Getting off the Hoek with Newton's laws

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Abstract

How to make sense of the notion of force-free motion which seems to be presupposed by Newton's first law? One can identify in the literature various different answers to this question, one among which is to be found in the writings of Torretti (1983). In a wonderful recent article, however, Hoek (2023) has proposed a radical revision to our understanding of Newton's first law, motivated on both exegetical and philosophical grounds. In light of this, one is left wondering whether this reconceptualisation of the content of Newton's first law obviates the need to provide a notion of force-free motion with which to undergird it. In this note, I'll argue that this is not the case: one can (and should!) endorse Hoek's understanding of the first law, while nevertheless seeking to define force-free motions in one of the various ways which have been proposed in the literature.

1 Introduction

In a wonderful recent article, Daniel Hoek (2023) has proposed, on the basis of sustained textual evidence, a compelling and quite radical reconceptualisation of the content of Newton's first law of motion (henceforth N1L). A standard orthodox statement of N1L is this:

N1L_O: Force-free bodies travel with uniform velocities.

('O' for 'orthodox'.) Such a statement would have it that N1L has to do *only* with the motions of unforced bodies. Hoek, on the other hand, argues that N1L should be taken to be a more general principle, having to do also with the circumstances under which bodies *deviate* from uniform motion. In order to motivate his alternative reading of N1L, Hoek turns to the Cohen and Whitman translation of Newton's *Principia*, according to which N1L reads as follows:

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> Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by the forces impressed. (Newton, 1999, p. 416)

Note that this is already a more general statement that $N1L_O$, as one can see from the 'except insofar as' clause, which identifies possible circumstances under which bodies do *not* move uniformly. Taking his cue from the above translation, Hoek arrives at the following alternative statement of N1L:[1]

$N1L_H$: Bodies travel with uniform velocities unless impressed forces act on them, and when forces do act on them they diverge from the trajectories that they would have followed if their velocities were to have been uniform only to the extent that these forces compel them to do so.

('H' for 'Hoek' or 'heterodox'. In a previous draft, I formulated $N1L_H$ as follows: 'Bodies travel with uniform velocities, unless and to the extent that they are compelled to change their state of motion by an impressed force.' This has the merit of being simpler than the formulation above, but the demerit of inviting a reading in which the conjunction is split into two—but where then the second conjunct ('Bodies travel with uniform velocities to the extent that they are compelled to change their state of motion by an impressed force.') is garbled. As such, I have plumped for the less elegant formulation above.)

In addition to the textual evidence, Hoek suggests that there are a number of conceptual advantages which speak in favour of $N1L_H$ over $N1L_O$. In particular, $N1L_H$ affords a solution to two long-standing issues in the interpretation of Newton's laws, which Hoek calls the 'independence problem' ('Since N1L is a special case of N2L, why state it as a separate law?'), and the 'triviality problem' ('Since there are no force-free bodies, N1L is trivially true.'). Hoek is completely correct to state that $N1L_H$ avoids both of these issues whereas $N1L_O$ does not: clearly, on his reading, N1L is not a special case of N2L (hence dispatching the independence problem); moreover, $N1L_H$ does not quantify over force-free bodies (hence dispatching the triviality problem).

All this being said, there remain a few aspects of Hoek's approach to N1L which are worth teasing out in more detail. First: one might ask how Hoek's approach bears on more orthodox approaches to understanding N1L as (e.g.) offering an operational definition of/prescription for the identification of force-free bodies: although Hoek's discussion might suggest that these approaches can now be discarded, I'll argue here that this isn't the case: even reading Newton's first law as $N1L_H$ rather than $N1L_O$,

[1] This is my formulation of $N1L_H$, but Hoek has confirmed in personal correspondence that he agrees with it.

one still requires an independent characterisation of inertial motion. Second: it's worth drawing attention to certain extra dynamical assumptions when engaging with Hoek on Newton's laws. And third: it is also worth reconsidering the significance of Newton's third law on Hoek's approach; this I will do both with reference to Torretti (1983) and with reference to recent work on reformulations of Newtonian gravity by Saunders (2013).

