

Quantum Measurement Without Collapse or Many Worlds: The Branched Hilbert Subspace Interpretation

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Abstract

The interpretation of quantum measurements presents a fundamental challenge in quantum mechanics, with concepts such as the Copenhagen Interpretation (CI), Many-Worlds Interpretation (MWI), and Bohmian Mechanics (BM) offering distinct perspectives. We propose the Branched Hilbert Subspace Interpretation (BHSI), which describes measurement as branching the local Hilbert space of a system into decoherent subspaces. We formalize the mathematical framework of BHSI using branching and the engaging and disengaging unitary operators to relationally and causally update the states of observers. Unlike the MWI, BHSI avoids the ontological proliferation of worlds and copies of observers, realizing the Born rule based on branch weights. Unlike the CI, BHSI retains the essential features of the MWI: unitary evolution and no wavefunction collapse. Unlike the BM, BHSI does not depend on a nonlocal structure, which may conflict with relativity. We compare CI, MWI, and BHSI in the double-slit experiment, Bell tests, Wigner and his friend, the black hole information paradox, and the delayed choice quantum eraser. Additionally, we examine the environmental scale of branching in MWI versus BHSI (maximal vs. minimal) and investigate whether recohering branches can be realized. Overall, BHSI offers a minimalist, unitarity-preserving, collapse-free, and probabilistically inherent alternative interpretation of quantum measurements.

Keywords: Bohmian Mechanics, Born Rule, Branched Hilbert Subspace Interpretation, Copenhagen Interpretation, Many-Worlds Interpretation, No-Hiding Theorem

1. Introduction

The interpretation of quantum mechanics (QM) has been debated since its inception in the 1920s. The theory's mathematical formalism, such as unitary evolution, superposition, and entanglement, yields strikingly non-classical predictions, yet its physical meaning remains contested. The Copenhagen Interpretation (CI; Bohr, Heisenberg, Born, Pauli; 1920s-1950s, [1-3]) provides a mathematically simple framework that aligns with lab observations. However, it faces criticism for its undefined wave function collapse, the straightforward postulation of the Born rule [4], the cornerstone of QM probabilistic predictions, and the subjective boundary separating quantum and classical regimes. The Many-Worlds Interpretation (MWI; Everett, DeWitt, Deutsch, Wallace; 1957-present, [5-7]) addresses the measurement problem by postulating that all possible quantum measurement outcomes occur in separate, non-interacting branches of reality (each branch is a world with a copy of the observer), thereby offering a compelling solution by eliminating wavefunction collapse. Still, it encounters significant challenges regarding its ontological excess, the lack of a convincing explanation for the Born rule, and the preferred basis issue [8-11]. Bohmian Mechanics (BM, Bohm, Bell, Goldstein; 1952-present; [12-14]), also known as the de Broglie-Bohm pilot-wave theory, resolves the wave

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collapse issue of CI within a single world, but it relies on hidden variables (actual particle positions), and its explicit nonlocality structure may conflict with relativity.

We propose an alternative approach: the Branched Hilbert Subspace Interpretation (BHSI), in which measurement splits the local Hilbert space into multiple branches instead of partitioning the universe into parallel worlds in the global Hilbert space. Since each possible outcome exists and evolves within one branch, no wave function collapses. The observer's state is updated relationally and causally, resulting in one outcome per observation. The Born rule [4] can be realized by assigning weight (probability) to each branch based on the initial state represented on the basis chosen by the observer. With only one observer in a single world, it does not face the ontological challenge of explaining probability in the MWI.

We formalize the mathematical framework of BHSI by defining branching and the engaging and disengaging (EGD) unitary operators. We explain how the EGD operator updates the observer's state. We compare BHSI with CI and MWI by exploring their implications for interference (double-slit experiment [15-17]), nonlocality (Bell tests [18-19]), causal dominance (Wigner's friend) [5,20-21], black hole radiation with the No-Hiding Theory (NHT) [22-23], and the delayed choice quantum eraser [24-25]. The last section discusses the nature of branching and compares the environmental scale of quantum decoherence [26-28] in MWI vs BHSI (the maximal vs. the minimal). Overall, BHSI can be viewed as a lightweight version of MWI, and it is interesting to investigate whether BHSI could potentially recohere branched local subspaces.

