of homogeneous density ρ in a frame of reference relative to which the universe as a whole has a translational acceleration $\mathbf{a}_\mathrm{U}$ and is rotating with an angular velocity $\omega_U(t)$, Assis calculates the induced acceleration on a body to be (Assis, 1989, 2014):

$$\mathbf{a}_1 = \mathbf{a}_U - \omega_U \times (\omega_U \times \mathbf{r}_1) + 2\omega_U \times \mathbf{v}_1 + \frac{d\omega_U}{dt} \times \mathbf{r}_1 \tag{20}$$

with $\mathbf{r}_1$ being the body's position and $\mathbf{v}_1$ being its velocity. These are the expected terms for the linear, centrifugal, Coriolis and Euler accelerations of classical mechanics respectively. Thus the Weberian model of Schrödinger and Assis correctly predicts classical inertial forces without reference to absolute space.

3.3. Treder's Riemann potential

Another little-known approach, which differs somewhat from that of Schrödinger and Assis, is the inertia-free mechanics of Hans-Jürgen Treder. Treder was a

[14] In Assis's paper, the Weber potential is given as $U_{ij} = \frac{k}{r_{ij}}\left(1 - \frac{\xi}{2c^2}\dot{r}_{ij}^2\right)$, with $k = H_g m_i m_j$ and $\xi = 6$. With respect to the notation we have chosen to use in this paper, $\xi = 2\delta$ and $H_g = \gamma$

prominent theoretical physicist in the German Democratic Republic, known primarily for his contributions to gravitational physics although in his later years he would gain wider recognition for his work in the history and philosophy of science. Despite their significance, Treder's papers, which were published in German, remained relatively unknown to his colleagues on the other side of the iron curtain, and by the time of Germany's reunification, Treder had largely retired, and the absence of English translations further restricted the reach of his ideas. His most important work on the subject of relational inertia was his monograph "*Die Relativität der Trägheit*" (Treder, 1972).

The essential difference from the theories of Schrödinger and Assis is that Treder uses the *Riemann* potential rather than the Weber potential:

$$V_{ij} = \delta \frac{\gamma m_i m_j}{c^2 r_{ij}} \mathbf{v}_{ij}^2 - \frac{\gamma m_i m_j}{r_{ij}}, \tag{21}$$

which is identical to the Weber potential apart from the velocity term $\mathbf{v}_{ij}^2$ which replaces the $\dot{r}_{ij}^2$ of Schrödinger and Assis. This potential was, like the Weber potential, originally introduced by Riemann (1876) in his theory of electromagnetism. The difference is that $\mathbf{v}_{ij} = \dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j$ encodes the relative velocity between particles i and j rather than the rate of change of their relative distances $\dot{r}_{ij}$. This choice comes with some advantages as well as drawbacks as we shall see.

On one hand, Treder's potential does not leads to the anisotropy of inertial mass, which was potentially a serious issue encountered by all the models which we have examined so far. In Treder's theory, the inertial mass becomes a scalar expression given by:

$$m_i^* = \frac{2\delta\gamma\varphi_i}{c^2} m_i, \qquad \varphi_i = \sum_{j\neq i} \frac{m_j}{r_{ij}}. \tag{22}$$

Unlike the Schrödinger-Assis model, the calculation of an expression for γ is quite straight forwards in Treder's theory, since we can observe immediately that the Lagrangian for a single particle $L_i = \sum_{j\neq i} L_{ij}$ reduces to the classical Lagrangian with kinetic and gravitational potential energy terms provided that:

$$\gamma_i = \frac{c^2}{2\delta\varphi_i} \tag{23}$$

where $\varphi_i = \sum_j m_j/r_{ij}$ is the total gravitational potential with respect to our test particle i, and γ_i is the gravitational coupling strength at the location of the particle i.

