

Dutch Strategies, Accuracy-Dominance, Lockean Beliefs, and their Plannings

Patrick Neal Rooyackers

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1 Introduction

2 Suppose we consider an agent with both numerical credences and all-or-nothing
3 beliefs. This agent might also have a plan about how she is going to update
4 her beliefs upon receiving new evidence. What rational requirements on such
5 a plan can be justified from an epistemic value point of view? Plan Almost
6 Lockean Revision is the claim that it is rationally required that one’s planned
7 beliefs are exactly one’s sufficiently high conditional credences. We start by
8 reviewing arguments available for Plan Lockean Revision in the current liter-
9 ature, ultimately concluding that they are non-optimal. We provide a better
10 argument to the effect that the belief updating rule that is expected to be the
11 best according to one’s current credences is exactly Plan Almost Lockean Re-
12 vision, that is, we prove a Qualitative Greaves-Wallace Theorem. Furthermore,
13 building on the work of (Rothschild, 2021), we investigate the dutchbookability
14 of Lockean betting behavior for all-or-nothing beliefs and their plannings, ul-
15 timately proving a qualitative version of the dutch strategy theorem which leads
16 to the development of novel dutch-strategy/accuracy-dominance arguments for
17 Lockean norms on belief/belief-planning pairs.

2 The Almost Lockean Thesis

19 Suppose we consider an agent with both numerical credences and all-or-nothing
20 beliefs in finitely-many propositions. We can thus represent the agent’s doxastic
21 state (at time t) as a pair (B_t, c_t) ¹, consisting of her set of beliefs and her cre-
22 dences. Question: How must one’s credences and beliefs be related in order for
23 them to rationally cohere? Kyburg (1974) and Foley (1992) suggested the fol-
24 lowing answer: one’s beliefs must be exactly one’s high credences. This is called
25 the Almost Lockean Thesis². So, according to the Almost Lockean Thesis, if

¹Henceforth, we drop the time indices. All unindexed pairs are considered synchronic.

²Strictly speaking, our Almost Lockean Thesis differs from (Foley, 1992)’s Lockean Thesis (with threshold t), that $B = \{p : c(p) \geq t\}$, by saying nothing about the belief/disbelief status of propositions at the threshold. Hence the “Almost” qualifier, which we adopt from (Rothschild, 2021).

1 you believe that Amy is coming to your party tonight, then you should have
2 high enough confidence that Amy is coming to your party tonight and, if you
3 have high enough confidence that Amy is coming to your party tonight, then
4 you should believe that Amy is coming to your party tonight. More precisely,

5
6 Almost Lockean Thesis (with threshold $t > \frac{1}{2}$): (B, c) is such that:

7 (1): If $p \in B$, then $c(p) \geq t$.

8 (2): If $p \notin B$, then $c(p) \leq t$.

9
10 But why think it a requirement of rationality? Dorst (2019), building off the
11 insights of Easwaran (2015) and Hempel (1962), develops an expected-accuracy
12 argument in its favor. Before detailing this argument, we first have to under-
13 stand how we are going to measure the accuracy of our all-or-nothing beliefs.

14 **3 Accuracy for Beliefs and Dorst’s Expected- 15 Accuracy Argument**

16 We start with a very basic and plausible idea. It seems good, from an epistemic
17 point of view, to believe true things and bad, from an epistemic point of view,
18 to believe false things. Furthermore, it seems good, from an epistemic point
19 of view, to disbelieve false things and bad, from an epistemic point of view, to
20 disbelieve true things. We can formalize this intuition numerically by assigning
21 a (non-negative) real number T as the value of believing true propositions and
22 disbelieving false propositions; and, assigning the (non-positive) real number F
23 as the disvalue of believing false propositions and disbelieving true propositions
24 at different possible worlds.³ We call this property Extensionality. Precisely,

25
26 Def⁴: accuracy-measure A is said to be Extensional iff

$$A(p \in B, w) = \begin{cases} T & \text{if } p \text{ is true at } w. \\ F & \text{if } p \text{ is false at } w. \end{cases} \quad (1)$$

27
28 and

$$A(p \notin B, w) = \begin{cases} F & \text{if } p \text{ is true at } w. \\ T & \text{if } p \text{ is false at } w. \end{cases} \quad (2)$$

29
30 Great, we now have a way to measure the accuracy of your all-or-nothing beliefs,
31 or lack thereof, in particular propositions. But, how are we going to measure the
32 accuracy of your entire belief-set? Answer: we measure it additively over propo-
33 sitions. That is, we look at the value of your either believing or disbelieving

³The possible worlds, the w 's, are going to be understood as the logically possible disjunctive normal forms of our agent's finitely-many propositions that they have doxastic attitudes towards.

⁴For the sake of simplicity, we are going to ignore including the attitude of suspended judgement in this paper. So, when I write " $p \notin B$ ", I mean that our agent disbelieves p . See (Dorst, 2019) for some discussion on accuracy and suspended judgement.

1 different propositions and sum those values over all such propositions. Precisely,

2
3 Def: accuracy-measure A is said to be Additive iff the accuracy of belief-set
4 B at world w is such that $A(B, w) = \sum_{p \in B} A(p \in B, w) + \sum_{p \notin B} A(p \notin B, w)$.

5
6 Finally, our last legitimacy condition on qualitative accuracy-measures that we'll
7 need is as follows:

8
9 Def: accuracy-measure A is said to be Variable Conservative iff $T > 0 > F$
10 and $|F| > T$.

11
12 This is the first and only condition that we impose that actually constrains
13 how T and F are related. The first part of Variable Conservativeness says that
14 it is *strictly* good to believe (disbelieve) true (false) things and *strictly* bad to
15 believe (disbelieve) false (true) things. The second part says that the disvalue
16 of believing (disbelieving) false (true) things is greater than the value of believ-
17 ing (disbelieving) true (false) things. One motivation for why we might want
18 the second part of Variable Conservativeness is because it helps us rule out the
19 rational permissibility of believing both p and $\neg p$. Because, after all, if $|F| \leq T$,
20 then $A(p \in B, w) + A(\neg p \in B, w) \geq 0$, so why not then believe both p and
21 $\neg p$. Further discussion of this point can be found in (Steinberger, 2019) and
22 (Hewson, 2020).

23
24 Great, having finally finished our discussion of how we are going to legitimately
25 measure the accuracy of one's all-or-nothing beliefs, without further ado, here
26 is the key technical result for Dorst's expected-accuracy argument:

27
28 **Easwaran-Dorst Theorem:** Let c be a probabilistic credence function. Let
29 qualitative accuracy-measure A satisfy Additivity, Extensionality, and Variable
30 Conservativeness. Then, belief set B maximizes expected accuracy with re-
31 spect to c iff the pair (B, c) satisfies the Almost Lockean Thesis with threshold
32 $t = \frac{-F}{T-F}$.

33
34 In detail, assuming that the relevant credence function is probabilistic, Dorst's
35 expected-accuracy argument for the Almost Lockean Thesis with threshold t is:

- 36
37 (1): Qualitative Veritism⁵: only qualitative accuracy is intrinsically epistem-
38 ically valuable for all-or-nothing beliefs.
39 (2): Legitimate qualitative accuracy-measures are Extensional, Additive, and

⁵Of course, a more neutral term in place of "accuracy-measure" might be epistemic-utility function or scoring rule for which (Dorst, 2019)'s argument for the Almost Lockean Thesis might avoid commitment to Qualitative Veritism and proceed under something like "whatever is of epistemic value, it is only legitimately measured by an Additive, Extensional, and Variable Conservative function." Nevertheless, in the Jamesian spirit of giving the "first and great[est]" importance to "Believe truth! Shun error!" (James, 1897), I remain sympathetic to Veritism and continue to use the accuracy terminology.

1 Variable Conservative with threshold t .⁶
2 (3): Rational (B, c) 's are such that B maximizes expected accuracy with respect
3 to c .
4 (4): **Easwaran-Dorst Theorem.**
5 (5): Therefore, the Almost Lockean Thesis with threshold t is rationally re-
6 quired.
7
8 Now, Dorst goes on to show that Extensionality can be dropped from his theo-
9 rem entirely. Doing so permits contextual and proposition-wise dependencies for
10 T and F , which results in an Almost Lockean Thesis with variable/contextual
11 thresholds.⁷ Furthermore, he shows that even Additivity can be significantly
12 weakened while still retaining the Almost Lockean Thesis. Here, we extend mat-
13 ters in a different direction: by investigating rational norms for all-or-nothing
14 belief updating. In more detail, how should an agent rationally change their
15 all-or-nothing beliefs upon coming to possess new evidence E ? Let us look at
16 this.

17 4 Plan Almost Lockean Revision

18 The literature of proposed norms for rational belief updating is extensive⁸, but
19 we will be focused on just one proposal. Shear and Fitelson in their excellent
20 “Two Approaches to Belief Revision” suggest the following dynamics for updat-
21 ing all-or-nothing belief upon coming to possess new evidence E . Namely, they
22 propose the following as a candidate for a requirement of epistemic rationality
23 on belief updating:

24
25 Actual Almost Lockean Revision (with threshold $t > \frac{1}{2}$): For (B_{t_1}, c_{t_1}) and
26 (B_{t_2}, c_{t_2}) , if the agent actually receives total evidence E between times t_1 and
27 t_2 and $c_{t_1}(E) > 0$, then:

- 28 (1): If $p \in B_{t_2}$, then $c_{t_1}(p|E) \geq t$.
29 (2): If $p \notin B_{t_2}$, then $c_{t_1}(p|E) \leq t$.

30
31 In words: one’s updated beliefs must be exactly one’s high conditional cre-
32 dences. In the current literature, the best argument for Actual Almost Lockean
33 Revision is the one provided by Shear and Fitelson:

34
35 “Thus, Lockean revision, as we have explored it, is entailed by the more general

⁶An accuracy-measure is said to have threshold t when $t = \frac{-F}{T-F}$.