2. Mathematical Framework

In this section, we present the fundamental concepts of BHSI: branching local Hilbert spaces, updating (engaging and disengaging) the observer's state, and the Born rule.

2.1. The Branching, Engaging, and Disengaging Operators

Assuming the observer chooses to measure an observable \hat{G} , the following linear combination on the G -basis describes the initial quantum state ([2, p.29]):

$$|\Psi\rangle = \sum_{i=1}^D c_i |g_i\rangle, \quad \hat{G}|g_i\rangle = g_i |g_i\rangle, \quad \langle g_i | g_j \rangle = \delta_{i,j}, \quad \sum_{i=1}^D |c_i|^2 = 1, \quad \prod_{i=1}^D c_i \neq 0 \quad (1)$$

The initial Hilbert space is D -dimensional, corresponding to the D possible outcomes of the measurement, each with a non-zero probability. The *branching operator* \hat{B} is a unitary operator that splits the D -dimensional Hilbert space \mathcal{H}^D into D branches:

$$\hat{B} \equiv \sum_{k=1}^D |g_{B,k}\rangle \langle g_k|, \quad \hat{B}^\dagger \hat{B} = I, \quad \hat{B} \hat{B}^\dagger = I_B, \quad \langle g_k | \Psi_B \rangle = \langle g_k | \hat{B}^\dagger \hat{B} | \Psi \rangle = \langle g_k | \Psi \rangle = c_k \quad (2)$$

$$\hat{B}(|\Psi\rangle \otimes |E\rangle_L) = |\Psi_B\rangle = \sum_{k=1}^D c_k |g_k\rangle |E_k\rangle_L = \sum_{k=1}^D c_k |g_{B;k}\rangle, \quad |g_{B;k}\rangle \equiv |g_k\rangle |E_k\rangle_L \quad (3)$$

$$\hat{B}(\mathcal{H}_S \otimes \mathcal{H}) = \bigoplus_{k=1}^D \mathcal{H}_{S,k}(\text{span } c_k |g_{B;k}\rangle), \quad |\langle g_k | \Psi \rangle|^2 = |\langle g_{B;k} | \Psi_B \rangle|^2 = |c_k|^2$$

Note that the states $|g_{B,k}\rangle$ are mutually decoherent, evolving in different branches, and the surrounding environment $|E\rangle_L$ is involved to make them decoherent. The *engaging and disengaging* (EGD) operator $\Sigma_\beta \equiv \Gamma_\beta T_\beta \Lambda_\beta$ is a product of three unitary operators². The first operator is the engaging operator Λ_β . It updates the observer's state from $|\text{ready}\rangle$ in the environment \mathcal{H}_E to $|\text{reads}\rangle$ and entangles the observer's state with the β^{th} subspace.

$$\Sigma_\beta \equiv \Gamma_\beta T_\beta \Lambda_\beta, \quad \Lambda_\beta : |\text{ready}\rangle_o \in \mathcal{H}_E \mapsto |\text{reads } g_\beta\rangle_o \in \mathcal{H}_{S,\beta} \quad (4)$$

The operator product $\Lambda_\beta \hat{B}$ randomly engages the observer with one branch:

$$\Lambda_\beta \hat{B} : \mathcal{H}_S \otimes \mathcal{H}_L \mapsto \mathcal{H}_B = \bigoplus_{k=1}^D \left\{ \mathcal{H}_{S,k}(\text{span } c_k |g_{B,k}\rangle) (|\text{reads } g_\beta\rangle_o)^{\Delta(k,\beta)} \right\}, \quad \beta \in \{1, 2, \dots, D\} \quad (5)$$

To simplify the expression, we have used the following notation:

$$\Delta(k, \beta) = \delta_{k,\beta} = \begin{cases} 1, & \text{if } k = \beta \\ 0, & \text{if } k \neq \beta \end{cases} \quad (\text{discrete case}), \quad (|\text{reads}\rangle_o)^{\Delta(k,\beta)} = \begin{cases} |\text{reads}\rangle_o, & \text{if } k = \beta \\ 1, & \text{if } k \neq \beta \end{cases} \quad (6)$$