Just like in the models of Schrödinger and Assis, the inertial constant $\delta = 3/2$ is fitted to the observed perihelion shift of Mercury, which is achieved, like in the models of Schrödinger and Assis by considering a system of two masses orbiting one-another superimposed upon an otherwise homogeneous universe. The resulting expression for the gravitational constant in Treder's model is therefore:

$$\gamma_i = \frac{c^2}{3\varphi_i} \tag{24}$$

The major drawback of Treder's approach is that by using relative velocities rather than the rate of change of relative distances, Treder's theory is no longer rotation invariant as the Lagrangian looses its invariance under rotations of the reference system. This is because in a rotating frame of reference, the difference in velocities $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$ is not generally constant, for instance if we choose a frame of reference which is rotating with angular velocity $\boldsymbol{\omega}$, the relative velocity transforms as:[15]

$$\mathbf{v}_{ij} \to \mathbf{v}_{ij} - \boldsymbol{\omega} \times \mathbf{r}_{ij} \tag{25}$$

Remark. It should be noted that for all three models seen in this section, it is implicitly assumed that elementary matter does not vibrate at high speeds. If we supposed, in accordance with the hypothesis of Reissner and Barbour (and later Cook, see section 4.2) that elementary particles are rotating or vibrating at high speeds, then the velocity dependent term in the Weber or Riemann potentials would produce an additional attraction between masses. If this is the case, the above approaches might be interpreted as idealisations of the Reissner-Barbour theory, in which internal motion is ignored, but its effect is reintroduced into the dynamics by adding a velocity-independent term in the potential.

4. Sciama's Maxwellian analogy

4.1. Sciama's model

Whereas all of the models considered so far operate within the framework of classical mechanics—i.e. they involve instantaneous action at a distance between masses—, Sciama's model is not entirely classical as it borrows results and concepts from special relativity and Maxwellian electromagnetism. Despite this, Sciama's "toy-model", which employs a vector-potential, is not

[15] It should be noted that this issue may potentially be resolved by introducing a term of the form $-\boldsymbol{\Omega} \times \mathbf{r_{ij}}$ into the Lagrangian, in a manner analogous to Lynden-Bell (1995). Lynden-Bell did not work with the Riemann potential in his papers, however the correct modification of Treder's Lagrangian has recently been carried out by Braun (2024) who has shown that the modified Riemann potential enables calculation of the correct classical inertial forces (eq. 20).

a consistent relativistic theory. This did not matter to Sciama since it was only constructed for illustrative purposes and was meant to precede a complete theory which Sciama thought would have to involve a rank-2 tensor potential like general relativity (Sciama, 1953a).

Sciama gives his potentials in terms of integrals of the matter density $\rho = \rho(r)$ divided by the distance r from the point we are calculating the potentials at. The integration is implicitly carried out over a three dimensional hypersurface V corresponding to the past light cone of the observable universe relative to the point in question. Although this is not specified in the equations given it is mentioned in the text. The potentials provided are:

$$\Phi = -\int_V \frac{\rho}{r} dV, \qquad \mathbf{A} = -\int_V \frac{\mathbf{v}\rho}{cr} dV, \tag{26}$$

By idealising the universe as perfectly homogeneous with a radius approximated as $c\tau$, Sciama gets:

$$\Phi = -2\pi\rho c^2\tau^2 \tag{27}$$

If a test-mass now moves with a small non-relativistic velocity $\mathbf{v}$ with respect to this universe, the vector potential in the rest frame of this particle will be given by:

$$\mathbf{A} = \frac{\Phi}{c}\mathbf{v}, \tag{28}$$

The analogues of Maxwell's electric and magnetic fields are given by:

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{H} = \nabla \times \mathbf{A} \tag{29}$$

Inertial motion of the particle is then recovered in the condition that $\mathbf{E} = 0$, which, in the case of this perfectly homogeneous universe, applies when $\frac{\partial \mathbf{v}}{\partial t} = 0$, i.e. when the test particle is not accelerating with respect to the universe.

