Relativistic Length and Space Contractions for Light-Speed Systems in Discrete Space-Time

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Abstract

In this paper we argue that because photons experience change, such as coming into and going out of existence and interacting with other particles, that photons must experience time...we then calculate how much time photons experience. We start with a discussion of the philosophical position advocated by Aristotle and most philosophers: in the absence of time, no change, no interaction, no "becoming" is possible for an entity. We then review the laws of special relativity, first reviewing the conventional laws of relativity if both space and time are continuous, and then show how the laws change if both space and time are discretized (atomized, granular...). We show that if space and time are discretized, then the Lorentz factor describing length contraction and time dilation no longer diverge to infinity as the velocity of a system approaches and reaches the speed of light. In doing so, we derive equations that provide the finite Lorentz factors and time durations experienced by light-speed particles. We then discuss how these results significantly strengthen the case that space and time are discretized.

Keywords: time, change, discrete space, discrete spacetime, relativity, light

1 Introduction

From our everyday experiences to our most sophisticated experiments, we see that light interacts with things. In other words, light can and does change. Aristotle and most philosophers since then have stated that some flow of time, however small, is required for change to occur [1].[1] Thus, photons (of which light is composed) must

[1] Aristotle "neither does time exist with no change" [1], David Hume "tis impossible to conceive...a time when there was no succession or change" [2], Henri Bergson, Alfred North Whitehead, Martin Heidegger

experience time, and not just the time of their reference frame but also the time of our sub light-speed reference frame. This, however, violates the widely-accepted laws of special relativity. When questioned about photons' experiencing time in the context of special relativity, contemporary physicists will state one of two things, either that special relativity is inapplicable to light-speed travel, or that the laws predict that light-speed systems do not experience our sub light-speed time and space. If the latter is true, then light could not change or interact, and the commonly observed optical and photonic phenomena such as reflection, refraction, light generation and detection, electron-positron generation from a high energy photon, and countless other phenomena would not be possible. The objective of this paper is to show that this is not true, and that photons experience a calculable duration of the time that is experienced by sub light-speed reference frames, and similarly for space. Certainly, the time and space experienced by sub light-speed systems will be greatly contracted from the photons' perspective, but it will be shown time and space are not contracted to zero. The necessary condition though, is that space and time are discretized, i.e., atomized.

In the modern era, relativity theory dominates the discussion on space and time, but this has not always been the case. Philosophers have debated these topics from the time of some the very first philosophers, starting with Parmenides and Zeno of Elea in the 5th century BC with their famous paradoxes, up to modern times with famous works that include Henri Bergson's works [5], Bertrand Russell's [6], Sydney Shoemaker's work on changeless time [4], quantum gravity and time [7, 8], and Roger Penrose's cycling of time [9] to name but a few of the practically countless number of works on the topic. Many of these works focus on the inevitability/necessity of time, change, and the connection between time and change [1], [2], [3], [4]. Several of these works argue that change does not occur, that time is nonexistent or an emergent property; others argue the converse. In this paper, we largely steer clear of this debate and assume: (1) time exists, (2) the experience of time is required for change to occur, (3) we observe that photons undergo change. From this starting point we show that photons experience time in their reference frame...but only if spacetime is discretized.

This paper—with its objective of demonstrating a connection between light's time experience and the discretization of spacetime—is a small but important side topic of our larger works on discrete spacetime (DST) that has as their overarching objective the development of a model of DST (the *Isotropic Model*) that is mathematically consistent, does not suffer from any of the problems typically associated with DST (see [10]), explains one or more heretofore unexplained observed physical phenomena, and is falsifiable. In this paper, we argue that the observed nature of light, with all the ways it dynamically interacts with the universe, is the experimental (observable, measurable...) proof we need of the discretization of time and space. This paper is organized as follows. In Section 2, we review conventional special relativity in continuous space time (CST) only up to the point of deriving the equation describing time dilation (this is as far as we need to go in special relativity in CST for the purposes of

"change is time" [3], and most famously Sydney Shoemaker [4] have all discussed the possibility of time without change, but no one has considered the possibility of the converse, namely *change without time*.

this paper). We then discuss the *Isotropic Model of DST* (the Crouse Model of [10]) in Section 3. In Section 4, we derive time dilation in DST and discuss some results.

