Analogue Gravity and the de Broglie wave: a Missed Opportunity

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Abstract

In a small book entitled *Ondes et Mouvements* [1], published in February 1926, Louis de Broglie described the wave, now known as the de Broglie wave, as a modulation or beating effect of undulatory form induced in the structure of the particle by the failure of simultaneity. Considered in this way, the de Broglie wave is neither ontologically distinct, nor in any way separate, from the particle, but like the Fitzgerald-Lorentz contraction is a distortion in the structure of the particle itself. So understood, the de Broglie wave is a physically real phenomenon, capable of describing for the particle, a well-defined and physically realistic trajectory. In comparison, and as I argue in this paper, the wave functions that emerge as solutions to the Schrödinger and Klein-Gordon equations are better regarded as mathematical constructs, albeit constructs of significant utility, identifying the wave number and frequency that the particle *would have* at each point of space if it *were in fact* at that point of space. A particular concern of this paper will be to show that the de Broglie wave would emerge as such a distortion of structure in certain sonic quasiparticles proposed in the context of analogue gravity for the purpose of simulating the Lorentz transformation.

Keywords Analogue gravity · quantum gravity · Minkowski spacetime · Lorentz transformation · de Broglie wave · wave function

1 Introduction

This presentation[1] concerns an opportunity - a missed opportunity - to employ the methods of analogue gravity to demonstrate the physical origin of the de Broglie wave as that origin was described by de Broglie himself in a small book entitled *Ondes et Mouvements* [1] published in Paris in February, 1926.

In *Ondes et Mouvements*, which de Broglie completed little more than a year after his better known thesis [2], he described the wave, not as a true

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wave, but as a modulation or beating effect of undulatory or sinusoidal form, induced in the underlying structure of the particle by the failure of simultaneity. Understood in this way, the de Broglie wave is neither ontologically distinct, nor in any way physically separate, from the particle but, like the Fitzgerald-Lorentz contraction, is a distortion in the structure of the particle itself.

In the concluding paragraph of the thesis, de Broglie had acknowledged that he had provided only "vague" and "rather tentative" definitions of the de Broglie wave and the "periodic phenomenon" from which it emerges. *Ondes et Mouvements* may be seen as his attempt to remedy these deficiencies in the thesis, and in this I believe he was eminently successful. I will argue that in *Ondes et Mouvements,* he provided the only interpretation of the de Broglie wave that makes physical sense.

But unfortunately for the development of quantum mechanics, and as I will also explain in this paper, *Ondes et Mouvements* [1] was very soon sidelined by the publication of Schrödinger's important papers on wave mechanics (Schrödinger [3]) and Born's proposal that the wave functions from the Schrödinger equation are objectively probabilistic (Born [4]).

To show how the de Broglie wave might arise as the undulatory distortion proposed by de Broglie, I will discuss two sonic analogues that employ the methods of analogue gravity to simulate, not the Hawking radiation that has been the primary interest of analogue gravity, but the effects of the Lorentz transformation. In these analogues, the role of light is played instead by sound. One such analogue is described in a paper by Barceló and Jannes [5], and the other is due to Todd and Menicucci [6]. These analogues show how changes predicted by the Lorentz transformation might be simulated in a universe in which everything within the universe, including particles and forces and observers and measuring devices, is formed from sonic waves[2].

These curious universes have been referred to as "fishbowl universes", it being possible to contemplate two kinds of observer, one within the fishbowl where everything is constructed from effects that evolve at the speed of sound, and the other who is outside the fishbowl and like some supernatural being is able to look into the bowl to observe and understand the strange workings of (sonic) special relativity. Of course this god-like observer may simply be the post-doc who is running the experiment, but from their privileged position outside the bowl, this *external* observer will perceive how changes of length, time and simultaneity ensure that the speed of light and the laws of physics are the same for all *internal* observers.

Neither of the two papers actually mentions the de Broglie wave. Yet the de Broglie wave is also a consequence of the Lorentz transformation, specifically of the failure of simultaneity. I will show that if the sonic analogue of a massive particle were constructed in the manner described in these sonic analogues, it

[2] For discussions of the implications of these sonic analogues for the fundamentality of the speed of light, see Cheng and Read [7] and Shanahan [8].

would engender the de Broglie wave in precisely the manner described in *Ondes et Mouvements.*

It is important to notice that this way of understanding the de Broglie wave is not an alternative to some orthodox or "standard" explanation of the wave. Standard quantum mechanics (SQM) has no such explanation. If you were to consult a standard text or ChatGPT or Wikipedia, you would be referred to the concept of wave-particle duality and would learn that a particle acts sometimes as a particle and sometimes as a wave, and that the wave serves, in some mysterious manner, as a wave of probability.

You might also be told (quite correctly) that this wave or wave-like phenomenon has a frequency ω_E directly related to the energy E of the particle by the Planck-Einstein relation,

$$E = \hbar\omega_E = \hbar\gamma\omega_o, \tag{1}$$

and a wave number κ_{dB} similarly related to the momentum p of the particle by the de Broglie relation,

$$p = \hbar\kappa_{dB} = \hbar\gamma\,\omega_o\frac{v}{c^2}, \tag{2}$$

where $\hbar$ is the reduced Planck's constant, ω_o is the characteristic frequency of the particle at rest, v is the velocity of the particle, and c is the speed of light in vacuum.