1. Inhomogeneous universe: gravity. By considering a mass M superposed upon this otherwise homogeneous model universe and at rest relative to it, Sciama finds that the fields are given by:

$$\mathbf{E} = -\frac{M}{r^2}\hat{\mathbf{r}} + \frac{\phi + \Phi}{c^2}\frac{\partial \mathbf{v}}{\partial t} \tag{30}$$

where $\phi = M/r$ is the potential generated by the test particle. Setting $\mathbf{E} = 0$ now gives:

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{c^2}{\phi + \Phi}\frac{M}{r^2}\hat{\mathbf{r}} \tag{31}$$

Which reduces precisely to Newton's law of gravitation provided that $\frac{-c^2}{\phi+\Phi}$ is just the gravitational constant γ. Therefore we will have achieved the proper unification of inertia and gravity provided that:

$$\gamma = -\frac{c^2}{\Phi_U} \tag{32}$$

where Φ_U is the total Newtonian potential of the universe.[16]

2. Uniform rotation. So far, Sciama's model has shown that it is possible, in a slightly inhomogeneous, non-rotating universe to interpret Newton's laws of gravity and inertia as arising from the balance between the effects of a local inhomogeneity and the resistance to acceleration provided by the rest of the universe. However two shortcomings remain:

1. There so far has not been any justification for introducing the two potentials Φ and $\mathbf{A}$ as a unified entity, therefore Sciama might as well have chosen different coupling constant in front of each potential in which case the derived relation for the gravitational constant $\gamma = -c^2/\Phi$ would be made meaningless.
2. Sciama has not shown that his theory will actually generalise to non-inertial frames of reference.

These questions appear to be at least partially answered by an analysis of Sciama's treatment of the centrifugal force in a uniformly rotating frame. Although, as we shall see, the method he uses is *ad hoc*, and directly undermines Mach's principle, it is nonetheless highly suggestive since it indicates that a proper relativistic extension of his model might be able to resolve these issues adequately.

Sciama starts out by giving the scalar and vector potentials (Φ and $\mathbf{A} = (A_x, A_y, A_z)$ respectively) in a non-rotating frame as:

$$A_x = A_y = A_z = 0; \qquad \Phi = -1$$

where units are chosen such that $\gamma = c = 1$. Now Sciama argues that in a frame rotating around the z axis with angular velocity ω, the potentials will transform as:

$$\begin{aligned} A_x &= \omega y \\ A_y &= -\omega x \\ A_z &= 0 \\ \Phi &= -(1 + \omega^2 r^2)^{1/2} \end{aligned}$$

[16] It is worth noting that Braun (2024) has recently shown that Sciama's model as we have seen it so far can essentially be derived from Treder's theory.

where $r^2 = x^2 + y^2$. Whereas the modifications of the $\mathbf{A}$ fields follow naturally from Sciama's definition of this potential as $\mathbf{A} = -\int_V \frac{\mathbf{v}\rho}{cr} dV$, the modification given of the scalar potential Φ does not. Instead, Sciama explains that this follows from "the transformation properties of four-vectors", citing a paper by Nathan Rosen (Rosen, 1947) as justification. Although it is true that if $A_\mu = (\Phi, \mathbf{A})$ is taken to be a four-vector in Minkowski space, then the given transformations applied to the components of $\mathbf{A}$ imply that Φ must also transform as shown above, however there are two issues with using this result: (1) Sciama's model is only allegedly concerned with the classical limit so there is no justification for the use of results particular to special relativity, and more importantly, (2) the transformation of Φ does not follow from the integral expression given for Φ in equation 26, therefore the modification of Φ should be interpreted as an effect of an absolute Minkowski space, not of the masses of the universe, thus undermining Sciama's Machian intentions altogether.

Nonetheless, it is conceivable that a relativistic modification of Sciama's Maxwellian analogy would include relativistic gamma factors in the integrals for Φ and $\mathbf{A}$—for instance by integrating over four-momentum rather than $(\rho, \rho\mathbf{v})$—and thereby generate an appropriate modification of the scalar potential.[17]

The gravilectric field generated by the rotating universe according to Sciama's transformed potentials is:

$$\mathbf{E} = \Phi_U \frac{\omega^2 \mathbf{r}/c^2}{(1 + \omega^2 r^2)^{1/2}} + \frac{\Phi_U}{c^2}\mathbf{a} \qquad (33)$$

where $\mathbf{a}$ is the acceleration of the universe with respect to our frame of reference. Setting Sciama's condition for inertial motion, i.e. $\mathbf{E} = 0$, we get:

$$\mathbf{a} = -\frac{\omega^2 \mathbf{r}/c^2}{(1 + \omega^2 r^2)^{1/2}} \qquad (34)$$

which for non-relativistic speeds reduces to the classical centrifugal acceleration $\mathbf{a} = -\omega^2 \mathbf{r}$. In this case the sign is negative since we have actually calculated the acceleration of the universe with respect to our test particle, which will be a centripetal acceleration.