2 Special Relativity in Continuous Spacetime

The concepts of length contraction and time dilation in the theory of special relativity (SR) can be derived in a several ways. One way is to look at the space time coordinates in two reference frames, namely (x, t) and (x', t'), assume that there is a linear transformation between the two sets of coordinates, and invoke the invariance of the square of the spacetime interval, namely $ds^2 = c^2t^2 - x^2 - y^2 - z^2 = ct'^2 - x'^2 - y'^2 - z'^2$. Alternatively—and what is done in most elementary texts on the subject—one can look at a light pulse on a train and invoke the constancy of the speed of light (as Einstein told us). The latter approach, simple as it may be, is a much better approach to achieve the objectives of this paper, having greater transparency and defensibility of the assumptions on which it relies. It also is superior because it better helps us glean the impacts of the discretization of space and time on physical phenomena. Because of this, it is beneficial to review the derivation of time dilation in SR in CST before introducing special relativity in DST.

The derivation of time dilation is easy once the constancy of the speed of light is invoked. Typically, one studies two reference frames, a rest reference frame (RF1) with spacetime coordinates (x, t), and a moving reference frame (RF2) with spacetime coordinates (x', t'). One then considers a light pulse on a moving train (with a velocity v) observed by two sets of observers, one set in RF1 (alongside and stationary with respect to the tracks) and the other set in RF2 (in the train)—see Fig. 1. As is seen in the figure, the set of observers on the train see the light pulse traveling at a speed $c(\approx 3 \times 10^8 m/s)$ in a straight vertical direction from the floor of the train to the detector on the ceiling—a distance h covered over a duration in time of $\Delta t' = h/c$. Observers in RF1, however, will see the pulse of light propagating diagonally, propagating a horizontal distance of $x = v\Delta t$ and a vertical distance $h = c\Delta t'$.[2] Because the speed of light must be the same for observers in RF1 and RF2, thus the length of the diagonal path of the light (as observed by observers in RF1) is $d = c\Delta t$. Using the Pythagorean theorem on the triangle in Fig. 1b, we obtain:

$$c\Delta t = \sqrt{(c\Delta t')^2 + (v\Delta t)^2} \tag{1}$$

which we then use to derive the relation between time durations in the moving frame ($\Delta t'$) relative to time durations in the rest frame (Δt):

$$\Delta t = \frac{1}{\sqrt{1-\beta^2}}\Delta t' = \gamma_{CST}\Delta t' \tag{2}$$

with $\beta = v/c$ and $\gamma_{CST} = 1/\sqrt{1-\beta^2}$ is the Lorentz factor in CST. One therefore sees that over the course of their journey, people on the train traveling at a velocity v will age slower than those at the station. For example, if the train is traveling at

[2]Note that length contraction does not occur in any direction perpendicular to direction of travel.

$v = \sqrt{3/4}c = 0.87c$, then $\Delta t = 2\Delta t'$ and the people on the train will age half as much as those waiting at the station. This effect becomes even more extreme as $v \to c$ as γ becomes much larger than one, diverging to infinity as v approaches c (nature's speed limit).

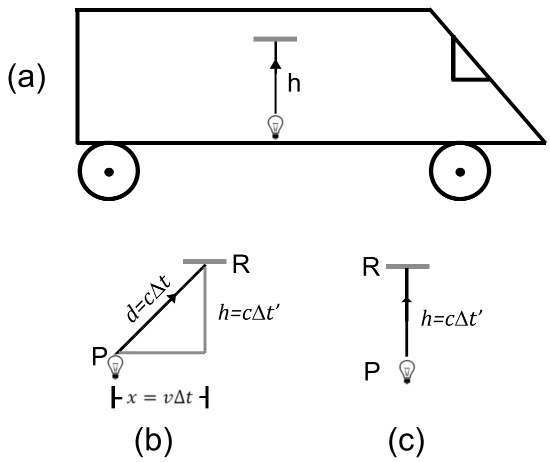

Fig. 1 **(a)**: The thought experiment used to derive time dilation. It considers a light clock from the perspective of two sets of observers in different reference frames, the rest reference frame RF1 and the moving reference frame RF2. **(b):** The photon's trajectory as observed by observers in RF1 alongside the track. **(c):** The photon's trajectory as observed by observers in RF2 inside the train.