2 Defining inertial motion

One of the challenges for the orthodox reading of Newton's laws, and in particular N1L, regards the definition of force-free motion—in the absence of such a definition, $N1L_O$ would seem to be radically under-specified.

In the literature, there is a range of different possible ways of cashing out what it is for a body to be force-free—these have been canvassed recently in e.g. (Read, 2023, ch. 1). To name here a few such approaches:[2] one could for example take a 'geometrical' approach to the definition of force-freeness, according to which a particle is force-free just in case its trajectory is straight according to the affine connection of Galilean spacetime (Friedman, 1983).[3] Alternatively, one could take a more empiricist/operationalist line, according to which e.g.:

1. Force-free bodies are those sufficiently far removed from all other matter in the universe (see e.g. (Brown, 2005)).[4]

2. Force-free bodies are identified via a generalised Humean strategy (see e.g. (Huggett, 2006)).

3. Force-free bodies are to be identified as those bodies which move on uniform trajectories in those frames of reference in which Newton's third law (henceforth

[2] Hoek discusses such approaches and others at (Hoek, 2023, pp. 72–3).

[3] On this approach, Hoek writes the following:

> The geometric approach rests on the observation that the weak First Law implies the existence of infinite, straight spacetime trajectories through any point and in any direction (Earman and Friedman 1973; Anderson 1990; Brown 2006). By treating this as a *consequence* rather than a *presupposition* of the First Law, the law is transformed into a load-bearing principle in the theory of spacetime structure. (Hoek, 2023, p. 73)

[4] This is somewhat akin to the claim made by Poincaré that "the variation of the acceleration of a body depends only on the position of the body and of neighbouring bodies" (Poincaré, 2017, p. 93). This could be read as implying that a non-accelerating body is one which has no neighbouring bodies—i.e., is far removed from other bodies. My thanks to Daniel Hoek for drawing this connection to my attention.

N3L)—that for every force exerted by one body on another, there is an equal and opposite force exerted by the second body on the first—is satisfied (see e.g. (Torretti, 1983)[5]).

I won't go into these options any more right now, although I will discuss (3) further in §4. Suffice it to say that all of these approaches are related to attempts to reconstrue $N1L_O$ as a disguised operational definition of/prescription for the identification of the inertial (i.e., $N1L_O$-satisfying) frames.

Now, on the face of it, moving from $N1L_O$ to $N1L_H$ might invite the thought that the need to define force-free motion is obviated, since the latter does not refer to/quantify over force-free bodies as does the former. To say this would, however, be too fast—for although it is indeed true that $N1L_H$ doesn't make reference to force-free bodies in the same way as does $N1L_O$, it still appeals to the concept of inertial motion insofar as it makes reference to 'uniform velocities'—and so one might still desire an independent characterisation of what said inertial motion amounts to. Indeed, this is especially evident given the counterfactual formulation of $N1L_H$ above, on which of course we still need to understand just what *are* the trajectories that bodies "would have followed if their velocities were to have been uniform". Note moreover that N2L is unable to do this, for that law gives a handle on what the divergences from uniform motion are, but it does not allow one to understand what those divergences are *from*, for which one still requires such an independent standard of force-free motion. Thus: even though the above options for notions of inertial motions were articulated originally out of discussions to do with the content of $N1L_O$, there is therefore a good case to be made that these definitions remain important even given Hoek's preferred formulation of Newtonian mechanics.[6]

3 Velocity-dependent functions

The next point which I want to make has to do with rendering explicit some assumptions which one might (at least initially) think are made by Hoek (2023) in giving various

[5]Here, Torretti follows the lead of Stein (1974).