3. Discussion. In contrast to the models of Schrödinger, Assis and Treder, in which the ratio of the inertial to gravitational coupling was postulated to be of order 1 but then fitted precisely to produce the perihelion precession predicted by general relativity, in Sciama's model although the ratio in coupling strengths between the inertial $\mathbf{A}$ and gravitational Φ potential is initially justified only by analogy with Maxwell's equations, the particular ratio between them is justified by the treatment of $A_\mu = (\Phi, \mathbf{A})$ as a relativistic four-vector. As we saw, just such a relativistic treatment was necessary in order to derive the centrifugal force, which suggests that a fully relativistic extension of Sciama's theory would fulfil Reissner's hypothesis, that is, it would predict the existence of gravity as a necessary consequence of the relativisation of inertia (in this case, the relativistic treatment of inertial forces in analogy with electromagnetism), although it does not rely on the particular zitterbewegung hypothesis of Reissner and Barbour (i.e. Reissner's second postulate, see section 2.1). This opens an intriguing question concerning a possible relationship between Sciama's model and those of Reissner and Barbour. As we will see in the next subsection, just such a connection is made in the model proposed by Cook (1976).

4.2. Cook's zitterbewegung

In a highly interesting paper of 1976 (Cook, 1976), written one year after the publication of Barbour's 1975 paper, Richard J. Cook—a physicist at a US Air Force Academy who's main claim to fame would later be to figure out how to directly observe quantum jumps in a single trapped atom (Cook and Kimble, 1985) which turned out to be useful for making very accurate atomic clocks—published an article independently rediscovering the idea of Reissner (1915) and Barbour (1975) of deriving gravity as a side-effect of relativised inertia arising from the "zitterbewegung" of elementary particles. We have chosen to include cook's model here rather than in section 2 since Cook's method more closely resembles Sciama's and is developed in part out of some of Sciama's work. Importantly however, Cook's model does not come from a consideration of rotation, but only of the inertial force induced by a linear acceleration of the reference frame.

Cook's model is derived from a consideration of what he calls the "Einstein-Sciama" force (eq. 36), which he borrows from Dennis Sciama's 1969 book "The physical foundations of general relativity" (Sciama, 1969), in which Sciama leverages his analogy with electromagnetism to derive an expression for the mutual inertial force between two masses m_i and m_j in relative acceleration[18]:

$$\mathbf{F}_{ij} = \frac{\gamma m_i m_j \mathbf{a}_{ij}}{c^2 r} \qquad (35)$$

[17] See Fay (2025) for a recent attempt along these lines.

[18] See section 5 for a derivation of this from Sciama's 1953 model.

which he generalises as:

$$\mathbf{F} = \frac{\gamma m}{c^2} \int \frac{\rho(\mathbf{r})\mathbf{a}(\mathbf{r})}{|\mathbf{r}_m - \mathbf{r}|} dr^3 = 1 \tag{36}$$

Cook then remarks that this force will be identifiable with the inertial force under linear acceleration of the reference frame provided that the matter distribution is such that:

$$\gamma\Phi = -c^2 \tag{37}$$

where $\Phi = -\int \frac{\rho(\mathbf{r})}{|\mathbf{r}_m - \mathbf{r}|} dr^3$, and like Sciama (1953b), notes that this equation is in good agreement with experimental values.