After recording the outcome, operator T_β changes the observer's state to $|\text{ready}\rangle$, then operator Γ_β disengages him from the branch, ensuring he is prepared for the next engagement.

$$T_\beta : |\text{reads}\rangle_o \mapsto |\text{ready}\rangle_o; \quad \Gamma_\beta T_\beta : \mathcal{H}_B \mapsto \mathcal{H}_f = \left\{ \bigoplus_{k=1}^D \mathcal{H}_{S,k}(\text{span } c_k |g_{B,k}\rangle) \right\} \otimes |\text{ready}\rangle_o \quad (7)$$

Let $U(t)$ be the time evolution operator of the system, which can be relativistic or not:

$$U(t) |\Psi(0)\rangle = |\Psi(t)\rangle, \quad |\Psi\rangle \equiv |\Psi(0)\rangle, \quad c_k \equiv c_k(0), \quad U(t) |\Psi_B(0)\rangle = |\Psi_B(t)\rangle \quad (8)$$

The branching operator \hat{B} commutes with the time evolution operator:

$$\hat{B} \hat{U}(t) \{|\Psi\rangle\} = \hat{B} \sum_{k=1}^D c_k(t) |g_k\rangle = \sum_{k=1}^D c_k(t) |g_{B;k}\rangle \quad (9)$$

$$= \hat{U}(t) \hat{B} \{|\Psi\rangle\} = \hat{U}(t) \{|\Psi_B\rangle\} = \sum_{k=1}^D c_k(t) |g_{B;k}\rangle \quad (10)$$

Altogether, a measurement process can be described as a unitary transformation \hat{M}_β (β is a random choice):

² They act like the unitary NOT gate, flipping between the observer's states [24, p.233].

$$\hat{M}_\beta \equiv \Sigma_\beta \hat{B} = \Gamma_\beta T_\beta \Lambda_\beta \hat{B}, \quad \hat{M}_\beta^\dagger \hat{M}_\beta = \hat{B}^\dagger \Sigma_\beta^\dagger \Sigma_\beta \hat{B} = I, \quad \beta \in \{1, 2, \dots, D\} \quad (11)$$

$$\Gamma_\beta T_\beta \Lambda_\beta \hat{B} \{ |\Psi\rangle \otimes |\text{ready}\rangle_o |E\rangle_L \} = \Gamma_\beta T_\beta \left\{ \sum_{k=1}^D c_k |g_{B,k}\rangle \otimes (|\text{reads}\rangle_o)^{\Delta(k,\beta)} \right\} = |\Psi_B\rangle \otimes |\text{ready}\rangle_o \quad (12)$$

2.2. The Measurement Process and the Born Rule in BSHI

The initial Hilbert space is D -dimensional, as Eq. (1) describes. We discuss three cases.

Case 1: $D = 1$. The initial normalized state contains only one basis state.

$$|\Psi\rangle = |g_1\rangle \quad (13)$$

Since this reflects the observer's measurement basis, the observer consistently records g_1 , with $P(g_1) = 1$, by unitarily branching, engaging, and disengaging. Only one branch exists, containing $|g_{B,1}\rangle$ after the measurement. There is no loss of information or gain of entropy.

Case 2: $D \geq 1$. Before the observation, the system (S), the local environment $|E\rangle_L$, and the state of the observer or the apparatus (O) are in the following pre-measurement state:

$$|\Psi_o\rangle = |\Psi\rangle \otimes |\text{ready}\rangle_o |E\rangle_L, \quad |\Psi\rangle = \sum_{k=1}^D c_k |g_k\rangle \quad (14)$$

According to Eq. (5), branching the system causes its local Hilbert space to split into D parallel subspaces, each spanning a basis state. The observer engages with one branch, which has an associated weight (chance) based on the initial state, thereby realizing the Born rule:

$$\mathcal{H}_S \otimes \mathcal{H}_L \rightarrow \bigoplus_{k=1}^D \mathcal{H}_{S,k} [\text{span } c_k |g_{b,k}\rangle (|\text{reads } g_\beta\rangle_o)^{\Delta(k,\beta)}], \quad P(\beta) = |c_\beta|^2 = |\langle g_\beta | \Psi \rangle_S|^2 \quad (15)$$

