Are Symmetry Principles Meta-laws?

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Forthcoming in *Synthese*. Please cite the published version.

Abstract

Noether's first theorem demonstrates that continuous symmetries give rise to conserved quantities (under appropriate conditions). This fact tempts many to hold that symmetry principles explain conservation laws. Yet there is a puzzle: the derivation goes both ways. So why does symmetry explain conservation when the derivation is bidirectional? Lange (2007, 2009) provides an answer: symmetry principles are *meta-laws*, and meta-laws explain first-order laws just as first-order laws explain facts. Using a "non-standard" Lagrangian, Smith (2008) claims that conservation of angular momentum can hold without rotational symmetry, providing a counter-example to Lange. In this paper, I show that Smith's non-standard Lagrangian fails to serve as a counterexample. However, that doesn't leave Lange's account unchallenged. I argue that the debate between Lange and Smith ultimately revolves around an ambiguity which, once clarified, leads to a dilemma. *Which* symmetry principle explains? Is it the symmetry of the action or the symmetry of equations of motion? If the former, then the symmetry is no more stable than conservation laws. Hence, we lose the desired explanatory direction. If the latter, the symmetry lacks explanatory relevance and fails to exhibit greater stability than conservation laws. However one disambiguates 'symmetry', it remains mysterious why symmetry principles explain conservation laws.

1 Introduction

In 1918 Emmy Noether established something remarkable: under appropriate conditions, continuous symmetries of the action give rise to conserved quantities. This is now known as Noether's first theorem. Noether's theorem has tempted many to think that *symmetry principles explain conservation laws.*[1] However, the mathematical relationship is in fact bidirectional: one can derive a conserved quantity from a symmetry, and conversely, a symmetry from a conserved quantity.[2] This poses a puzzle: if the derivation goes both ways, why do we so often regard symmetry principles as explaining conservation laws, rather than the reverse?

Marc Lange (2007, 2009) develops an account of laws that he claims can effectively ground the explanatory direction. For Lange, symmetries are *meta-laws* that possess a stronger type of necessity than conservation laws. The necessity is stronger if it is more *nomically stable*, where nomic stability is a measure of persistence under counterfactual perturbations. Crudely, symmetries explain conservation laws because *symmetries would still have held, had the first-order laws been different.* Sheldon Smith (2008) argues that Lange's account fails. Using a "non-standard" Lagrangian, Smith claims that conservation of angular momentum can hold without rotational symmetry. Hence, symmetry cannot be more nomically stable than conservation laws.

In this paper, I argue that Smith's counterexample cannot be used against Lange because the conserved quantity in the non-standard Lagrangian is not angular momentum. This doesn't mean that Lange's account stands unchallenged, however. I argue that the debate between Lange and Smith ultimately revolves around an ambiguity which, once clarified, leads to a dilemma. In particular, we must ask *which* symmetry principle explains. Is it the symmetry of equations of motion or the symmetry of the action? If the symmetry refers to the symmetry of the action, then we fail to achieve the desired explanatory direction. If the symmetry refers to the symmetry of equations of motion (or the symmetry of any other first-order laws), then the symmetry is explanatorily irrelevant to and is no more stable than conservation laws. Whichever way one disambiguates

[1] E.g., Wigner (1964), Zee (1986).

[2] The bijection in the Lagrangian framework only holds for appropriately defined equivalence classes of both symmetries and conservation laws; see Martinez Alonso (1979) and independently Olver (1986); for further discussion, see Brown (2022). I thank the anonymous reviewer for prompting this clarification. In a Hamiltonian framework, the converse theorem can be shown in a more straightforward way; see Butterfield (2006, Section 5) for Noether's theorem and its converse in a Hamiltonian framework.

'symmetry', it remains mysterious why symmetry principles explain conservation laws.

Many have responded to Lange (2007, 2009, 2011b) by either accommodating meta-laws within other accounts of laws (Yudell 2013, Duguid 2023) or proposing alternative interpretations of symmetry principles (Hicks 2019, Friend 2024). In contrast to these existing responses, I emphasize the need to make a finer distinction between different symmetries *before* determining their modal status and explanatory power.

While the paper primarily addresses the Lange-Smith debate, it also makes two independent contributions. First, the discussion of symmetries shows that we need an account of explanation that resolves the dilemma between explanatory asymmetry and explanatory relevance. Second, the analysis of Smith's non-standard Lagrangian raises interesting questions about the physical equivalence of the Euler-Lagrange equations and Hamilton's equations of motion.

The paper is structured as follows. Section 2 explains Lange's account of symmetries as meta-laws. Section 3 introduces Smith's alleged counterexample to Lange and argues that this counterexample does not undermine Lange's account. In Section 4, I make a distinction between the symmetry of the action and the symmetry of the equations of motion. I introduce two criteria for explanation: *stability* and *relevance*. I then pose a dilemma. *Which* symmetry explains? If we say it's the symmetry of the action, then the symmetry fails *stability*. If we say it's the symmetry of equations of motion or any other first-order laws, then the symmetry fails *stability* and *relevance*. Either way, we cannot achieve the desired explanatory direction. Section 5 is the conclusion.[3]

2 Symmetry as a Langean Meta-law

Symmetries are operations on objects that leave some salient feature of the object unchanged. The object can be a material or geometrical object as well as a physical law. *Symmetries of laws* can refer to many different operations. While space-time translations, rotations, and Galilean transformations are continuous, time reversal and parity reversal are discrete. For a system described by dynamical laws, the objects that the symmetry operation acts on ("laws") can be ambiguous. Sometimes *laws* refer to the equations of

[3]For readers interested in the technical aspects of Noether's theorem, my analysis of the "non-standard" Lagrangian in Section 3 offers a fresh contribution to that literature, independent of the metaphysical debate. Those who are more interested in the metaphysical concerns may wish to skip the technical details in Section 3 and proceed to Section 4.

motion and other times the action (functional). Feynman et al. (1963), for instance, often refers to physical laws as the action. The equations of motion are so closely related to the action that in many systems one implies the other. We will see however, a distinction between a *symmetry* of equations of motion and a *symmetry* of the action is important in understanding the explanatory direction.