Next however, instead of deriving his gravity from the scalar potential Φ, who's effects are not strictly necessary in any case for a non-relativistic description of inertial forces, Cook seeks to derive gravitational effects out of this purely inertial force law, in a manner essentially analogous to the models of Reissner and Barbour, in particular Cook seeks to show:

> that the acceleration of particles within matter gives rise to an Einstein-Sciama force outside of matter, that is similar in many respects to the Newtonian force of gravity.

Which is analogous to postulate 2 of Reissner. In order to do this, Cook considers a particle m_1, located at $\mathbf{r}_1$ and confined within a sphere of radius ϵ around the origin which is vibrating with a velocity v_1 assumed to be random and statistically isotropic". After some manipulation, Cook arrives at the following expression for the time-averaged force $\langle F^i \rangle$ on a particle at some distance $r_2 >> \epsilon > r_1$ away:

$$\langle F^i \rangle = -\frac{\gamma m_1 m_2 r_2^i}{r_2^3} \frac{\langle v_1^2 \rangle}{3c^2} \tag{38}$$

where, $\mathbf{r}_2 = (r_2^i, r_2^j, r_2^k)$ such that $r_2^i = |\mathbf{r}_2| = r_2$ when i is taken to be the coordinate in the direction of $\hat{\mathbf{r}}$. The factor of 3 in the denominator come out because of the isotropy of the particle's average velocity $\langle v_1^2 \rangle$. At this point it is worth noting that the result is entirely analogous to the force calculated by both Reissner (1995) and Barbour (1975), however since Cook has already used Sciama's result that $\gamma = -c^2/\Phi$, it is not as easy to identify this result with Newtonian gravity, since even if—with Julian Barbour—we suppose that $\langle v_1^2 \rangle = c^2$, Cook's expression will only correspond to $1/3$ of the force required for Newtonian gravity.

However, in a fascinating twist, Cook then goes on to show that the difficulty is eliminated when quantum mechanics is used to calculate the inertial-gravitational force. Although we will not show Cook's calculation here, his method essentially involves replacing the classical numbers a_1^i and $r_1^j a_1^i$ by expectation values of the corresponding Dirac operators of a generic spin-$\frac{1}{2}$ particle. The operator associated with the square of the velocity is $v^2 = v^i v^i = c^2 \beta_i \beta_i$, however the anticommutation relation $\beta_i\beta_j + \beta_j\beta_i = 2\delta_{ij}$ implies that $\beta_i\beta_i = \delta_{ii} = 3$, giving the paradoxical result that $v^2 = 3c^2$ which thus yields a gravitational force of the correct magnitude:[19]

$$F^i = -\frac{\gamma m_1 m_2 r_2^i}{r_2^3} \tag{39}$$

Since "matter may be regarded as composed of neutrons, protons and electrons", Cook now feels justified in "identifying the inertial-gravitational force with the Newtonian force of gravity" (Cook, 1976).

Apart from being briefly cited by Barbour and Pfister (1995, p.142) in the notes to Barbour's translation of Hans Reissner's papers, Cook's paper has received very little attention, even though Cook appeared to be on his way to fulfilling exactly the condition that Barbour had given for "an ideal theory" the previous year, that "the four universals of Nature—gravity, inertia, special relativity and quantum mechanics—would clearly be linked in an intimate and inseparable manner" (Barbour, 1975). Barbour's own turn away from this class of relational theories at this time was, as we saw, due to his becoming persuaded that they were inconsistent with null-results in measurements of the anisotropy of inertial mass. Cook on the other hand has taken the side of Dicke (1961) on this issue, however, Cook himself went on to have a career in laser and atomic physics and didn't develop the idea further. In correspondence, Cook has mentioned that until now he was not even aware of the models of Reissner and Barbour, and that the only person who had taken his idea seriously at the time was Edwin Jaynes, who is well known for his information-theoretic approach to thermodynamics.