So far in this review of time dilation, nothing new is introduced that is not in elementary textbooks on special relativity. However, upon being introduced to time dilation and Eq. (2) clever students often ask the question "What happens when the reference frame (or particle) is traveling at a velocity of c?". Most textbooks avoid the question entirely, but a few make the bold but incorrect statement that these particles (such as photons and possibly neutrinos) do not experience the time that we, in our sub light-speed reference frame, experience. We dispensed this fallacy quickly but adequately in the *Introduction* where we deferred to Bergson, Russell, Whitehead... and their respected opinions on the matter. We thus strive to solve this problem, namely the inadequacies of our existing theoretical approaches describing particles that travel at the speed of light. But how does one solve this problem? Certainly, the constancy of the speed of light in the two reference frames is unassailable. It is the only other part of the derivation that we take issue with and discuss in the next section, namely the use of the Pythagorean theorem as the distance formula.

3 Discrete Spacetime

It will be seen in this paper that the key to allowing light to experience time is the discretization of space and time. Therefore, a brief review of this topic is provided in this work. Readers interested in a comprehensive discussion of DST, its history, models, governing equations and predictions are referred to [10].

The debate on whether space and time are continuous or discrete (i.e., atomized, granular) was introduced by some of the very first philosophers: Zeno of Elea and Parmenides in ∼500BC and it has continued to generate strong interest by philosophers, mathematicians, and the occasional layperson. This interest is justified, since the topic and the paradoxes it spawned (i.e., the well known Zeno's paradoxes) have "an onion-like quality; as one peels away outer layers by disposing of the more superficial difficulties, new and more profound problems are revealed." [11, 43]. Good reviews on the topic of discrete spacetime are [12], [10] and [13]; a good review of Zeno's paradoxes is [11]. Most models of DST include as their starting points the incorrect assumption that space is arranged in some fixed lattice (usually cubic) with a cell-to-cell (i.e., periodicity) of a hodon χ typically assumed to be equal to the Planck length ($\chi = l_p = 1.62 \times 10^{-35} m$) and with time occurring in snapshots with a regularity of a chronon τ typically assumed to be equal to the Planck time ($\tau = \tau_p = 5.39 \times 10^{-44} s$) [13]. Unfortunately, these models with their fixed lattices are wrong because they suffer from one (usually more) of the many problems often associated with DST, with the one most cited by physicists being that the models do not adhere to the laws of special relativity.[3] Summarized in the next section is a DST model that does *not* suffer from these problems and that will be used in this paper to study light-speed travel: the *Isotropic Model of DST* described in [10].

3.1 The Isotropic Model of Discrete Spacetime

For the purposes of this paper, we describe below only the aspects of the *Isotropic Model of DST* that are necessary to show how DST allows light to experience time.

1. Rather than space being quantized in a lattice arrangement or some aperiodic structure (e.g., Penrose tiles), the model has *translations in any direction* being quantized, with any step being of extent χ. The model also has time being discretized with a step size of τ. χ and τ typically taken to be equal to the Planck length ($1.61 \times 10^{-35}\ m$) and Planck time ($5.39 \times 10^{-44}\ s$) respectively.[4]
2. Starting from a point in space (*Point A*), a *Point C* is reached when the leading edge (α_4 in Fig. 2) of the translating point (*fourth blue point along the diagonal*) overlaps the trailing edge (θ in Fig. 2) of the endpoint *(third green point along the vertical*).

From these two aspects of the model, we get a new distance formula, namely the length of the hypotenuse ($n\chi$) given the lengths of the base ($m\chi$) and height ($n'\chi$):

$$n = \left\lfloor \sqrt{m^2 + n'^2} \right\rfloor \tag{3}$$

[3]The major problems involve: spatial anisotropy (imposed by a grid) of physical phenomena, causality of instant signal propagation across a hodon, non conservation of energy of particle motion in DST, the Weyl-tile argument, and nonadherence to the laws of special relativity [10].