Given the abstract definition and broad application, a taxonomy of symmetries is needed in order to identify their distinctive roles in physics. In the Standard Model of particle physics, symmetry groups are related to the properties of elementary particles; in Dirac's theory of constrained Hamiltonian systems, gauge symmetries are related to constraints that the phase space variables must satisfy. Given the purpose of this paper, however, I limit my discussion to a particular set of relata: continuous symmetries of spacetime and conservation laws. I focus exclusively on non-relativistic classical particle mechanics, where the assumptions underlying Noether's theorem (and its standard interpretations) are most straightforwardly satisfied.[4]

The connection between spacetime symmetries and conservation laws was hinted at by physicists such as Lagrange (1811) and Hamilton (1834) way before Noether, but Noether's first theorem establishes the connection systematically. Textbooks in classical mechanics commonly say that time translation symmetry "leads to" the conservation of energy, or that rotational symmetry "results in" the conservation of angular momentum per Noether's theorem. Here, the derivation often involves introducing a generic action, calculating the variation of the action under infinitesimal symmetry transformations, and then using the Euler-Lagrange equation to show that only boundary terms survive. If the transformations leave the action invariant (up to a boundary term), we obtain quantities that are constant along solutions of the equations of motion.[5] While the converse derivation is well-known and often introduced in a Hamiltonian framework, the explanatory direction, if indicated, is always established in one way but not the other. That there exists an explanatory direction enjoys a broad consensus among many

[4] I thank the anonymous reviewer for pointing out this qualification.

[5] In this paper, I call a transformation a symmetry of the action if it leaves the action invariant up to a boundary term. Including a boundary term makes the symmetry condition weaker. It turns out, we only need the weaker condition to derive conserved quantities. Interestingly, this is related to the gauge freedom associated with the Lagrangian, i.e. the fact that the equations of motion are invariant under a shift of a Lagrangian by a total time derivative of an arbitrary smooth function (or a total divergence). Others, e.g., Brown and Holland (2004) and Brown (2022), have referred to the weaker condition as a "quasi-symmetry" to distinguish it from a "strict" symmetry that leaves the action strictly invariant.

physicists. For instance, Eugene Wigner in his paper "Symmetry and Conservation Laws" wrote,

> "...the conservation laws for energy and for linear and angular momentum are direct consequences of the symmetries..." (Wigner, 1964, p.959)

Whereas Wigner did not mention Noether, Anthony Zee in his book "Fearful Symmetry" endorsed this direction of explanation appealing to Noether's theorem:

> "Conservation of energy and momentum had been known for centuries, but physicists never linked them explicitly with symmetries...For years, I did not question where these conservation laws came from; they seemed so basic that they demanded no explanation. Then I heard about Noether's insight and I was profoundly impressed." (Zee, 1986, p.120-121)

Noether's theorem and its converse allows us to derive conservation laws from symmetry principles and symmetry principles from conservation laws. Yet, since explanations are unidirectional, this raises a puzzle. Why do symmetry principles *explain* conservation laws? Houtappel, Van Dam, and Wigner (1965) suggest that symmetry might be a "superlaw". More recently, Lange (2007, 2009) develops an account of laws that he claims can effectively ground the explanatory asymmetry. For Lange, symmetries are *meta-laws* that possess a stronger type of necessity than conservation laws.

To fully appreciate Lange's argument, a short detour through his counterfactual account of laws is needed. On Lange's view, laws persist under various counterfactual suppositions in a way that accidental facts do not. This is not simply to say that laws persist under *more* counterfactual suppositions. If a counterfactual antecedent is logically contradictory to a law, then the law would not hold under that particular counterfactual perturbation. For example, had copper been an electrical insulator, then the law that copper conducts electricity would not hold. The remedy is to impose a "logical consistency requirement" to restrict the counterfactual antecedents to those ones that are logically consistent with the conjunction of the laws.

However, it is blatantly circular to distinguish laws from the accidents by stating that the laws are truths preserved under all counterfactual suppositions *logically consistent with the laws.* To remove the circularity, Lange introduces the idea of sub-nomic facts. Sub-nomic facts can either be laws or accidental facts but they do not declare their nomological status. For instance, "it is a law that F=ma" is *not* a sub-nomic fact, but "F=ma" is. According to Lange, laws form *the largest non-maximal sub-nomically stable set.* The set of laws is *non-maximal* because if a set of sub-nomic facts contains at least one accident, then it is sub-nomically stable only if it is the maximal set of all sub-nomic facts.[6] Roughly speaking, a set of sub-nomic facts is *sub-nomically stable* if and only if the set's members would still have held under every sub-nomic supposition consistent with the set.[7] A more precise definition is stated as follows (Lange 2009, p.29 and Lange 2007, p.471):

> *A sub-nomically stable set.* A non-empty and logically closed set Γ of sub-nomic facts is sub-nomically stable if and only if the sub-nomic facts in Γ are invariant under all sub-nomic counterfactual suppositions (and all nested sub-nomic counterfactuals) that are logically consistent with Γ.

Given any sub-nomic claim p that belongs to a stable set Γ and any sub-nomic claim q logically consistent with Γ, if q were true, then p would still hold. The notion of sub-nomic stability provides a delicate way to distinguish laws from accidents without presupposing which facts are laws. The sub-nomic claim "copper conducts electricity" belongs to a sub-nomically stable set, but the sub-nomic claim "all coins in my pocket are copper" belongs to a sub-nomically unstable set. With this notion of stability, a hierarchy is constructed to distinguish claims with various modal forces. On the top, there is the set of broadly logical truths, and at the bottom the set of all truths that are nomic or sub-nomic (Lange, 2009, p.116).[8] There can be multiple strata in the middle, which makes room for laws and meta-laws.

Meta-laws are laws of laws. In particular, the relation between meta-laws and laws mirrors the relation between first-order laws and accidental facts (Lange, 2007, p.478).

[6] There is a snowball effect – if one accidental fact is included in the set, then more accidental facts need to be included to guarantee the set's sub-nomic stability. The process continues and the set ends up containing all sub-nomic truths. Such a maximal set cannot be a set of laws. For more details, see Lange (2009).

[7] In his 2007 paper, Lange uses the term "non-nomic stability" for first-order laws in order to make a distinction from "nomic stability" that applies to meta-laws. Later in his 2009 book, he replaces the term "non-nomic stability" with "sub-nomic stability". They are both defined in the same manner, but I adopt the later terminology for its interpretive clarity.

[8] Lange (2009) introduces different kinds of hierarchies. One such hierarchy is the "bossing around" picture, where sub-nomic facts are governed by laws, which, in turn, are governed by meta-laws, and so on (p.19). Additionally, there is a pyramid representing exclusively sub-nomic truths, where meta-laws are not present (p.41). For the purposes of this paper, I employ the one that includes both nomic and sub-nomic facts (p.116).

Analogous to sub-nomic stability, *nomic stability* is defined as the invariance of the members of a non-empty and closed set of nomic or sub-nomic facts under all nomic and sub-nomic counterfactual suppositions that are logically consistent with the set. Meta-laws form *the largest non-maximal set* that possesses *nomic stability*, which is a "stronger" type of necessity than *sub-nomic stability.* For Lange, this helps to resolve the puzzle of explanatory direction.[9]

A conservation law is a *sub-nomic* claim that concerns particulars (e.g., the fact that the total energy of an isolated physical system stays constant over time), while a symmetry principle is a *nomic* claim that concerns laws (e.g., the fact $F = ma$ is invariant under a time translation). Symmetry principles possesses a stronger type of necessity, making them more stable under counterfactual perturbations: *had dynamical laws been different, symmetry principles would still have held.* On the other hand, conservation laws might not have held under different dynamical laws.