⁷This is a very important development. Perhaps believing some truths is more epistemically valuable than believing other truths. Perhaps it is better to believe truths with higher “informational content” as advocated in (Levi, 1967) and more recently by (Dorst & Manderlker, 2021) and (Skipper, 2023). Perhaps such values depend upon its relevance to a question under discussion (Levi, 1967) and (Yalcin, 2016). Even considerations of verisimilitude can be accommodated within this framework.

⁸To name just a few: (Alchourrón, Gärdenfors, & Makinson, 1985) (Lin & Kelly, 2012) (Leitgeb, 2017) (Shear & Fitelson, 2018) (Kelly & Lin, 2021) (Mierzewski, 2022).

1 norm requiring that agents have belief sets that maximize expected epistemic
 2 value at any given time. Assuming conditionalization as the rational procedure
 3 for credal updating, the norm entails that Lockean revision is the unique proce-
 4 dure that will guarantee that agents maximize overall expected epistemic value.”

5
 6 In premise form, we have:

- 7 (1): Actual Conditionalization⁹.
 8 (2): Almost Lockean Thesis with threshold t .
 9 (3): Therefore, Actual Almost Lockean Revision with threshold t .

10
 11 Can we do better than this argument? Clearly, this argument appeals to Ac-
 12 tual Conditionalization in the first premise, so, we might ask, is this argument
 13 epistemic-value-theoretic through-and-through? In other words, do there exist
 14 good epistemic value arguments for Actual Conditionalization? The prevailing
 15 answer in the literature seems to be “no” (Pettigrew, 2016, Chapter 15)¹⁰.

16
 17 In order to do better, we need the notion of a belief plan $\beta : \epsilon \rightarrow \{\text{Belief Sets}\}$
 18 which is a function from an evidential partition to the set of belief-sets. The
 19 notion of a belief plan can be interpreted in, at least, three ways:

- 20 (a): The dispositional interpretation: If an agent were to receive total evidence
 21 E , then the agent will adopt beliefs $\beta(E)$.
 22 (b): The planning interpretation: The agent plans to adopt beliefs $\beta(E)$ upon
 23 receiving total evidence E .
 24 (c): The suppositional interpretation: The agent’s suppositional/conditional
 25 beliefs are $\beta(E)$ upon supposing exactly that E .

26
 27 We say a little about each interpretation. The dispositional interpretation is
 28 assumedly deterministic along the lines discussed by (Pettigrew, 2020). There
 29 is no chance that you might adopt another belief-set different from $\beta(E)$. Plan-
 30 ning is to be understood as some kind of mental state involving intentionality,
 31 that is, it is about what you intend to do (or how you intend to be) upon
 32 receiving new evidence E . The suppositional interpretation is indicative (as op-
 33 posed to subjunctive) and is to be understood along the lines discussed by (Eva,

⁹Actual Conditionalization says that it is a requirement of epistemic rationality that if you receive exactly evidence E between times t_1 and t_2 , then your credal pair (c_{t_1}, c_{t_2}) is such that if $c_{t_1}(E) > 0$, then $c_{t_2} = c_{t_1}(\cdot|E)$.

¹⁰Although, see (Gallow, 2019) for an attempted accuracy-theoretic argument for Actual Conditionalization from value-change. (For a better argument see (Rooyackers, ms).) I thank ??? for suggesting this paper to me. We note here that an entirely analogous change-of-value argument can be developed for Actual Almost Lockean Revision, namely: given the Easwaran-Dorst conditions, the belief set which maximizes $\sum_{w \in E} A(B, w)c(w)$ satisfies the Almost Lockean Thesis with respect to $c(\cdot|E)$ (if defined). Let $B_{c(\cdot|E)}$ satisfy the Almost Lockean Thesis with respect to $c(\cdot|E)$.

Proof: The Easwaran-Dorst Theorem gives us that $\text{Exp}[A(B_{c(\cdot|E)})|c(\cdot|E)]$
 $= \sum_{w \in W} A(B_{c(\cdot|E)}, w)c(w|E)$
 $= \sum_{w \in E} A(B_{c(\cdot|E)}, w)c(w|E)$
 $= \frac{1}{c(E)} \sum_{w \in E} A(B_{c(\cdot|E)}, w)c(w)$ is maximal over all belief sets B and scaling by $\frac{1}{c(E)}$ (a constant not depending upon B) does not affect the expected accuracy ordering. \diamond .

1 Shear, & Fitelson, 2022), minus the part about E being interpreted as evidence
2 received. After all, supposing that E does not amount to possessing evidence
3 that E ; we can and do engage in suppositional reasoning without treating the
4 supposition as evidence that we came to possess.

5
6 It is important to note that, while these interpretations are distinct, they are not
7 necessarily competing. Plans can (and often do) go awry and what one plans to
8 believe upon receiving evidence E need not be the same as their beliefs under
9 the supposition that E . Except where explicitly remarked, we take the view
10 that any interpretation is fitting. Great, now, consider the following candidate
11 requirement of epistemic rationality:

12
13 Plan Almost Lockean Revision (with threshold $t > \frac{1}{2}$): the pair (β, c) is such
14 that, for every $E \in \epsilon$ with $c(E) > 0$:

15 (1): If $p \in \beta(E)$, then $c(p|E) \geq t$.

16 (2): If $p \notin \beta(E)$, then $c(p|E) \leq t$.

17
18 In words: one's planned beliefs must be one's high conditional credences on
19 the relevant evidence. But why think it a requirement of rationality? Well, it
20 turns out that we can develop an expected-accuracy argument in its favor.¹¹

21 5 Accuracy for Belief-plannings and the Quali- 22 tative Greaves-Wallace Theorem

23 In order to develop our expected-accuracy argument for Plan Almost Lockean
24 Revision, we have to first say how we are going to measure the accuracy of our
25 belief-plannings. Here is how we do that:

26
27 Def: $A(\beta, w) = A(\beta_{E_w}, w)$.

28
29 In words, the accuracy of your belief-plan at world w is the accuracy of the
30 planned belief-set that you would adopt at world w . In this way, we can reduce
31 the problem of measuring the accuracy of belief-plannings to that of measuring
32 the accuracy of belief-sets. Great, now that we know how to legitimately mea-
33 sure the accuracy of one's all-or-nothing belief-plannings, without further ado,
34 here is the key technical result for our expected-accuracy argument:

35
36 **Qualitative Greaves-Wallace Theorem**¹²: Let c be a probabilistic credence
37 function. Suppose accuracy measure A is Additive, Extensional, and Variable

¹¹In the appendix, we develop another (Shear & Fitelson, 2018)-style argument for Plan Almost Lockean Revision.

¹²In the statement of this theorem, and throughout the rest of this paper, we are going to ignore the concerns raised in (Schoenfeld, 2017), as similar concerns arise for our qualitative cases of interest. The relevant adjustments to all the results in this paper are exactly the expected ones.

1 Conservative. Then, (β, c) satisfies Plan Almost Lockean Revision iff β maxi-
 2 mizes expected accuracy with respect to c .

3 Proof:

4 Maximized Expected Accuracy \Rightarrow Plan Almost Lockean Revision:

5 Assume belief plan β maximizes expected accuracy. Let $p \in \beta_E$. Let $\beta' = \beta$
 6 everywhere on the evidential partition except on E , where $\beta'_E = \beta_E - p$. Thus,

7 $EA(\beta) - EA(\beta')$

$$8 = \sum_{w \in E} c(w)[A(\beta_E, w) - A(\beta'_E, w)] \geq 0$$

$$9 = \sum_{w \in E} c(w)[A(p \in \beta_E, w)]$$

$$10 = \sum_{w \in E \cap p} c(w)T + \sum_{w \in E \cap \neg p} c(w)F$$

$$11 = c(E \cap p)T + c(E \cap \neg p)F \geq 0$$

$$12 \Rightarrow c(p|E)T + c(\neg p|E)F \geq 0 \text{ by dividing both sides by } c(E) > 0.$$

$$13 = c(p|E)T + (1 - c(p|E))F \geq 0$$

$$14 \Rightarrow c(p|E) \geq \frac{-F}{T-F}.$$

15 An analogous argument gives: if $p \notin \beta_E$, then $c(p|E) \leq \frac{-F}{T-F}$.

16
 17 Plan Almost Lockean Revision \Rightarrow Maximized Expected Accuracy:

18 Assume Plan Almost Lockean Revision. So, we have:

19 1.) If $p \in \beta_E$, then $c(p|E) \geq \frac{-F}{T-F}$.

20 2.) If $p \notin \beta_E$, then $c(p|E) \leq \frac{-F}{T-F}$.

21 Now, assume (for contra.) that β doesn't maximize expected accuracy.

22 Thus, there exists a β' st. $EA(\beta') > EA(\beta)$ and β' maximizes expected accuracy.

23 $\Rightarrow \exists p$ for some E st.

24 1*) $[p \in \beta'_E \text{ and } p \notin \beta_E]$ or

25 2*) $[p \notin \beta'_E \text{ and } p \in \beta_E]$.

26 Case 1*): We have that $p \in \beta'_E$ and β' maximizes expected accuracy \Rightarrow (by the
 27 first part of the proof) $c(p|E) \geq \frac{-F}{T-F}$. Also, by condition 2.) and $p \notin \beta_E$, we

28 have that $c(p|E) \leq \frac{-F}{T-F}$. Thus, $c(p|E) = \frac{-F}{T-F}$.