All this is standard fare, but these are not explanations. You would not learn what this wave actually is, nor why a particle sometimes behaves as a wave and sometimes as a particle, nor why the wave is superluminal, nor why it emerges only when the particle is moving. The answers to all these questions *are* apparent in *Ondes et Mouvements.*

Since 1926, the interpretation of the de Broglie wave as a relativistically-induced beating effect has been discovered and rediscovered many times.. It now has a modest literature[3]. Thus a question I will need to address in this paper is why, if the interpretaion presented in that literature is physically reasonable and has no apparent alternative, it has not yet achieved the status of orthodoxy.

But before proceeding further, I should explain what I mean by a modulation or beat induced by the failure of simultaneity.

2 Beats, simultaneity and the de Broglie wave

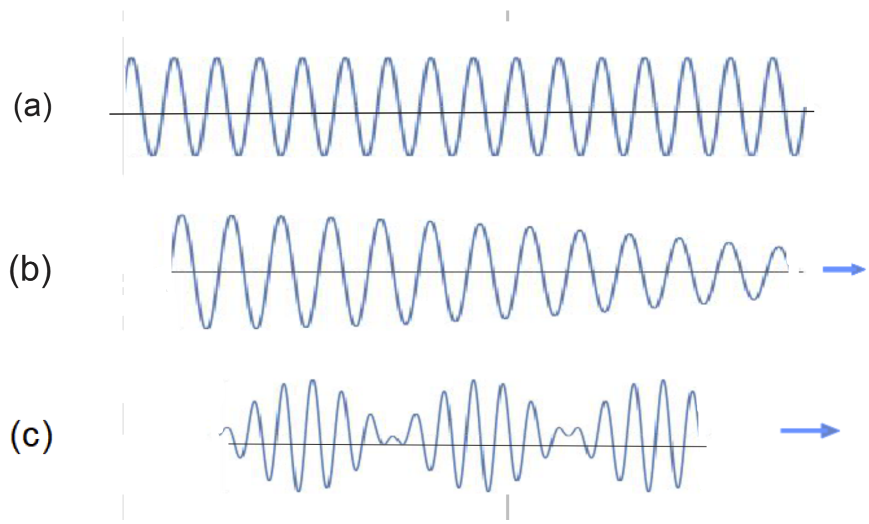
A beat or phase modulation[4] is a periodic variation in intensity caused by interference between two waves of different frequency. Its occurrence in music

[3]See the listing attempted in Ref. [9], as well as two more recent papers of my own, published last year, one in *Foundations of Physics* [8], the other pursuant to a presentation at a conference at the Sorbonne commemorating the centenary of de Broglie's first papers on the wave [13].

[4]I have not burdened this presentation with the mathematical analysis of standing waves and beats, other that that due to de Broglie himself, some of which is presented in a simplified form in Sect. 3.

was studied by the ancient Greeks (see Lindsay [10]), while its origin in interference, in both sound (Rayleigh [11]) and light (Brewster [12]), has been well-understood mathematically since at least the 19th century.

An illustration of how such a modulation is induced by a failure of simultaneity is provided by the standing wave of Fig. 1. In its rest frame, every part of the standing wave is oscillating in unison as in Fig. 1(a). But to an observer for whom the frame of the wave is moving to the right at a relativistic velocity, as in Fig. 1(b), the standing wave is experiencing the changes described by the Lorentz transformation. These include the failure of simultaneity. To the stationary observer, those parts of the wave to the right are rising and falling later than those to the left.



(a) a standing wave (b) the same wave moving to the right and progressively retarded in phase in that direction as a consequence of the failure of simultaneity (c) the same wave now moving at a sufficiently greater velocity that the progressive retardation of phase is observed as a sinusoidal beat or phase modulation having the velocity and other characteristics of a de Broglie wave.

If the inertial frame of the wave is moving even faster relative to the observer, as in Fig. 1(c), this progressive retardation in phase will be observed as a sinusoidal wave advancing through the underlying wave structure, and having the velocity and other characteristics of the de Broglie wave.

As will be discussed in the next section, the wave described by de Broglie in *Ondes et Mouvements* [1] is the realization in three dimensions of the progressive retardation of phase depicted in Fig. 1(c). And as will then be seen in Sect. 4, the quasiparticles of the sonic universes of Barceló and Jannes [5] and Todd and Menicucci [6] are able to simulate the Lorentz transformation because they too are structured as standing waves from which the de Broglie wave emerges as a modulation as the particle moves.

The simple standing wave of Fig. 1 already suffices to explain a number of features of the de Broglie wave that would be anomalous in a true wave. One is the velocity of the modulation, which becomes infinite as the particle comes to rest because all parts of the wave are then oscillating in unison. The progression of phase from one peak to the next becomes instantaneous, and the modulation disappears.

It should also be apparent from the lowermost drawing why the manner in which a massive particle interferes is determined by the wavelength of its de Broglie wave. It is in its sinusoidally distorted form that the moving particle approaches and interacts with a device such as a (stationary) beam splitter. Whether its interference at the beam splitter with another such wave is constructive or destructive, or somewhere in between, will depend on the degree to which the modulations are in or out of phase at the beam splitter.

There are two reasons why interference is not also observed at the Compton wavelength, one being that the Compton wavelength is much smaller than the separations of typical scattering elements, the other being that the Compton wavefronts are distributed in a spheroidal manner about the centre of the particle and unlikely to have any coherent relationship with the spacings of those scattering elements.

These drawings also reveal the relevance of the de Broglie wave to the covariance ensured by the Lorentz transformation. It is the full modulated wave structure, rather than the de Broglie wave considered *solus,* that is the covariant relativistic object. The Fitzgerald-Lorentz contraction appears in the carrier wave (3), while the dilation of time and failure of simultaneity predicted by the Lorentz transformation are described by the modulation, that is to say, by the de Broglie wave (5).