To show this, Lange (2007) uses an interesting example. Imagine a world where the dynamical laws are such that everything remains at rest if no force acts on it. Following Wigner (1954), the equations of motion, applying to every particle α would be

$$m_\alpha \dot{x}_\alpha = -\frac{\partial f}{\partial x_\alpha}, \quad m_\alpha \dot{y}_\alpha = -\frac{\partial f}{\partial y_\alpha}, \quad m_\alpha \dot{z}_\alpha = -\frac{\partial f}{\partial z_\alpha}, \tag{1}$$

where f is some potential. Call this law $F = mv$, in contrast to Newton's second law $F = ma$. Note that familiar symmetries still hold. Take, for instance, a spatial shift $x_\alpha \rightarrow x'_\alpha = x_\alpha + \epsilon_\alpha$, where ϵ_α is a constant. The equations of motion and the "forces" are unaffected by the spatial shift. The symmetries in space and time, however, are not accompanied by all conservation laws. While a conserved quantity corresponding to the total momentum exists (if f is independent of displacements),[10] there is no conserved energy or angular momentum (Wigner 1954).[11] It showcases that time translation symmetry and rotational symmetry are more fundamental – they can hold without the

[9]It's helpful to note that Lange takes counterfactuals as primitive, i.e. the ontological bedrock that grounds laws. Although counterfactual fundamentalism is itself subject to debate, for present purposes, it suffices to state the idea of Langean meta-law without delving into the specifics of the metaphysics of counterfactuals. See Woodward, Loewer, Carroll, and Lange (2011).

[10]In this system, the conserved momentum in the x direction would be $Q = mx_1 + mx_2 + ... + mx_n$.

[11]Noether's theorem guarantees that a conserved quantity follows from a symmetry transformation under a Lagrangian framework. However, as Wigner (1954) points out, this system cannot be formulated in a Lagrangian or Hamiltonian framework.

presence of conservation of energy and angular momentum.[12] Furthermore, this example does not violate Lange's logical consistency requirement, because the energy and angular momentum *could* be conserved (if the universe contained just a single inertial particle with constant mass) even though they *would not* be conserved generally. While conservation laws and dynamical laws possess *sub-nomic stability*, symmetry principles as meta-laws possess *nomic stability*. Had the dynamical law been $F = mv$, symmetry principles would still hold but not conservation laws.[13]

For people who endorse symmetry principles' elite status and believe that conservation laws hold *by virtue of* continuous symmetries, it makes sense that conservation laws fail to hold when (global) spacetime symmetries are violated (e.g., in general relativity). Lange's exotic route to the explanatory direction might strike them as odd, since it says that symmetry does *not* suffice to bring about conservation laws. However, this is exactly how an "asymmetric" relation can be found. Contrasting the presence of symmetries and the absence of conservation laws helps to signify the (nomic) stability of symmetries and in turn elevates their modal status, allowing them to explain conservation laws. To obtain energy conservation, we still need other conditions regarding the dynamics of the system, e.g., the energy being represented by the Hamiltonian, the presence of a time-independent Lagrangian etc. For Lange, the explanatory direction of symmetries is not undermined by this, because symmetry principles are treated as the "covering law". It is enough to show that if further conditions are satisfied, then the symmetry principle explains the

[12]What about mass conservation, which is not associated with a symmetry? Lange thinks that it follows from the fact that momentum is conserved in all inertial frames, and is thus still an explandum. See Lange (2009, p. 220, Note 25). I thank the anonymous reviewer for raising this question.

[13]One might wonder if we can find a parallel counterfactual that obeys conservation laws yet breaks symmetry principles. Lange has contemplated this possibility: "had it been a non-vacuous law that each body always moves at $5m/s$ in the $+x$ direction", then symmetry principles would not hold (Lange, 2007, p.477). However, the very fact that such a law is not invariant under any rotational transformation violates the logical consistency requirement. Thus, it is not a legitimate counterfactual supposition that undermines the nomic stability of symmetries. Recall that in the case of first-order laws, the statement "had copper been an electrical insulator, then the law that copper conducts electricity would not hold" cannot undermine the sub-nomic stability of the law, exactly because the requirement of logical consistency (i.e., restricting the counterfactual antecedents to those that are logically consistent with the conjunction of the laws) shields it from such an attack. The upshot is that there is a nomic counterfactual supposition (e.g. $F = mv$) *logically consistent* with conservation laws under which some conservation laws fail to hold, but there seems to be no counterfactual supposition *logically consistent* with symmetry principles yet "breaks" symmetry principles. One might still look at the requirement of logical consistency suspiciously – it seems that once we lock in our meta-law candidate, the requirement of logical consistency would protect it from any counterexamples. Lange thinks this is not the right way to understand the requirement though. In the $F = mv$ case, the requirement is fulfilled because conservation laws *could* hold (e.g., if there were only two particles sitting at rest in the whole universe) but *would not* (generally) hold.

conservation law. Lange (2007, p.478) writes,

> When a conservation law is explained by a symmetry principle, the symmetry principle functions as the "covering law" and the fundamental dynamical law functions as the "initial condition". That is, the dynamical law is governed by the symmetry principle; the symmetry would still have held even if the dynamical law had not.

A conservation law, thus interpreted, is an "output" obtained from Hempel's deductive-nomological model of explanation. It is "governed" by the corresponding symmetry principle when the right "initial condition" is inserted. Had the "initial condition" been different, e.g. had the dynamical laws been $F = mv$, conservation laws would not hold. Lange stresses that symmetries are not "byproducts" of laws, but "requirements" that (together with the dynamics) restrict what types of force laws could exist. It is not an accident that the laws of electromagnetism, laws of gravitational force and other familiar force laws all obey symmetry principles. Symmetry principles help to explain conservation laws, just as first-order laws help to explain accidental facts.

3 A Counterexample?

Lange's argument is ingenious and intricate, but Smith (2008) presents a counterexample. Smith shows that there is a system in which the Lagrangian does not possess rotational symmetry and the conserved angular momentum does not correspond to (and thus cannot be explained by) rotational symmetry. He concludes as follows: if the symmetry of laws refers to the symmetry of the action, then the conservation of angular momentum can hold without the corresponding rotational symmetry. Lange's view, according to Smith, fails.

In this section, I argue that Smith's counterexample fails for technical reasons. Readers who are interested in the technical aspects of Noether's theorem should read on, since this result has independent value. Readers who are interested solely in the metaphysical debate may wish to take my conclusion for granted and proceed to Section 4.