The MWI features parallel decoherent worlds within the global Hilbert space. In contrast, the BSHI operates minimally, involving only the system's local Hilbert space, minimal environment, and the observer's state. The observer (or apparatus) becomes entangled with one branch and reads the corresponding outcome with a specific probability. Each branch contains only a single basis state, and its evolution adheres to unitarity, similar to the $D = 1$ case. Therefore, wave collapse in the CI is circumvented without the necessity of many worlds in the MWI. After the measurement, the observer disengages from the branched system state, as illustrated by Eq. (7):

$$|\Psi_f\rangle = |\Psi_B\rangle \otimes |\text{ready}\rangle_o = \left\{ \sum_{k=1}^M c_k |g_{b,k}\rangle \right\} \otimes |\text{ready}\rangle_o \quad (16)$$

Case 3: $D = 2$. This is a specific example of Case 2: the initial state consists of only two basis states. We aim to use this case to compare step-by-step with the MWI. Assume that Bob is observing a qubit. Before the measurement, we have:

$$\text{MWI: } |\Psi_o\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle) |B\rangle |E\rangle, \quad |\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (17)$$

$$\text{BSHI: } |\Psi_o\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle) |\text{ready}\rangle_o |E\rangle_L, \quad |\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (18)$$

The equation appears similar. The difference is that in MWI, the environment encompasses the entire world, including Bob, while BHSI's minimal environment is $|E\rangle_L$, including Bob's state. After branching, their states have the following forms:

$$\text{MWI: } |\Psi_f\rangle = \alpha_0 |0\rangle |B_0\rangle |E_0\rangle + \alpha_1 |1\rangle |B_1\rangle |E_1\rangle, \quad \langle B_0 | B_1 \rangle \approx 0, \langle E_0 | E_1 \rangle \approx 0 \quad (19)$$

$$\text{BHSI: } |\Psi_B\rangle = \sum_{k=0}^1 \alpha_k |k_B\rangle (|\text{reads } k\rangle_o)^{\Delta(k,\lambda)}, \quad \lambda \in \{0,1\}, \quad P(\lambda) = |\alpha_\lambda|^2 \quad (20)$$

$$\text{Or: } |\Psi_B\rangle = \alpha_0 |0_B\rangle (|\text{reads } 0\rangle_o)^{\Delta(0,\lambda)} + \alpha_1 |1_B\rangle (|\text{reads } 1\rangle_o)^{\Delta(1,\lambda)}, \quad \lambda \in \{0,1\}, \quad P(\lambda) = |\alpha_\lambda|^2 \quad (21)$$

The BHSI borrows the branching idea from the MWI. However, instead of updating the universal wave function in the global Hilbert space, the BHSI only updates the minimal environment with Bob's state in one of the local spaces (see Fig. 1). After the branching, in the MWI, each branch is a real world with a real Bob, which is the end of the observation. In contrast, in the BHSI, Bob in the local Hilbert space is not a real person but rather the state of Bob as represented through the engaged part of his apparatus. After reading, Bob becomes disengaged, as described by Eq. (7). The final state contains two decoherent branches:

$$\text{BHSI: } |\Psi_B\rangle = \alpha_0 |0_B\rangle + \alpha_1 |1_B\rangle \quad |\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (22)$$

Assuming Bob reads 1 ($\lambda = 1$). During the entire measurement process, Bob experiences three stages (before, during, and after the measurement), as described by Eqs (11-12):

$$|\Psi\rangle \otimes |\text{ready}\rangle_o |E\rangle_L \rightarrow \alpha_0 |0_B\rangle + \alpha_1 |1_B\rangle |\text{reads } 1\rangle_o \rightarrow |\Psi_B\rangle \otimes |\text{ready}\rangle_o \quad (23)$$