5. Discussion and conclusion

Before discussing the differences between the models presented in this paper, we should first address the difference between all these approaches and Einstein's general relativity. As we saw in section 1.2, Einstein's own intention with his theory was to "find a more

[19] Cook also remarks that "Paradoxical properties of the Dirac velocity operator, such as $v^2 = 3c^2$, are known to be a result of the zitterbewegung or rapid oscillatory motion of the Dirac electron."

profound explanation" for the equivalence of gravitational and inertial mass (Einstein, 1996). However, as has been emphasised especially by Sciama (1953b) and Cook (1976), the principle of equivalence is not given an explanation in this theory since, as Cook has put it:

> General relativity is certainly not capable of providing a more fundamental understanding of the principle of equivalence, for no theory can explain its postulates, except by argument in a circle.

The same issue is recognised by Sciama (1953b) who insists that in his own model, unlike general relativity, "[t]he principle of equivalence is a consequence of the theory, not an initial axiom". This issue is related to the fact that Einstein's theory is unable to derive a relation between the gravitational constant and the amount of matter in the universe without additional assumptions; on the other hand, as we have seen, such a relation was an intrinsic feature of all the models we have discussed in this paper which gave an expression of the form $\gamma \sim -c^2\Phi^{-1}$ (although in Reissner's model this was less clear due to the choice of units). An expression of this form implies not only that gravity and inertia should be described using similar conceptual apparatus (as is the case in Einstein's theory), but that one is *reduced* to the other such that they arise as twin aspects of a unified law. This was not achieved in Einstein's theory since the inertia of a given mass does not necessarily originate in mass-mass interactions, as Sciama (1953b) put it: "Einstein showed that his field equations imply that a test-particle in an otherwise empty universe has inertial properties"; moreover, matter will have inertial properties regardless of the existence or strength of the gravitational interaction. Einstein's theory on its own does not imply a relation between γ and Φ^{-1}, although, interestingly, such a relation does arise in cosmology if one places constraints on the geometry of the universe (Antoniou and Fay, 2025).

On the other hand, in all of the models we have examined in this paper (apart from Reissner's first model), the strength of the gravitational interaction is fixed by the requirement that the inertial interaction is of the correct magnitude to reproduce classical predictions in the case of a homogeneous cosmic mass-distribution. Therefore, these models can to a certain degree be interpreted as instantiating Reissner's hypothesis (3). The models all share a common guiding idea, but they differ markedly in how they put this idea into practice and in the assumptions—both stated and unstated—that shape their approaches. In what follows, we will unpack these differences in detail. As we'll see, the contrasts in their underlying hypotheses are mirrored in the assumptions that enter into their respective derivations of expressions for γ. For convenience, at this stage, we will divide the models we have discussed into four classes[20]:

- (1) includes the models of Reissner (1915), Barbour (1975) and Cook (1976) which are distinguished by their independent discovery of the hypothesis that gravity may arise due to the "zitterbewegung" of elementary particles.
- (2) includes the models of Schrödinger (1925) and Assis (1989) which use the Weber potential.
- (3) is the model of Treder (1972) using the Riemann potential.
- (4) is the model of Sciama (1953b) which employs an analogy with Maxwellian electromagnetism.

1. Zitterbewegung. The first category we have identified is clearly the purest expressions of "Reissner's hypothesis". Gravity, in all three of these models, is derived as a side-effect of relative inertia arising due to the rotational or vibratory motion of elementary particles, their "zitterbewegung". This hypothesis, which we may call the *Reissner-Barbour-Cook hypothesis*, and which Reissner designates as his postulate 2, is the surest and most direct way to fulfil what we have called "Reissner's hypothesis", i.e. that gravity can be explained as a consequence of the relativity of inertia (hypothesis 3). In these models, the strength γ of the gravitational interaction is unambiguously determined by the size and structure of the cosmos, provided that the internal motion in bodies is irregularly distributed. This remaining question is most satisfactorily resolved in Cook's model which appeals to the quantum mechanical nature of matter. Cook's model is consistent with the observed strength of the gravitational constant provided that $\gamma\Phi = -c^2$, a result which was also derived by Sciama.