[6]At one point towards the end of his article, Hoek writes: "Far from being a trivial truth, the strong First Law [i.e., $N1L_H$] is arguably false in light of General Relativity: in curved spacetimes, particles change direction without the action of any impressed force" (Hoek, 2023, p. 74). It is not obvious that this is so, since gravitating but otherwise force-free bodies in general relativity *follow geodesics*—they do not "change direction". So, as far as I can tell, $N1L_H$ can hold just as well in general relativity as in Newtonian gravitation. This, indeed, all carries over to Newton–Cartan theory (see (Malament, 2012, ch. 4)), which is good news for Hoek, considering that Newton–Cartan spacetime is often regarded as being the "most appropriate spacetime setting" for Newtonian gravitation (see Knox (2014)).

different statements of his preferred form of N1L. Hoek's initial statement of N1L_H—reproduced above—can be schematised as follows:[7]

$$\Delta(\text{uniform motion}) \Rightarrow \text{force}. \tag{1}$$

Now, if one reads 'uniform motion' as 'velocity', then one can rewrite the above as:

$$\Delta(v) \Rightarrow \text{force}. \tag{2}$$

Now, on the assumption that masses (and forces) aren't functions of absolute velocities (i.e. $m \neq m(v)$), one can infer from this that:

$$\Delta(mv) \Rightarrow \text{force}, \tag{3}$$

which states that any change in the momentum of a body is due to a force. At first blush, one might read Hoek (2023) as equivocating between the first and last of these statements of N1L_H: he writes explicitly that his version of N1L "says that every change in a body's *momentum* is due to impressed forces" (Hoek, 2023, p. 62). But recall now that it has long been appreciated that Newton's derivation of Corollary V (seemingly) goes through only if he assumes that masses and forces are not functions of absolute velocities—see (Brown, 1993) and (Barbour, 1988); the same assumption is required here in order to move from the first to the third of these formulations.[8]

At this point, however, it is very important to be clear that there is a more charitable reading of Hoek available here. Newton himself defined 'quantity of motion' in terms of (speaking anachronistically) *momentum*—in which case one should have *begun* with (3) above, and (rather) the velocity-independence of masses is required to move to (2), rather than the other way around! On this reading, the velocity-independence of mass is not an illicit presupposition of Hoek's analysis. I commend this reading of Hoek

[7] This schematisation of N1L_H might be taken to imply that it is the contrapositive of N1L_O, and hence logically equivalent to it. That would not be a correct reading, for as Hoek writes:

> But [N1L_O] and [N1L_H] are not contrapositives: [N1L_O] fails to entail [N1L_H], because [N1L_O] leaves open the possibility of an unforced change in the state of motion of a body subject to impressed forces (it could simultaneously be subject to influences other than impressed forces, or undergo random changes, for no reason at all). (Hoek, 2023, p. 62)

As such, one should not read '$\Rightarrow$' as a material implication. On N1L_H, the change in motion must be *due to* forces, rather than just *accompanied by* forces; it's of course very well known that these causal connections are not fully captured if one uses the material implication—see e.g. Henderson (1954). (I'm grateful to Caspar Jacobs and Josef Vacha for discussions here.)

[8] More on this assumption in the next section.

(2023)—nevertheless, I think it's helpful to think through how dynamical assumptions about (say) velocity-independence enter the picture here.

To close this section and explore this point a little further, note that one can understand N2L as the converse of $N1L_H$ (but *not* as the converse of $N1L_O$), so reading:

$$\text{force} \Rightarrow \Delta(\text{uniform motion}). \tag{4}$$

Again, if one reads 'uniform motion' as 'velocity', then one can rewrite the above as:

$$\text{force} \Rightarrow \Delta(\text{velocity}). \tag{5}$$

On the assumption that $m \neq m(v)$, the above then implies that

$$\text{force} \Rightarrow \Delta(mv). \tag{6}$$

So to derive the 'change of momentum' version of N2L, one again needs to same dynamical assumptions as discussed above—unless, again with Newton, one begins by identifying 'quantity of motion' with momentum, in which case the derivation proceeds the other way around.