[4]Different methods of calculating χ and τ can be found in [10, 14–18].

Another way to express Eq. (3) is that n is the smallest integer that satisfies the relation:

$$n > \sqrt{m^2 + n'^2} - 1 \tag{4}$$

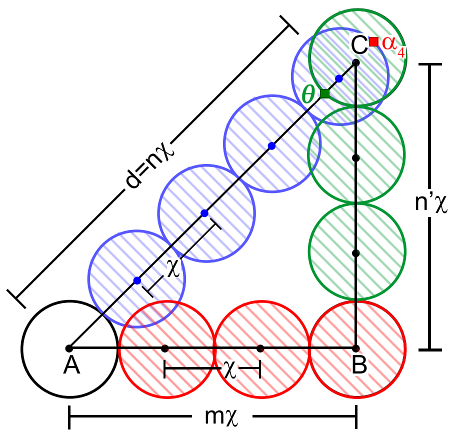

Fig. 2 For an arbitrarily large triangle, the distance formula Eq. (3) is easily derived by determining how many translations are required along $\overline{AC}$ such that the leading edge of the translated point along the hypotenuse (denoted by α_n) overtakes the trailing edge (denoted by θ) of the sphere that defines point C.

The new distance formula has profound impacts on relativity theory, including a Lorentz factor γ that depends not only on the relative velocity of two reference frames but also on distances and times, disagreements amongst observers even in the same reference frame on lengths and times measured in a moving reference frame, positional and temporal uncertainty upon the cessation of motion, and other intriquing issues. Additionally, important mathematical issues are raised with an distance formula of the sort Eq. (3), the most important being its non adherence to the triangle inequality. Each of these things is worthy of extensive discussion and in some cases require major changes in our thinking about space, time, and motion; in this paper, however, we focus on just one of these things: the Lorentz factor and its application to light-speed motion.

3.2 The Lorentz Factor in Discrete Spacetime

Armed with the new distance formula applicable for DST (Eq. (3)) we can redo the derivation of the Lorentz factor.[5] However, the derivation assuming DST is a bit more

[5]The issues of a modified distance formula and time dilation in DST was discussed first in [19] and later by [20]. The distance formulas developed by both are the same and both works have equal merit, yet the two works are different. Crouse comprehensively developed the formula and discussed how the new model addresses all the problems typically associated with DST. Gudder's work has the same starting point—measurements and logical positivism—but he quickly gets to the new version of the distance formula and, without further discussion of the physical or philosophical issues associated with DST, he quickly moves on

complicated: one needs to look at an array of light-clocks on the train (traveling at an average velocity v_{avg}) rather than just a single clock that was used to derive Eq. (2). As shown in Fig. 3, an array of light-clocks is used, with each subsequent clock having a longer length...longer by an amount χ. For each clock with a defined n', we have two equations to solve for the two unknowns n and m—the two equations being Eq. (4) and the equation (or correspondence) that provides the distance traveled $m\chi$ as a function of time in the rest reference frame $n\tau$—we discuss this (n, m) correspondence next.

