# Comment on “Aharonov-Bohm Phase is Locally Generated Like All Other Quantum Phases”

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**Abstract**

Marletto and Vedral [Phys. Rev. Lett. 125, 040401 (2020)] propose that the Aharonov-Bohm (AB) phase is locally mediated by entanglement between a charged particle and the quantized electromagnetic field, asserting gauge independence for non-closed paths. Using quantum electrodynamics (QED), we critically analyze their model and demonstrate that the AB phase arises from the interaction with the vector potential $\mathbf{A}$, not from entanglement, which is merely a byproduct of the QED framework. We show that their field-based energy formulation, intended to reflect local electromagnetic interactions, is mathematically flawed due to an incorrect prefactor and involves fields inside the solenoid, failing to support local mediation of the phase. Its equivalence to $q\mathbf{v} \cdot \mathbf{A}$ holds only in the Coulomb gauge, undermining their claim of a gauge-independent local mechanism. Furthermore, we confirm that the AB phase is gauge-dependent for non-closed paths, contradicting their assertion. Our analysis reaffirms the semi-classical interpretation, where the AB phase is driven by the vector potential $\mathbf{A}$, with entanglement playing no causal role in its generation.

## 1 Introduction

The Aharonov-Bohm (AB) effect [1] demonstrates that a charged particle’s wavefunction acquires a phase due to the vector potential $\mathbf{A}$, even in regions where electromagnetic fields vanish. For a closed path, the phase is:

$$\phi_{AB} = \frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{q\Phi}{\hbar}, \tag{1}$$

where $q$ is the charge of the particle and $\Phi$ is the magnetic flux. This phenomenon, often interpreted as evidence of the physical significance of gauge potentials, has prompted extensive discussions regarding its local versus nonlocal nature, given that the phase depends on the flux enclosed by the path, which may suggest a non-local interaction. In their recent work, Marletto and Vedral [2] present a quantum field theory (QFT) model to address the AB effect, proposing that the phase is locally mediated through entanglement between the charged particle and the quantized electromagnetic field or photons. They further claim that this phase is gauge-independent, even for non-closed paths, and detectable via local measurements, challenging the traditional view that the phase is inherently tied to the gauge-dependent vector potential and measurable only upon path closure.

In this comment, we carefully examine Marletto and Vedral's model, focusing on their derivation of the interaction energy, their field-based energy formulation, the role of entanglement, and the gauge properties of the phase. Using quantum electrodynamics (QED), we demonstrate that the AB phase arises from the coupling between the charge's current and the solenoid's current via the photon propagator, with the vector potential $\mathbf{A}$ serving as an effective description. We find that the phase remains gauge-dependent for non-closed paths, contrary to their assertion, and that their field-based energy does not mediate the AB phase. Additionally, we argue that entanglement, while present in the QED framework, is not the primary driver of the phase, which is fundamentally governed by the interaction with the vector potential $\mathbf{A}$. Our analysis seeks to clarify the mechanisms underlying the AB effect and reaffirm its conventional interpretation within QED.

## 2 Marletto and Vedral's Model

Marletto and Vedral [2] model a charged particle (charge $q$, mass $m$) in a superposition of paths around a solenoid, using qubits for the charge ($|0\rangle_C, |1\rangle_C$) and solenoid ($|0\rangle_S, |1\rangle_S$). The electromagnetic field is quantized with photon operators $a_k, a_k^\dagger$. Their Hamiltonian in the Coulomb gauge is:

$$
\begin{aligned}
H_{AB} &= E_C q_z^{(C)} + E_S q_z^{(S)} + \int d^3\mathbf{k} \hbar\omega_k a_k^\dagger a_k \\
&+ \int d^3\mathbf{k} g_k \frac{q}{m} \mathbf{p} \cdot \mathbf{u}_k \left( a_k e^{i\mathbf{k}\cdot\mathbf{r_c}} + a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{r_c}} \right) q_z^{(C)} \\
&+ \int d^3\mathbf{k} \int d^3\mathbf{x} g_k \mathbf{j} \cdot \mathbf{u}_k \left( a_k e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right) q_z^{(S)}, \qquad (2)
\end{aligned}
$$