4 The role of the third law

The final point which I want to make in this note has to do with the role of N3L on Hoek's preferred understanding of the content of Newtonian mechanics. One question which has sometimes been asked in the foundations of Newtonian mechanics is whether N3L is presupposed by N1L—that is, whether frames in which N1L is satisfied are frames in which N3L is satisfied (for the classic discussion of this question, see (Torretti, 1983); the issue is also discussed in (Read, 2023, ch. 1)).[9] This question remains meaningful when one moves from understanding N1L as $N1L_H$ rather than as $N1L_O$—to see this, consider some frame of reference in which $N1L_H$ holds, then boost to some frame of reference accelerating uniformly with respect to the original frame. In that new frame, there will be changes of motions of bodies not caused by any physical force (although, of course, those changes will still be correlated with fictitious

[9] "We saw on pages 19–20 that in the Newtonian theory the Third Law is quite essential for distinguishing the family of inertial frames from any family of frames travelling past it with the same constant acceleration" (Torretti, 1983, p. 51). In this passage, Torretti goes on to acknowledge that "the Third Law is not required for singling out the inertial frames in Einstein's theory", thereby anticipating the same point made in (Read, 2023, ch. 1).

forces)—the situation here is analogous to the failure of $N1L_O$ when one moves to an arbitrarily-moving frame of reference.[10]

As a result of this, questions regarding the extent to which N3L affords a means of identifying operationally the frames of reference in which N1L obtains remain pertinent in the context of $N1L_H$ as well as the context of $N1L_O$. This, of course, is option (3) in §2. Moreover, even if one chooses to define force-free motions in some other way—perhaps via the geometrical strategy of Friedman (1983) as also mentioned in §2—it remains perfectly legitimate (and interesting) to ask about the extent to which the N3L-satisfying frames are 'adapted' to this spacetime structure.

For the sake of clarity in the remainder of this section, let me switch back now to understanding N1L as $N1L_O$. In (Read, 2023, ch. 1), I presented some simple problem cases for this 'operational identification via N3L' strategy, and I stick by those. There's another straightforward way to see this point, though, which is to note that Saunders' 'vector relationalism'—his reformulation of Newtonian gravity in terms of relational quantities, which has the Maxwell group as its dynamical symmetries (see Saunders (2013))—commits explicitly to N3L, yet has no standard of inertial (non-rotational) motion. Hence, the inference from N3L to $N1L_O$ must fail.[11]

Here is a further way to see the point. As already discussed, Barbour (1988) and Brown (1993) have pointed out that the assumption of the velocity-independence of forces and masses is (seemingly) needed for Newton's derivation of the Galilean invariance of his laws (i.e., his derivation of Corollary V) to go through. What happens, though, if one makes the weaker assumption that forces be constrained to satisfy N3L, while still allowing that they (and the inertial masses) might be functions of absolute velocities? If *no* restrictions are placed on forces and masses with respect to their velocity-dependence, then the symmetries of the laws will in general (i.e., save in special cases) be those of the Newton group (see Pooley (2013) for an explicit statement of the Newton group), because boosts will generally not preserve the laws; on the other hand, velocity-independence of forces and masses underwrites Galilean invariance. So what happens in this intermediate case?

[10] Of course, if one is already taking motions to be with respect to (say) Newtonian absolute space, then *ipso facto* $N1L_H$ obtains in all frames of reference. Evidently, this is not the reading of $N1L_H$ which I have in mind here.

[11] How can this be so, given the stress which both Stein (1974) and Torretti (1983) place upon the significance of N3L as restricting to the inertial frames (special problem cases such as those discussed in (Barbour, 1988, pp. 577–8) and (Read, 2023, ch. 1) notwithstanding)? The point is that Saunders (2013) is concerned only with relational quantities to begin with; for that reason, global non-rotational accelerations can remain symmetries of his theory, despite the imposition of N3L. Nevertheless, N3L does underwrite a standard of absolute rotational acceleration in Saunders' theory, as I discuss further below.