To find the (n, m) correspondence, we need to discuss movement and velocity in CST and in DST. In CST, v_{avg} can be equal to the instantaneous velocity at any time of a particle's travel, and the distance traveled Δx and travel time Δt do not have to be integer multiples of χ and τ respectively. DST, however, provides more flexibility in how movement of the particle occurs as a function of time. In DST, motion occurs by having the particle undergo Q number of χ movements in each $N\tau$ segment of time of a *total* travel time of $P\tau$, with Q, N and P being integers, $Q \leq N$, and with $\frac{Q\chi}{N\tau} = \frac{Q}{N}c \approx v_{avg}$. These Q movements can occur in different ways in each $N\tau$ time segments that make up the total travel time of the particle. For example, the Q movements can all occur during the first Q ticks in a N-tick sequence...if this occurs for all of the N-tick sequences that compose the total travel time $P\tau$, then $m = \lceil \frac{Q}{N} n \rceil$ and we therefore have the m to n correspondence we need, along with Eq. (4), to solve for each $(n, m; n')$ tuple. Another example is where the Q number of χ movments all occur during the last Q ticks in a N-tick sequence...if this occurs for all of the N-tick sequences that compose the total travel time $P\tau$, then $m = \lfloor \frac{Q}{N} n \rfloor$. More generally, these Q movements can occur randomly over each N-tick sequence while still maintaining the same v_{avg}; if this is the case, the best we can do is to express m as a function of n, namely $m = m(n)$, namely, have a look-up table for the motion of the particle. In all cases, as n becomes large, we have $m\chi \to v_{avg}\Delta t$. In the example show below in Fig. 4 and Table 1 we set $v_{avg} = 0.5c$ and we make the arbitrary choice of having the particle move every second tick in each two-tick sequence (i.e., $Q = 1$ and $N = 2$ and P should be larger than 2), namely, having the distance traveled m in Fig. 4 as $m = \lfloor \frac{v_{avg}}{c} n \rfloor \chi = \lfloor \frac{n}{2} \rfloor \chi$. With our relation associating m to n being set, we can solve Eq. (4) for $(n, m; n')$—results are shown in Fig. 4 and Table 1.

to relativity and quantum field theory in DST. Concerning time dilation and light-speed travel in DST, both works end up with similar results but similar to their treatment of the distance formula, Crouse develops time dilation with far greater care and discussion whereas Gudder moves on quickly to quantum field theory.

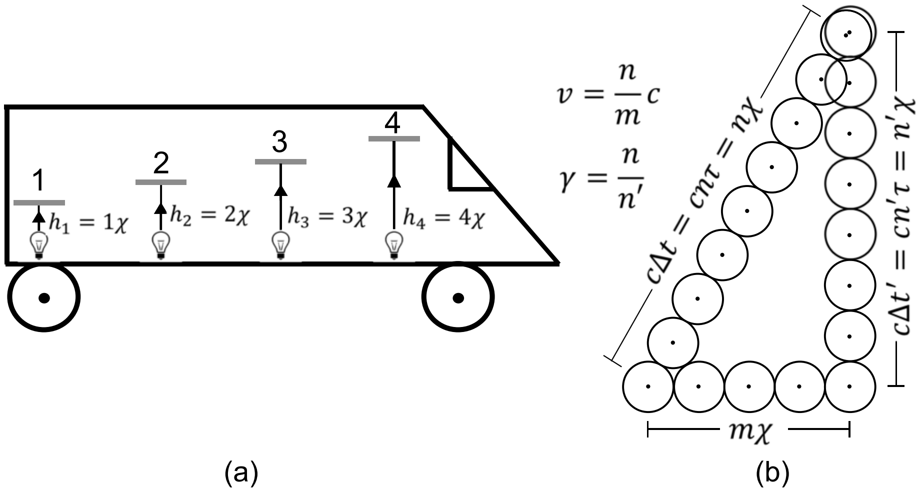

Fig. 3 **(a):** An array of light clocks on a train traveling at a velocity $v_{avg} = 0.5c$. The clocks vertical lengths as integer multiples of χ. **(b):** The photon's trajectory (the hypotenuse) as observed by observers in RF1 alongside the tracks. In this case, observers in RF1 (trackside) observe the photon traversing 8 hodons while observers in RF2 (in the train) observe the photon traversing 7 hodons–the triplet $(m, n, n') = (4, 8, 7)$ satisfies Eq. (4) with $v_{avg} = 0.5c$.