Above all, these drawings illustrate in one dimension the ontological parsimony of this understanding of the de Broglie wave. The wave requires no additional structure or new physics. There is no need to rationalize the superluminality of the wave, or to consider the ontological status of a wave that might otherwise seem to emerge from nowhere. The de Broglie "wave", so called, is simply, as stated above, a distortion predicted in well known manner by the Lorentz transformation, and entirely consistent with classical wave theory.

3 *Ondes et Mouvements*

In missing the de Broglie wave, the originators of the two sound analogues were in illustrious company. Its existence was missed by Einstein in 1905 [14], by Minkowski in 1908 [15], and again by Einstein when he presented his general theory in 1916 [16].

When in 1923 de Broglie did propose this wave, it proved elusive in another sense. He first described the wave - in a short paper [17] published prior to his thesis - as fictitious (*un onde fictive*). In a subsequent note [18], it had become a "phase wave", and by the time of the famous thesis [2], this phase wave had acquired physical reality, albeit as a curiously superluminal wave that seemed to be related in some way to a spatially extended "periodic phenomenon", which de Broglie described as surrounding the particle in its rest frame.

But in the thesis, de Broglie derived this phase wave, not by relating it to the periodic phenomenon surrounding the particle, but from his "theorem of the harmony of phases", according to which, the superluminal wave maintains consistency of phase with the subluminal particle as it moves.

As mentioned above, it is evident from the last paragraph of his doctoral submission that de Broglie was aware of the provisional state of these proposals [2]:

> I have left the definitions of phase waves and the periodic phenomena for which such waves are a realization .... deliberately vague. The present theory is, therefore, to be considered rather tentative as physics and not an established doctrine.

But with his doctorate in hand, de Broglie embarked upon *Ondes et Mouvements* [1], in which what had been "deliberately vague" and prudently "tentative" the previous year now took a more decided form. In *Ondes et Mouvements*, the periodic phenomenon is clearly a standing wave, from which the de Broglie wave emerges as what is also very clearly now, not a true wave, but the relativistically induced phase modulation of the underlying wave structure.

De Broglie showed that under a Lorentz boost,

$$
\begin{aligned}
x' &= \gamma\,(x - vt)\,, \\
y' &= y, \\
z' &= z, \\
t' &= \gamma\left(t - \frac{vx}{c^2}\right),
\end{aligned}
$$

a spatially extended "periodic phenomenon" of any form,

$$f(x', y', z')\, e^{i\omega_0 t'},$$

that is oscillating in three dimensions at some frequency ω_0 becomes,

$$f\left(\gamma(x - vt), y, z, \right) e^{i(\omega_E t - \kappa_{dB} x)},$$

in which the spatial factor $f(x', y', z')$ is now the carrier wave,

$$f\left(\gamma(x - vt)\right) \tag{3}$$

which is moving in the x-direction at velocity v and as can be seen from the inclusion of the Lorentz factor γ, has contracted in that direction in accordance with the Fitzgerald-Lorentz contraction.

What is of particular relevance here is that the oscillatory factor $e^{i\omega_0 t'}$ is now,

$$e^{i\omega_0\gamma(t-vx/c^2)}, \tag{4}$$

and describes a progressive loss of phase (the modulation of beating effect discussed above) evolving through the carrier wave (3) at the superluminal velocity,

$$v_{dB} = \frac{dx}{dt} = \frac{c^2}{v}.$$

With the assistance of Eqns. (1) and (2), Eqn. (4) can be rewritten in terms of the Einstein frequency ω_E and de Broglie wave number κ_{dB} as,

$$e^{i(\omega_E t - \kappa_{dB} x)}, \tag{5}$$

and is now more readily recognizable as the de Broglie wave.

Combining wave factors (3) and (5), the full modulated wave is,

$$f\left(\gamma(x-vt), y, z\right) e^{i(\omega_E t - \kappa_{dB} x)}. \tag{6}$$

But it is not to every conceivable structure that the Lorentz transformation can be *validly* applied. One example of an object to which the transformation cannot be applied and which accordingly could not exist in our universe is a perfectly rigid object. Such a structure has spatial extension and could also be oscillatory, but the displacement of one end would be transmitted instantaneously to the other, which would be contrary to the limiting role of the speed of light. The question of what kinds of structures may have physical existence in the actual universe has a direct bearing on how an object might be plausibly analogized, and this a question to which I will return in Sect. 6.

Before leaving the present section, I mention a more specific model discussed by de Broglie in *Ondes et Mouvements* in which the "periodic phenomenon" has the idealized form of a spherical standing wave,

$$\varphi(\mathbf{r}', t') = \frac{A}{|\mathbf{r}'|}\sin(\kappa_0 r')\, e^{i\omega t'}, \tag{7}$$

where,

$$\mathbf{r}' = \sqrt{x'^2 + y'^2 + z'^2}.$$

Under a Lorentz transformation, and switching now to cartesian coordinates, this wave structure becomes[5],

$$\varphi(x,y,z,t) = \frac{A}{\sqrt{\gamma^2(x-vt)^2 + y^2 + z^2}}\sin(\kappa_0\sqrt{\gamma^2(x-vt)^2 + y^2 + z^2}\,)\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{8}$$

[5] I have simplified slightly de Broglie's equations and have expressed the final wave factor in Eqn. (8) in the more usual form of the de Broglie wave.

where the previously spherical structure has contracted in the direction of motion and acquired the form of an oblate spheroidal wave.