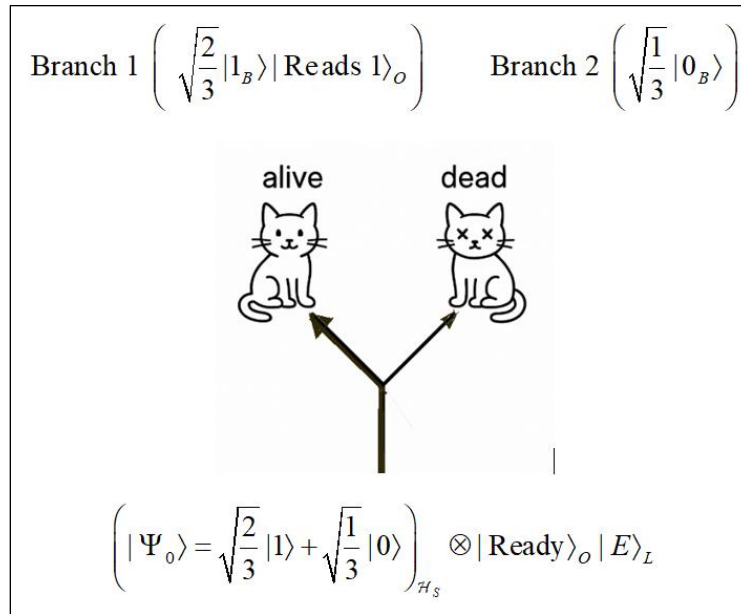


Fig. 1: The Branched Local Hilbert Subspaces

The branched local Hilbert spaces are eventually relocated into the environment at large by unitary transformations, complying with the No-Hiding Theorem (NHT, [23]):

$$U_E : |\Psi_B\rangle \otimes |E\rangle \rightarrow |E'\rangle \quad (24)$$

2.3. The Observer's Local View of the Measurement:

In quantum measurements or quantum computing, the observer must repeatedly measure the same initial states. Each time, he reads one possible outcome, with the probability predicted by the Born rule, which leads to the following density matrix [29, p.53]:

$$\rho = \sum_{k=1}^D |g_k\rangle |c_k|^2 \langle g_k|, \quad \sum_{k=1}^D |c_k|^2 = 1 \quad (25)$$

Locally, the observer sees that the initial pure state, Eq. (1), with zero von Neumann entropy [29, p.179], becomes a mixed state, and its von Neumann entropy is increased to:

$$S(\rho) = -\text{Tr}(\rho \ln \rho) = -\sum_{k=1}^D \{|c_k|^2 \ln |c_k|^2\} > 0 \quad (26)$$

The observer concludes that his measurement is irreversible because the system's entropy increases and certain information is lost. However, in the entire Hilbert space encompassing all branches, there is no loss of information or gain in entropy. This is quite similar to the MWI, except that MWI consists of many independent, equally real worlds, while BHSI features numerous independent local Hilbert subspaces with predictable weights (probabilities).

3. Comparison of CI, MWI, BM, and BHSI

Feature	Copenhagen (CI)	Many-Worlds (MWI)	Bohmian Mechanics (BM)	BHSI
1. Wave Collapse? Unitarity?	Yes. Non-unitary	No. Fully unitary by splitting the global Hilbert space	No. Fully unitary (wavefunction guides particles)	No. Fully unitary by splitting the local Hilbert space
2. Ontology: Number of Worlds and "Me"	A single world, a single "Me."	Many real worlds, each with a "Me."	A single world, a single "Me."	A single world, a single "Me."
3. Probability: The Born Rule	Fundamental postulate (no deeper explanation)	Emergent from decision theory? (self-locating uncertainty?)	Explained by the equilibrium distributions of hidden variables	Interpreted as the weights of local Hilbert branches.
4. The Role of the Observer	Passive, external to the system, and causes collapse	Branching, then following one world, and all worlds are real.	Passive (particles have definite positions at all times)	Branching, engaging, then disengaging from one Hilbert branch.

5. Determinism	Indeterministic (collapse introduces randomness)	Deterministic (but observers experience subjective randomness)	Deterministic (hidden variables define definite trajectories)	Deterministic (but observers experience local randomness)
6. Information Loss	Yes (collapse destroys superpositions permanently)	No (information persists in different worlds)	No (global wave function guides particles deterministically)	No (information persists in different Hilbert subspaces)
7. Can Branches Recombine?	N/A (only one world exists)	No (recoherence leads to identity crises)	N/A (only one world exists)	Yes? In theory, it is possible.
8. Locality of Physical Laws	Local (except for nonlocal collapse)	Local (no signal between branches)	Nonlocal (built-in by the global wave function)	Local (no faster-than-light action)

Table 1. Comparison of the Four Interpretations of Measurements

4. Comparing BHSI with MWI and CL by Examples

The BHSI is proposed as a “cost-effective” version of the MWI to avoid the collapse issue in the CI without the ontological excess of MWI. This section uses several examples to illustrate the similarities and differences between the three interpretations.