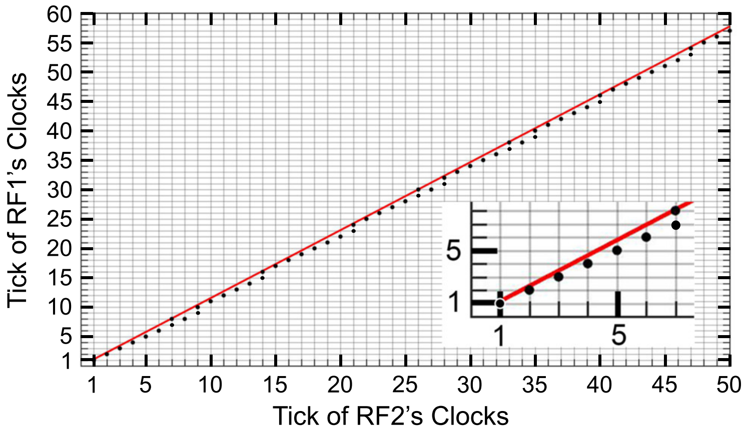

Fig. 4 The correspondence between the ticks of the clocks in RF1 to the ticks of the clocks in RF2. The RFs have a relative velocity of $0.5c$. The red line shows $\gamma_{CST} = 1.15$. **Inset**: No time dilation occurs for the first seven ticks (each of duration τ), but then the clocks on the train start trailing the clocks at the station. (from [10])

Table 1 The height, base and hypotenuse (relative to χ) of the triangles traced out by the photons in the light-clocks, the velocity (relative to c), and γ. The target v_{avg} is $0.5c$ The correspondence between the ticks of the clocks in RF1 and RF2 is also given.

Height (n') or tick of RF2's clock	Base (m)	Hypotenuse (n) or tick of RF1's clock	$\frac{v}{c}$	$\gamma(v, n')$
1	0	1	0	1
2	1	2	0.5	1
3	1	3	0.33	1
4	2	4	0.5	1
5	2	5	0.4	1
6	3	6	0.5	1
7	3	7	0.43	1
7	4	8	0.5	1.14
8	4	8	0.5	1
9	4	9	0.44	1
9	5	10	0.5	1.11
10	5	11	0.45	1.10
11	6	12	0.5	1.09
12	6	13	0.46	1.08
13	7	14	0.5	1.08
14	7	15	0.47	1.07
14	8	16	0.5	1.14
15	8	17	0.47	1.13
16	8	17	0.47	1.06
16	9	18	0.5	1.13
17	9	19	0.47	1.12
18	10	20	0.5	1.11
19	10	21	0.48	1.11
20	11	22	0.5	1.10
21	11	23	0.48	1.10
21	12	24	0.5	1.14
22	12	25	0.48	1.14
23	12	25	0.48	1.09
23	13	26	0.5	1.13
24	13	27	0.48	1.13
25	14	28	0.5	1.12
26	14	29	0.48	1.12
27	15	30	0.5	1.11
28	15	31	0.48	1.11

4 Time Dilation at the Speed of Light

An important behavior of the *Isotropic Model of DST* and its distance formula (Eq. (4)) is that the hypotenuse ($n\chi$) *can equal the base* ($m\chi$) as long as $n\chi$ and $m\chi$ are larger than some particular value of the height $n'\chi$.[6] This capability of DST is not shared with CST and is important for light-speed travel. From the perspective of observers in RF1 (trackside), light-speed travel has the lateral distance traveled by the train (the base of the triangle) being equal to the diagonal distance traveled by the

[6]When this occurs, the triangle inequality is violated. However, it is seen in this work that this violation *must* occur if light is allowed to interact with sub light-speed particles.

photon (the hypotenuse). To derive the condition (i.e., how large n has to be relative to n') required for this to occur, we set $m = n$ in Eq. (4) and simplify, obtaining the following relation between the ticks of the moving clock (n') and the ticks of the stationary clock (n):

$$n > \frac{n'^2 - 1}{2} \tag{5}$$

where n is the smallest integer that satisfies Eq. (5) and with the requirement that $n \geq n'$. Examples include $n = m = n' = 1$, $n = m = n' = 2$ and, as shown in Fig. 5, the case when $n = m = 5$ and $n' = 3$. Another way to express Eq. (5) is using the floor operation:

$$n = \left\lfloor \frac{n'^2 - 1}{2} + 1 \right\rfloor \tag{6}$$

For large n' values, Eq. (5) and (6) converge to:

$$n \xrightarrow{n'>>1} \frac{n'^2}{2} \tag{7}$$

The Lorentz factor γ is:

$$\gamma = \frac{n}{n'} = \frac{\left\lfloor \frac{n'^2-1}{2} + 1 \right\rfloor}{n'} \xrightarrow{n'>>1} \frac{n'}{2} = \sqrt{\frac{n}{2}} \tag{8}$$