Let's examine Smith's case.[14] The system involves a simple two-dimensional harmonic

[14] This example is also discussed by Brown and Holland (2004) and Butterfield (2006) as an example where the variational symmetry and the dynamical symmetry come apart. Smith (2008) also uses other

oscillator, originally introduced by Morandi et al. (1990). The standard Lagrangian has conserved angular momentum associated with rotational symmetry, but for the "non-standard" Lagrangian, the corresponding Noether transformation is not a rotation but a "squeeze". The symmetry one associates with a given conservation principle can depend on the choice of Lagrangian. A two-dimensional harmonic oscillator (with $m = 1$ and $\omega = 1$) can be described by the standard Lagrangian

$$L = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2 - q_1^2 - q_2^2), \tag{2}$$

as well as the "non-standard" Lagrangian

$$\tilde{L} = \dot{q}_1\dot{q}_2 - q_1 q_2. \tag{3}$$

This freedom is possible because two Lagrangians are solution-equivalent, that is, their Euler-Lagrange equations admit the same solutions. However, they are *not* gauge-equivalent. Gauge-equivalent Lagrangians, formed by adding to the Lagrangian a total time derivative of an arbitrary smooth function, are always included in modern derivation of conservation laws (and they have the same conserved charge).[15] However, the non-standard Lagrangian (3) is not gauge-equivalent to (2) and has different kinetic and potential terms. This poses an interesting interpretive question: which Lagriangian actually describes the system? Smith (2008) argues that there is no reason to privilege one over the other.

In their original paper, Morandi et al. (1990) point out that the non-standard Lagrangian is invariant under "squeeze" transformations but not rotations, so the conserved angular momentum is associated with invariance under squeeze. The same conclusion is drawn by Brown and Holland (2004) and Smith (2008). Weaponizing it as an argument against Lange, Smith (2008) states, "conservation of angular momentum need not be associated with invariance under rotations" (p.338). If it is true that there is no philosophically defensible reason to privilege the standard Lagrangian, then we seem to be able to form a counterfactual that overthrows the 'rotational symmetry-meta-law': had

examples (e.g., Lagrangians for damped systems) to show caveats about symmetries of Lagrangian. Here I focus on the example that serves as the strongest objection to Lange.

[15]See footnote[5] for the distinction of quasi-symmetry and strict symmetry of the action, which concerns gauge-equivalent Lagrangians.

the system been governed by the non-standard Lagrangian $\tilde{L}$, rotational symmetry would not have held.

This would be a strong objection to Lange, but the conclusion is drawn prematurely.[16] In the standard Lagrangian (2), the Hamiltonian (with $\dot{q}$ in L expressed in terms of the canonical momentum) takes the usual form

$$H = \frac{1}{2}(p_1^2 + p_2^2 + q_1^2 + q_2^2). \tag{4}$$

The angular momentum is conserved and associated with the rotational symmetry. This can be written in terms of the Poisson bracket in a Hamiltonian framework, which provides a simpler and clearer visualization of Noether's theorem and its converse.[17] The angular momentum J is a constant of motion because J Poisson-commutes with H:

$$\frac{dJ}{dt} = \{J, H\} = 0, \tag{5}$$

where $J = q_1 p_2 - q_2 p_1$ and the Poisson bracket of any two functions $f(q^i, p_i, t)$ and $g(q^i, p_i, t)$ is defined (with the usual summation convention) as

$$\{f, g\} := \frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i}. \tag{6}$$

The Poisson bracket has nice algebraic properties that directly follow from its definition. Geometrically, equation (5) says that J is invariant under the flow generated by H on

[16] A defender of Lange might find a way to accommodate such systems within Lange's counterfactual framework even if the counterexample were to work. The response works as follows: the meta-law of rotational symmetry is not meant to be universal (i.e., applying to all force laws or all Lagrangians), but only to some subset of them. Rotational symmetry possesses *nomic stability* in a subset of Lagrangians, instead of all (possible) Lagrangians. There is a sense in which this response deflates the metaphysical oomph of a symmetry-meta-law. It seems too cheap to simply restrict the scope whenever a counterexample surfaces. Since Lange does not commit to any specific symmetry-meta-law but rather describes the conditions under which symmetry principles are meta-laws, it might be better to simply acknowledge that rotational symmetry is not a meta-law. (Lange, personal communications, November 2023)

[17] One can also adopt a Lagrangian framework, using the vanishing of the Lie derivative of the Lagrangian along the lift to the tangent bundle to obtain a constant of the motion, but this approach as Butterfield (2006) notes is known to have no straightforward converse (see Brown (2022) for a "non-straightforward" converse in the Lagrangian framework originally developed by Martinez Alonso (1979) and Olver (1986) independently and see Butterfield (2006) for a geometrical perspective of Lagrangian and Hamiltonian formulations). The reason, I suspect, is tied to the gauge freedom of the Lagrangian and the larger symmetry group associated with canonical transformations in the Hamiltonian. For the purpose of this paper, I use the Hamiltonian formulation to provide a simpler and clearer construction of Noether's theorem and its converse.

phase space. Since Poisson brackets are anti-symmetric, H is also invariant under the flow generated by J, which corresponds to rotational symmetry. So we have

$$\frac{dJ}{dt} = \{J, H\} = 0 \text{ iff } 0 = \{H, J\} = \frac{dH}{ds}. \tag{7}$$

This is the Hamiltonian version of Noether's theorem – the quantity is conserved if and only if the transformation that it generates is a symmetry of the Hamiltonian. A symmetry of a Hamiltonian is a canonical transformation on phase space that preserves the Poisson brackets (or equivalently, the structure of Hamilton's equations).

In the case of the non-standard Lagrangian, the associated Hamiltonian is

$$\tilde{H} = p_1 p_2 + q_1 q_2. \tag{8}$$

It's easy to show that the J associated with the standard Lagrangian does not Poisson-commute with $\tilde{H}$. This is one way of suggesting that the non-standard Lagrangian is not invariant under rotation. Let's denote the conserved quantity, the infinitesimal generator for the non-standard Lagrangian, $\tilde{J}$ and we find

$$\tilde{J} = q_1 p_1 - q_2 p_2 \tag{9}$$

such that

$$\left\{\tilde{J}, \tilde{H}\right\} = 0 = \left\{\tilde{H}, \tilde{J}\right\}. \tag{10}$$

On the left, $\tilde{J}$ is invariant under the flow generated by $\tilde{H}$, and $\tilde{H}$ is the generator of time translation, which means $\tilde{J}$ is conserved. On the right, $\tilde{H}$ is invariant under the flow generated by $\tilde{J}$. $\tilde{J}$ generates infinitesimal transformation $q \to q' = q + \delta q$, where δq is a variation with the form $\delta q = \epsilon \left\{q, \tilde{J}\right\}$. To see what kind of transformation $\tilde{J}$ gives rise to, we calculate

$$\left\{q_1, \tilde{J}\right\} = q_1, \text{ and } \left\{q_2, \tilde{J}\right\} = -q_2. \tag{11}$$

