Unitarity and Reality of the Quantum State: A New ψ-Ontology Theorem without Assumptions

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn

May 13, 2025

Abstract

We present a new ψ-ontology theorem demonstrating that the quantum wave function is ontic (real) rather than epistemic (representing knowledge) in single-world unitary quantum theories (SUQTs). By leveraging a protocol of repeated reversible measurements on a single quantum system, we show that any two distinct quantum states produce different statistical distributions of (erased) measurement outcomes. This theoretical distinguishability implies that different quantum states correspond to different physical realities, supporting the ontic nature of the quantum state. Unlike previous ψ-ontology theorems, such as the Pusey-Barrett-Rudolph theorem, our proof relies solely on the unitary evolution and Born rule of SUQTs, without additional assumptions like preparation independence. This strengthens its implications for quantum foundations, particularly in restricting non-ψ-ontic interpretations like QBism without assuming an underlying ontic state and its dynamics. The theorem applies to any pair of distinct states in a finite-dimensional Hilbert space, with extensions to infinite-dimensional systems, offering a robust and general argument for the reality of the quantum state.

1 Introduction

The nature of the quantum wave function—whether it describes an objective physical reality (ontic) or merely an observer's knowledge (epistemic)—remains a cornerstone debate in quantum foundations [1]. Resolving this question is crucial for interpreting quantum mechanics and understanding the physical world. ψ-ontology theorems, such as the seminal Pusey-Barrett-Rudolph (PBR) theorem [2], argue that the wave function is ontic by showing that distinct quantum states yield different physical predictions. However, existing theorems often rely on assumptions about the ontic state and its dynamics, like preparation independence, which may limit their generality or invite debate over their physical validity.

In this paper, we propose a new ψ-ontology theorem within the framework of single-world unitary quantum theories (SUQTs), where systems evolve unitarily, measurement outcomes are unique, and probabilities obey the Born rule. Our approach uses a protocol of repeated reversible measurements on a single quantum system, enabled by SUQTs' unitary dynamics, to distinguish any two distinct quantum states via their predicted measurement statistics. Unlike the PBR theorem, our proof requires no additional assumptions beyond the standard postulates of quantum mechanics in SUQTs, making it more general. The theorem applies to any pair of distinct states in a finite-dimensional Hilbert space, with extensions to infinite-dimensional systems, reinforcing the ontic nature of the wave function.

The paper is structured as follows: Section 2 outlines the reversible measurement protocol. Section 3 presents the rigorous mathematical proof of the ψ-ontology theorem. Section 4 explores implications for SUQTs, addresses objections, and evaluates the theorem's impact on non-ψ-ontic interpretations. Section 5 summarizes the findings and suggests directions for future research.

2 Reversible Measurement Protocol

Consider a quantum system in a Hilbert space $\mathcal{H}$ of finite dimension d (e.g., $d = 2$ for a qubit). The system is prepared in a state $|\Psi\rangle$, which is either $|\psi\rangle$ or $|\phi\rangle$, where $|\psi\rangle, |\phi\rangle \in \mathcal{H}$ are distinct ($|\psi\rangle \neq |\phi\rangle$). Alice measures the system in an orthonormal basis $\{|m_k\rangle\}_{k=1}^{d}$, entangling it with her measurement apparatus. A superobserver reverses the measurement, restoring the system and apparatus to their initial states. This process repeats multiple times to generate a sequence of measurement outcomes, whose statistical distribution we analyze.

In SUQTs, the evolution is governed by unitary operators, and measurements are reversible [3]. The initial state is:

$$|\Psi\rangle |\text{ready}\rangle_A , \tag{1}$$

where $|\text{ready}\rangle_A$ is the apparatus's initial state. The measurement applies a unitary U_A^m:

$$U_A^m |m_k\rangle |\text{ready}\rangle_A = |m_k\rangle |m_k\rangle_A , \tag{2}$$

where $|m_k\rangle_A$ is the apparatus state recording outcome k. For a general state $|\Psi\rangle = \sum_k c_k |m_k\rangle$, the measurement yields:

$$U_A^m |\Psi\rangle |\text{ready}\rangle_A = \sum_k c_k |m_k\rangle |m_k\rangle_A . \tag{3}$$

The Born rule gives the probability of outcome k:

$$P(k) = |\langle m_k|\Psi\rangle|^2 = |c_k|^2. \tag{4}$$

The superobserver applies the inverse unitary $U_A^{m\dagger}$, restoring the state:

$$U_A^{m\dagger} U_A^m |\Psi\rangle |\text{ready}\rangle_A = |\Psi\rangle |\text{ready}\rangle_A . \tag{5}$$

This reversibility ensures each measurement is independent, producing a theoretical sequence of outcomes $\{k_1, k_2, \dots, k_N\}$ over N repetitions, with probabilities converging to $P(k)$ as $N \to \infty$.