Example 4.1. The Double-Slit Experiment: It is the most popular experiment to explain the particle-wave duality of QM [15-16], including photons, electrons, and large C₆₀ molecules [17]. When a particle hits the screen, the local Hilbert space in BHSI splits into uncountable infinite branches (in theory), and the observer reads it at one position x .

$$|\Psi_B\rangle = \int dx' |x'\rangle \langle x' | \Psi_B \rangle [|\text{reads } x\rangle_O]^{\Delta(x,x')}, \quad \Delta(x,x') = \begin{cases} 1, & \text{if } x = x' \\ 0, & \text{if } x \neq x' \end{cases} \quad (\text{continuous case}) \quad (27)$$

$$|\langle x | \Psi_B \rangle|^2 = |\Psi(x)|^2 = |\Psi_I(x) + \Psi_{II}(x)|^2 \quad (28)$$

Because of the limitations of the experimental equipment, the integral in Eq. (27-28) should be replaced by a discrete summation over tiny pieces Δ_k :

$$|\Psi_B\rangle = \sum_k \Delta_k |x_k\rangle \langle x_k | \Psi_B \rangle [|\text{reads } x_k\rangle_O]^{\Delta(k,k')}, \quad P(\Delta_k) = |\Psi_I(x_k) + \Psi_{II}(x_k)|^2 \Delta_k \quad (29)$$

The BHSI and MWI rely on branching to maintain unitarity and interference without total information loss. In the BHSI, the observer disengages with the system after reading, and the interference or probability distribution (the Born rule) can be assigned naturally; however, in the MWI, the environment coherent with each piece Δ_k is a whole world with a real observer. In a typical double-slit experiment, tens of thousands of photons hit the screen, and each photon updates thousands of branches. Because of the ontological issue, there is no convincing

interpretation of probability in MWI yet: Many minds? Indexicalism? Decision theory? A rational bet on a particular result? Or Envariance [8,9]?

The CI can explain the interference by simply assuming the Born rule. Still, each particle's hit causes a wave collapse (FTL action), breaking unitarity and causing information loss.

Example 4.2. The Bell Tests of Entanglement: Applying the Born rule, all three interpretations can explain the violation of the Bell inequality [18-19] without spooky actions at a distance between the paired particles or the two observers. However, the costs are different. In CI, the measurements by Alice and Bob cause two wave collapses (FTL actions), leading to information loss. MWI and BHSI have no collapse and no total information loss. But, MWI ends with four Hilbert branches of worlds per photon pair, each containing Alice and Bob, while BHSI ends with four local Hilbert branches without multiple Alice and Bob.

MWI: Alice and Bob update four worlds per photon pair, each containing an Alice and a Bob:

$$A_1 : |0\rangle_a |Alice_{0,A}\rangle |Bob_{0,A}\rangle |E_{0,A}\rangle, \quad A_2 : |1\rangle_a |Alice_{1,A}\rangle |Bob_{1,A}\rangle |E_{1,A}\rangle \quad (30)$$

$$B_1 : |0\rangle_b |Alice_{0,B}\rangle |Bob_{0,B}\rangle |E_{0,B}\rangle, \quad B_2 : |1\rangle_b |Alice_{1,B}\rangle |Bob_{1,B}\rangle |E_{1,B}\rangle \quad (31)$$

BHSI: Alice and Bob update four branches per photon pair in their local Hilbert space,

$$A_1 : |0_B\rangle_a (| \text{reads } 0 \rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle_a, \quad A_2 : |1_B\rangle_a (| \text{reads } 1 \rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle_a, \quad \alpha \in \{0,1\} \quad (32)$$

$$B_1 : |0_B\rangle_b (| \text{reads } 0 \rangle_O)^{\Delta(\beta,0)} \rightarrow |0_B\rangle_b, \quad B_2 : |1_B\rangle_b (| \text{reads } 1 \rangle_O)^{\Delta(\beta,1)} \rightarrow |1_B\rangle_b, \quad \beta \in \{0,1\} \quad (33)$$

Typically, millions of photon pairs are measured by Alice and Bob in a Bell test.