Equation (8) yields the important result that the Lorentz factor γ is unity for time durations of the one or two chronons but quickly grows to exceptionally large values—so large that no discrepancies with experimentally measured Lorentz factors would be discernable at the smallest time durations measurable with existing technology (i.e., attosecond time-scale measurements). The fact that no length and time contraction occurs for $n = n' = 1$ is important so that the foundational concept of the immutability of the atoms of space and time are upheld—the atom of space always has a length χ and the atom of time always has a length of τ regardless of the speed of the reference frame from which an observer is measuring their length and duration.[7]

We now can quantify the time experienced by the photon. From Eq. (6) we see that for the first two ticks of the clocks in RF1 and RF2 match, but then the clocks on the light-speed train run progressively slower according to the equation Eq. (5), which for large n' values can be expressed as:

$$n' \approx \sqrt{2n} \tag{9}$$

Expressing the equation above in terms of elapsed times, we have:

$$t_{photon} \approx \sqrt{2\tau t_{rest}} \tag{10}$$

[7] By atoms we mean the minimal spatial translation and minimal temporal duration of the *Isotropic Model of DST*.

Table 2 Clock tick associations, length associations, Lorentz factors for $v = c$.

RF1 Tick n Length m	RF2 Tick n' Length m'	$\gamma_n = \gamma_m$ $\frac{n}{n'} = \frac{m}{m'}$
1	1	1
2	2	1
3	2	1.5
4	2	2
5	3	1.67
6	3	2
7	3	2.33
8	4	2
9	4	2.25
10	4	2.5
11	4	2.75
12	4	3
13	5	2.6
14	5	2.8
15	5	3
16	5	3.2
17	5	3.4
18	6	3
$\vdots$	$\vdots$	$\vdots$
n	$\sqrt{2n}$	$\sqrt{n/2}$

where t_{photon} is the time experienced by the photon, t_{rest} is the time experienced by the at-rest reference frame, and τ is typically taken to be the Planck time $\tau_p = 5.39 \times 10^{-44}$ s.

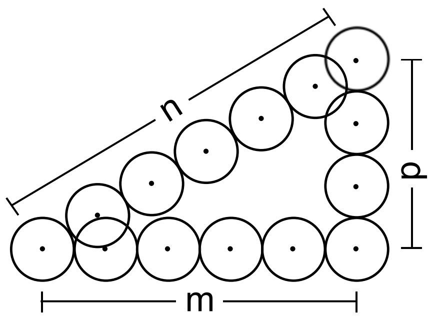

Fig. 5 An example of the length of the hypotenuse equaling the length of the base of the triangle for $n = m = 5$ and $n' = 3$. With the new distance formula for DST, for any value of n' there will always be a value for n and m such that $n = m$, specifically, a value calculable using Eq. (5).

5 Discussion

Let us consider a couple of examples. A photon traveling from the Sun to the Earth travels a distance (in our reference frame) of $d = 1.47 \times 10^{11}$ m, doing so in a time (in our reference frame) of $t = d/c = 490$ s. With us letting the chronon τ be equal to the Planck time τ_p, namely, $\tau = \tau_p = 5.39 \times 10^{-44}$ s, a time of 490 s gives $n = t/\tau = 9.09 \times 10^{45}$ atoms of space (hodons) as being traversed by the photon from our at-rest perspective. Using Eq. (5), (6) or (9) to solve for n' (the number of chronons required for the trip), we get that $n' = 1.35 \times 10^{23}$. The Lorentz factor ($\gamma = n/n'$) is incredibly large, $\gamma = 6.74 \times 10^{22}$, the Sun to Earth transit time and length in the photon's reference frame can then be calculated as being 0.007 attoseconds and 2 picometers respectively. In spite of this seemingly small transit time and travel distance experienced by the photon, there is an incredibly large number of opportunities during the photon's travel, namely $\approx 10^{23}$, for it to interact with ions, dust, satellites and anything else. Even if we consider inter-atomic distances, say $5\mathring{A}$, we still obtain very large values for n and n' of $n = 3.09 \times 10^{25}$ and $n' = 7.86 \times 10^{12}$; these values may be too large to have a measurable effect on the optical or atomic properties of materials that we are able measure today, but it is not unrealistic to think that measurements with 10^{-12} precision will be possible with future technology. But importantly, one effect that is measurable to all of us at any moment that demonstrates light's time experience is light's ability to interact with the particles that make up the universe...and us. We can *see*, light bends in water, light is produced by the Sun...all of these effects require light to experience time, and we have shown in this work that for light to experience time, space and time must be discrete.