The idealized model described by Eqn. (7) lacks the asymmetries that in an actual particle are suggested by properties such as charge, spin and parity, but displays in a conveniently simplified form, properties that I will argue in Sect. 6 are essential to the structure of all massive particles, namely that the particle be not only spatially extended and oscillatory, but that it comprise underlying influences and effects evolving at velocity c.

This model particle is also consistent with the assertion in Sect. 1 of this paper that the de Broglie wave is not in any way separate or ontologically distinct from the particle, but a distortion induced by the failure of simultaneity in the structure of the particle itself.

4 The fishbowl universes

Interest in sonic analogues as a means of investigating Hawking radiation ([19] [20]) seems to have been sparked by Unruh's suggestion in 1981 that the behaviour of a quantum field in a classical gravitational field might be modelled by sound waves in a convergent fluid flow (Unruh [21]).

The idea is to create what has been referred to as a "dumb hole", which in the parlance of analogue gravity is a region from which sound cannot escape, and which is formed by having the medium carrying that sound move at a speed greater than that of the sound itself. Unruh showed that near its event horizon, the metric of such a model would correspond with the Schwarzschild metric. He suggested that while the possibility of actually constructing such an analogue might be "extremely slim", it would present a simpler task than that of creating an actual black hole or of finding a small black hole near the Earth!

Analogues employing various species of sonic waves in various media have since been proposed (see the review by Barceló et al [22]) and an impressive number have actually been constructed, beginning with that of Steinhauer [23] in 2016, who reported the observation of Hawking radiation in the sonic analogue of a one-dimensional black hole formed in a Bose-Einstein concentrate.

Analogue gravity now has a significant literature. Reviews have been written and books and anthologies have been published. Arguments both for and against the confirmatory value of these analogues can be found in this literature, of which two interesting examples are Dardashti et al [24] (for) and Crowther et al [25] (against).

However, my concern here is not with black holes and Hawking radiation, but with a small subset of this literature relating to what might be termed analogue special relativity or analogue Minkowski spacetime, and in particular, as mentioned above, the sonic analogues proposed by Barceló and Jannes [5] and Todd and Menicucci [6].

Imagine a universe, or simply a closed laboratory (a fishbowl universe), in which the velocity c_s of sound is measured by devices formed from material in which all physical influences evolve at the velocity of sound. In the analogue of Barceló and Jannes, the arms of a Michelson-Morley interferometer comprise equally spaced quasiparticles, which in a collective oscillation produce waves that evolve outwardly in all directions at the velocity c_s. When this sonic interferometer moves at a velocity $v < c_s$ with respect to the surrounding medium, it experiences a relativistic contraction with a sonic Lorentz factor,

$$\gamma_s = (1 - \frac{v^2}{c_s^2})^{-\frac{1}{2}}, \tag{9}$$

based on the speed c_s of sound rather than the speed c of light.

Todd and Menicucci [6] show how *all* three of the curious changes predicted by the Lorentz transformation, namely the contraction, the dilation of time and the failure of simultaneity, might be analogized by a chain of sound clocks, these being akin to the light clocks described by Einstein except that the return journey between opposed mirrors is made by sound waves rather than electromagnetic waves.

These curious fishbowl universes differ considerably of course from the universe we know, and the significance of these disanalogies will be discussed in Sect. 6. But for the purposes of the present section, all that need be noticed is the central assumption of these analogue universes, which is that everything within the actual universe, including observers and their measuring instruments, can be plausibly analogized by structures comprising counter-propagating sound waves of velocity c_s. As the quasiparticles of Barceló and Jannes propagate outwardly, each will be the recipient of incoming waves from other quasiparticles, while in the chains of sound clocks described by Todd and Menicucci, these counter-propagating waves comprise sequences of sound pulses making return trips between opposed reflectors.

The massive quasiparticles of these fishbowl universes thus comprise structures that, when at rest in the medium, are akin to standing waves, or sufficiently so that they transform between one inertial frame and another in the manner described by de Broglie in *Ondes et Mouvements* [1]. It follows therefore that while the de Broglie wave may not have been within the contemplation of the originators of these analogues, this wave-like phenomenon should emerge in the quasiparticles of each analogue if it were actually built and tested, a possibility that I will also say something about in Sect. 6.

Why then was the de Broglie wave not noticed in the formulation of these analogues? One reason, of course, is that the objective was merely to analogize the Lorentz transformation - and in that objective these sonic analogues have succeeded very well. But it is also relevant to notice here that these analogues were effectively pre-quantum and would have inevitably missed the de Broglie wave for very much the same reason that it was missed by Einstein in 1905 [14]. In each analogue, it was important for the authors to show that their measuring devices, that is to say, the sonic interferometer of Barceló and Jannes, and the chain of sound clocks of Todd and Menicucci, were plausible simulations of the corresponding devices of the actual universe. In deriving the Lorentz transformation, they were thus concerned, not with the microscopic structure of matter, but with the plausibility of devices capable of simulating the macroscopic measuring rods and clocks considered by Einstein in 1905.

The derivations thus proceeded, as Einstein did in 1905, from a consideration of the classical rather than the quantum. Todd and Menicucci [6] inform the reader that their sound clocks are separated by "spacing arms", while Barceló and Jannes [5], at 194, employ "emergent vector fields and sources to produce a rigid bar". In each case, the derivation involved the consideration, not of changes in the frequencies and wave numbers of counterpropagating waves, but of the different times taken by light, propagating longitudinally and transversely, with respect to the direction of motion of the macroscopic measuring device in question.