Indeed, the infinitesimal transformation that $\tilde{J}$ generates looks like a "squeeze" – simultaneously scaling up q_1 and scaling down q_2 (or the other way around) by e^λ where λ is an arbitrary constant:

$$(q_1, q_2) \to (e^\lambda q_1, e^{-\lambda} q_2), \ \lambda \in \mathbb{R}. \tag{12}$$

According to Morandi et al. (1990), in the case of the non-standard Lagrangian, the angular momentum is associated with invariance under squeeze. However, this is not strictly correct because we only know from (10) that $\tilde{J}$ is conserved, but $\tilde{J}$ is not angular momentum. Brown and Holland (2004, p.7) explain that "[t]he shared Euler-Lagrange equations ensure conservation of angular momentum", which nevertheless corresponds to different transformations in the two Lagrangians. However, as they themselves point out, the dynamical symmetry associated with equations of motion is different from the variational symmetry associated with Lagrangians. It is the latter that ensures conservation laws via Noether's theorem. A more careful treatment is needed in interpreting the conserved quantity in the non-standard Lagrangian.

It is true that the two different Lagrangians share the same Euler-Lagrange equations. Interestingly, however, they have different Hamilton's equations of motion![18] For the two-dimensional harmonic oscillator with $m = 1$ and $\omega = 1$ (and here I switch to subscripts for both q and p to be clearer), Hamilton's equations associated with the standard Lagrangian (2) are

$$\dot{q}_1 = p_1 \quad \text{and} \quad \dot{q}_2 = p_2; \tag{13}$$

$$\dot{p}_1 = -q_1 \quad \text{and} \quad \dot{p}_2 = -q_2. \tag{14}$$

Hamilton's equations associated with the non-standard Lagrangian (3) are

$$\dot{q}_1 = p_2 \quad \text{and} \quad \dot{q}_2 = p_1; \tag{15}$$

$$\dot{p}_1 = -q_2 \quad \text{and} \quad \dot{p}_2 = -q_1. \tag{16}$$

The two Lagrangians with different Hamilton's equations (notice the difference in indices) nevertheless arrive at the same Euler-Lagrange equations! For both Lagrangians, plugging $m = 1$ and $\omega = 1$ back, we find $m\ddot{q}_1 = -m\omega^2 q_1$ and $m\ddot{q}_2 = -m\omega^2 q_2$, which are the familiar Euler-Lagrange equations for a two-dimensional harmonic oscillator. Perhaps this is why Morandi et al. (1990) claim "they are indeed only different descriptions of the *same* physical system" (p.205 emphasis in original). What does it mean to say two physical systems are "the same"? Hamilton's equations describe a system in terms

[18]Hamilton's equations can be written in an elegant way in terms of Poisson brackets as $\dot{q}^i = \{q^i, H\}$, $\dot{p}_i = \{p_i, H\}$. Using the properties of Poisson brackets, we can derive the equations of motion in a purely algebraic way (by linearity and the Leibniz rule).

of $2N$ first-order ordinary differential equations, whereas the Euler-Lagrange equations describe a system of N second-order ordinary differential equations (where N denotes the configurational degrees of freedom). In this case, we obtain different sets of first-order differential equations, which nevertheless correspond to the same set of second-order equations.

It would be natural at this point to figure out how the two coordinate systems are related, if they in fact describe the same physical system.[19] We introduce new variables Q and P to denote the coordinate system for the "non-standard" Lagrangian and leave q and p for the standard Lagrangian. Comparing (2) and (3), we find

$$Q_1 = \frac{1}{\sqrt{2}}(q_1 + iq_2), \tag{17}$$

$$Q_2 = \frac{1}{\sqrt{2}}(q_1 - iq_2). \tag{18}$$

The two systems are related by an analytic continuation. Moreover, the conserved quantity $\tilde{J}$ in the non-standard Lagrangian is also related to the conserved quantity J in the standard Lagrangian in an interesting way:

$$\begin{aligned} \tilde{J} &= P_1 Q_1 - P_2 Q_2 \\ &= -i(q_1 p_2 - q_2 p_1) \\ &= -iJ. \end{aligned} \tag{19}$$

Are two systems related by an analytic continuation to be regarded as physically identical? Do two quantities related by multiplication with the imaginary i represent the same physical quantity? Smith (2008) argues that if the two Lagrangians describe the same physical system, there is no reason to privilege one over the other. But that assumption – the sameness of the physical system – turns out to be non-trivial. It depends, in part, on whether one treats the Euler–Lagrange equations alone as physically significant, or whether one takes Hamilton's equations to be physically significant as well. There might be an independent reason from quantum mechanics to take Hamilton's equations "physically".[20] Smith (2008) holds that one should not "appeal to quantum mechanics

[19] I thank Raphael Flauger for suggesting the subsequent line of thought.

[20] In fact, Morandi et al. (1990) discusses the non-standard Lagrangian and its implications both in classical mechanics and quantum mechanics. In comparison of the two, they suggest that the non-standard Lagrangians describes "the same classical system, [yet] *genuinely different quantum systems*"

to select the privileged *classical* Lagrangian" (p.343 emphasis in original). It seems that for Morandi et al. (1990), Brown and Holland (2004) and Smith (2008), to be physically equivalent is to have the same Euler-Lagrange equations.

But should one care about differences in Hamilton's equations within classical mechanics? I leave the ontological question open, since my argument does not rely on any particular answer. Still, it's worth seeing what follows on either view. Suppose one holds that the standard and non-standard Lagrangians are merely alternative formulations of the same classical system. Then any symmetry present in one must be present – albeit in disguise – in the other. In that case, "squeeze invariance" is just a re-description of rotational invariance, and the non-standard Lagrangian cannot be used to show that angular momentum conservation can hold without rotational symmetry.

But suppose instead that the two Lagrangians describe different physical systems. That opens the door to further insight. Classically, it is natural to have variables expressed only in terms of real numbers in Euler-Lagrange equations in order to have meaningful physical interpretations. If we require coordinates q^i, p_i and Q^i, P_i to take only real values, then there's reason to think that the two Lagrangians – differing as they do in their symmetries – do *not* describe the same physical situation. The symmetry associated with the standard Lagrangian is *compact* O(2) and the symmetry associated with the non-standard Lagrangian is *non-compact* U(1). In one system, all observables return to the same state after rotation by 2π; in the other system, they do not. There is no *real canonical* transformation between the two systems.[21] One might use this result to suggest that these two systems are not physically equivalent.