where $\mathbf{p}$ is the charge's momentum operator, $\mathbf{r_c}$ is its position, and $\mathbf{j}(\mathbf{x} - \mathbf{r_s})$ is the solenoid's current density centered at $\mathbf{r_s}$. The coupling constant is $g_k = \sqrt{\frac{\hbar}{2\epsilon_0\omega_k V}}$, and $\mathbf{u}_k$ is the photon polarization vector satisfying $\mathbf{k} \cdot \mathbf{u}_k = 0$. The vector potential is:

$$
\mathbf{A}(\mathbf{x}) = \int d^3\mathbf{k} g_k \mathbf{u}_k \left( a_k e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}} \right). \qquad (3)
$$

They compute the phase from the transition amplitude:

$$\langle 1|_C\langle 0|_F\langle 1|_S \exp\left(-\frac{i}{\hbar}H_{AB}\tau\right)|1\rangle_C|0\rangle_F|1\rangle_S = \exp\left\{-i\left(\xi + \phi(\mathbf{r_c}, \mathbf{r_s})\right)\right\}. \quad (4)$$

where $\tau$ is set to 1. The phase $\phi(\mathbf{r_c}, \mathbf{r_s})$ is computed from the interaction Hamiltonian using the second-order term of the time-ordered exponential.

The interaction terms are:

$$\begin{aligned} H_{\text{int}} &= \int d^3\mathbf{k} g_k \frac{q}{m}\mathbf{p}\cdot\mathbf{u}_k\left(a_k e^{i\mathbf{k}\cdot\mathbf{r_c}} + a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{r_c}}\right) q_z^{(C)} \\ &+ \int d^3\mathbf{k}\int d^3\mathbf{x} g_k \mathbf{j}\cdot\mathbf{u}_k\left(a_k e^{i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}\right) q_z^{(S)}. \end{aligned} \quad (5)$$

The second-order term of the transition amplitude gives:

$$\phi = \frac{q}{m}\frac{\mu_0}{4\pi\hbar}\int d^3\mathbf{x}\frac{\mathbf{p}\cdot\mathbf{j}(\mathbf{x}-\mathbf{r_s})}{|\mathbf{r_c}-\mathbf{x}|}. \quad (6)$$

The corresponding interaction energy is:

$$\mathcal{E} = \phi\hbar = \frac{q}{m}\mathbf{p}\cdot\left(\frac{\mu_0}{4\pi}\int d^3\mathbf{x}\frac{\mathbf{j}(\mathbf{x}-\mathbf{r_s})}{|\mathbf{r_c}-\mathbf{x}|}\right). \quad (7)$$

Using $\mathbf{p} = m\mathbf{v}$, this becomes:

$$\mathcal{E} = q\mathbf{v}\cdot\left(\frac{\mu_0}{4\pi}\int d^3\mathbf{x}\frac{\mathbf{j}(\mathbf{x}-\mathbf{r_s})}{|\mathbf{r_c}-\mathbf{x}|}\right). \quad (8)$$

The vector potential in the Coulomb gauge is defined as

$$\mathbf{A}(\mathbf{r_c}) = \frac{\mu_0}{4\pi}\int d^3\mathbf{x}\frac{\mathbf{j}(\mathbf{x}-\mathbf{r_s})}{|\mathbf{r_c}-\mathbf{x}|}, \quad (9)$$

so:

$$\mathcal{E} = q\mathbf{v}\cdot\mathbf{A}. \quad (10)$$

For a solenoid with flux $\Phi = B_0\pi a^2$, $\mathbf{A} = \frac{\Phi}{2\pi(x^2+y^2)}(-y, x, 0)$, and with $\mathbf{v} = v\hat{y}$:

$$\mathcal{E} = \frac{qv\Phi x}{2\pi(x^2+y^2)} = \frac{qvB_0a^2x}{2(x^2+y^2)}, \quad (11)$$

yielding the accumulating phase during a time interval:

$$\phi = \int\frac{\mathcal{E}}{\hbar}dt = \frac{q}{\hbar}\int\mathbf{A}\cdot\mathbf{v}\,dt = \frac{q}{\hbar}\int\mathbf{A}\cdot dl, \quad (12)$$

which matches the semi-classical AB phase.