In 1905, Einstein was unaware of the de Broglie wave. While he realized that if light is to have the velocity c for all observers, solid matter must change in the manner of counterpropagating light rays, he was unable to take the further step of proposing that in some sense solid matter must in fact comprise counterpropagating wave-like influences of velocity c.

When Einstein did learn from de Broglie's thesis of the wave-like behaviour of matter, he famously declared, in a latter to Paul Langevin, that de Broglie had "lifted a corner of the great veil" (as cited in Ref. [26]).

5 Schrödinger's wave functions

Unfortunately, for the orderly development of quantum mechanics, *Ondes et Mouvements* [1] was overtaken in early 1926 by rapidly developing events, these being (as mentioned earlier) the publication in quick succession of Schrödinger's papers on wave mechanics (Schrödinger [3]) and Born's proposal that the wave functions from the Schrödinger equation are objectively probabilistic (Born [4]). This was unfortunate, at least, for any prospect of a physically realistic interpretation of quantum mechanics consistent with the suggestion in *Ondes et Mouvements* that the de Broglie wave is a physically real phenomenon that describes for the particle, a well-defined and physically realistic trajectory.

Schrödinger's papers attracted immediate interest. He was able to explain the observed energies of the Hydrogen atom and harmonic oscillator, as also the Stark and Zeeman effects. Crucially, he demonstrated the equivalence of his wave mechanics and the earlier matrix mechanics of Heisenberg, Born and Jordan (Schrödinger [27]).

The first of Schrödinger's papers on wave mechanics (Schrödinger [28]) was published just three weeks prior to *Ondes et Mouvement*s and was apparently written in ignorance of that work. Indeed, there is no reference to *Ondes et*

Mouvements or the interpretation of the de Broglie wave as a modulation in any of the papers that Schrödinger submitted to Annalen der Physik during 1926 and 1927. While Schrödinger acknowledged in his papers his intellectual debt to de Broglie, the de Broglie wave seems to have remained for Schrödinger the "intentionally vague" and "tentative" superluminal wave of de Broglie's thesis of 1924 [2].

It was inevitably the wave function rather than the de Broglie wave that now became the focus of enquiry, and it would seem that from this time de Broglie's own efforts were concentrated on reconciling his ideas with those of Born and Schrödinger. His double solution paper of 1927 (de Broglie [29]) is very much concerned with the interpretation of Schrödinger's wave functions, which de Broglie treated as both the source of a guiding function and a means of determining the "probability of presence" of the electron in a manner which, as he put it in that paper, "approaches the one brilliantly upheld by Born" (and see also de Broglie [30]).

Yet despite the predictive value of those wave functions, they have seemed sufficiently mysterious as to encourage a debate as to whether they are ontic or merely epistemic in their significance, that is to say, whether they are physically real waveforms, or merely a means of calculating energies and momenta, for which, as Schrödinger demonstrated, they are highly successful. And of course, it is the objectively probabilistic Born interpretation of these wave functions that has been the source of the measurement problem of quantum mechanics.

Let us suppose then that de Broglie was correct in the interpretation of the de Broglie wave he presented in *Ondes et Mouvements* and consider what this interpretation might have to say regarding the physical meaning of these mysterious wave functions.

It would seem that as originally conceived, the Schrödinger and Klein-Gordon equations were intended as equations for the de Broglie wave (see Jammer [31], at p. 255 et seq. and Bloch [32]). As the story goes, Schrödinger led a colloquium on de Broglie's thesis, and during the ensuing discussion, one of his audience, apparently the Dutch physical chemist Peter Debye, commented that talk about a wave was a bit silly in the absence of an equation for the wave.

We can imagine that Schrödinger was a little peeved at this, but he did then develop a wave equation - apparently during a romantic sojourn in the Swiss alps (Moore [33], at p. 140 et seq.). He first derived the relativistic equation, which is to say, the aforesaid Klein-Gordon equation, named after two subsequent discoverers. But on encountering difficulties with the relativistic equation, he turned instead to the nonrelativistic equation, which is the equation now known as the Schrödinger equation, and it was this that Schrödinger presented in the papers on wave mechanics that he submitted to *Annalen der Physik* during 1926 and 1927 (collected in Ref. [3]).

But it would be incorrect to suppose that the Schrödinger and Klein-Gordon equations are equations for the wave contemplated by de Broglie. In his thesis of

1924 [2], and even more clearly in *Ondes et Mouvements* [1], de Broglie described the de Broglie wave as centred upon the position of the moving particle. As the particle moved, the spatial evolution of its wave would thus define a trajectory for the particle.

However, it is apparent from Schrödinger's papers, as also from the equation itself, that the wave for which Schrödinger constructed the Schrödinger equation was not the localized travelling wave contemplated by de Broglie, but what would be better described as a mathematical construct - an artificial wave having at every point within its domain, the frequency and wave number that *would be* associated with a particle of a specified energy if it *were in fact* at that point of space.

That this is so can be understood by deriving the Schrödinger equation directly from the corresponding classical equation of motion,

$$E = \frac{p^2}{2m} + V. \tag{10}$$

Employing the Planck-Einstein and de Broglie relations (Eqns. (1) and (2), respectively) to make the substitutions,

$$E \to i\hbar\frac{\partial\psi}{\partial t}, \quad \textit{and} \quad \mathbf{p} \to i\hbar\nabla\psi, \tag{11}$$

we obtain,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = 0, \tag{12}$$

which is the Schrödinger equation (and by making the same substitutions in the corresponding relativistic equation of motion we obtain directly, in the same manner, the Klein-Gordon equation).