If we consider the analytic continuation between two coordinate systems, then, topologically, rotations and squeeze transformations take different directions on a complex cylinder (Figure 1). Using equation (17) and (18) to relate two systems, we see that a "squeeze" transformation *could* take the form of "rotation" in q^i, p_i coordinates if the "squeeze" parameter λ is not constrained to real values, i.e., $(q_1, q_2) \to (e^{i\theta}q_1, e^{-i\theta}q_2)$, for $\theta \in \mathbb{R}$. This would set Q^1 and Q^2 to rotate in opposite directions. In this sense,

(p.209 emphasis in original). But we see explicitly that the "i" independently appears at the classical level and that two systems with different symmetries are not identical either classically or quantum-mechanically.

[21] A transformation is canonical *iff* it leaves the Poisson brackets invariant, which requires $\{q^i, p_j\}_{Q,P} = \delta^i{}_j$, where the subscript Q, P means computing the Poisson bracket of coordinates q^i, p_i in terms of coordinates Q^i, P_i. The transformations here involve imaginary numbers, though the analysis remains within the classical regime.

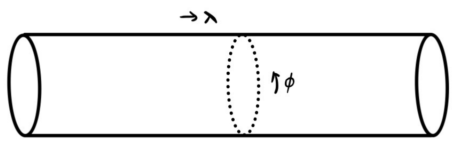

Figure 1: squeeze(λ) and rotation(ϕ) illustrated topologically

the non-standard Lagrangian passes Lange's logical consistency test. For it to be a real "squeeze" though, $\lambda \in \mathbb{R}$ is required. More importantly, the two Noether charges are not identical but instead different real sections of a complex quantity. It would be mistaken to view $\tilde{J}$ as the same conserved quantity as J. The infinitesimal transformation that $\tilde{J}$ generates is not rotation for $\lambda \in \mathbb{R}$, and, accordingly $\frac{d\tilde{J}}{dt} = 0$ does not stand for the conservation of angular momentum.[22]

One might think that the invariance under squeeze transformation still *implies* the conservation of angular momentum given $\tilde{J} = -iJ$ in (19), even though $\tilde{J}$ is not itself angular momentum.[23] Hence, the non-standard Lagrangian still showcases that the conservation of angular momentum can hold without rotational symmetry. However, as far as $\frac{d\tilde{J}}{dt} = 0$ implies the conservation of angular momentum, the invariance under squeeze also implies the existence of rotational symmetry in the exact same manner! A squeeze transformation *could* take the form of a rotation if λ is not constrained to real values.

So long as one holds that the two Lagrangians describe the same physical system, the conservation of angular momentum and rotational symmetry go hand in hand – merely represented differently in each formulation. On the other hand, if one takes the two Lagrangians to describe different physical systems, then the quantity conserved under the non-standard Lagrangian is not angular momentum. Either way, the non-standard Lagrangian cannot be used to establish that angular momentum conservation is possible without rotational symmetry. The upshot is that no genuinely asymmetric mathematical relation can be extracted from this example.

I've argued that Smith's non-standard Lagrangian cannot serve as a counterexample

[22] One might still wonder, why different Hamilton's equations can "share" the same Euler-Lagrange equations, especially considering the mathematical theorem that *for any hyper-regular Lagrangian the Euler-Lagrange equations are equivalent to Hamilton's equations.* A short answer is that the Hessian matrices and Legendre transforms differ in these two Lagrangians. This observation provides further clarification of the alternative interpretation of physical equivalence, which pertains to the same Euler-Lagrange equations *and* the same Legendre transforms.

[23] I thank Niels Linnemann for raising this point.

to Lange's account.[24] The technical analysis above yields three payoffs. First, the analysis clarifies and re-establishes the symmetric nature of the relation: if the transformation is a "squeeze", then the conserved quantity $\tilde{J}$ is *not* angular momentum; if the conservation of $\tilde{J}$ were to imply the conservation of angular momentum, then invariance under "squeeze" would also imply rotational symmetry. Hence, the non-standard Lagrangian cannot show the asymmetric relation that the conservation of angular momentum holds without rotational symmetry. Second, the analysis helps to disambiguate different notions of symmetry (which I will discuss in detail in the following section), thereby indicating a cross-talk in the existing debate. Finally, the analysis raises an interesting question about the physical equivalence of equations of motion in the form of first-order and second-order differential equations. The arguments in Section 4 will draw on the first two payoffs from this discussion. I leave this third and final issue as an open question.

4 Which Symmetry?

For symmetry principles to explain conservation laws, two conditions must be satisfied. First, symmetry principles must be more stable than conservation laws. Call this *stability*. This is established in Section 2 and connected to the idea that explanation is unidirectional. Second, symmetry principles must be explanatorily *relevant*. 1+1=2 is more stable than $F = ma$, yet 1+1=2 clearly doesn't *explain* $F = ma$. Intuitively, this is because the truth of 1+1=2 is *irrelevant* to the truth of $F = ma$. Call this *relevance*. For my purposes, I need not provide complete necessary and sufficient conditions for explanatory relevance. Instead, I assume that a candidate explanans satisfies *relevance* only if the explanandum deductively follows from the explanans, given the appropriate initial conditions. This is appropriate because Lange (2007) uses Hempel's deductive-nomological model of explanation and mirrors it one level up for meta-laws. It allows us to see how 1+1=2 is explantorily *irrelevant* to $F = ma$: there is no deductively valid inference from 1+1=2 to $F = ma$.

This section argues that the Lange-Smith debate revolves around an ambiguity. When we talk about symmetries as explaining conservation laws, *which* symmetry are we talking about? After distinguishing three different symmetries that we might be concerned

[24] While there might be other counterexamples, it is beyond the scope of this paper to discuss the consequence of every "non-standard" Lagrangian.

with, I proceed to show that none of these can vindicate the claim that symmetries *explain* conservation laws. Specifically, I show that none can simultaneously satisfy both conditions required for explanation, i.e., *stability* and *relevance*.

The first candidate for which symmetry is being said to explain conservation laws is the symmetry of the *action*. If the symmetry refers to the symmetry of the action, however, then the symmetry is not more stable than conservation laws. Why? Because the symmetry of the action is derivable from conversation laws as per Noether's (converse) theorem, even in the case of the non-standard Lagrangian. The two relata always go hand in hand. This result was established in Section 3. Since the symmetry of the action is no more stable than conservation laws, it follows that on this interpretation, symmetries do not *explain* conservation laws.

The second candidate is the symmetry of the *equations of motion*. Below I suggest that there is good reason to think that this interpretation of symmetry too cannot vindicate the claim that symmetries explain conservation laws. Specifically, I argue that there is reason to think that symmetries of the equations of motion are both explanatorily irrelevant to conservation laws and that such symmetries are not more stable than conservation laws. Since there is thus good reason to think that the conditions of both relevance and stability are unsatisfied, I conclude that symmetries of equations of motion are unlikely to explain conservation laws. I take these two issues in turn.