3 Proof of the ψ-Ontology Theorem

We prove that any two distinct quantum states $|\psi\rangle, |\phi\rangle \in \mathcal{H}$, with $|\psi\rangle \neq |\phi\rangle$, produce different measurement statistics in the reversible measurement protocol, implying that the wave function is ontic.

Theorem 1. *In an SUQT, for any two distinct states $|\psi\rangle, |\phi\rangle \in \mathcal{H}$ ($|\psi\rangle \neq |\phi\rangle$), there exists a measurement basis such that the probability distributions of outcomes differ, i.e., $P_\psi(k) \neq P_\phi(k)$ for some outcome k. This implies that $|\psi\rangle$ and $|\phi\rangle$ correspond to distinct physical realities, supporting the ontic nature of the wave function.*

Proof. Let $|\psi\rangle = \sum_{k=1}^{d} c_k |m_k\rangle$ and $|\phi\rangle = \sum_{k=1}^{d} d_k |m_k\rangle$ in an orthonormal basis $\{|m_k\rangle\}_{k=1}^{d}$, with

$$\sum_k |c_k|^2 = \sum_k |d_k|^2 = 1. \tag{6}$$

In the reversible measurement protocol, Alice measures in the basis $\{|m_k\rangle\}$, producing:

- For $|\psi\rangle$: Probabilities
$$P_\psi(k) = |\langle m_k|\psi\rangle|^2 = |c_k|^2. \tag{7}$$

- For $|\phi\rangle$: Probabilities
$$P_\phi(k) = |\langle m_k|\phi\rangle|^2 = |d_k|^2. \tag{8}$$

The states are distinguishable if $P_\psi(k) \neq P_\phi(k)$ for some k, i.e., $|c_k|^2 \neq |d_k|^2$. Since $|\psi\rangle \neq |\phi\rangle$, their coefficients differ in general. If $|c_k|^2 = |d_k|^2$ for all k in this basis, the states may produce identical distributions. To ensure distinguishability, consider a general measurement basis $\{|n_l\rangle\}_{l=1}^{d}$. The probabilities are:

$$P_\psi(l) = |\langle n_l|\psi\rangle|^2 = \left|\sum_k c_k \langle n_l|m_k\rangle\right|^2, \quad P_\phi(l) = |\langle n_l|\phi\rangle|^2 = \left|\sum_k d_k \langle n_l|m_k\rangle\right|^2. \tag{9}$$

Since $|\psi\rangle \neq |\phi\rangle$, their density operators differ:

$$|\psi\rangle\langle\psi| \neq |\phi\rangle\langle\phi|. \tag{10}$$

The distinguishability is maximized by choosing a basis aligned with the eigenvectors of the difference operator:

$$\Delta = |\psi\rangle\langle\psi| - |\phi\rangle\langle\phi|. \tag{11}$$

Since $\Delta \neq 0$, it has at least one non-zero eigenvalue. Let $|n_l\rangle$ be an eigenvector with eigenvalue $\lambda_l \neq 0$:

$$\Delta |n_l\rangle = \lambda_l |n_l\rangle, \quad \lambda_l = \langle n_l| |\psi\rangle\langle\psi| - |\phi\rangle\langle\phi| |n_l\rangle = P_\psi(l) - P_\phi(l). \tag{12}$$

If $\lambda_l \neq 0$, then

$$P_\psi(l) \neq P_\phi(l). \tag{13}$$

Since $|\psi\rangle \neq |\phi\rangle$, such an eigenvector exists, ensuring a basis where the distributions differ.