29 Case 2*): Similarly, we find $c(p|E) = \frac{-F}{T-F}$.

30 But $c(p|E) = \frac{-F}{T-F}$ for all p that β' and β disagree about at some $E \Rightarrow$

31 $EA(\beta') = EA(\beta)$, a contradiction, because

32 $EA(\beta') - EA(\beta)$

$$33 = \sum_{E \in \epsilon} \sum_{w \in E} c(w)[A(\beta'_E - \beta_E, w) - A(\beta_E - \beta'_E, w)]$$

$$34 = \sum_{E \in \epsilon} [\sum_{w \in E} c(w)A(\beta'_E - \beta_E, w) - \sum_{w \in E} c(w)A(\beta_E - \beta'_E, w)]$$

$$35 = 0 \text{ as } \sum_{w \in E} c(w)A(\beta'_E - \beta_E, w) = 0 \text{ and } \sum_{w \in E} c(w)A(\beta_E - \beta'_E, w) = 0 \text{ for every}$$

36 E by backtracking the first part of the proof (for all the relevant p) starting

37 with $c(p|E) = \frac{-F}{T-F}$.

38 \diamond^{13} .

39
 40 With this technical result in hand, and assuming that the relevant credence

¹³Perhaps the Lockean threshold might depend upon the evidential context, that is, upon which evidence is received. This can be done by having the value of believing truths and epistemic disvalue of believing falsehoods depend on the evidential context. In this case, it is easy enough to show that an analogous theorem to the one above holds by replacing "T" and "F" with "T_E" and "F_E" respectively where appropriate.

1 function is probabilistic, we can develop our expected-accuracy argument for
2 Plan Almost Lockean Revision:

3
4 (1): Qualitative Veritism: only qualitative accuracy is intrinsically epistemi-
5 cally valuable for all-or-nothing belief-plannings.

6 (2): Legitimate qualitative accuracy-measures are Extensional, Additive, and
7 Variable Conservative with threshold t .

8 (3): Rational (β, c) 's are such that β maximizes expected accuracy with respect
9 to c .

10 (4): **Qualitative Greaves-Wallace Theorem.**

11 (5): Therefore, Plan Almost Lockean Revision with threshold t is rationally
12 required.

13

14 6 Lockean Betting for All-or-Nothing Beliefs

15 There is a long and venerable tradition in statistics, decision theory, and philoso-
16 phy of connecting¹⁴ credences/previsions to betting behavior (Ramsey, 1926) (de
17 Finetti, 1937, 1974) (Lehman, 1955) (Kemeny, 1955) (Hajek, 2005) (Schervish,
18 Seidenfeld, & Kadane, 2009).¹⁵ Only recently have all-or-nothing beliefs been
19 connected to such betting behavior (Rothschild, 2021). We follow Rothschild
20 in describing this kind of Lockean betting.¹⁶ Consider an agent who has all-or-
21 nothing beliefs B , never mind if they also have credences. They might; they
22 might not. If $p \in B$, then the agent will (or ought) to buy a \$1 bet on p for
23 \$ t (or less). And, if $p \notin B$, then the agent will (or ought) to sell a \$1 bet on
24 p for \$ t (or more). Generalizing this to bets with non-negative stakes other
25 than \$1 is straightforward. If $p \in B$, then the agent will (or ought) to buy a
26 \$ $x \geq 0$ bet on p for \$ tx (or less). And, if $p \notin B$, then the agent will (or ought)
27 to sell a \$ $x \geq 0$ bet on p for \$ tx (or more). Now, with the notion of Lockean
28 betting in-hand, we can ask when some belief-set B is strongly dutchbookable
29 at threshold t , that is, when there exists a collection of bets that B is willing (or
30 ought) to buy/sell¹⁷ at threshold t and that, when taken together additively,
31 guarantee a net loss at every possible world. That is, when can a Lockean bet-
32 tor be bilked out of money? And, furthermore, how is strong dutchbookability
33 related to strong accuracy-dominance¹⁸ for Lockean bettors? These questions

¹⁴This connection being descriptive as in de Finetti's Radical Operationalism or normative as in (Christensen, 1996)'s "sanction as fair" view.

¹⁵To name just a few.

¹⁶We note that (de Finetti, 1981)'s worry about truthful credence elicitation due to strategic pricing in a Prevision Game also arises in the context of Lockean betting if we allow varying thresholds between agents. See (Seidenfeld, 2021) for a discussion of strategic pricing in a Prevision Game in the credal case and strategic concerns in a Forecasting Game.

¹⁷If the dutchbook involves both buying and selling, it is said to be a two-way dutchbook. If it just involves buying, then it is said to be a one-way dutchbook. We focus on two-way dutchbookability in this paper. The terminology is (Rothschild, 2021)'s.

¹⁸Belief-set B is said to be strongly accuracy-dominated with respect to legitimate accuracy-measure A iff there exists some belief-set B' such that $A(B, w) < A(B', w)$ for every possible

1 are answered by (Rothschild, 2021). We quickly rehearse these answers. But
2 first, a very important definition:

3
4 Def: B is said to be Almost Lockean Complete at threshold t iff there ex-
5 ists a probabilistic credence function c st. for every proposition p [if $p \in B$, then
6 $c(p) \geq t$ and if $p \notin B$, then $c(p) \leq t$].

7
8 The idea here is that such Almost Lockean Complete beliefs can be represented
9 as being Almost Lockean with respect to some probabilistic credences. It is
10 important to note that Almost Lockean Completeness, or the lack thereof, is a
11 property of one’s all-or-nothing beliefs. It has nothing to do with one’s confi-
12 dences. This is unlike the Almost Lockean Thesis, which explicitly relates one’s
13 all-or-nothing beliefs and their actual credences. Back to (Rothschild, 2021):

14
15 **Rothschild’s Equivalence Theorem**¹⁹: B is two-way strongly dutchbook-
16 able at threshold t iff B is strongly accuracy-dominated with respect to some
17 Additive accuracy-measure A with threshold t .

18
19 **Qualitative Dutch Book Theorem**: B avoids two-way strong dutchbooka-
20 bility at threshold t iff B is Almost Lockean Complete at threshold t .

21
22 The proof of the Qualitative Dutch Book Theorem makes use of the following
23 very important and technical Lemma found in (Rothschild, 2021) as “Theorem
24 2”. We mention it explicitly because it will also be useful in the proofs of our
25 subsequent theorems.

26
27 **Farkas-Rothschild Lemma**²⁰:

28 Let A be any $m \times n$ matrix, then:

29 $(\nexists \vec{x} \geq 0 : A\vec{x} < 0) \iff (\exists \vec{y} \geq 0 : \vec{y} \neq 0 \text{ and } \vec{y}A \geq 0)$.

30
31 Now, having finished our brief rehearsal of (Rothschild, 2021), we proceed to
32 extend his results to the diachronic-ish case of all-or-nothing belief updating,
33 but first a discussion of: why the “-ish” in “diachronic-ish”?

world w . In words, B' is more accurate than B at every possible world.

¹⁹de Finetti (1974) was the first to explicitly investigate the relationship between dutch-
bookability and accuracy-dominance with respect to the (Brier, 1950)-score in the credal case,
which was later implicitly extended to include the class of proper scoring rules by (Savage,
1971). In this sense, Rothschild’s Theorem can be understood as a qualitative version of de
Finetti’s work.

²⁰The appearance of this lemma in our dutchbookability inquiries is not entirely unexpected
because its proof involves the Separating Hyperplane Theorem, which is known to be useful
in investigating dutchbookability (Pettigrew, 2020).

1 7 Diachronic Dutchbooks and Beliefs

2 A genuinely diachronic book is a collection of bets offered at different times.
3 For our purposes, the times $t_1 < t_2$ will suffice. Now, in the credal case,
4 it is well-known that rationally requiring the avoidance of strong diachronic-
5 dutchbookability is an exceedingly strong demand; so strong in fact that one
6 cannot rationally change their credences, even if one comes to possess new evi-
7 dence.²¹ Surely, this result refutes rationally requiring the avoidance of strong
8 diachronic-dutchbookability in the credal case. We show that a similar result
9 holds for the all-or-nothing belief case with Lockean betting. In particular,
10 we show that requiring the avoidance of strong diachronic-dutchbookability at
11 threshold $t > \frac{1}{2}$ would prevent Lockean bettors from changing their mind about
12 p in the sense that one cannot rationally believe p at t_1 and then believe $\neg p$
13 at t_2 , even if one comes to possess new evidence that $\neg p$.²² Let (B_{t_1}, B_{t_2}) in-
14 volve such a mind change about p . Now, consider the following bets (which
15 are to be sold to our Lockean bettor): a \$1 bet at t_1 on p for \$ t and a \$1
16 bet at t_2 on $\neg p$ for \$ t . These two bets, taken together, constitute a strong
17 diachronic-dutchbook against such (B_{t_1}, B_{t_2}) . Thus, requiring the avoidance of
18 strong diachronic-dutchbookability is not the way to go in the qualitative case
19 either.

20

21 These observations independently motivate the introduction of belief plans as
22 the proper target for dutchbook theorems in the context of investigating Lockean
23 betting behavior; thus making the forthcoming dutchbook arguments properly
24 strategic (Teller, 1973) (Skyrms, 1987) (Lewis, 1999), fundamentally synchronic
25 (van Fraassen, 1989) (Levi, 2002), and “irreducibly modal” (Pettigrew, 2020)
26 just as in the credal case. Such planned beliefs are to be connected, descriptively
27 or normatively, with Lockean betting behavior in the same way that planned
28 credences are connected with betting behavior: via conditional/called-off bets.
29 In detail, a conditional bet gives a net payoff of \$0 if the condition²³ is not ful-
30 filled and works like a normal bet otherwise. The notion of conditional Lockean
31 betting is now immediate. If $p \in \beta_E$, then the agent will (or ought) to buy a \$1
32 E -conditional bet on p for \$ t (or less). And, if $p \notin \beta_E$, then the agent will (or
33 ought) to sell a \$1 E -conditional bet on p for \$ t (or more). Generalizing this to
34 conditional bets with non-negative stakes other than \$1 is straightforward.