Why think that symmetries of the equations of motion are explanatorily irrelevant to conservation laws? Well, if we take the equations of motion as the initial conditions and the *symmetry* of the equations of motion as the covering law, conservation laws do not follow deductively. To appreciate this, note that from Noether's theorem we have $dL = (eom)\epsilon + \frac{d}{dt}Q$, where L stands for Lagrangian, eom stands for equations of motion, ϵ stands for an infinitesimally small quantity, and Q is the Noether charge or the conserved quantity. Here, to deductively obtain a conservation law, i.e., $\frac{d}{dt}Q = 0$, we need to assume the symmetry condition $dL = 0$ and the on-shell condition $eom = 0$. But this means the relevant symmetry $dL = 0$ is the symmetry of the *action* not the symmetry of the equations of motion (recall that the action is the integral of a Lagrangian). It is true that we still need the equations of motion to hold, i.e., $eom = 0$ in order to obtain conserved quantities. After all, for a quantity to be conserved is for it to be constant *along its trajectory of motion*. However, we do not need the *symmetry* of equations of motion to deductively obtain conservation laws! Hence, just as the truth of 1+1=2 is explanatorily irrelevant to the truth of $F = ma$, so too is the symmetry of equations of motion irrelevant to conservation laws. Or so we might think.[25]

A defender of Lange might reply as follows: sure, it is the symmetry of the action which is playing a direct role in the deduction of conservation laws in Noether's theorem. However, the symmetry of the action may itself be derivable from the symmetry of the equations of motion. This is plausible, one might think, if the symmetry of the equations of motion is more stable and necessary than the symmetry of the action. That is, the symmetry of equations of motion holds even in cases where the symmetry of the action and conservation laws do not. If further conditions are satisfied, such as the system permitting a Lagrangian formulation, the symmetry of the equations of motion implies both the symmetry of the action and the conservation laws. This, I think, is Lange's argument in the strongest form. It finds support in examples like Wigner's $F = mv$ case. Hence, even if the symmetry of the equations of motion isn't playing a direct role in the derivation it may be necessary for the derivation to go through. According to this reply, the previous charge of explanatory irrelevance is too strong.

This response, however, rests upon a conjecture. While there is reason to think that the symmetry of the equations of motion holds more generally than the symmetry of the action – and that there is often a correlation between the symmetry of the equations of motion and conservation laws – it is still unclear how the symmetry of the equations of motion *implies* both the symmetry of the action and the corresponding conservation laws.[26] Hence, we currently lack grounds for treating symmetries of the equations of motion as explanatorily prior to conservation laws. Beyond worries about the explanatory

[25]The symmetry of the equations of motion and the symmetry of the action both play important, yet different roles, as Brown and Holland (2004) point out. Their terminology is slightly different from mine. They call the symmetry of the equations of motion "dynamical symmetry" and the symmetry of the action "variational symmetry". It might be proper to define variational symmetry as a symmetry of the Lagrangian rather than a symmetry of the action (see Butterfield 2006). They typically carry the same meaning, although there is a caveat (due to a Jacobian factor). It is more general to formulate Noether's theorem in terms of the action, which is what Noether (1918) originally did. The symmetry of equations of motion is particularly important in physics because it maps solutions to solutions, but it is not directly related to a conserved quantity. If the equations of motion of a system follow from the action, then most of the symmetries of the equations of motion are also symmetries of the action, but not all. The most common example of a dynamical symmetry in absence of variational symmetry is the scale transformation. It leaves the equations of motion invariant, but the action obtained from the rescaled Lagrangian differs from the original action by a multiplicative constant.

[26]See Brown and Holland (2004, Section 5.2) for the connection between the symmetry of the action and the symmetry of the equations of motion.

relevance of symmetries of equations of motion, however, we can also raise concerns about the superior *stability* of symmetries of equations of motion, relative to conservation laws.

To show that symmetries of equations of motion are no more stable than conservation laws, we need find a case where a conservation law holds but the symmetry of equations of motion does not. Interestingly, Chen's 2022 "Wentaculus" account provides the basis for such a case. Wentaculus is a package of density matrix realism combined with a nomological interpretation of the initial quantum state of the universe. The main idea involves taking the actual quantum state of the universe as objective and impure (mixed), using the density matrix formalism. The initial quantum state of the universe at t_0 is given by the (normalized) projection onto the Past Hypothesis subspace, which is a particular low-dimensional subspace in the total Hilbert space. This is known as the initial projection hypothesis (IPH). The fundamental universal density matrix W then evolves according to the von Neumann equation. Here the modal status of the initial quantum state of the world is on a par with laws and the density matrix plays a dynamical role. On the Everretian formalism, W always evolves deterministically according to the von Neumann equation.[27] Everretian Wentaculus is not time-translation invariant because the past hypothesis applies at a particular time. The guidance equation in Bohmian Wentaculus also explicitly violates time-translation symmetry.[28]

If we accept Chen's package, then the fundamental equations of motion explicitly violate time-translation symmetry.[29] In both Bohmian and Everretian cases, however, energy is still conserved (as far as energy in "standard" quantum mechanics is conserved).[30] Chen's Wentaculus or the nomological interpretation may be controversial, but one need not endorse this particular view to see the purpose of this example though. The upshot is that the conservation of energy is associated with the symmetry of the action rather than the symmetry of the equations of motion. The example illustrates that, at least in

[27]The quantum state at a later time is given by $W_{IPH}(t) = e^{-iHt/\hbar} W_{IPH}(t_0) e^{iHt/\hbar}$, where $W_{IPH}(t_0)$ is specified by the IPH.

[28]For the Bohmian dynamical law, the guidance equation formulated in terms of the particle configuration Q and the density matrix is given by

$$\frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i} W_{\mathrm{IPH}}(q,q',t)}{W_{\mathrm{IPH}}(q,q',t)}(Q) = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i}\langle q|\, e^{-i\hat{H}t/\hbar}\hat{W}_{\mathrm{IPH}}(t_0)e^{i\hat{H}t/\hbar}\,|q'\rangle}{\langle q|\, e^{-i\hat{H}t/\hbar}\hat{W}_{\mathrm{IPH}}(t_0)e^{i\hat{H}t/\hbar}\,|q'\rangle}(q=q'=Q). \tag{20}$$

[29]While the fundamental dynamical law violates time-translation symmetry, the derived laws can still be time-translation invariant. See also Chen (2022) for a relevant discussion on time asymmetry.