Clearly, the classical equation of motion (Eqn. (10)) does not identify a particular trajectory. It is a general rule governing all possible (non-relativistic) trajectories for a mass of a particular energy in a given potential. It is thus nomological in its primary significance, yet at the same time epistemic, for if the vector momentum of an object is known at some point within a known potential field, its trajectory may then be deduced.

So too, the Schrödinger and Klein-Gordon equations should be regarded as nomological and epistemic. As de Broglie suggested in his double solution paper of 1927 [29], a solution to the Schrödinger equation may be thought of as representing, not a single trajectory, but the trajectories of a "swarm of particles" filling the entire domain of the problem. As de Broglie also showed, an individual trajectory can be deduced from its wave function by simply reversing the second of the substitutions (11) to obtain a guidance equation, which might simply have the form (see for example de Broglie [34], at p.94),

$$\mathbf{p} = i\hbar\frac{\nabla\psi}{\psi}.$$

It should not be surprising then that these wave functions have seemed so inscrutable. A wave function has by construction, at each point of space, the frequency and wave number that in accordance with the Planck-Einstein and de Broglie relations, correspond with the energy and momentum, respectively, that the particle would have at that same point of space. But being an equation for a wave, the wave equation is agnostic as to the existence of this particle. Occupying, in the manner of a wave, the entire space available to a wave, a wave function will display a symmetry of structure from which it may not even be apparent that the particle has a trajectory, let alone a well-defined and physical reasonable trajectory of the kind supposed in physically realistic interpretations of quantum mechanics.

The distinction that I am making here between wave function and de Broglie wave can be illustrated in a rather stark manner by imagining a molecule, perhaps of oxygen, that is somewhere in the room, but we don't know where. Because the Schrödinger wave function covers all possible locations, it must encompass in like manner the entire room. But wherever the molecule might actually be, its de Broglie wave will be a microscopic wave centred on the current position of the particle with an amplitude increasingly attenuated with distance from that position.

While de Broglie supposed the existence of both wave and particle, Schrödinger took the position at the time that the physical entity is simply the wave, that is to say, the wave function. For Schrödinger, the wave was the particle! In his address to the Solvay meeting of 1927, he explained this as as follows:

> I myself have so far found useful the following perhaps somewhat naive but quite concrete idea. The classical system of material points does not really exist, instead there exists something that continuously fills the entire space and of which one would obtain a 'snapshot' if one dragged the classical system with the camera shutter open through *all* its configurations, the representative point in q-space spending in each volume element $d\tau$ a time that is proportional to the instantaneous value of $\psi\psi^*$. (see Bacciagaluppi and Valentini [35], at p. 411)

It is possible to discern in these differing interpretations, a progression of ontologies from de Broglie, who supposed a localized wave surrounding a localized particle, to Schrödinger who was willing to forsake the particle in favour of an all-encompassing wave function, and from thence to Born, who was unwilling to discard the particle, but willing to abandon the certainties of physical reality in favour of a wave function that would serve as a probability function for the particle. In this confused melee of competing possibilities, there could be wave and particle, or simply the wave, which could be interpreted deterministically or probabilistically, and if probabilistically, objectively so or subjectively so.

In the aftermath of the Solvay conference of 1927, it was Born's objectively probabilistic interpretation of the wave function that eventually achieved orthodoxy. But one possibility that seems to have been overlooked at the time was that de Broglie had correctly interpreted the nature of the de Broglie wave in *Ondes et Mouvements*, but was incorrect in his insistence that wave and particle are separate entities.

It is that possibility that is suggested by the fishbowl universes of Barceló and Jannes [5] and Todd and Menicucci [6] and it is to these that I now return.

6 Analogy and disanalogy

I suggest that in *Ondes et Mouvements*, de Broglie presented the only interpretation of the de Broglie wave that makes physical sense. The question I now consider is whether, by facilitating an argument by analogy, these ingenious sonic analogues of Barceló and Jannes [5] and Todd and Menicucci [6] might provide the means of resuscitating that explanation.

The idea would be to simulate the Lorentz transformation of a sonic quasiparticle and look for the scattering of that particle in a manner consistent with its de Broglie wave, this being the way in which the actual wave was originally confirmed by the experiments of Davisson and Germer [36] and Thompson [37].

While such an argument from analogy is rarely conclusive [6], the literature suggests that it may well be persuasive depending on the existence or otherwise of significant disanalogies and the degree to which the *source* (the established phenomenon) and the *target* (the hypothesized phenomenon) correspond, see generally Bartha [41]. Analogical reasoning may be particularly plausible, it would seem, when, as with these fishbowl universes, source and target have a common mathematical structure, as is the case, for example, in those situations, ubiquitous in Nature, where two otherwise dissimilar systems exhibit harmonic motion (an example cited by Crowther et al [25]).

The formulation of an appropriate analogue and *a fortiori*, its experimental realization, would address two possible obstacles to the interpretation of the de Broglie wave proposed in *Ondes et Mouvements*. By showing how the de Broglie wave might emerge from a thoroughly wave-structured particle, it would provide a practical demonstration that, contrary to de Broglie's position on this particular issue (see, for instance, de Broglie [42]), wave and particle need not be separate physical entities. By showing that the wave thus emerging is neither fictitious nor probabilistic, but the consequence of a well-recognized and well-understood process of interference, it might encourage the adoption of a physically realistic solution to the measurement problem.