Example 4.3. Wigner's Friend Thought Experiment [5, 19-20] is a compelling example involving mixed observers. Setup: The Friend (F) observes a qubit state: $(|0\rangle + |1\rangle)/\sqrt{2}$ in a Lab; simultaneously, Wagner (W), outside, observes F and the qubit. What occurs?

CI: F collapses the qubit, and W sees what F sees. One collapse. Why? F is the preferred observer (he measures the qubit), and F is a classical object that cannot entangle with a qubit.

MWI: F updates two worlds in the global Hilbert space, each containing an F and a W:

$$H_1 : |0\rangle |F_0\rangle |W_0\rangle |E_0\rangle, \quad H_2 : |1\rangle |F_1\rangle |W_1\rangle |E_1\rangle \quad (34)$$

At the same time, W also updates two worlds, each containing an F and a W, too:

$$H_3 : |0\rangle |F_0\rangle |W_0\rangle |E_0\rangle, \quad H_4 : |1\rangle |F_1\rangle |W_1\rangle |E_1\rangle \quad (35)$$

There is no collapse, no preferred observer, and F can be entangled with a qubit. Moreover, we can set $H_1 = H_3$ and $H_2 = H_4$, because H_1 & H_3 (H_2 & H_4) are physically indistinguishable,

leading to one branching, two worlds. No matter whether it is two or four worlds, there is no identity conflict. If F and W shake hands, they must see the same result and in the same world.

BHSI: Friend updates two decoherent local branches, engages one, and then disengages:

$$H_1 : |0_B\rangle(|\text{reads } 0\rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle, \quad H_2 : |1_B\rangle(|\text{reads } 1\rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle, \quad \alpha \in \{0,1\} \quad (36)$$

Because Friend measures the qubit, his branching is dominant; the local Hilbert subspaces must be updated synchronously with his, so Wagner's two branches should synchronize with Friend's:

$$H_3 : |0_B\rangle(|F \text{ reads } 0\rangle|\text{reads } 0\rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle, \quad H_4 : |1_B\rangle(|F \text{ reads } 1\rangle|\text{reads } 1\rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle \quad (37)$$

Wagner will see an outcome of 0/1 if his friend engages with H_1/H_2 . Like the MWI, the process is unitary, with no information loss or collapse; the friend's state can be entangled with a qubit, with no preferred observer but a causally dominant branching. Similar to the CI, only one world with one Wagner and one Friend; they see the same result and can always shake hands.

Example 4.4. *The Black hole information paradox:* Hawking's semi-classical calculations suggest that black hole evaporation via Hawking radiation is thermal and random [22]. If so, it destroys information about the infalling matter, violating unitarity. MWI and BHSI have their own branching structure (global vs. local) for modeling Hawking radiation, which is consistent with the No-Hiding theory (NHT, [23]). However, the Hawking radiation in the CI causes collapses and information loss, violating the NHT.

Example 4.5. *The delayed choice quantum eraser experiments* [24-25]: In the MWI and BHSI, whether an observer sees the interference depends on which Hilbert branches (already recorded when signal photons hit the screen) the observer later chooses based on the path of the idle photons (which-way information kept or erased). There is no collapse or retrocausality, but many worlds in MWI and many local subspaces in BHSI. In the CI, reality is only settled when the measurement is fully completed, so there is no retrocausality, even though partial collapses may occur when signal photons hit the screen.