The above derivation in the Coulomb gauge is a specific case. It can be shown that the interaction energy calculated from the second-order term of the transition amplitude in QED with a quantized electromagnetic field, which determines the AB phase, always takes the form $q\mathbf{v} \cdot \mathbf{A}$, where $\mathbf{A}$ is the classical vector potential in the chosen gauge. This holds for all gauges because the second-order amplitude, governed by the photon propagator and conserved currents, consistently yields the effective vector potential interaction.

# 3 Critique of the Claims

## 3.1 Marletto and Vedral's Field-Based Energy Proposal

Marletto and Vedral propose an alternative formulation of the interaction energy as a field overlap integral:

$$\mathcal{E}_{\text{field}} = \frac{1}{2} \int_V \left( \frac{\mathbf{B}_0 \cdot \mathbf{B_c}}{\mu_0} + \epsilon_0 \mathbf{E_s} \cdot \mathbf{E_c} \right) d^3\mathbf{r}, \tag{13}$$

where $\mathbf{B}_0$ is the solenoid's magnetic field, $\mathbf{B_c}$ is the field generated by the moving charge, and $\mathbf{E_s}$ and $\mathbf{E_c}$ are the electric fields of the solenoid and the particle, respectively. They claim this energy reflects a local electromagnetic (EM) field interaction that mediates the AB phase, independent of the vector potential $\mathbf{A}$. We demonstrate that this formulation is flawed, both mathematically and physically, and that the AB phase is fundamentally driven by $\mathbf{A}$, not local EM fields, which vanish along the charge's path. Furthermore, we provide a detailed proof that their field-based energy is equivalent to the standard interaction energy $q\mathbf{v} \cdot \mathbf{A}$ only in the Coulomb gauge, explicitly highlighting where this gauge is used.

### 3.1.1 Mathematical Critique of the Field-Based Energy

The $\frac{1}{2}$ prefactor in Eq. (13) is inappropriate for the interaction energy between two distinct sources (the solenoid and the charged particle). In electromagnetic theory, the interaction energy between two systems with magnetic fields $\mathbf{B}_0$ (solenoid) and $\mathbf{B_c}$ (charged particle) and electric fields $\mathbf{E_s}$ and $\mathbf{E_c}$ is given by:

$$\mathcal{E}_{\text{field}} = \frac{1}{\mu_0} \int_V \mathbf{B}_0 \cdot \mathbf{B_c} d^3\mathbf{r} + \epsilon_0 \int_V \mathbf{E_s} \cdot \mathbf{E_c} d^3\mathbf{r}. \tag{14}$$

The $\frac{1}{2}$ prefactor, typically used for the total field energy of a single system, leads to an underestimation of the interaction energy by a factor of 2, making Marletto and Vedral's formulation quantitatively incorrect.

### 3.1.2 Equivalence to $q\mathbf{v} \cdot \mathbf{A}$ in the Coulomb Gauge

It can be demonstrated that the proposed field-based energy is equivalent to the standard interaction energy $q\mathbf{v} \cdot \mathbf{A}$ only in the Coulomb gauge.

For a solenoid with steady current, the electric field $\mathbf{E_s} = 0$, simplifying the interaction energy to:

$$\mathcal{E}_{\text{field}} = \frac{1}{\mu_0} \int_V \mathbf{B}_0 \cdot \mathbf{B_c} d^3\mathbf{r}, \tag{15}$$

where $\mathbf{B}_0 = B_0\hat{z}$ for $r < a$ (inside the solenoid, with radius $a$) and $\mathbf{B}_0 = 0$ for $r > a$, and $\mathbf{B_c}$ is the magnetic field produced by the charged particle (charge $q$, velocity $\mathbf{v} = v\hat{y}$) at position $\mathbf{r_c} = (x, y, z)$.