35

²¹In detail, it can be shown that a credal pair (c_{t_1}, c_{t_2}) avoids strong diachronic-dutchbookability iff c_{t_1} is probabilistic and $c_{t_1} = c_{t_2}$. See (Pettigrew, 2020) for an especially clear presentation of this result. The idea is that if $c_{t_1} \neq c_{t_2}$, you, as the bookie, could sell a bet on p for higher at one time and buy a bet on p for cheaper at the other time. And, this gives a strong dutchbook against such (c_{t_1}, c_{t_2}) .

²²Demonstrating this is all we need to unseat the standard of avoiding strong diachronic-dutchbookability. However, for the curious, see the Appendix for a full characterization of avoiding two-way strong diachronic-dutchbookability in the qualitative case.

²³How, exactly, the relevant condition is to be understood depends upon how one understands belief plans. The condition might be “coming to possess evidence E”, “supposing that E”, “E being true”, etc.

1 Now, suppose we consider an agent with both all-or-nothing beliefs and their
 2 plannings. Never mind if our agent has credences or credal plannings. They
 3 might; they might not. We can thus represent such an agent's doxastic state
 4 with a pair (B, β) . With the notions of Lockean betting and conditional Lockean
 5 betting in-hand, we can investigate when such Lockean bettors can be bilked
 6 out of money, and, furthermore, how the strong dutchbookability of pairs (B, β)
 7 is related to strong accuracy-dominance. We begin with the latter question and
 8 answer the former in the next section. But first, we must discuss how we are
 9 going to measure the accuracy of belief/belief-planning pairs.

10
 11 Def: accuracy-measure A is said to be Fully-Additive iff $A[(B, \beta), w] = A(B, w) +$
 12 $A(\beta_{E_w}, w)$ and A is additive.²⁴

13
 14 In words, an accuracy-measure being Fully-Additive means that the accuracy
 15 of a belief/belief-planning pair at world w is the sum of the accuracy of one's
 16 beliefs at world w and the accuracy of your planned beliefs at world w . And,
 17 further, that the accuracy-measure of your beliefs and their plannings is also
 18 additive. In this way, Full-Additivity allows us to jointly measure the accuracy
 19 of such (B, β) pairings.

20
 21 **Qualitative SSK Theorem**²⁵: (B, β) is two-way strongly dutchbookable at
 22 threshold t iff (B, β) is strongly accuracy-dominated with respect to some Fully-
 23 Additive accuracy-measure A with threshold t .

24
 25 Proof: -omitted, it is almost entirely analogous to the proof of **Rothschild's**
 26 **Equivalence Theorem** for the case of just beliefs.

27 \diamond .

28 8 The Qualitative Dutch Strategy Theorem

29 In this section, we develop the promised dutch strategy theorem for pairs of all-
 30 or-nothing beliefs and their plannings, the so-called Qualitative Dutch Strategy
 31 Theorem. But first, a few definitions.

32
 33 Def: belief plan β is said to be Almost Lockean Complete at threshold t iff
 34 there exists a probabilistic credence function c st. for every proposition p and
 35 for every piece of evidence E if $c(E) > 0$, then [if $p \in \beta_E$, then $c(p|E) \geq t$ and
 36 if $p \notin \beta_E$, then $c(p|E) \leq t$].

²⁴We stress that A 's being Fully-Additive does *not* imply that it is then Extensional. It might be that $A(p \in B, w) \neq A(p \in \beta_E, w)$. This observation matters in the proof of the Qualitative SSK Theorem. Its origin lies in the fact that we allow varying stakes between plain and conditional bets on the same proposition.

²⁵SSK refers to (Schervish, Seidenfeld, & Kadane, 2009) who, among other things, investigate when strong dutchbookability (their *incoherence*₁) and scoring-dominance (their *incoherence*₃) coincide in a very general setting involving infinitely-many and even randomized marginal/conditional forecasts.

1
2 Def: the pair (B, β) is said to be jointly Almost Lockean Complete at threshold
3 t iff B is Almost Lockean Complete at threshold t with respect to c and β is
4 Almost Lockean Complete at threshold t with respect to the same c . The idea
5 being that there exists a c that “works for both”.

6
7 Given this notion of Joint Almost Lockean Completeness, a question immed-
8 iately presents itself. Is Joint Almost Lockean Completeness substantive, in
9 the sense that it rules out belief/belief-planning pairs that are both individually
10 Almost Lockean Complete? It turns out that the answer is yes! Here’s a sketch
11 of how the proof would go. Pick some B such that B contains $\neg p \& q$ (with p and
12 q logically independent) but doesn’t contain q and is Almost Lockean Complete
13 (at threshold $t < 1$). This implies that any Almost Lockean representer of B
14 is such that $c(p) = 1$ (and which implies that you believe p). Now, pick an
15 evidential partition with an $E \in B$ that is compatible with p but doesn’t entail
16 it. So, for any such c we have that $c(p|E) = 1$ (which implies that any β such
17 that (B, β) is Jointly Almost Lockean Complete is such that $p \in \beta_E$). Now, just
18 pick some β that is Almost Lockean Complete but $p \notin \beta_E$.

19
20 **Qualitative Dutch Strategy Theorem:** (B, β) avoids two-way strong dutch-
21 bookability at threshold t iff (B, β) is jointly Almost Lockean Complete at
22 threshold t .

23
24 Proof: We begin by defining a $E_k \in \epsilon$ conditional two-sided betting matrix
25 A_{E_k} for β_{E_k} with a fixed Lockean threshold t (in which the unconditional bets
26 on B are given by taking E_k as the tautology). We define:

$$27$$

$$28 \quad a_{i,j}^{E_k} = \begin{cases} 1-t & \text{if } w_i \in E_k \text{ and } p_j \in \beta_{E_k} \text{ and } w_i \in p_j \\ -t & \text{if } w_i \in E_k \text{ and } p_j \in \beta_{E_k} \text{ and } w_i \notin p_j \\ -(1-t) & \text{if } w_i \in E_k \text{ and } p_j \notin \beta_{E_k} \text{ and } w_i \in p_j \\ t & \text{if } w_i \in E_k \text{ and } p_j \notin \beta_{E_k} \text{ and } w_i \notin p_j \\ 0 & \text{if } w_i \notin E_k \end{cases} \quad (3)$$

29
30
31 Let $\vec{x} \geq 0$ be the column vector of betting stakes over all the propositions
32 for the unconditional bets. Furthermore, let $\vec{x}_{E_k} \geq 0$ be the column vector
33 of betting stakes over all the propositions for the E_k -conditional bets. Now,
34 given Payoff Additivity, a straightforward computation shows that (B, β) being
35 strongly two-way dutchbookable at threshold t is equivalent to there existing
36 $\vec{x}, \vec{x}_{E \in \epsilon} \geq 0$ st:

37

$$1 \quad [\mathbf{A} \quad \mathbf{A}_{E_1} \quad \dots \quad \mathbf{A}_{E_{|\epsilon|}}] \begin{pmatrix} \vec{x} \\ \vec{x}_{E_1} \\ \dots \\ \vec{x}_{E_{|\epsilon|}} \end{pmatrix} < 0.$$

2
3 Now, by the Farkas-Rothschild Lemma, we know that the non-existence of
4 such stake vectors, that is, the non-dutchbookability of (B, β) , is equivalent
5 to there existing a row vector $\vec{c} = [c(w_1), \dots, c(w_{|W|})]$ of probabilistic credences
6 over worlds such that:

$$7 \quad \vec{c} [\mathbf{A} \quad \mathbf{A}_{E_1} \quad \dots \quad \mathbf{A}_{E_{|\epsilon|}}] \geq 0.$$

8
9
10 Now, a little computation shows that this expression is equivalent to $\exists c$ st.
11 [if $p \in B$, then $c(p) \geq t$ and if $p \notin B$, then $c(p) \leq t$.] and if $c(E_k) > 0$, then [if
12 $p \in \beta_{E_k}$, then $c(p|E_k) \geq t$ and if $p \notin \beta_{E_k}$, then $c(p|E_k) \leq t$]. That is, the pair
13 (B, β) is jointly Almost Lockean Complete at threshold t . This little computa-
14 tion being:

15
16 The j th coordinate of $\vec{c} [\mathbf{A} \quad \mathbf{A}_{E_1} \quad \dots \quad \mathbf{A}_{E_{|\epsilon|}}]$ for the \mathbf{A}_{E_k} -part is equal to
17 $\sum_{i \in |W|} c(w_i) a_{i,j}^{E_k}$, so if $\sum_{i \in |W|} c(w_i) a_{i,j}^{E_k} \geq 0$, then [if $p_j \in \beta_{E_k}$, then $c(p_j \cap$
18 $E_k)(1 - t) + c(\neg p_j \cap E_k)(-t) \geq 0$] and [if $p_j \notin \beta_{E_k}$, then $c(p_j \cap E_k)(t - 1) +$
19 $c(\neg p_j \cap E_k)(t) \geq 0$]. Furthermore, if $c(E_k) > 0$, we can divide these inequalities
20 by $c(E_k)$. Finally, using the fact that c is a probability function (or, rather, can
21 be normalized into one without disrupting the relevant inequalities), we arrive
22 at our result.

23 \diamond^{26} .