[6] Even so, some such arguments seem compelling: Galileo inferred the existence of mountains on the moon from his observation that, as on the Earth, points of light appear ahead of the advancing edge of sunlight [38]; Darwin drew support for the hypothesis of natural selection from the analogy of artificial selection [39]; and Priestley argued from the absence of a electric field within a uniformly charged spherical shell that, as in the analogous case with the gravitational field, electrostatic charge must follow an inverse square law [40]. (For these and other illustrations, see Bartha [41]).

For such an analogue to be plausible, there must be reason to assume that in the actual universe, as in the fishbowls, a massive particle may be treated, for the purposes of the Lorentz transformation, as comprising a superposition of waves of a single fundamental velocity. I will argue that this is implied by two well-established and fundamental principles of physics, the aforesaid Lorentz transformation and the Planck-Einstein relation (Eqn. (1) above).

The Lorentz transformation implies that whatever the structure of a massive particle might be, the various effects from which that structure is constituted, whether they be internal forces, topologies, or whatever, must evolve at the velocity c. If there were some fundamental effect or influence that evolved at a velocity other than c, it would have its own Lorentz factor γ and corresponding Lorentz transformation, and neither the structure of matter, nor the laws of physics, could survive unchanged from one inertial frame to the next.

As discussed earlier, it is implicit in Einstein's various thought experiments involving moving trains and railway platforms and the like that this must indeed be the nature of solid matter (see also Shanahan [8] and [43]). Velocities that differ from c, those for example of sound waves, refracted light and massive objects may be explained as the net effect of underlying influences that *do* evolve at velocity c.

Meanwhile, the Planck-Einstein relation,

$$E = \hbar\omega,$$

suggests that whatever the standard model might ultimately have to say regarding the structures of the elementary particles, these consistory influences of velocity c must have, in the rest frame of the particle, the characteristic frequency ω_o of the species of particle in question.

There are also disanalogies that should be considered. One is that sound requires a medium, which is to say, an analogue of the *luminiferous aether* and "absolutely stationary space" that Einstein dismissed as superfluous in 1905 [14]. Barceló and Jannes [5] and Todd and Menicucci [6] reject this as a significant disanalogy on the basis that the two relativities, that of Einstein and that of Lorentz, are mathematically and empirically equivalent. In this, I believe they are correct.

Two further disanalogies, both of which were referred to earlier in this paper, are not so much reasons to doubt the analogy, but the reasons it is useful. One is that, as Unruh said in 1981, "all the basic physics [of sound] is completely understood" [21]. The other is that the relatively low velocity of sound makes it possible to consider the workings of the Lorentz transformation from the standpoint of an external observer. It follows that if a sonic de Broglie wave were observed, there could be little doubt as to its origin whereas, at the mysterious level of the quantum, there must be at least a theoretical possibility that the de Broglie wave has an origin as yet unknown.

It is apparent from the literature of analogue gravity that to a significant degree the expertise necessary to construct and test a sonic analogue of the de Broglie wave does already exist. For instance - and at the risk of revealing an experimental naiveté - a simple analogue, in which the carrier wave varies in only one dimension, might simply consist of:

(a) opposed plates of the kind used in Steinhauer's experiment [23], which by vibrating at the same frequency at opposite ends of a tubular "fishbowl" would create a standing wave;

(b) a means of varying the frequency of vibration of either or both plates so as simulate a moving quasiparticle and accompanying beat, that is to say a de Broglie wave, as contemplated in one spatial dimension in Fig. 1;

(c) some way of inserting an appropriate scattering device; and,

(d) a means of confirming the path taken by the scattered wave.

The scattering element (c) might comprise, for example, parallel wires or thin rods set in the path of the wave, angled so as to simulate the scoring of a diffraction grating, and having spacings of an order of magnitude adapted to the scattering of the particle at its de Broglie wavelength rather than at its Compton wavelength.

7 Concluding discussion

On comparing the situation of the de Broglie wave with that of Hawking radiation, one might question whether the interpretation in *Ondes et Mouvements* [1] should need further demonstration. Whereas Hawking radiation is empirically inaccessible, controversial and predicted from a relatively abstruse mathematical analysis, the de Broglie wave is evidenced routinely in interferometry and scattering experiments, while the manner in which a beat emerges as a consequence of interference is well-known and understood, and not at all controversial.

And, as I have stressed above, the issue is not as to which of two competing theories provides the better explanation of the de Broglie wave. For the de Broglie wave, there is only the one physically reasonable explanation in suit, unless at least the imprimatur of orthodoxy is to be accorded to a superluminal wave of unknown origin and ontology that seems to arise out of nowhere as the particle moves. The difficulty here is of a different kind, namely that the explanation presented in *Ondes et Mouvements* would be an embarrassment to a quantum theory that has insisted for nearly a century that, prior to measurement, a particle has no location or trajectory.

Yet it was from the prediction and experimental confirmation of the de Broglie wave that it became possible to formulate a quantum mechanics in which all particles, whether massive or massless, are treated as evolving and interacting in the manner of waves. I suggest that in the absence of a physically reasonable explanation of this wave-like behaviour, no quantum effect can be properly understood including indeed the Hawking radiation which was the original motivation for these sonic analogues.