[30]This is because the expected value of energy, given by $tr(HW_t)$, is constant over time, assuming the Hamiltonian is time-independent.

principle, symmetry of the equations of motion and conservation laws can come apart.[31]

Altogether, if we interpret the claim that it is the symmetries of equations of motion that explain conservation laws, then we have two problems. First, symmetries of equations of motion do not seem explanatorily relevant to conservation laws. Second, symmetries of equations of motion do not seem more stable than conservation laws.

Lastly, consider a third candidate. According to this interpretation, the symmetries that explain conservation laws are the symmetries that all first-order laws must obey. This might be the most direct interpretation of Lange's own view, since Lange never explicitly endorses any particular symmetry-meta-law. He thinks it is scientists' job to investigate which symmetry principle can be a meta-law (Lange, 2007, p.460). How does this last interpretation fare? I raise two concerns.

First, we might worry that such a symmetry is too vague to provide explanatory relevance. In some occasions (Lange, 2009, p.18, p.110), it seems that the symmetry that all first-order laws must obey would take the form of the principle of relativity: laws of physics are invariant in different inertial frames of reference. The principle of relativity states that laws are invariant in different inertial frames but it does not explain the *content* of the laws themselves. So even if the principle of relativity satisfies *stability*, it does not satisfy *relevance*: we cannot deductively obtain conservation laws from the principle of relativity.

My second worry is this: the symmetry of all first-order laws might cast its net too wide, leaving its claim to superior stability subject to counterexample. Even if one argues that the explanatory program operates at a coarser level of grain, and my fine distinctions miss the forest for the trees, counterexamples still abound. Consider the case of parity symmetry. Before the Wu experiment confirmed its violation in weak interactions – a hypothesis proposed by Lee and Yang (1956) – most physicists held that parity symmetry must hold for all fundamental laws and interactions. As Weinberg (2004) reminds us, many of the symmetry principles we once cherished – like parity symmetry, isospin symmetry, and Gell-Mann's SU(3) – turned out to be mere approximations.

Perhaps one might retreat to a narrower reading of this third candidate: continuous spacetime symmetries (which all laws of physics obey). But this too is on shaky ground. In general relativity, continuous symmetries of spacetime – and the corresponding con-

[31] My purpose in invoking this example is not to defend Chen's package, but to use it to sharpen the distinction between the two kinds of symmetries.

servation laws – are contingent. Not all solutions of the Einstein field equations possess Killing vector fields. In their absence, there is neither spacetime symmetry nor conserved quantities, at least not in any global sense. Again, *stability* fails.[32]

In this section, I've examined three different candidates for which symmetry is being said to explain conservation laws: the symmetry of the action, the symmetry of equations of motion, and the symmetry that all first-order laws must obey. However, none of them appears to satisfy both *stability* and *relevance*. In sum, we lack any clear way to vindicate the claim that symmetries explain conservation laws.

5 Conclusion

Lange (2009, 2011b) argues that his account of meta-laws can ground the explanatory direction of symmetry principles to conservation laws. Existing responses to Lange either attempt to devise alternative accounts of meta-laws (Yudell 2013, Duguid 2023) or offer competing interpretations of symmetry principles, e.g., as "maxi-laws" (Hicks 2019) or as "consequences of the structure of world-making relations" (Friend 2024). If my argument succeeds, then one should make a finer distinction between different kinds of symmetry principles *before* determining their modal status and explanatory power.

In an effort to disambiguate the term "symmetry principles" in the debate between Lange (2007) and Smith (2008), I have shown that once the symmetry is defined and identified clearly, the claim that symmetry explains conservation laws collapses. There is good reason to look under the hood to find out exactly *which symmetry* is concerned in each case. It becomes clear, however, that there is a dilemma for anyone who claims that symmetries are more explanatorily fundamental. If the symmetry principle refers to the symmetry of the action, then the symmetry is no more stable than conservation laws. We cannot cook up an asymmetric relation for a preferred explanatory direction from the mathematical formalism alone in equation (7) or (10), just as we cannot tell whether

[32] This is what I envision Lange could say in response: *There are accidental symmetries as well as meta-law-like symmetries. Accidental symmetries are by-products of the dynamics, but symmetry-meta-laws are requirements of the dynamics. If a particular symmetry is shown by physicists to be wrong or only approximate, then it is not a meta-law.* See Lange (2011a) for a similar response concerning conservation laws. I think the tension still exists though. If Lange's project is to be understood semantically, i.e., specifying conditions under which symmetries can be regarded as meta-laws, then there is a epistemic gap between the semantic and the empirical. See a similar worry in Van Fraassen (1989, 1993). I will defer the discussion on the metaphysics of laws to another time.

the length of the flagpole explains the length of the shadow or vice versa purely based on the trigonometric equation, which treats two lengths on a par. If the symmetry principle refers to the symmetry of equations of motion or the symmetry of any other laws, then the symmetry is explanatorily irrelevant to and is no more stable than conservation laws.

We seem to have ended up exactly where we began. Do symmetries explain conservation laws and if so, how? Perhaps, it would be easier at this point to join Brown and Holland (2004) and Albert (2015) in rejecting the claim that symmetry actually explains. For Albert, "what *actually* explains [the conservation of energy is] the fundamental physical laws of the *actual world*" (2015 p.14 emphasis in original). For Brown and Holland, "the real physics is in the Euler-Lagrange equations of motion for the fields, from which the existence of dynamical symmetries and conservation principles, if any, jointly spring" (2004 p.10). Brown (2022) has recently shown, on *pragmatic* grounds, why physicists often treat symmetries as explanatorily prior (even though they are not *fundamentally* explanatory).

Are symmetry principles meta-laws? Even if we found a symmetry-meta-law candidate that is nomically stable under counterfactual perturbations, it might not be explanatory relevant to conservation laws. What this suggests is that we must take a closer look at the connection between modality and explanation. If the motivation to elevate the modal status of symmetry principles stems from a desire to capture their explanatory power, then we must also provide a robust account of explanation. Here, Humean perspectives may prove fruitful, offering an account of explanation that is flexible enough to handle the graded modality. For now, I leave this as an open question, deferring its resolution to future work.

Acknowledgment

I thank Craig Callender, Nancy Cartwright, Eddy Keming Chen, Kerry McKenzie, and Faron Ray for extensive comments and support. I owe much to Harvey Brown, Marc Lange, two anonymous reviewers, and the 2023 Du Châtelet Prize Committee for their generous reading and feedback. I am grateful to David Albert, Katherine Brading, Josh Hunt, Jenann Ismael, and Niels Linnemann for discussions of an earlier draft, as well as to Raphael Flauger and Guanhao Sun for conversations about the non-standard Lagrangian. I thank the audiences at the 28th Biennial Meeting of the Philosophy of Science Association (Pittsburgh) and the 21st European Conference on Foundations of Physics (Bristol) for their helpful questions.

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