24
25 With this theorem in hand, we could proceed to develop a dutch strategy argu-
26 ment in the usual way, that is, just as in the credal case.²⁷ But, instead
27 of rehearsing this well-trodden ground, we make a few observations about the
28 conclusion of such dutch strategy arguments.²⁸ We stress that the Qualitative
29 Dutch Strategy Theorem cannot, by itself, provide an argument for Plan Al-
30 most Lockean Revision. Why? Just because some agent's pair (B, β) is made
31 jointly Almost Lockean Complete at threshold t by some credence function c
32 does not mean that c represents that agent's actual credence function, if they
33 have one. Thus, in this sense, we see that the requirement that rational be-

²⁶Adapting this proof to the case of evidentially-varying thresholds $t_{E \in \epsilon}$ is straightforward, just swap the unindexed t 's in \mathbf{A}_{E_k} for the relevantly indexed threshold t_{E_k} . Also, observe that a necessary condition for (B, β) to avoid two-way weak dutchbookability and weak accuracy-dominance is for $E \in \beta_E$ and β_E to be Almost Lockean Complete for every $E \in \epsilon$. Finally, it can be analogously shown that (B, β) is not one-way strongly dutchbookable at threshold t iff (B, β) is jointly Lockean Compatible at threshold t : just keep $a_{i,j}^{E_k}$ the same as above except for setting $a_{i,j}^{E_k} = 0$ if $[w \notin E_k \text{ or } p_j \notin \beta_{E_k}]$. See (Rothschild, 2021) for the notion of Lockean Compatibility.

²⁷See (Christensen, 1996) and (Pettigrew, 2020) for such a development.

²⁸These observations also apply to the conclusion of our subsequent accuracy-dominance argument.

1 lief plans maximize expected accuracy with respect to one’s actual credences
2 is a stronger claim. This is to be expected. After all, that claim is genuinely
3 inter-theoretic across different types of actual doxastic attitudes while our de-
4 velopment of Lockean betting behavior and its susceptibility to dutchbooks only
5 required our relevant agent to have all-or-nothing beliefs and their plannings.

6
7 We now make explicit an obvious corollary of the results of this section. This
8 corollary can be usefully deployed in developing an accuracy-dominance argu-
9 ment for the rational requirement of having jointly Almost Lockean Complete
10 belief/belief-planning pairs. We develop this accuracy-dominance argument in
11 the upcoming section. Here is that corollary:

12
13 **Qualitative SSK-BP Theorem**²⁹: (B, β) is strongly accuracy-dominated
14 with respect to some Fully-Additive accuracy-measure A with threshold t iff
15 (B, β) is not jointly Almost Lockean Complete at threshold t .

16
17 Proof: Follows immediately from the Qualitative SSK Theorem and the Quali-
18 tative Dutch Strategy Theorem. \diamond .

19 9 A Newly Conceptualized Accuracy-Dominance 20 Argument

21 It is worth stressing that the accuracy-dominance argument that we could de-
22 velop for (joint) Almost Lockean Completeness from the Qualitative SSK-BP
23 Theorem is different from the usual accuracy-dominance arguments for Proba-
24 bilism and Plan Conditionalization. Whereas the latter arguments show that
25 failing to satisfy Probabilism or Plan Conditionalization opens one up to strong
26 accuracy-dominance for every “legitimate” accuracy-measure, the former does
27 not. What it shows is that there exists a “legitimate” qualitative accuracy-
28 measure such that one’s Almost Lockean Incomplete beliefs/belief-plannings
29 perform poorly in the sense that they are strongly accuracy-dominated.³⁰ I am
30 inclined to think this enough to establish one’s irrationality. But how is this pos-
31 sible? Pettigrew (2016), for the credal case, has helpfully listed and investigated
32 different ways of understanding the notion of “legitimacy” for accuracy-measures
33 and the proper formulation of the relevant accuracy-dominance condition essen-
34 tial to all accuracy-dominance arguments. This list consists of exactly three
35 positions: Epistemicism, Supervaluationism, and Subjectivism. Here we claim
36 that this list is incomplete. That is, we outline a newly conceptualized accuracy-
37 dominance principle; one that allows the usual accuracy-dominance arguments
38 for Probabilism and Plan Conditionalization to go through as well as allowing
39 both Rothchild’s argument and our new argument for (joint) Almost Lockean

²⁹SSK again refers to (Schervish, Seidenfeld, and Kadane, 2009). BP refers to (Briggs and Pettigrew, 2018). We note that the result found in the Briggs-Pettigrew paper follows from a special case of SSK’s Corollary 2 in conjunction with the Greaves-Wallace Theorem.

³⁰I thank ???? for suggesting that I address this matter.

1 Completeness.

2

3 Here is the idea. If we understand “legitimate” as “permissible to evaluate
4 with”, then it seems bad for there to be a manner of evaluation with which you
5 perform relatively poorly. More precisely, it seems bad to be able to permis-
6 sibly assess yourself in such a way that you perform relatively poorly, at least
7 when there exists a way to guarantee avoiding such a bad evaluation by having
8 different doxastic attitudes. We might call this the Evaluationist View of un-
9 derstanding what we mean when we impose properties on accuracy-measures.
10 More precisely, the proposed norm is this:

11

12 **Evaluationist Non-Vacuous Dominance:** doxastic attitude D is irrational
13 if

14 (1): There exists a legitimate accuracy-measure A and there exists a D' which
15 strongly accuracy-dominates D according to A . And,

16 (2): There exists a D' such that D' is not strongly accuracy-dominated accord-
17 ing to any legitimate accuracy-measure A .

18

19 A few remarks: Condition (2) is a non-vacuity condition. It helps to ensure
20 that some doxastic attitude cannot be accessed relatively poorly in a legitimate
21 way. After all, if every doxastic attitude is accuracy-dominated according to
22 some legitimate manner of evaluation, then it hardly seems reasonable to flag
23 every attitude as (equally) irrational³¹. Finally, it is worth keeping in mind
24 that, according to **Evaluationist Non-Vacuous Dominance**, (1) and (2) are
25 offered as a sufficient condition for irrationality and not as a necessary condi-
26 tion³². It says nothing about the rational status of situations in which condition
27 (2) fails, if there be such. Having developed our new dominance principle, we
28 now contrast it with Epistemicism and Subjectivism.³³

29

30 Epistemicism is the view that there exists exactly one correct accuracy-measure
31 (perhaps relative to a context), but that we do not know which accuracy-measure
32 it is. All we know is that the true accuracy-measure is among our collection of
33 legitimate accuracy-measures. The weakest corresponding Epistemicist Domi-
34 nance Principle is that it is a requirement of rationality to avoid being accuracy-
35 dominated with respect to the one true accuracy-measure. My complaint against
36 this Epistemicist View is that it needlessly exposes Epistemicists to the risk of

³¹In this way, our proposed dominance norm addresses (Pettigrew, 2016)’s concerns in the “Name Your Fortune” decision problem without going all the way to his Undominated Dominance Principle.

³²For this reason, it is not a good reason to reject our proposed dominance norm on the grounds that something stronger, such as avoiding being weakly-dominated, possibly under some additional conditions, is necessary for being rational.

³³I leave out an extended discussion of Supervaluationism because I don’t have anything new to add on the topic, except to say that I’m sympathetic with (Pettigrew, 2016)’s concerns about its extremeness in the sense that it makes evaluations of rationality require unanimity of all legitimate accuracy-measures and ignores other possible, and seemingly relevant, evaluative asymmetries, such as the one made explicit in our new Evaluationist View.

1 being irrational because they don't know which qualitative accuracy-measure is
2 the correct one. After all, why risk the irrationality of being accuracy-dominated
3 when it could be avoided by accepting a different formulation of the relevant
4 dominance principle?³⁴ The Evaluationist View does not come with this bug,
5 or the wishful thinking of there being exactly one true qualitative accuracy-
6 measure. See (Pettigrew, 2016) for further concerns regarding the Epistemicist
7 View.

8
9 We now contrast our proposed Evaluationist View with Subjectivism. The Sub-
10 jectivist understands "legitimate" as "permissible to adopt". The Subjectivist
11 might object to the Evaluationist View by saying that this argument is not con-
12 vincing to one who adopts a non-dominated accuracy-measure. My response³⁵
13 is that it's unclear why such an adoption confers sole normative authority to
14 the adopted accuracy-measure, especially when it comes to matters of being in
15 contrast to matters of doing (Konek & Levinstein, 2017). According to Eval-
16 uationism, epistemic value theory was never in the business of describing or
17 normatively confining peoples' preferences about epistemic matters; it's in the
18 business of evaluating their doxastic attitudes according to purely epistemic
19 values/concepts, whether they correctly adopt such purely epistemic values or
20 not.³⁶ After all, it doesn't seem correct to have to first learn about someone's
21 personal epistemic preferences in order to be able to give them an evaluation of
22 how well their doxastic attitudes are representing the world. In this way, Sub-
23 jectivism also needlessly limits the scope/applicability of accuracy-dominance
24 arguments for epistemic norms, for what if the relevant agent adopts an illegit-
25 imate accuracy-measure? Subjectivism, unlike Evaluationism, then blocks our
26 usual accuracy-dominance arguments from applying to such an agent, and this
27 seems unfortunate. At the very least, Evaluationism is not easily dismissed as
28 a plausible view of epistemic value theory; a view that gets us more of what
29 we want. Finally, in detail, our newly conceptualized accuracy-dominance ar-
30 gument for joint Almost Lockean Completeness at threshold t is as follows:

- 31
32 (1): Qualitative Veritism (for beliefs and their plannings).
33 (2): Legitimate accuracy-measures for (B, β) 's are Fully-Additive and Exten-
34 sional with threshold t .
35 (3): **Evaluationist Non-Vacuous Dominance.**
36 (4): **Qualitative SSK-BP Theorem.**
37 (5): Therefore, rational (B, β) 's are jointly Almost Lockean Complete at thresh-
38 old t .

³⁴A similar point has been raised in favor of money-pump arguments for preference transi-
tivity. Of course, such arguments might fail for other reasons.

³⁵See (Pettigrew, 2016) for other concerns about the Subjectivist View.

³⁶We note that this Evaluationist understanding of epistemic value theory is unlike tradi-
tional decision theory in which the preferences one has do confer normative authority. Maybe
this is just another non-problematic point of disanalogy (Konek & Levinstein, 2017).

