A No-Go Theorem for ψ-ontic Models? No, Surely Not!

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Abstract

In a recent reply to my criticisms (Found. Phys. 55:5, 2025), Carcassi, Oldofredi, and Aidala (COA) admitted that their no-go result for ψ-ontic models is based on the implicit assumption that all states are equally distinguishable, but insisted that this assumption is a part of the ψ-ontic models defined by Harrigan and Spekkens, thus maintaining their result's validity. In this note, I refute their argument again, emphasizing that the ontological models framework (OMF) does not entail this assumption. I clarify the distinction between ontological distinctness and experimental distinguishability, showing that the latter depends on dynamics absent from OMF, and address COA's broader claims about quantum statistical mechanics and Bohmian mechanics.

1 Introduction

Last year, Carcassi, Oldofredi, and Aidala (COA) argued that the ψ-ontic models framework (OMF) defined by Harrigan and Spekkens [1] cannot be consistent with quantum mechanics (QM) [2]. This was a surprising claim. I subsequently presented a critical analysis of their no-go result [3], arguing that COA implicitly assume all ontic states can be distinguished by experiments with certainty—an assumption neither part of OMF nor consistent with QM. In their recent reply [4], COA conceded that their result relies on this distinguishability assumption but contended it is inherent to OMF, thereby upholding their theorem. Here, I demonstrate that this is not the case, refining my critique with additional clarity and responding to their latest arguments.

2 The Ontological Models Framework

OMF has two fundamental assumptions [5]. First, if a quantum system is prepared such that QM assigns a wave function to it, then after preparation, the system possesses a well-defined set of physical properties or an underlying ontic state, represented by a mathematical object λ. A wave function or pure state corresponds to a probability distribution $p(\lambda|P)$ over the ontic states λ associated with a specific preparation P, quantifying the likelihood of each ontic state given that preparation. The probability distributions for two different wave functions may overlap (in ψ-epistemic models) or not (in ψ-ontic models). A mixture of pure states $|\psi_i\rangle$ with probabilities p_i is represented by $\sum_i p_i p(\lambda|P_{\psi_i})$.

Second, when a measurement is performed, the behavior of the measuring device is determined by the ontic state λ and the device's physical properties. For a projective measurement M, the probability of outcome k is $p(k|\lambda, M)$, and consistency with QM requires:

$$\int d\lambda\, p(k|\lambda, M)p(\lambda|P) = p(k|M, P), \tag{1}$$

where $p(k|M, P)$ is the Born probability. Neither assumption states that all ontic states can be distinguished experimentally with certainty.

3 COA's Argument and Their Reply

COA's no-go theorem hinges on comparing the information entropy of a mixed state in ψ-ontic models and QM [2]. For a mixture $\rho = \frac{1}{2}(|\psi\rangle\langle\psi| + |\phi\rangle\langle\phi|)$, QM yields the von Neumann entropy:

$$H_{\rm QM}(\rho) = -\frac{1+p}{2}\ln\frac{1+p}{2} - \frac{1-p}{2}\ln\frac{1-p}{2}, \tag{2}$$

where $p = |\langle\psi|\phi\rangle|$. In ψ-ontic models, where $|\psi\rangle$ and $|\phi\rangle$ correspond to distinct ontic states λ_ψ and λ_ϕ, COA compute the Shannon entropy as:

$$H_{\rm OM}(\rho) = -\frac{1}{2}\ln\frac{1}{2} - \frac{1}{2}\ln\frac{1}{2} = 1, \tag{3}$$

which differs from $H_{\rm QM}(\rho)$ unless $p = 0$ (orthogonal states). They conclude that ψ-ontic models cannot replicate QM.

In my critique [3], I argued this relies on assuming λ_ψ and λ_ϕ are distinguishable with certainty, a premise not in OMF. COA's reply [4] admits this assumption but asserts it is embedded in OMF, as statistical mixtures are "classical probability distributions" implying equal distinguishability.

4 Refutation of COA's Argument

However, OMF does not assume all ontic states are experimentally distinguishable with certainty, as evident from its two assumptions. COA's claim that "statistical mixtures are modeled as classical probability distributions" implies distinguishability is difficult to follow, and it is unclear whether they intended it as a formal justification. They argue that $\sum_i p_i p(\lambda|P_{\psi_i})$ treats mixtures as Kolmogorovian probability measures, where all states are distinguishable [4]. However, this misinterprets OMF.