The total variation distance quantifies distinguishability:

$$D(P_\psi, P_\phi) = \frac{1}{2} \sum_l |P_\psi(l) - P_\phi(l)|. \tag{14}$$

For distinct states, $D(P_\psi, P_\phi) > 0$ in some basis, as identical distributions in all bases would imply $|\psi\rangle\langle\psi| = |\phi\rangle\langle\phi|$, contradicting $|\psi\rangle \neq |\phi\rangle$. Over N repetitions, the empirical distributions converge to P_ψ and P_ϕ, making the states distinguishable with high probability as $N \to \infty$.

If the quantum state were not ontic, intrinsic to the system, but subjective or epistemic, related to the observer, $|\psi\rangle$ and $|\phi\rangle$ could not always yield different statistics for repeated measurements on the system. The existence of a basis where $P_\psi \neq P_\phi$ implies the ontic nature of the quantum state. $\square$

4 Discussion

Our theorem demonstrates that in SUQTs, any two distinct quantum states produce different measurement statistics in the reversible measurement protocol, implying that the quantum state is ontic. Below, we address potential objections and discuss implications in detail.

4.1 Deterministic Hidden-Variable Theories

An objection arises in deterministic hidden-variable theories, such as de Broglie-Bohm theory, where the system's complete state includes a hidden variable $\lambda \in \Lambda$ (e.g., particle positions) that determines measurement outcomes. For a single system in state $|\psi\rangle = \sum_k c_k |m_k\rangle$, a fixed λ could consistently produce the same outcome (e.g., $k = 1$) across repeated measurements, yielding a sequence $\{1, 1, \ldots\}$ instead of the Born rule probabilities $P_\psi(k) = |\langle m_k|\psi\rangle|^2$.

We address this within the single-system reversible measurement protocol (Section 2). Each cycle involves Alice's measurement with apparatus A, applying U_A^m : $|m_k\rangle |\text{ready}\rangle_A \to |m_k\rangle |m_k\rangle_A$, followed by the superobserver's reversal using $U_A^{m\dagger}$, restoring $|\psi\rangle |\text{ready}\rangle_A$. The reversal, being a distinct process from the measurement, involves a separate system (e.g., the superobserver's apparatus) with independent hidden variables λ_R.

In de Broglie-Bohm theory, the configuration is $\lambda = (q_s, q_A, q_R)$, initially distributed as $\mu_\psi(q_s, q_A, q_R) = |\langle q_s|\psi\rangle|^2 |\langle q_A|\text{ready}\rangle_A|^2 |\langle q_A|\text{ready}\rangle_R|^2$. For cycle i, The measurement evolves $\lambda_0^{(i)} = (q_s^0, q_A^0, q_R^0) \to \lambda_1^{(i)} = (q_s^1, q_A^1, q_R^0)$, with q_A^1 indicating outcome k_i. The reversal, driven by $U_A^{m\dagger}$ and coupled to λ_R, restores the wave function, evolving $\lambda_1^{(i)} \to \lambda_2^{(i)} = (q_s^2, q_A^2, q_R^2)$, where $q_s^2 \sim |\langle q_s|\psi\rangle|^2$. Since λ_R is independent of q_s^0 and q_A^0, the guidance equation resamples q_s^2 from $\mu_\psi(q_s)$, breaking correlations with q_s^0. Then the sequence $\{k_1, k_2, \ldots, k_N\}$ will reflect $P_\psi(k) = |\langle m_k|\psi\rangle|^2$. For distinct states $|\psi\rangle \neq |\phi\rangle$, the distributions $\mu_\psi \neq \mu_\phi$ produce:

$$P_\psi(k) = |\langle m_k|\psi\rangle|^2 \neq P_\phi(k) = |\langle m_k|\phi\rangle|^2, \tag{15}$$

in some basis. In general deterministic theories, the reversal's independent hidden variables resample λ_s from μ_ψ, preserving the Born rule. The wave function's role in driving the statistics implies its ontic status, as it determines the system's distinct behaviors.

4.2 Alice's Outcome Statistics

An objection to the ψ-ontology theorem (Section 3) asserts that the statistics of Alice's measurement outcomes in the reversible measurement protocol (Section 2) lack physical significance because they are erased by the superobserver's unitary reversal ($U_A^{m\dagger}$). Since quantum mechanics prohibits permanently recording the sequence $\{k_1, k_2, \ldots, k_N\}$, and the outcomes occur sequentially without coexisting at any instant, the statistical distribution $P_\psi(k)$ is argued to be merely an illusion, undermining its comparison with $P_\phi(k)$ for a distinct state $|\phi\rangle$. This challenges the theorem's assertion that differing statistics imply $|\psi\rangle \neq |\phi\rangle$ represent distinct physical realities.