10 Non-Deterministic Belief Planning

In this section, we consider the case of non-deterministic all-or-nothing belief planning and develop a qualitative dutch strategy theorem for such belief/belief-planning pairs along the lines developed by (Pettigrew, 2020) for the credal case. In this spirit, consider an agent with both all-or-nothing beliefs and their possibly non-deterministic but finitely-many plannings. Never mind if our agent also has credences or credal plannings. They might; they might not. We can thus represent such an agent’s doxastic state with a pair (B, β_R) with a possibly non-deterministic belief-plan $\beta_R = (\beta_{R_{E_1}}, \dots, \beta_{R_{E_{|\epsilon|}}})$ where $\beta_{R_{E_k}} = \{\beta_{R_{E_k}^1}, \dots, \beta_{R_{E_k}^{|\beta_{R_{E_k}}|}}\}$. The interpretation, under a non-deterministic dispositional reading, being that our agent might adopt any belief-set $\beta_{R_{E_k}^i}$ in $\beta_{R_{E_k}}$ in response to receiving total evidence E_k .³⁷ Now, let $R_{E_k}^i$ be the proposition that our agent actually adopts beliefs $\beta_{R_{E_k}^i}$ in response to receiving total evidence E_k . At this point, it is useful to remind ourselves that our agent has qualitative attitudes towards propositions in some set \mathcal{F} , and $R_{E_k}^i$ need not be in \mathcal{F} . Question: when can such pairs (B, β_R) be two-way strongly dutchbooked at threshold t with a book consisting only of plain and $R_{E_k}^i$ -conditional bets? The answer is given by the following Qualitative Non-Deterministic Dutch Strategy Theorem, but first, a few definitions:

Def: β_R is said to be Almost Lockean Complete at threshold t iff there exists a credence function c st. for every $p \in \mathcal{F}$ if $c(R_{E_k}^i) > 0$, then [if $p \in \beta_{R_{E_k}^i}$, then $c(p|R_{E_k}^i) \geq t$ and if $p \notin \beta_{R_{E_k}^i}$, then $c(p|R_{E_k}^i) \leq t$].

Def: the pair (B, β_R) is said to be jointly Almost Lockean Complete at threshold t iff B is Almost Lockean Complete at threshold t with respect to c and β_R is Almost Lockean Complete at threshold t with respect to the same c . The idea again being that there exists a c that “works for both”.

Qualitative Non-Deterministic Dutch Strategy Theorem: (B, β_R) avoids two-way strong dutchbookability at threshold t iff (B, β_R) is jointly Almost Lockean Complete at threshold t .

Proof: This theorem follows as a corollary from the Qualitative Dutch Strategy Theorem. The idea is to transform the possibly non-deterministic pair (B, β_R) into the deterministic pair (B, β) in which β is a function taking “evidence” $R_{E_k}^i$ to belief-set $\beta_{R_{E_k}^i}$. Now, just apply the Qualitative Dutch Strategy Theorem to our now deterministic pairing and we’re done.

◇.

³⁷Note that the planning and suppositional interpretations also generalize easily to the non-deterministic case.

1 11 Rothschild’s Question

2 In this section, we answer an implicit open question of (Rothschild, 2021): when
 3 does there exist a qualitative accuracy-measure such that B is weakly accuracy-
 4 dominated? Rothschild (2021) showed that B ’s being Almost Lockean Complete
 5 is not enough to avoid weak accuracy-dominance. He provides an explicit ex-
 6 ample of a collection of beliefs that is Almost Lockean Complete, but is still
 7 weakly accuracy-dominated. More recently, Hewson (2021) improves on this by
 8 showing that a belief-set being Closed-under-single-premise-Entailment is nec-
 9 essary for avoiding weak accuracy-dominance.³⁸ What this means is that if you
 10 believe something, then you should believe whatever is (properly) entailed by
 11 your belief, on pain of being weakly accuracy-dominated. After all, it would
 12 seem odd if you believed that both Bob and Alice are coming to your party
 13 tonight, but you failed to believe that Bob is coming tonight. More precisely,

14
 15 Def: B is Closed-under-single-premise-Entailment iff if $p \in B$ and p properly
 16 entails q , then $q \in B$.

17
 18 In this section, we will show that (Hewson, 2021)’s result about Closure-under-
 19 single-premise-Entailment follows from our general characterization of quali-
 20 tative weak dutchbookability and weak accuracy-dominance. Without further
 21 ado, here are our results:

22
 23 **Qualitative Weak Dutchbook Theorem:** B avoids two-way weak dutch-
 24 bookability at threshold $t < 1$ iff B is Almost Lockean Complete with threshold
 25 $t < 1$ with respect to a regular probability function.³⁹

26 Proof:

27 “ \Rightarrow ”: Assume B avoids two-way weak dutchbookability at threshold $t < 1$. We
 28 use the following variant of Farkas’s Lemma (Wikipedia, 2024):

29
 30 Let \mathbf{A} be any $m \times n$ matrix, then:

$$31 (\exists \vec{x} \geq 0 : \mathbf{A}\vec{x} \leq \vec{b}) \iff (\exists \vec{c} \geq 0 : \mathbf{A}^T \vec{c} \geq 0 \text{ and } \vec{b}^T \vec{c} < 0).$$

32
 33 Now, B ’s avoiding two-way weak dutchbookability gives us the left-hand side
 34 of the above Lemma for the following collection of \vec{b} ’s: $\vec{b}_i = (0, \dots, -1, \dots, 0)$ with -1
 35 in the i th coordinate. Thus, we have a bunch of corresponding \vec{c}_i ’s such that

³⁸Of course, this observation is not a complaint against Almost Lockean Completeness as a rational norm. If it is a complaint, it is a complaint against the position that Almost Lockean Completeness is the *only* rational norm for all-or-nothing beliefs, with the upshot that Almost Lockean Completeness should be supplemented with further rational norms. Norms that hopefully also have an accuracy-centered justification.

³⁹A credence function is said to be regular when it assigns positive credence to every possible world. The reason why the theorem has the $t < 1$ restriction is because any non-tautological belief-set is weakly dutchbookable when $t = 1$. Such belief-sets are willing to buy a \$1 bet on a contingent proposition for \$1. Also, this characterization of avoiding qualitative weak dutchbookability is not entirely unexpected because, in the credal case, weak dutchbookability is only avoided when the relevant credences are regular and probabilistic.

1 the i th coordinate of \vec{c}_i is non-zero (and thus positive) because $\vec{b}^T \vec{c} < 0$. Fur-
 2 thermore, the right-hand side gives us that B is Almost Lockean Complete with
 3 respect to the \vec{c}_i 's because $\mathbf{A}^T \vec{c} = ((\mathbf{A}^T \vec{c})^T)^T = (\vec{c}^T \mathbf{A})^T \geq 0 \iff \vec{c}^T \mathbf{A} \geq 0$
 4 and this last condition is exactly the Almost Lockean Completeness condition.
 5 Finally, we note that if B is Almost Lockean Complete with respect to the \vec{c}_i 's,
 6 then it is Almost Lockean Complete with respect to convex combinations of
 7 them. Choosing a positive convex combination then gives us a regular proba-
 8 bility function for which B is Almost Lockean Complete.
 9 “ \Leftarrow ”: Assume that B is Almost Lockean Complete with respect to a regular
 10 probability function c . Suppose, for contradiction, that B is two-way weakly
 11 dutchbookable. Now, observe that c accepts every bet that B accepts because
 12 B is Almost Lockean Complete wrt. c , so if B is weakly dutchbookable, then c
 13 is weakly dutchbookable. But, because c is regular, it is not weakly dutchbook-
 14 able. Thus, B is not even two-way weakly dutchbookable.
 15 \diamond .

16
 17 **Qualitative Weak Accuracy-Dominance Theorem:** B is Almost Lockean
 18 Complete with threshold $t < 1$ with respect to a regular probability function iff
 19 there does not exist a conditionally⁴⁰ legitimate qualitative accuracy-measure
 20 such that B is weakly accuracy-dominated.

21 Proof:

22 “ \Rightarrow ”: We prove the contrapositive. Suppose that B is weakly accuracy-dominated
 23 by some B' with respect to qualitative accuracy-measure A . Now, suppose, for
 24 contradiction, that B is Almost Lockean Complete with threshold $t < 1$ with
 25 respect to a regular probability function c . Thus, we know that $EA(B|c) \geq$
 26 $EA(B'|c)$ by (a conditional version of) the Easwaran-Dorst Theorem. But, c 's
 27 regularity and B 's being weakly accuracy-dominated by B' , that is: $A(B', w) \geq$
 28 $A(B, w)$ for every $w \in W$ and $A(B', w) > A(B, w)$ for some $w \in W$, implies that
 29 $A(B', w)c(w) \geq A(B, w)c(w)$ for every $w \in W$ and $A(B', w)c(w) > A(B, w)c(w)$
 30 for some $w \in W$. Which, when summed over worlds, gives us that $EA(B'|c) >$
 31 $EA(B|c)$, a contradiction.

32 “ \Leftarrow ”: We prove the contrapositive. Assume that B is not Almost Lockean Com-
 33 plete with threshold $t < 1$ with respect to a regular probability function. Then,
 34 the above Theorem gives us that B is two-way weakly dutchbookable. Now,
 35 just apply a conditional version of Rothschild's Equivalence Theorem (which
 36 generalizes to the weak case in the correct direction) and we're done.
 37 \diamond .

38
 39 Theorem: If B avoids two-way weak dutchbookability at threshold $t < 1$, then

⁴⁰We are here modifying our notion of legitimacy for qualitative accuracy-measures. In-
 stead of Variable Conservativeness, we are considering a weaker claim of Conditional Variable
 Conservativeness: If $T_p \neq 0$, then $F_p \neq 0$ and Variable Conservativeness holds, but if $T_p = 0$,
 then $F_p = 0$ also and if $F_p = 0$, then $T_p = 0$. In words, if correctly believing p adds no
 epistemic value, then incorrectly believing p adds no epistemic disvalue and vice versa. But,
 if correctly believing p does add epistemic value, then incorrectly believing p adds epistemic
 disvalue, in a Variable Conservative way.