5. Branching and Possible Debranching: MWI vs. BHSI

In MWI, each branch is a whole, independent, and real world. Within a world, "objects" have definite macroscopic states by fiat [Eq. (1), 8]:

$$|\Psi_{\text{WORLD}}\rangle = |\Psi_{\text{OBJ. } 1}\rangle |\Psi_{\text{OBJ. } 2}\rangle \cdots |\Psi_{\text{OBJ. } N}\rangle |\Phi\rangle \quad (38)$$

The product state is only for relevant variables for the macroscopic description of the objects. There might be some entanglement between weakly coupled variables, which should belong to $|\Phi\rangle$. The universe is expressed as a superposition of all existing worlds:

$$|\Psi_{\text{UNIVERSE}}\rangle = \sum_i^M \alpha_i |\Psi_{\text{WORLD } i}\rangle, \quad \sum_i^M \alpha_i = 1 \quad (39)$$

Determining how to configure a world with about 10^{80} particles is hard (preferred basis?), and nobody knows how big the total M is, except that it is exponentially growing (just one double-slit experiment will add millions of branches). As described in Case 3, Section 2.2, when measuring a qubit, one of the branches in Eq. (39) (where the observer lives) is entangled with the two-qubit states described in Eqs. (17) and (19), resulting in two independent worlds, each having a Bob. Although mathematically possible, recohering the two branches in Eq. (19) or any two in (39) is ontologically forbidden (it causes identity crises) and practically impossible for any two worlds.

Contrary to MWI, the branches in the BHSI are local Hilbert subspaces. Based on the quantum decoherence theory [26-28], the branching operator in Eq. (3) can be understood as follows:

$$\hat{B} : \left(\sum_{k=1}^N c_k |g_k\rangle \right) \otimes |E\rangle_L \rightarrow \sum_{k=1}^N c_k |g_k\rangle |E_k\rangle_L \equiv \sum_{k=1}^N c_k |g_{B;k}\rangle, \quad {}_L\langle E_i | E_k \rangle_L \approx \delta_{i,k} \quad (40)$$

Here, $|E\rangle_L$ represents the minimal local environment, which directly interacts with the quantum system and contains about $10 \sim 100$ particles. Thus, the nature of branching is the same for MWI and BHSI. The difference lies in the size of their respective environments: a whole world versus the local environment (maximal versus minimal, or 10^{80} versus 10^2). Therefore, controlled recoherence in BHSI is mathematically, ontologically permissible, and practically conceivable. In theory, one may construct a debranching operator for the recoherence of decohered branches:

$$B^\dagger (\alpha_1 |\psi_{B,1}\rangle + \alpha_2 |\psi_{B,2}\rangle) = B^\dagger (\alpha_1 |\psi_1\rangle |E_1\rangle_L + \alpha_2 |\psi_2\rangle |E_2\rangle_L) = (\alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle) \otimes |E\rangle_L \quad (41)$$

This suggests a potential test to differentiate between MWI and BHSI. Experiments such as delayed choice and quantum eraser [24-25], quantum error correction [30], or trapped ions entangled with photons [31] could be utilized for this purpose. However, the possibility of practically debranching the branched local Hilbert spaces before they are relocated within the environment remains an open question.

6. Conclusion and Discussion

While sidestepping the concept of many worlds and observers, the Branched Hilbert Subspace Interpretation (BHSI) retains all the benefits of the Many-Worlds Interpretation (MWI), such as unitarity, no collapse, and deterministic evolution. It also preserves all the advantages of the Copenhagen Interpretation (CI), offering one world and a single observer while circumventing wave collapse. Compared to CI and MWI, BHSI provides a balanced perspective:

- Ontological simplicity: No parallel worlds or preferred basis.
- Unitarity: No collapse, no information loss.
- Casual dominance: Whoever decoheres the system first defines the branching structure.
- The Born rule: Probability can be assigned to branches, not simply assumed.

- Testability: Predicts standard quantum results with fewer metaphysical commitments.

The BHSI remains a developing framework that requires further mathematical refinement and empirical engagement through quantum experiments and thought scenarios. Nevertheless, it offers a promising middle ground for those uncomfortable with wavefunction collapse in the CI and skeptical of the MWI's ontological commitments. BHSI provides a conceptually coherent and physically grounded approach to quantum measurement by preserving unitarity and causal consistency within a single-world ontology.

Abbreviations

BHSI	Branched Hilbert Subspace Interpretation
BM	Bohmian Mechanics
CI	Copenhagen Interpretation
EGD	Engaging and Disengaging
FTL	Faster Than Light
MWI	Many-Worlds Interpretation
NHT	No-Hiding Theorem
QM	Quantum Mechanics

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