This objection can be answered by noting that although the outcomes are erased sequentially and do not coexist at any instant, the statistics are well-defined predictions derived from the unitary evolution and Born rule, and they are physically significant as consistent predictions of the wave function that characterize the system's behavior across repeated, independent measurement cycles. In each cycle, Alice applies the unitary U_A^m, producing a definite outcome k_i with probability $P_\psi(k_i)$. The superobserver's reversal ($U_A^{m\dagger}$) restores the system to $|\psi\rangle\,|\text{ready}\rangle_A$, ensuring that each subsequent measurement is independent and governed by the same probabilities. The statistics $P_\psi(k)$ do not require the outcomes to coexist or be recorded; rather, they represent the expected frequencies of outcomes that would be observed if the sequence were hypothetically collected, providing a complete description of the system's measurement behavior in the protocol.

To illustrate, consider a classical analogy: a fair coin tossed repeatedly, with each outcome (heads or tails) erased immediately after being recorded by an observer who then resets the coin to its initial state. The probability of heads (0.5) characterizes the coin's behavior, predicting that over many tosses, approximately half would yield heads if recorded. This probability remains meaningful even if each toss's outcome is erased and the tosses occur sequentially, as it consistently governs the expected frequency of outcomes. Similarly, in the quantum protocol, the system is reset to $|\psi\rangle$ after each measurement, and $P_\psi(k)$ predicts the frequency of outcome k across cycles, as if the sequence $\{k_1, k_2, \ldots, k_N\}$ could be observed. The reversibility of the protocol ensures that each cycle is a fresh probe of the same quantum state, making the statistics a robust characterization of the system's behavior, independent of whether the outcomes persist or coexist.

4.3 Infinite-Dimensional Systems

The theorem assumes a finite-dimensional Hilbert space for simplicity, but a possible objection is its applicability to infinite-dimensional systems, such as position or momentum states in quantum mechanics. We extend the proof to infinite-dimensional Hilbert spaces with discrete measurement outcomes, ensuring the theorem's generality.

Consider a quantum system in an infinite-dimensional Hilbert space $\mathcal{H}$, with states $|\psi\rangle = \sum_{k=1}^{\infty} c_k |m_k\rangle$ and $|\phi\rangle = \sum_{k=1}^{\infty} d_k |m_k\rangle$, where $\{|m_k\rangle\}$ is a countable orthonormal basis (e.g., Fock states for a harmonic oscillator), and

$$\sum_{k=1}^{\infty} |c_k|^2 = \sum_{k=1}^{\infty} |d_k|^2 = 1, \quad \langle\phi|\psi\rangle = \sum_{k=1}^{\infty} d_k^* c_k \neq 0, \quad |\psi\rangle \neq |\phi\rangle . \tag{16}$$

The reversible measurement protocol applies as in the finite-dimensional case, with Alice measuring in the basis $\{|m_k\rangle\}$, yielding probabilities:

$$P_\psi(k) = |c_k|^2, \quad P_\phi(k) = |d_k|^2. \tag{17}$$

If $|c_k|^2 \neq |d_k|^2$ for some k, the distributions differ. If $|c_k|^2 = |d_k|^2$ for all k, consider a new basis $\{|n_l\rangle\}_{l=1}^{\infty}$. The probabilities are:

$$P_\psi(l) = |\langle n_l|\psi\rangle|^2, \quad P_\phi(l) = |\langle n_l|\phi\rangle|^2. \tag{18}$$

The operator $\Delta = |\psi\rangle\langle\psi| - |\phi\rangle\langle\phi|$ is non-zero, and in a separable Hilbert space, it has a spectral decomposition. Since $|\psi\rangle \neq |\phi\rangle$, there exists an eigenvector $|n_l\rangle$ with eigenvalue:

$$\lambda_l = P_\psi(l) - P_\phi(l) \neq 0, \tag{19}$$

ensuring $P_\psi(l) \neq P_\phi(l)$. The total variation distance remains:

$$D(P_\psi, P_\phi) = \frac{1}{2} \sum_{l=1}^{\infty} |P_\psi(l) - P_\phi(l)| > 0. \tag{20}$$

For measurements with finitely many outcomes (e.g., a coarse-grained position measurement), the sum is finite, and convergence of empirical distributions holds as in the finite-dimensional case. For continuous outcomes, one must consider probability density functions, but discrete measurements (common in experiments) suffice to distinguish the states, preserving the theorem's conclusion.