1 B is Closed-under-single-premise-Entailment.

2 Direct Proof:

3 We prove the contrapositive. Suppose that B is not Closed-under-single-premise-
4 Entailment. Thus, there exists a p and q such that p properly entails q , $p \in B$,
5 and $q \notin B$. So, our agent is then willing to buy a \$1 bet on p for \$ t and sell a
6 \$1 bet on q for \$ t . These bets constitute a weak dutchbook against B . See the
7 following table:

8
9
10

Dutchbook	$p \& q$	$\neg p \& q$	$\neg p \& \neg q$
Payoff on p	$1-t$	$-t$	$-t$
Payoff on q	$t-1$	$t-1$	t
Total Payoff	0	-1	0

11

12 \diamond .

13

14 Another Proof:

15 Suppose that B avoids two-way weak dutchbookability at threshold $t < 1$. Then,
16 B is Almost Lockean Complete wrt. a regular probability function c , so B must
17 be Closed-under-single-premise-Entailment. Because if not, then there exists
18 propositions p and q such that p properly entails q and you believe p but you
19 don't believe q , so $c(p) = c(q) = t$ which, because c is a probability function,
20 entails c 's non-regularity.

21 \diamond .

22

23 Theorem: If there does not exist a legitimate ($t < 1$) qualitative accuracy-
24 measure such that B is weakly accuracy-dominated, then B is Closed-under-
25 single-premise-Entailment.

26 Direct Proof:

27 We prove the contrapositive. Assume B is not Closed-under-single-premise-
28 Entailment. Then, the above Theorem gives us that B is two-way weakly dutch-
29 bookable at threshold $t < 1$. Now just apply the weak version of Rothschild's
30 Equivalence Theorem and we're done.

31 \diamond .

32

33 Another Proof:

34 We prove the contrapositive. Assume that B is not Closed-under-single-premise-
35 Entailment. Then, B is not Almost Lockean Complete with threshold $t < 1$ wrt.
36 a regular probability function. Thus, Section 3's Theorem gives us that there
37 exists a legitimate ($t < 1$) qualitative accuracy-measure such that B is weakly
38 accuracy-dominated.

39 \diamond .

40

41 To close this section, we offer the following conjecture: B is Almost Lockean
42 Complete at threshold $t < 1$ and Closed-under-single-premise-Entailment

1 iff B is Almost Lockean Complete at threshold $t < 1$ wrt. a regular probability
2 function.

3 12 A Response to Hewson

4 In this section, we extend our results about weak dutchbookability and weak
5 accuracy-dominance to the diachronic-ish case, that is, for belief and belief-
6 planning pairs. In so doing, we attempt to address a concern raised by (Hewson,
7 2021), who shows that Closure-under-single-premise-Entailment is not necessary
8 for belief-plannings to avoid weak accuracy-dominance. Basically, it turns out
9 that your planned beliefs can fail to be Closed-under-single-Entailment while
10 still avoiding being weakly accuracy-dominated for any legitimate accuracy-
11 measure. We begin with a characterization of avoiding weak accuracy-dominance
12 for belief and belief-planning pairs.

13
14 **Qualitative Weak Accuracy-Dominance Theorem for Belief-Plannings:**
15 (B, β) is jointly Almost Lockean Complete with $t < 1$ with respect to a regular
16 probability function iff there does not exist a conditionally legitimate accuracy-
17 measure such that (B, β) is weakly accuracy-dominated.

18 Proof:

19 “ \Rightarrow ”: Parody the proof of the left-to-right direction of the **Qualitative Weak**
20 **Accuracy-Dominance Theorem** using both the (conditional version of the)
21 **Easwaran-Dorst Theorem** and the (conditional version of the) **Qualitative**
22 **Greaves-Wallace Theorem** (along with the full-additivity of the qualitative
23 accuracy-measure).

24 “ \Leftarrow ”: Suppose that there does not exist a conditionally legitimate accuracy-
25 measure such that (B, β) is weakly accuracy-dominated. Then, (B, β) avoids
26 weak dutchbookability, because if not, the planning version of (conditional)
27 Rothschild’s Equivalence Theorem gives us that (B, β) is weakly dutchbook-
28 able. Now, consider the following variant of Farkas’s Lemma (Wikipedia, 2024):

29
30 Let A be any $m \times n$ matrix, then:

$$31 (\exists \vec{x} \geq 0 : A\vec{x} \leq \vec{b}) \iff (\exists \vec{c} \geq 0 : A^T \vec{c} \geq 0 \text{ and } \vec{b}^T \vec{c} < 0).$$

32
33 An entirely analogous argument to the **Qualitative Weak Accuracy-Dominance**
34 **Theorem** can be made using this Lemma and the planning version of \vec{x} and A
35 as found in the **Qualitative SSK-BP Theorem**. (Observe that this argument
36 again uses the linearity of the Lockean Condition on the right-hand-side to deal
37 with positive convex combinations of probability functions.)

38 \diamond .⁴¹

39
40 **Weak Dutch Strategy Theorem for Belief-Plannings:** (B, β) avoids weak

⁴¹In comparison with the credal case, as found in (Nielsen, 2021), we note a trivial corollary of the above result: If there does not exist a legitimate accuracy-measure such that (B, β) is weakly accuracy-dominated, then β is Blackwell, that is, $E \in \beta_E$ for every $E \in \epsilon$.

1 dutchbookability at threshold $t < 1$ iff (B, β) is jointly Almost Lockean Com-
2 plete at threshold $t < 1$ wrt. a regular probability function.

3 Proof:

4 “ \Rightarrow ”: proven in previous Theorem.

5 “ \Leftarrow ”: Suppose that (B, β) is jointly Almost Lockean complete wrt. a regular
6 probability function c . Then, (B, β) avoids weak dutchbookability at threshold
7 $t < 1$, because, if not, then you could (by joint Almost Lockean Completeness)
8 weakly dutchbook the (c, P) where P is the conditionalizing plan on c . But this
9 is a contradiction because you cannot weakly dutchbook such a pair.

10 \diamond .

11
12 Armed with these results, we can, at least, partially respond to (Hewson, 2021)’s
13 concern that avoiding weak accuracy-dominance fails to secure Closure-under-
14 single-premise-Entailment for our planned beliefs. While we can’t get Closure-
15 under-single-premise-Entailment, we can still get something close.

16
17 Def: β is E -Closed-under-single-premise-Entailment iff for every $E \in \epsilon$ if $p \in \beta_E$,
18 p properly entails q , and $q - p$ contains an E -world, then $q \in \beta_E$.

19
20 Corollary: If there does not exist a legitimate accuracy-measure such that
21 (B, β) is weakly accuracy-dominated, then β is E -Closed-under-single-premise-
22 Entailment.

23 Proof: trivial.

24
25 The reason this result helps address (Hewson, 2021)’s concerns is that if your
26 evidence requires you to discount the consideration of non- E worlds in evalu-
27 ating your epistemic performance (in this case your E -planned beliefs), then it
28 seems permissible to then discount non- E worlds in your reasoning in the sense
29 that possessing evidence E makes certain non-logically-equivalent propositions
30 evidentially equivalent, for you, in the sense that matters epistemically. So,
31 perhaps, moving to E -coherence, that is, evaluating β_E only at E -worlds (as
32 demanded by Full-Additivity), does not come with the serious *epistemic* cost
33 that Hewson (2021) thinks it does.

34 13 Some Leitgebian Thoughts on Thresholds

35 So far, all of the qualitative norms that we have argued for in this paper have
36 been parameterized by the relevant threshold t . But why think that such a
37 threshold even exists?⁴² After all, if no threshold exists, then our proposed
38 norms seem to be useless and without content. Luckily, an idea from (Leitgeb,
39 2017) rescues us here. Leitgeb (2017), among other things, proposed that what

⁴²Further questions abound. Where does the threshold come from? If it is determined (or at least constrained) by contextual features, what exactly are these contextual features and how, exactly, is it so determined? Unfortunately, I do not have answers to any of these questions (if there even are answers).

1 makes a pair (B, c) rational is that there exists a threshold t such that the pair
2 satisfies the (Almost) Lockean Thesis with respect to that t . A similar existen-
3 tializing strategy also works for all of our qualitative norms on all-or-nothing
4 beliefs and their plannings. In fact, we can *argue* for this position, (even in the
5 absence of a detailed account of how t is contextually determined). The idea is
6 that if all-or-nothing beliefs and their plannings are to play their characteristic
7 normative roles in sanctioning bets as fair or not and in representing the world
8 for better or worse, then such a threshold must exist because, if a threshold
9 doesn't exist, then your all-or-nothing beliefs and their plannings fail to sort
10 permissible and impermissible gambles and they cannot be evaluated as repre-
11 senting the world more or less accurately. In short, if there is no threshold, then
12 your beliefs cannot play their characteristic normative roles, but beliefs do play
13 these characteristic normative roles, hence a threshold must normatively exist.

14 **14 Conclusion**

15 Moral of the story: rational beliefs and their plannings are appropriately Lock-
16 ean, or so says accuracy-first epistemology. This means that any account of
17 rational beliefs and their plannings that disagrees with the relevant Lockean
18 norms discussed in this paper is incorrect. To be clear, we have not shown
19 that further norms on your beliefs that go beyond our Lockean norms are in-
20 correct. However, given our characterization theorems, it might be difficult to
21 justify them from an accuracy-first standpoint, as any such justification must
22 go beyond our proposed dominance reasoning.