A key point here is that while all ontic states are distinct in ontology—meaning they represent different physical realities—this does not imply they are distinguishable in experiments. In QM, empirical distinguishability is not an inherent property of the ontic states themselves but is determined by the dynamics of the system during measurement interactions. For example, non-orthogonal states, such as $|\psi\rangle$ and $|\phi\rangle$ with $\langle\psi|\phi\rangle \neq 0$, cannot be perfectly discriminated due to the linearity of quantum evolution, which limits the information accessible via measurement. COA's reply assumes that modeling statistical mixtures as classical probability distributions (e.g., $\sum_i p_i p(\lambda \mid P_{\psi_i})$) implies equal distinguishability of all ontic states in experiments [4]. However, this conflates ontological distinctness with empirical accessibility, a misstep not required by OMF. Their definition of a probability measure over ontic states does not account for the quantum constraint that non-orthogonal states lack perfect distinguishability, undermining their claim that this assumption is inherent to OMF.

In fact, if OMF implied this (experimental) distinguishability, it would be obviously inconsistent with QM, where non-orthogonal states are indistinguishable (in experiments). This basic principle of QM is widely recognized, making it surprising that COA's argument hinges on an assumption at odds with it. They did not pursue this simpler inconsistency proof, suggesting they may not fully endorse this interpretation of OMF.

Finally, it is worth pointing out that OMF, with its two fundamental assumptions, does not include the dynamics for the ontic states and thus is not a complete theory. Rather, it serves as a general framework designed to clarify the relationship between the underlying ontology and the predictive formalism of QM, or to explore how the latter might emerge from objective physical features of the ontic states. Consequently, one cannot compute information entropy directly from an epistemic state in OMF without dynamics, as entropy depends on distinguishability. For instance, consider non-orthogonal states $|\psi\rangle$ and $|\phi\rangle$ with overlapping $p(\lambda|P_\psi)$ and $p(\lambda|P_\phi)$. In QM, their indistinguishability under linear dynamics ensures the von Neumann entropy reflects this, differing from the Shannon entropy of a classical mixture where states are fully distinguishable. Without dynamics, OMF cannot resolve this, but with the Schrödinger dynamics, it aligns with QM.

5 Discussion

COA's reply suggests that my earlier critique overlooks their broader challenge: that OMF must reproduce all results of quantum statistical mechanics, thermodynamics, and information theory, not just entropy [4, Section 3]. However, my argument does not deny this requirement but highlights that their no-go theorem fails because it imposes an extraneous distinguishability assumption not inherent to OMF. Without this assumption, OMF remains open to dynamics (e.g., Schrödinger evolution) that align it with QM's predictions, including statistical mechanics. COA's example of rotational symmetry in a spin-1/2 mixture further illustrates the need for dynamics—precisely my point—yet they do not show how their classicality assumption is necessitated by OMF itself. Thus, their critique mischaracterizes my position as incomplete rather than addressing the core flaw I identify.

COA also claim that Bohmian mechanics may not conform to OMF due to the nomological status of the wave function [4, Section 4]. However, this overlooks that OMF is agnostic about specific dynamics or metaphysical commitments. In Bohmian mechanics, the ontic state (particle positions guided by the wave function) satisfies OMF's assumptions: preparation yields a probability distribution over positions, and measurement outcomes depend deterministically on these states. Whether the wave function is nomological or ontological is irrelevant to OMF's structure, suggesting COA's exclusion of Bohmian mechanics reflects a misreading of the framework's flexibility.

6 Conclusion

Carcassi, Oldofredi, and Aidala's (COA) no-go theorem for ψ-ontic models hinges on an assumption of experimental distinguishability of all ontic states, a condition neither inherent to the ontological models framework (OMF) nor consistent with quantum mechanics (QM). Their reply fails to justify this assumption's place in OMF, conflating ontological distinctness with empirical accessibility—a distinction clarified by QM's dynamical constraints. With appropriate dynamics, such as Schrödinger evolution, ψ-ontic models can align with QM's predictions, undermining COA's claim of incompatibility. Their broader critiques, including Bohmian mechanics' exclusion, misread OMF's flexibility. Thus, far from a definitive no-go result, this analysis reaffirms the viability of ψ-ontic interpretations within a properly understood framework.

References

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