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References

[1] L. de Broglie, *Ondes et Mouvements*, Gauthier-Villars, Paris (1926)

[2] L. de Broglie, Doctoral thesis, *Recherches sur la théorie des quanta*. Ann. de Phys. (10) **3**, 22 (1925)

[3] E. Schrödinger, *Collected Papers on Wave Mechanics*, Minkowski Institute Press, Montreal, 2020

[4] M. Born, Quantenmechanik der Stossvorgange. Z. Phys. **38**, 803 (1926)

[5] C. Barceló, G. Jannes, A Real Lorentz–Fitzgerald contraction, Found. Phys. **38**, 191 (2008)

[6] S. L. Todd, N. C. Menicucci, Sonic relativity, Found. Phys. **47**, 1267 (2017)

[7] B. Cheng, J. Read, Why Not a Sound Postulate? Found. Phys. **51,** 71 (2021)

[8] D. Shanahan, The Lorentz Transformation in a Fishbowl: A Comment on Cheng and Read's "Why Not a Sound Postulate". Found. Phys. **53**, 55 (2023)

[9] D. Shanahan, Reverse Engineering the de Broglie Wave, International Journal of Quantum Foundations **9,** 44 (2023)

[10] R. B. Lindsay, *The Story of Acoustics*, J. Acoustic Soc. Amer. **39**, 629 (1966)

[11] J. W. Strutt, Lord Rayleigh, *The Theory of Sound*, Cambridge University Press (1877)

[12] D. Brewster, *A Treatise on Optics*, Longman, London (1831)

[13] D. Shanahan, The de Broglie wave as an undulatory distortion induced in the moving particle by the failure of simultaneity, Ann. Fond. Louis de Broglie **48**, 197 (2023)

[14] A. Einstein, Zur elektrodynamik bewegter Korper, Ann. Phys. **17**, 891 (1905)

[15] H. Minkowski, Raum und Zeit, Phys. Zeit. **10**, 104 (1909)

[16] A. Einstein, Die Grundlage der allgemeinen der Relativitätstheorie, Ann. Phys. **354**, 769 (1916)

[17] L. de Broglie, Ondes et Quanta, C. R. Acad. Sci. **177**, 507 (1923)

[18] L. de Broglie, Sur la fréquence propre de l'électron, C. R. Acad. Sci. **180**, 498 (1925)

[19] S. W. Hawking, Black Hole Explosions?, Nature **248**, 30 (1974)

[20] S. W. Hawking, Particle creation by black holes, Comm. Math. Phys. **43**, 199 (1975)

[21] W. G. Unruh, Experimental black-hole evaporation, Phys. Rev. Lett. **46**, 1351 (1981)

[22] C. Barceló, S. Liberati, M. Visser, Analogue gravity, Living Rev. Relativ. 14 (2011)

[23] J. Steinhauer, Observation of quantum Hawking radiation and its entanglement in an analogue black hole. Nature Physics, **12**, 959 (2016)

[24] R. Dardashti, S. Hartmann, K. P. Thébault, E. Winsberg, Hawking radiation and analogue experiments: A Bayesian analysis. Studies in History and Philosophy of Modern Physics **67**, 1 (2019)

[25] K. Crowther, N. Linneman, C. Wüthrich, What we cannot learn from analogue experiments, Synthese 198, Suppl. **16**, 3701 (2021)

[26] D. K. Buchwald et al (eds.), *The Collected Papers of Albert Einstein*, Vol. 14, Princeton (2015)

[27] E. Schrödinger, Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen, Ann. Phys. **79**, 734 (1926)

[28] E. Schrödinger, Quantisierung als Eigenvertproblem, Ann. Phys. **79**, 361(1926)

[29] L. de Broglie, La mécanique ondulatoire et la structure atomique de la matière et du rayonnement, J. Phys. Rad. **8**, 225 (1927)

[30] L. de Broglie, Interpretation of quantum mechanics by the double solution theory, Ann. Fond. Louis de Broglie, **12**, 1 (1987)

[31] M. Jammer, *The Conceptual Development of Quantum Mechanics*, McGraw-Hill, New York (1966)

[32] F. Bloch, Heisenberg and the early days of quantum mechanics, Physics Today, Dec. 1976

[33] W. Moore, *A Life of Erwin Schrödinger*, Cambridge University Press (1994)

[34] L. de Broglie, *Non-linear Wave Mechanics*, Elsevier, Amsterdam (1960)

[35] G. Bacciagaluppi, A. Valentini, *Quantum theory at the crossroads: Reconsidering the 1927 Solvay Conference*. Cambridge University Press. Cambridge, 2009

[36] C. Davisson, L. H. Germer, Diffraction of Electrons by a Crystal of Nickel, Phys. Rev. **30**, 705 (1927)

[37] G. P. Thompson, Diffraction of Cathode Rays by a Thin Film, Nature **119**, 890 (1927)

[38] G. Galilei, *Siderius Nuncius*, Baglioni, Venice (1610), trans. by S. Drake in *Telescopes, Tides and Tactics*, University of Chicago Press, London (1983)

[39] Letter to Henslow, May 1860, in C. Darwin, *More Letters of Charles Darwin*, Vol. I, Francis Darwin and Albert Seward (eds.) Appleton and Co., New York (1903)

[40] J. Priestley, *The History and Present State of Electricity*, London (1767)

[41] P. Bartha, Analogy and Analogical Reasoning, The Stanford Encyclopedia of Philosophy (Fall 2024 Edition), E. N. Zalta and U. Nodelman (eds.)

[42] L. de Broglie, On the true ideas underlying wave mechanics, C. R. Acad. Sci. **B277**, 71 (1973)

[43] D. Shanahan, What might the matter wave be telling us of the nature of matter? International Journal of Quantum Foundations, **5**, 165 (2019)