To prove that $\mathcal{E}_{\text{field}}$ is equivalent to the standard interaction energy $q\mathbf{v}\cdot\mathbf{A}$ in the Coulomb gauge, we evaluate Eq. (15) using a vector identity and explicitly note the use of the Coulomb gauge. The magnetic field $\mathbf{B_c} = \nabla \times \mathbf{A_c}$, where $\mathbf{A_c}$ is the vector potential of the charged particle. We use the vector identity:

$$\mathbf{B}_0 \cdot \mathbf{B_c} = \mathbf{B}_0 \cdot (\nabla \times \mathbf{A_c}) = \nabla \cdot (\mathbf{A_c} \times \mathbf{B}_0) + \mu_0 \mathbf{A_c} \cdot \mathbf{j_s}, \tag{16}$$

where $\mathbf{j_s}$ is the solenoid's current density, and we have used $\nabla \times \mathbf{B}_0 = \mu_0 \mathbf{j_s}$ inside the solenoid (since $\mathbf{E_s} = 0$). Integrating over the volume $V$:

$$\int_V \mathbf{B}_0 \cdot \mathbf{B_c} d^3\mathbf{r} = \int_{\partial V} (\mathbf{A_c} \times \mathbf{B}_0) \cdot d\mathbf{S} + \mu_0 \int_V \mathbf{A_c} \cdot \mathbf{j_s} d^3\mathbf{r}, \tag{17}$$

where the first term is a surface integral over the boundary $\partial V$. Since $\mathbf{B}_0 = 0$ outside the solenoid ($r > a$), the surface integral vanishes (since $V$ encloses the solenoid and extends to a region where $\mathbf{B}_0 = 0$). Thus:

$$\mathcal{E}_{\text{field}} = \frac{1}{\mu_0} \int_V \mathbf{B}_0 \cdot \mathbf{B_c} d^3\mathbf{r} = \int_V \mathbf{A_c} \cdot \mathbf{j_s} d^3\mathbf{r}. \tag{18}$$

Now we need the vector potential $\mathbf{A_c}$ of the charged particle. In the Coulomb gauge ($\nabla \cdot \mathbf{A_c} = 0$), the vector potential for a point charge $q$ moving with velocity $\mathbf{v}$ at position $\mathbf{r_c}$ is approximately (in the non-relativistic limit):

$$\mathbf{A_c}(\mathbf{r}) = \frac{\mu_0 q \mathbf{v}}{4\pi |\mathbf{r} - \mathbf{r_c}|}, \tag{19}$$

where $\mathbf{r}$ is the field point. Substituting into Eq. (18):

$$\mathcal{E}_{\text{field}} = \int_V \left( \frac{\mu_0 q \mathbf{v}}{4\pi |\mathbf{r} - \mathbf{r_c}|} \right) \cdot \mathbf{j_s}(\mathbf{r}) d^3\mathbf{r}. \tag{20}$$

The solenoid's vector potential at the charge's position $\mathbf{r_c}$ is:

$$\mathbf{A_s}(\mathbf{r_c}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j_s}(\mathbf{r})}{|\mathbf{r} - \mathbf{r_c}|} d^3\mathbf{r}, \tag{21}$$

which is also defined in the Coulomb gauge ($\nabla \cdot \mathbf{A_s} = 0$). Comparing this with Eq. (20), we see:

$$\mathcal{E}_{\text{field}} = q\mathbf{v} \cdot \left( \frac{\mu_0}{4\pi} \int \frac{\mathbf{j_s}(\mathbf{r})}{|\mathbf{r} - \mathbf{r_c}|} d^3\mathbf{r} \right) = q\mathbf{v} \cdot \mathbf{A_s}(\mathbf{r_c}). \tag{22}$$

Thus, in the Coulomb gauge, the field-based energy reduces to the standard interaction energy:

$$\mathcal{E}_{\text{field}} = q\mathbf{v} \cdot \mathbf{A_s}, \tag{23}$$

matching Eq. (10) in the original model, confirming the equivalence in the Coulomb gauge.

However, in non-Coulomb gauges ($\mathbf{A}'_\mathbf{s} = \mathbf{A_s} + \nabla\chi$), $\mathbf{A_s}$ includes additional terms, and thus the equivalence $\mathcal{E}_{\text{field}} = q\mathbf{v} \cdot \mathbf{A_s}$ does not hold. This means that the equivalence between the field-based energy and the standard interaction energy is gauge-specific, and the field-based energy is not consistent with the standard interaction energy in QED for non-Coulomb gauges.