The other models we have examined in this paper, in classes (2), (3) and (4) do not involve any interaction due to the internal motion of bodies. This may be viewed either as a strength or a weakness of these models. The issue is made clear if we place the Lagrangians for a pair of particles in Reissner (T_{ij}) and Schrödinger's (W_{ij}) models side by side for comparison. Given in the

[20] We are not considering the models of Reissner (1914) and Barbour (1974) which serve as preludes to the papers of 1915 and 1975 respectively.

same units these are:

$$T_{ij} = \frac{\delta\gamma}{c^2}\frac{m_i m_j}{r_{ij}}\dot{r}_{ij}^2, \quad W_{ij} = \frac{\delta\gamma}{c^2}\frac{m_i m_j}{r_{ij}}\dot{r}_{ij}^2 - \gamma\frac{m_i m_j}{r_{ij}} \tag{40}$$

respectively. According to the models in class (1), T_{ij} produces both inertial and gravitational effects due to the internal motion of particles; but since, T_{ij} is identical to the first term of Schrödinger's Lagrangian the same gravitational effects should arise in this theory, though they are not considered in his model, and instead masses are idealised as lacking internal motion. The same issue applies to all the other models in classes (2), (3) and (4). The results of Reissner, Barbour, and Cook suggest that this idealization—implicitly assumed in all the other models we have considered—is inappropriate. If, as Cook and Barbour propose, the internal motions occur at velocities on the order of c, then the predicted gravitational interaction is comparable in magnitude to the interaction arising from the second term in Schrödinger's Lagrangian. If we are inclined to think that the force derived by Reissner, Barbour and Cook is somehow based on fallacious reasoning their models should be rejected and these others would remain as plausible candidates of a unified theory of gravity and inertia. On the other hand, if we accept that the zitterbewegung of elementary particles does produce an attractive force between masses as these authors suggest, we will be forced to conclude that the models in classes (2), (3) and (4) are false, or at least incomplete. Alternatively, provided the idea of Reissner, Barbour and Cook is correct, these other models may be interpreted as idealisations according to which $\dot{r}_{ij}$ (or $\mathbf{v}_{ij}$) is taken to not include internal motion of bodies, but that the effect of this internal motion is expressed by introducing a separate term in the Lagrangian.

2. The perihelion fit. Models in classes (2) and (3) are closely related. In both cases, gravity and inertia are derived from separate terms in the Lagrangian, however a fixed proportion δ between the gravitational and inertial coupling is postulated, which thereby enables a calculation of the gravitational constant. In both cases these models are not generally interpreted as implementing Reissner's hypothesis (i.e. the explanation of gravity in terms of relative inertia), instead they are usually viewed the other way round, i.e. inertia is seen as being derived from gravity. The idea is that these authors postulate a different law of gravitation than than of Newton, which includes a velocity dependent term in the potential, and this enables a *physical* account of inertia. The specific values calculated for δ and γ do differ between classes (2) and (3) which give ($\delta = 3, \gamma = \frac{1}{2}c^2\Phi^{-1}$) and ($\delta = 3/2, \gamma = \frac{1}{3}c^2\Phi^{-1}$) respectively such that the values of the gravitational constant differ by a factor of 3/2 between the two classes of models.

3. Anisotropy and rotation-invariance. Model (3) of Treder is primarily motivated by the issue of the anisotropy of the inertial mass which was a feature of all the models in classes (1) and (2).[21] If we are inclined to agree with Cocconi and Salpeter (1958, 1960) this would imply that the models in classes (1) and (2) have been ruled out experimentally and model (3) would appear much more favourable. On the other hand, if we are inclined to agree with Dicke (1961) the anisotropy of inertia in these models may not be an issue. Although Treder was able to eliminate the issue of the inertial mass's anisotropy in his model, this came at the price of abandoning its rotation-invariance. Unlike the Lagrangian for gravity and inertia in classical mechanics, Treder's Lagrangian is invariant under arbitrary linear accelerations because it uses relative rather than absolute velocities in the inertial potential. However, these relative velocities are not invariant in a uniformly rotating reference frame.[22] In principle, the failure of rotation invariance is also an issue in Sciama's model (class 4), in fact Braun (2024) has recently shown that it is possible to derive a theory very similar to Sciama's from Treder's model. There are important differences in Sciama's model however.