6 Conclusion

In this paper, we discussed that in the absence of experiencing time, entities cannot change or interact with anything. Because we observe that light changes, light must experience time. While the laws of special relativity in continuous space time predict that photons do not experience our time in sub-light speed reference frames, it has been shown in this work that photons experience calculable durations of time if space and time are discretized. It was shown that time progresses tick for tick for the photon's clocks relative to our clocks for time durations of one or two chronons, but for larger time scales, the time experienced by the photon scales by the square root of twice the time elapsed for non light-speed entities. These results significantly strengthen the argument that space and time are discrete.

7 Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

References

[1] Coope, U.: Time for Aristotle: Physics IV. 10-14. Oxford University Press, New

York (2005)

[2] Hume, D.: A Treatise of Human Nature. Oxford University Press, New York (2000)

[3] Heidegger, M.: The concept of time. New Yearbook for Phenomenology and Phenomenological Philosophy **9**, 192–210 (2009)

[4] Shoemaker, S.: Time without change. The Journal of Philosophy **66**(12), 363–381 (1969)

[5] Bergson, H.: Duration and Simultaneity: Bergson and the Einsteinian Universe vol. 199. Clinamen Press, Geneva (1999)

[6] Russell, B.: Our Knowledge of the External World as a Field for Scientific Methid in Philosophy vol. 298. G. Allen & Unwin Limited, London (1926)

[7] DeWitt, B.S.: Quantum theory of gravity. i. the canonical theory. Physical Review **160**(5), 1113 (1967)

[8] Rovelli, C.: Forget time: Essay written for the fqxi contest on the nature of time. Foundations of Physics **41**, 1475–1490 (2011)

[9] Penrose, R.: Cycles of Time: an Extraordinary New View of the Universe. Random House, London (2013)

[10] Crouse, D., Skufca, J.: On the nature of discrete space-time: Part 1: The distance formula, relativistic time dilation and length contradiction in discrete space-time. Logique et Analyse **246**, 177–223 (2019)

[11] Salmon, W.C.: Zeno's Paradoxes. Hackett Publishing, Indianapolis (2001)

[12] Hagar, A.: Discrete or Continuous?: the Quest for Fundamental Length in Modern Physics. Cambridge University Press, New York (2014)

[13] Van Bendegem, J.P.: Finitism in geometry (2002)

[14] Sorkin, R.: On the entropy of the vacuum outside a horizon 10th int. In: Conf. on General Relativity and Gravitation (Padova, 4–9 July 1983) (1983)

[15] Riggs, S.: The origin of the planck length, planck mass and planck time: A new candidate for dark matter. Np: CreateSpace Independent Publishing Platform (2009)

[16] Ng, Y.J.: Limitation to quantum measurements of spacetime distances. arXiv preprint hep-th/9406110 (1994)

[17] Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. Macmillan, New York

(1973)

[18] Rovelli, C., Speziale, S.: Reconcile planck-scale discreteness and the lorentz-fitzgerald contraction. Physical Review D **67**(6), 064019 (2003)

[19] Crouse, D.T.: On the nature of discrete space-time: The atomic theory of space-time and its effects on pythagoras's theorem, time versus duration, inertial anomalies of astronomical bodies, and special relativity at the planck scale. arXiv preprint arXiv:1608.08506 (2016)

[20] Gudder, S.: Discrete spacetime quantum field theory. arXiv preprint arXiv:1704.01639 (2017)