23 **15 Gestures to the Future**

24 Future work in this area might try to further generalize our results to the case of
25 having all-or-nothing beliefs and their plannings towards infinitely many propo-
26 sitions, as done in the credal case in (Kelley, ms) and citations therein. A
27 good place to start might be looking at infinitary versions of Farkas's Lemma.
28 In addition, it might be desirable to develop a Qualitative General Reflection
29 Principle in order to weaken some assumptions in our dutch-strategy/accuracy-
30 dominance arguments along the lines of (Pettigrew, 2023) and (Staffel & De
31 Bona, 2023). Furthermore, it seems desirable to develop guidance value argu-
32 ments for our proposed qualitative norms, if possible, as done in the credal case
33 by (Schervish, 1989) (Levinstein, 2017) (Pettigrew, 2020) (Konek, 2022). Such
34 arguments would show how having Almost Lockean Incomplete beliefs and their
35 plannings are bad, in a relevant sense, at guiding our actions in more general
36 decision problems than betting scenarios. Also, it seems desirable to develop, if
37 possible, an accuracy-dominance argument for the Almost Lockean Thesis and
38 Plan Almost Lockean Revision. Finally, it might be interesting to develop a
39 notion of comparative/degrees of dutchbookability for all-or-nothing beliefs and
40 their plannings along the lines of (Schervish, Seidenfeld, & Kadane, 2002), (De

1 Bona & Staffel, 2017), and (De Bona & Staffel, 2021).

2 16 Appendix

3 16.1 (Shear & Fitelson, 2018)-style Argument for Plan 4 Almost Lockean Revision

5 Here is the analogous Shear and Fitelson Argument:

6 (1): Plan Conditionalization for credal plan $P : \epsilon \rightarrow \{\text{credence functions}\}$.

7 (2): Legitimate qualitative accuracy-measures are Additive, Extensional, and
8 Variable Conservative.

9 (3): $\text{Exp}[A(\beta)|P]$ is maximized over belief plans.

10 (4): Therefore, Plan Almost Lockean Revision.

11

12 We quickly show the validity of this argument:

13 Proof:

14 Condition (3) gives us that $E(A(\beta)|P) = \sum_{E \in \epsilon} \sum_{w \in E} P_E(w)A(B_E, w)$ is maxi-
15 mized, thus for every $E \in \epsilon$ we have that $\sum_{w \in E} P_E(w)A(B_E, w)$ is maximized.

16 Now, because P is a conditionalization plan for one's credences c , we have that
17 $P_E(w) = 0$ if $w \notin E$, so $\sum_{w \in E} P_E(w)A(B_E, w) = \sum_{w \in W} P_E(w)A(B_E, w) = \text{Exp}(A(B_E)|P_E)$,

18 and, according to the Easwaran-Dorst Theorem, this latter formula is maximized
19 when B_E satisfies the Almost Lockean Thesis with respect to P_E . Thus, we see
20 that $E(A(\beta)|P)$ is maximized only when this latter formula is maximized, and
21 because P_E is the conditionalization of c on E , this gives us Plan Almost Lock-
22 ean Revision.

23

24 We briefly argue in favor of condition (3). This condition says that your planned
25 credences must rationally think your planned beliefs among the epistemically
26 best belief plans. This condition is entirely analogous to the Weak Propriety
27 condition on credal accuracy measures and (Dorst, 2019)'s own proposed condi-
28 tion of maximizing the expected accuracy of one's beliefs from the perspective
29 of one's credences.

30

31 Now, this argument is accuracy-theoretic through-and-through because we have
32 many accuracy-only arguments for Plan Conditionalization. [As found in, say,
33 (Greaves & Wallace, 2006), (Schervish, Seidenfeld, & Kadane, 2009), (Briggs &
34 Pettigrew, 2018), and (Nielsen, 2021).] Still, it seems unfortunate that this argu-
35 ment appeals to Plan Conditionalization. After all, what if the agent doesn't
36 even have a credal plan?⁴³ Thus, perhaps, we might want an accuracy argument
37 for Plan Almost Lockean Revision without appealing to Plan Conditionaliza-
38 tion, as provided by our expected-accuracy argument.

⁴³This point may not be convincing under a dispositional interpretation of credal planning. Even if so, it is better to have more arguments for a position than less, and, further, have a plausible rational requirement that is provably equivalent to our candidate norm.

1 **16.2 Qualitative Diachronic Dutchbook Theorem**

2 We begin with the promised characterization of two-way strong diachronic-
3 dutchbookability at threshold t for Lockean bettors, but first a definition.

4
5 Def: (B_{t_1}, B_{t_2}) is said to be jointly Almost Lockean Complete at threshold
6 t iff $\exists c$ st. B_{t_1} and B_{t_2} is Almost Lockean Complete at threshold t with respect
7 to c . The idea again being that there exists a c that “works for both”.

8
9 **Qualitative Diachronic Dutchbook Theorem:** (B_{t_1}, B_{t_2}) avoids two-way
10 strong diachronic-dutchbookability at threshold t iff (B_{t_1}, B_{t_2}) is jointly Almost
11 Lockean Complete at threshold t .

12
13 Proof: We begin by defining our two-sided betting matrices \mathbf{A}_{t_1} and \mathbf{A}_{t_2} for
14 B_{t_1} and B_{t_2} respectively. We define:

15
16

$$a_{i,j}^{t_k} = \begin{cases} 1-t & p_j \in B_{t_k} \text{ and } w_i \in p_j \\ -t & p_j \in B_{t_k} \text{ and } w_i \notin p_j \\ -(1-t) & p_j \notin B_{t_k} \text{ and } w_i \in p_j \\ t & p_j \notin B_{t_k} \text{ and } w_i \notin p_j \end{cases} \quad (4)$$

17
18
19 Let $\vec{x}_{t_k} \geq 0$ be the column vector of betting stakes over all the propositions
20 for the bets given at time t_k . Now, given Payoff Additivity, a straightfor-
21 ward computation shows that (B_{t_1}, B_{t_2}) being two-way strongly diachronically-
22 dutchbookable at threshold t is equivalent to there existing $\vec{x}_{t_1}, \vec{x}_{t_2} \geq 0$ st:

23
24
$$[\mathbf{A}_{t_1} \quad \mathbf{A}_{t_2}] \begin{pmatrix} \vec{x}_{t_1} \\ \vec{x}_{t_2} \end{pmatrix} < 0.$$

25
26 Now, by the Farkas-Rothschild Lemma, we know that the non-existence of
27 such stake vectors, that is, the non-diachronic-dutchbookability of (B_{t_1}, B_{t_2}) ,
28 is equivalent to there existing a row vector $\vec{c} = [c(w_1), \dots, c(w_{|W|})]$ of probabilis-
29 tic credences over worlds such that:

30
31
$$\vec{c} [\mathbf{A}_{t_1} \quad \mathbf{A}_{t_2}] \geq 0.$$

32
33 Now, a little computation shows that this expression is equivalent to $\exists c$ st.
34 [if $p \in B_{t_1}$, then $c(p) \geq t$ and if $p \notin B_{t_1}$, then $c(p) \leq t$] and [if $p \in B_{t_2}$, then
35 $c(p) \geq t$ and if $p \notin B_{t_2}$, then $c(p) \leq t$]. That is, the pair (B_{t_1}, B_{t_2}) is jointly
36 Almost Lockean Complete at threshold t .

37 \diamond .

38
39 With this characterization in hand, we note a difference between the quali-
40 tative and credal case of diachronic-dutchbookability. Namely, there is more

1 wiggle room in the qualitative case than the credal case in the sense that B_{t_1}
2 need not equal B_{t_2} . After all, the diachronic belief pair $(\{p, T\}_{t_1}, \{T\}_{t_2})$ is
3 jointly Almost Lockean Complete for $t < 1$, just take any $c(p) = t$. It is the
4 “Almost”-part in Almost Lockean Complete which opens the door to avoiding
5 diachronic-dutchbookability while having different belief-sets at different times.
6 Nevertheless, even with this extra wiggle room, the requirement of avoiding
7 diachronic-dutchbooks in the qualitative case is implausibly demanding, as pre-
8 viously established.

9
10 Finally, for pedagogical purposes, we also include an elementary proof of one
11 direction of the Qualitative Dutch Strategy Theorem. A straightforward proof
12 of the other direction is still wanting.

13
14 Proposition: If (B, β) is jointly Almost Lockean Complete at threshold t , then
15 (B, β) avoids two-way strong dutchbookability at threshold t .

16
17 Proof: (We follow the proof strategy found in (Nielsen, 2021).) Assume the an-
18 tecedent. Let c be a probabilistic credence function making (B, β) jointly Almost
19 Lockean Complete at threshold t . Suppose, for contradiction, that (B, β) is two-
20 way strongly dutchbookable at threshold t . Given the Qualitative SSK Theorem,
21 this implies that (B, β) is strictly accuracy-dominated for some fully-additive
22 accuracy measure A , that is, $\exists(B', \beta')$ st. $A[(B', \beta'), w] > A[(B, \beta), w]$ for every
23 $w \in W$. Clearly, this implies that $\text{Exp}[A(B', \beta')|c] > \text{Exp}[A(B, \beta)|c]$. But, given
24 Qualitative Temporal Separability, $\text{Exp}[A(B, \beta)|c] = \text{Exp}[A(B)|c] + \text{Exp}[A(\beta)|c]$
25 and, given that B and β is Almost Lockean Complete at threshold t with respect
26 to c , the Easwaran-Dorst Theorem and the Qualitative Greaves-Wallace Theo-
27 rem imply that the pair (B, β) maximizes $\text{Exp}[A(B^*, \beta^*)|c]$ (when considered as
28 a function from pairs (B^*, β^*)), contradicting $\text{Exp}[A(B', \beta')|c] > \text{Exp}[A(B, \beta)|c]$.
29 \diamond .

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