### 3.1.3 Physical Critique of Local EM Field Mediation

Marletto and Vedral's claim that $\mathcal{E}_{\text{field}}$ reflects a local EM field interaction mediating the AB phase is also problematic. In the AB effect, the charged particle travels in a region where the EM fields vanish ($\mathbf{B} = \nabla \times \mathbf{A} = 0$, $\mathbf{E} = 0$ for $r > a$), but the vector potential $\mathbf{A}$ is non-zero. The field-based energy $\mathcal{E}_{\text{field}}$ involves $\mathbf{B}_0$, which exists only inside the solenoid ($r < a$), not at the charge's position ($r > a$). Thus, it cannot represent a local field interaction at the charge's location. The AB phase arises from the phase integral:

$$\phi = \frac{q}{\hbar} \int \mathbf{A} \cdot d\mathbf{l}, \tag{24}$$

which depends on local $\mathbf{A}$, not on local EM fields.

To sum up, the field-based energy proposed by Marletto and Vedral is equivalent to the standard interaction energy only in the Coulomb gauge. Moreover, even in this specific gauge, the equivalence does not support the claim of local EM field mediation of the AB effect, as no such fields exist in the charge's path. Thus, the field-based energy cannot replace the vector potential $\mathbf{A}$ as the local mediator of the AB effect.

## 3.2 Gauge Dependence of the AB Phase for Non-Closed Paths

Marletto and Vedral also assert that their derived phase is gauge-independent, even for non-closed paths, which we show to be incorrect by analyzing their phase expression. Their phase, derived from the interaction energy $\mathcal{E} = q\mathbf{v} \cdot \mathbf{A}$ (see (10) and (11)), is given by:

$$\phi = \int \frac{\mathcal{E}}{\hbar}\, dt = \frac{e}{\hbar} \int_{\mathcal{C}} \mathbf{A} \cdot d\mathbf{l}, \tag{25}$$

for a path $\mathcal{C}$ from $\mathbf{r}_1$ to $\mathbf{r}_2$. In their QED framework, this phase arises from the second-order amplitude involving the photon propagator:

$$S^{(2)} \propto \int d^4x_1 d^4x_2 j^\mu(x_1) D_{\mu\nu}(x_1 - x_2) j^\nu(x_2), \tag{26}$$

which reconstructs the classical vector potential $\mathbf{A}$. Under a gauge transformation $\mathbf{A}' = \mathbf{A} + \nabla\chi$, the phase becomes:

$$\phi' = \frac{e}{\hbar} \int (\mathbf{A} + \nabla\chi) \cdot \mathbf{v}\, dt = \phi - \frac{e}{\hbar} \int \nabla\chi \cdot d\mathbf{l} = \phi - \frac{e}{\hbar}[\chi(\mathbf{r}_2) - \chi(\mathbf{r}_1)]. \tag{27}$$

For non-closed paths ($\mathbf{r}_1 \neq \mathbf{r}_2$), $\chi(\mathbf{r}_2) - \chi(\mathbf{r}_1) \neq 0$ in general, making the phase gauge-dependent. Higher-order QED corrections, such as vertex corrections, modify the coupling constant ($\delta Z_e \sim \alpha \approx \frac{1}{137}$) but do not eliminate the $\mathbf{A}$-dependent term, preserving gauge dependence. Thus, Marletto and Vedral's phase is not gauge-independent for non-closed paths, contradicting their claim.

# 4 Entanglement is Not the Primary Cause of the AB Phase

Marletto and Vedral's central claim is that the AB phase arises from local entanglement between the charged particle and the photon field. While their QED model does indeed produce such entanglement, we demonstrate that this entanglement is merely a consequence of the interaction formalism rather than the physical mechanism responsible for the AB phase. The phase is determined by the vector potential $\mathbf{A}$, with entanglement playing no causal role.