4. Connection to special relativity. Sciama's model (class 4) is similar to the models of classes (2) and (3) insofar as it also employs an analogy with electromagnetism, however it differs from these two classes in that no perihelion fit is needed in order to derive a specific expression for γ. Instead, the proportionality between the inertial and gravitational coupling strengths is implicitly fixed by the analogy with Maxwellian electromagnetism. In particular, the scalar potential Φ which is responsible for gravity, and the vector potential $\mathbf{A}$ which is responsible for inertia are assumed to have the same coupling strength by analogy with their counterparts in Maxwell's theory. This proportionality however, is corroborated by Sciama's treatment of the centrifugal force. This is because, in Sciama's paper the centrifugal force arises due to the transformation of Φ under rotations of the reference system when $(\Phi, \mathbf{A})$ is treated as

[21] In this case we are referring to the models of Reissner (1915) and Barbour (1975) from calss (1), but not the model of Cook (1976) since the question of the anisotropy of inertial mass is not clearly posed in this case.

[22] The failure of rotation invariance in Treder's model can in principle overcome as was recently shown by Braun (2024).

a special relativistic four-vector. Although, for reasons we have discussed in section 4.1, this method undermines the relational aims of Sciama's model, it nonetheless implies that in a consistent relativistic extension of Sciama's model the coupling strengths of the scalar and vector potentials for gravity and inertia respectively should be identical.

5. Connection to quantum mechanics. Perhaps the most thought provoking model however is that of Cook (1976), which we classed alongside the models of Reissner and Barbour due to Cook's application of the *zitterbewegung* hypothesis. However, we will now discuss some unique features of Cook's model. First of all, it should be noted that Cook's starting point, which involves a consideration of what he called the "Einstein-Sciama force" between a pair of mutually accelerating particles (which he takes from Sciama (1969)) is directly derivable from Sciama's original 1953 model. In particular, if we ignore the influence of the scalar potential Φ and consider the field $\mathbf{E}$ at the location of a test mass m_i due to the motion of a mass m_j, we have:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \frac{\gamma m_j}{c^2}\frac{\partial \mathbf{v}_{ij}}{\partial t}, \qquad (41)$$

where $\frac{\partial \mathbf{v}_{ij}}{\partial t}$ is just the mutual acceleration $\mathbf{a}_{ij}$ between the two particles, such that the force on m_i, given by $\mathbf{F}_{ij} = m_i \mathbf{E}$ is the same as that which Cook uses (eq. 36).

Since Cook then endeavours to show that gravity can be calculated from this inertial force as a side effect of the zitterbewegung of elementary particles, this suggests that it may be possible to abandon Φ in Sciama's theory and deduce both gravity and inertia from the single vector potential $\mathbf{A}$. The most interesting result of Cook's calculations however is that the correct magnitude of the gravitational interaction is only derived when elementary particles are modelled quantum mechanically as fermions (which encompass the protons, neutrons and electrons of material bodies). Cook's method, which essentially constitutes a model of quantum gravity, therefore also implies Sciama's result that:

$$\gamma\Phi = -c^2, \qquad (42)$$

which as we saw follows from Sciama's electromagnetic analogy and appears to be implied by a relativistic treatment of the centrifugal force. Cook and Sciama's results are therefore very much reminiscent of Barbour's remark (Barbour, 1975):

> The ideal would therefore be a theory based on a generalization of [eq. 14] in which special relativity and quantum mechanics play an integral part, providing a natural framework for the necessary internal motion. In such a theory, the four universals of Nature—gravity, inertia, special relativity and quantum mechanics—would clearly be linked in an intimate and inseparable manner.

It is also worth noting that equation 42 was independently derived by Jordan (1939), based on the requirement that the total energy contribution of any given mass m at rest with respect to the universe ($mc^2 - m\gamma\Phi$) is zero. Later in Sciama's career, an expression of this form was also associated with the critical density condition in cosmology (Antoniou and Fay, 2025). The potential fruitfulness of these results, especially those of Cook (1976) calls for further work in this area.

Declaration of Interest

The authors declares that there is no conflict of interest.

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