4.4 Examples of SUQTs

SUQTs are defined as quantum theories where isolated systems evolve unitarily via the Schrödinger equation, measurement outcomes are unique, and probabilities obey the Born rule. Several non-ψ-ontic quantum theories, often considered epistemic, qualify as SUQTs because they use the wave function and its unitary evolution for predictions. These include consistent histories [5], ψ-epistemic models [6], pragmatist approaches [7], QBism [8], relational quantum mechanics [9], and perspectivalism [10, 11]. We analyze why these theories are SUQTs and why our theorem implies their wave functions are ontic, contrary to their conventional non-ψ-ontic interpretations.

4.4.1 Consistent Histories

The Consistent Histories (CH) interpretation [5] is a SUQT where the wave function evolves unitarily between events, and probabilities for consistent sets of histories are computed via the Born rule. Measurements are modeled as unitary interactions, with outcomes described by projectors on the Hilbert space, avoiding wave function collapse. CH employs a single framework rule, restricting reasoning to a single projective decomposition of the identity (PDI) to prevent paradoxes from combining incompatible quantum descriptions. While some interpretations of CH suggest an epistemic view of the wave function, treating it as a tool for assigning probabilities within frameworks, our ψ-ontology theorem demonstrates its ontic status.

4.4.2 ψ-Epistemic Models

ψ-epistemic models, such as Spekkens' toy model [6], aim to represent quantum states as overlapping distributions over ontic states. However, when these models use the wave function for predictions, they assume unitary evolution for isolated systems and Born rule probabilities, qualifying as SUQTs. The different statistics for distinct states refute the epistemic view, as overlapping ontic states would produce identical statistics. Below is a more detailed explanation.

Assume a single quantum system, prepared in either $|\psi\rangle$ or $|\phi\rangle$ ($|\psi\rangle \neq |\phi\rangle$), has the same (complete) ontic state $\lambda_0 \in \Lambda$ as required by a ψ-epistemic model, lying within the overlap region where $\mu_\psi(\lambda_0) > 0$, $\mu_\phi(\lambda_0) > 0$. Given that for both wave functions, the apparatus starts in the same ontic state, and the dynamics of the ontic state of the whole system are unique and identical, determined by the same Hamiltonian or its corresponding quantity in the model, one would expect the statistical distributions of measurement outcomes to be identical. Yet, the theorem demonstrates that SUQTs predict different statistics ($P_\psi(k) = |\langle m_k|\psi\rangle|^2 \neq P_\phi(k) = |\langle m_k|\phi\rangle|^2$) in some basis, indicating that the ψ-epistemic model is not consistent with the prediction of SUQTs.

4.4.3 Pragmatist Approaches and QBism

Pragmatist approaches [7] and QBism [8] treat the wave function as a tool for assigning probabilities based on an agent's knowledge. Both rely on unitary evolution for isolated systems and the Born rule for outcome predictions. In these interpretations, wave function collapse is considered epistemic, reflecting an update in the agent's knowledge rather than a physical change in the system. This epistemic collapse does not affect the reversible measurement protocol, as the protocol's unitary operations (U_A^m and $U_A^{m\dagger}$) are applied to the system-apparatus state, which evolves unitarily in SUQTs. The superobserver's reversal restores the system to its initial state $|\Psi\rangle\,|\text{ready}\rangle_A$, independent of any epistemic update by Alice. Thus, pragmatist approaches and QBism remain SUQTs, as their predictions rely on the unitary dynamics and Born rule. Our theorem shows that different wave functions yield distinct statistics, implying that they represent distinct physical realities, challenging the subjective or epistemic interpretations of these frameworks.

4.4.4 Relational Quantum Mechanics and Perspectivalism

Relational quantum mechanics [9] and perspectivalism [10, 11] define quantum states relative to observers or reference systems, with unitary evolution governing the system-apparatus interaction. Outcomes are unique relative to each observer, and probabilities follow the Born rule, fitting the SUQT framework. The distinct statistics for different wave functions (for the same observer Alice) suggest that these states are ontic within each relational context.