## 4.1 The Origin of the Phase

The interaction Hamiltonian in their model,

$$H_{\text{int}} = \frac{q}{m}\mathbf{p} \cdot \mathbf{A}(\mathbf{r}_c) q_z^{(C)} + \int d^3\mathbf{x}\, \mathbf{j}(\mathbf{x} - \mathbf{r_s}) \cdot \mathbf{A}(\mathbf{x}) q_z^{(S)}, \tag{28}$$

couples the charged particle's momentum $\mathbf{p}$ to the quantized vector potential $\mathbf{A}$. For a particle in a superposition of left ($|L\rangle_C$) and right ($|R\rangle_C$) paths around the solenoid, this interaction generates path-dependent phases:

$$\phi_L = \frac{q}{\hbar}\mathbf{v} \cdot \mathbf{A}(\mathbf{r_L})\tau, \quad \phi_R = \frac{q}{\hbar}\mathbf{v} \cdot \mathbf{A}(\mathbf{r_R})\tau, \tag{29}$$

where $\tau$ is the interaction time. The phase difference,

$$\Delta\phi = \phi_R - \phi_L = \frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l}, \tag{30}$$

matches the standard AB phase and depends *only* on the **A**-field.

### 4.2 Incidental Nature of Entanglement

The system's post-interaction state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( e^{i\phi_L} |L\rangle_C |\chi_L\rangle_F + e^{i\phi_R} |R\rangle_C |\chi_R\rangle_F \right) |1\rangle_S, \tag{31}$$

where $|\chi_L\rangle_F$ and $|\chi_R\rangle_F$ are photon states associated with each path. Tracing out the photon field yields the reduced density matrix for the charge:

$$\rho_C = \mathrm{Tr}_F \left( |\psi\rangle\langle\psi| \right) = \frac{1}{2} \left( |L\rangle\langle L| + |R\rangle\langle R| + e^{i\Delta\phi} \langle \chi_L | \chi_R \rangle_F |L\rangle\langle R| + \text{h.c.} \right). \tag{32}$$

Crucially, in the AB regime, the photon field perturbation is negligible ($|\chi_L\rangle_F \approx |\chi_R\rangle_F$), so $\langle \chi_L | \chi_R \rangle_F \approx 1$. Then the off-diagonal terms ($|L\rangle\langle R|$) retain the phase $\Delta\phi$ *independent* of the photon overlap. Thus, while entanglement exists, it does not influence the observable phase.

### 4.3 Semiclassical Consistency

The AB phase can be derived *without* invoking entanglement: In the path integral formulation, the phase arises from $\exp\left(\frac{iq}{\hbar} \int \mathbf{A} \cdot d\mathbf{l}\right)$, with no reference to photon states, and the semiclassical treatment [1] uses only the classical **A**-field. This reinforces that entanglement in Marletto and Vedral's model is not a physical requirement.

To sum up, while Marletto and Vedral's model formally introduces entanglement between the charge and photon field, this entanglement: (1) Does not determine the AB phase (which is fixed by **A**); (2) Has negligible effect on observables ($\langle \chi_L | \chi_R \rangle_F \approx 1$); and (3) Is absent in simpler derivations of the effect. Thus, the AB phase remains a manifestation of the vector potential's role, not quantum correlations with the photon field.

## 5 Conclusion

Our analysis of Marletto and Vedral's quantum field theory model of the Aharonov-Bohm (AB) effect reveals several critical points. Their assertion that the AB phase is locally mediated by entanglement and gauge-independent for non-closed paths does not hold under scrutiny. Through a quantum electrodynamics (QED) framework, we demonstrate that the phase originates from the coupling between the charged particle's current and the solenoid's current via the photon propagator, with the vector potential **A** providing an effective description of the phase shift. Contrary to their claims, we find that the phase is gauge-dependent for non-closed paths, aligning with the conventional understanding of the AB effect. Furthermore, their proposed field-based energy is irrelevant to the phase's generation. While entanglement appears in the QED description, it is incidental rather than causal, as the phase is fundamentally driven by the interaction of the charged particle with the quantized electromagnetic field. Our analysis reinforces the conventional explanation of the AB effect in the semi-classical picture.

## References


[1] Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in the quantum theory," *Phys. Rev.* **115**, 485–491 (1959).

[2] C. Marletto and V. Vedral, "Aharonov-Bohm phase is locally generated like all other quantum phases," *Phys. Rev. Lett.* **125**, 040401 (2020).