The situation has already been considered in a very illuminating passage from (Barbour, 1988, pp. 577–8): as Barbour writes, even assuming N3L, there is "no reason whatever why the *strength* of interaction (the impulses) between two bodies (to consider the simplest case) should be the same when their centre of mass moves through absolute space with a uniform velocity as when it is at rest"—this already serves to temper the thought that N3L alone can take us to the inertial frames. So, if we have some force function for the ith particle of the form

$$\sum_j F_{ij} = m_i a_i, \tag{7}$$

then scaling all the forces after a boost will not be a symmetry of the laws—assuming that we don't also scale the inertial masses. (And even in that case, it will only be a symmetry if we assume that all forces F_{ij} scale *in the same way*.) So, insisting on N3L but assuming that *only* forces are functions of absolute velocities (and in particular that masses are not) would again seem to make it the case that boosts aren't symmetries and that the symmetry group of the laws is therefore the Newton group.

But if we make it the case that masses scale with absolute velocities in the same way as forces, then all of the scale factors in the above equation will cancel and we'll have it the case that symmetries are still Galilean—this is just one of the special cases of the kind mentioned above, in which one secures Galilean invariance of the laws without necessarily assuming something so strong as velocity-independence of the forces and masses.[12] In other words, what we see here is that merely assuming N3L doesn't by itself secure Galilean invariance of the laws. But N3L plus the additional assumption that masses scale just as do forces with absolute velocities (and, indeed, that forces all scale *in the same way* with absolute velocities) does underwrite this.[13]

[12]Note that this actually tempers, to a very mild degree, the point made by Barbour (1988) and Brown (1993).

[13]These transformations would seem to be a hidden symmetry of Newtonian gravitation (for recent philosophical discussions of hidden symmetries in physics more generally, see Bielińska and Jacobs (2024); Read (2024)). If there were some way of detecting these absolute masses directly, then of course the Galilean symmetry might thereby be broken (my thanks to Bhanu Narra for raising this point). It's also straightforward to show that these 'scale' transformations are not variational symmetries, despite being dynamical symmetries—in which case, one would not expect there to be associated conserved quantities by Noether's first theorem. That said, it's also somewhat unclear whether Noether's first theorem is applicable here in any case, given that the theorem deals with transformations of the dependent and independent variables, rather than with transformations of parameters such as masses. (My thanks to Harvey Brown for discussions regarding these 'scale' transformations and Noether's theorem; for further recent philosophical work on Noether's theorems, see Read and Teh (2022).) In addition: one wonders what is the right thing to say about the invariant structure associated with these 'scale' transformations—but

While we're here, there's one final puzzle regarding such an approach to Newtonian gravity which I'd like to take the opportunity to shore up; this comes back again to the work of Saunders (2013). An earlier approach to reformulating Newtonian gravity in terms of relational quantities is due to Hood (1970).[14] But since (a) Saunders (2013) takes N3L to have been essential to his derivation, yet (b) Hood (1970) does *not* avail himself of N3L, there's a question regarding how Hood is also able to derive a theory with the Maxwell group as its dynamical symmetries. Thankfully, it's straightforward to identify the solution here, which is just this: Hood's theory is in fact invariant under the full *Leibniz* group of transformations; it is only when one assumes N3L (in the form of the antisymmetry of the force function between any two particles) that one restricts this to the Maxwell group of transformations—because N3L will (save in very special cases) distinguish between states which differ only in their overall rotation.[15]

5 Close

The alternative understanding of N1L proposed by Hoek (2023) is compelling, both exegetically and philosophically. That said, in this note I have tried to demonstrate that embracing Hoek's approach to Newton's laws does not—as one might initially worry—render null-and-void much of the extant philosophical work on the content of Newton's laws, for even granting Hoek's understanding of N1L, (i) one arguably still requires a way of cashing out inertial motion/force-freeness (§2), (ii) one still has to be sensitive to extra dynamical assumptions which might in play (§3), and (iii) questions regarding the relationship between N3L and the other laws of motion, and regarding how N3L can be put to operational/empirical work—questions which occupied giants such as Torretti (1983)—remain of philosophical interest (§4).

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I won't explore this question further here. And finally: if the world were partitioned into classes of bodies only interacting with other bodies within their class, then of course one could have one such 'scale' transformation per class (thanks to Josef Vacha for this last point).

[14] Another early discussion of these issues is by Rosen (1972); both approaches are mentioned—but not discussed—by Earman (1989).

[15] For reasons discussed in (Read, 2023, ch. 1).

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