4.4.5 Summary

The key criterion for SUQTs is the unitary evolution of the wave function for an isolated system, coupled with the Born rule. The above theories, despite their often non-ψ-ontic claims, meet this criterion, making our theorem applicable. The theorem's critical implication is that it challenges the non-ψ-ontic interpretation without assuming an underlying ontic state, as required by the PBR theorem [2]. In QBism and similar theories, which reject a realist ontology, the distinct statistics for different wave functions imply that the wave function encodes objective physical differences, establishing its ontic status.

4.5 Collapse Theories and the Many-Worlds Interpretation

In collapse theories (e.g., GRW [12]) and the many-worlds interpretation (MWI) [13] of quantum mechanics, which are not considered in the above ψ-ontology theorem for SUQTs, the wave function is explicitly ontic, and no proof is needed to establish its reality.

Moreover, there are also reasons why these theories assume the reality of the wave function. In collapse theories, if the wave function were not real, a post-measurement superposition (e.g., $\sum_k c_k |m_k\rangle |m_k\rangle_A$) and a collapsed state (e.g., $|m_j\rangle |m_j\rangle_A$) could represent the same physical state (a definite outcome j). This would render collapse unnecessary, as the superposition could already correspond to a single outcome. Similarly, in MWI, if the wave function were not real, different branches of a post-measurement superposition (e.g., $\sum_k c_k |m_k\rangle |m_k\rangle_A$) could represent a single world with a definite outcome, which would render the assumption of many worlds unnecessary.

5 Conclusions

We have presented a new ψ-ontology theorem showing that in single-world unitary quantum theories (SUQTs), any two distinct quantum states produce different measurement statistics in a reversible measurement protocol. This implies that the quantum state is ontic. The proof's generality, requiring no assumptions beyond unitary evolution and the Born rule, strengthens its implications for quantum foundations, particularly in restricting non-ψ-ontic interpretations like QBism without assuming an underlying ontic state and its dynamics. The theorem applies to finite and infinite-dimensional systems. Future work could explore further extensions to non-SUQT frameworks.

References

[1] S. Gao, The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics. Cambridge: Cambridge University Press. (2017).

[2] M. F. Pusey, J. Barrett, and T. Rudolph, On the reality of the quantum state, *Nature Physics* **8**, 475 (2012).

[3] C. Brukner, On the quantum measurement problem, *arXiv:1507.05255* (2015).

[4] S. Gao, A new EPR-Bohm experiment with reversible measurements, *PhilSci-Archive*, `https://philsci-archive.pitt.edu/20836/` (2022).

[5] R. B. Griffiths, The Consistent Histories Approach to Quantum Mechanics, The Stanford Encyclopedia of Philosophy (Summer 2024 Edition), Edward N. Zalta and Uri Nodelman (eds.), https://plato.stanford.edu/archives/sum2024/entries/qm-consistent-histories/.

[6] R. W. Spekkens, Evidence for the epistemic view of quantum states: A toy theory, *Phys. Rev. A* **75**, 032110 (2007).

[7] R. Healey, Quantum-Bayesian and pragmatist views of quantum theory, in *The Stanford Encyclopedia of Philosophy*, edited by E. N. Zalta (Spring 2017 Edition), `https://plato.stanford.edu/archives/spr2017/entries/quantum-bayesian/`.

[8] C. A. Fuchs, N. D. Mermin, and R. Schack, An introduction to QBism with an application to the locality of quantum mechanics, *Am. J. Phys.* **82**, 749 (2014).

[9] C. Rovelli, Relational quantum mechanics, *Int. J. Theor. Phys.* **35**, 1637 (1996).

[10] D. Dieks, Quantum mechanics and perspectivalism, *arXiv:1801.09307* (2018).

[11] D. Dieks, Quantum reality, perspectivalism and covariance, *Found. Phys.* **49**, 629 (2019).

[12] G. C. Ghirardi, A. Rimini, and T. Weber, Unified dynamics for microscopic and macroscopic systems, *Phys. Rev. D* **34**, 470 (1986).

[13] H. Everett III, “Relative state” formulation of quantum mechanics, *Rev. Mod. Phys.* **29**, 454 (1957).