Quantum Measurement Without Collapse or Many Worlds: The Branched Hilbert Subspace Interpretation

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Abstract

The interpretation of quantum measurements remains contested between collapse-based frameworks like the Copenhagen Interpretation and no-collapse approaches like the Many-Worlds Interpretation (MWI). We propose the Branched Hilbert Subspace Interpretation (BHSI) as a minimalist alternative that preserves unitarity while avoiding both wavefunction collapse and ontological excess. BHSI models measurement as the unitary branching of a system's local Hilbert space into decoherent subspaces, formalized through causal state updates using branching and disengaging operators. Unlike MWI, BHSI avoids parallel worlds by maintaining a single-world ontology in which branching is confined to observable subspaces; unlike Copenhagen, it eliminates collapse while recovering the Born rule through branch weights. Through physically meaningful subspace records, the framework resolves quantum paradoxes such as particle-wave duality, Wigner and his friend, black hole radiation, etc. It remains consistent with interference patterns, entanglement correlations, and information preservation. By comparing BHSI with QBism, Relational Quantum Mechanics, and modal interpretations, as well as analyzing quantum teleportation (where locally decoherent Hilbert subspaces are observed), we demonstrate its advantages as a causally sound and empirically grounded approach that reconciles unitary evolution with definite measurement outcomes without metaphysical proliferation.

Keywords: quantum foundations; measurement problem; unitary branching; Born rule, subspace decoherence.

1. Introduction

The interpretation of quantum mechanics (QM) has been debated since its inception in the 1920s. The theory's mathematical formalism, such as unitary evolution, superposition, and entanglement, yields strikingly non-classical predictions, yet its physical meaning remains contested. The Copenhagen Interpretation (CI; Bohr, Heisenberg, Born, Pauli; 1920s-1950s, [1-3]) provides a mathematically simple framework that aligns with lab observations. However, it faces criticism for its undefined wave function collapse, the straightforward postulation of the Born rule [4], the cornerstone of QM probabilistic predictions, and the subjective boundary separating quantum and classical regimes. The Many-Worlds Interpretation (MWI; Everett, DeWitt, Deutsch, Wallace; 1957-present, [5-7]) addresses the measurement problem by postulating that all possible quantum measurement outcomes occur in separate, non-interacting branches of reality (each branch is a world with a copy of the observer), thereby offering a compelling solution by eliminating wavefunction collapse. Still, it encounters significant challenges regarding its ontological excess, the lack of a convincing explanation for the Born rule, and the preferred basis issue [8-11]. Bohmian Mechanics (BM, Bohm, Bell, Goldstein;

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1952-present; [12-14]), also known as the de Broglie-Bohm pilot-wave theory, resolves the wave collapse issue of CI within a single world, but it relies on hidden variables (actual particle positions), and its explicit nonlocality structure may conflict with relativity.

We propose an alternative approach: the Branched Hilbert Subspace Interpretation (BHSI), in which measurement splits the local Hilbert space into multiple branches instead of partitioning the universe into parallel worlds in the global Hilbert space. Since each possible outcome exists and evolves within one branch, no wave function collapses. The observer's state is updated relationally and causally, resulting in one outcome per observation. The Born rule [4] can be realized by assigning weight (probability) to each branch based on the initial state represented on the basis chosen by the observer. With only one observer in a single world, it does not face the ontological challenge of explaining probability in the MWI.

We formalize the mathematical framework of BHSI by defining unitary branching and the engaging and disengaging operators. We explain how these operators decohere subspaces and update the observer's state. We compare BHSI with CI and MWI by exploring their implications for interference (double-slit experiment [15-17]), nonlocality (Bell tests [18-19]), causal dominance (Wigner's friend) [5,20-21], black hole radiation with the No-Hiding Theory (NHT) [22-23], and the delayed choice quantum eraser [24-25]. The differences between BHSI and other interpretations, such as QBism [26], Relational QM (RQM) [27], and Modal Interpretation [28-29], are also discussed. The last section discusses the nature of branching. It compares the environmental scale of quantum decoherence [30-32] in MWI vs BHSI (the maximal vs. the minimal). We explain that BHSI's locally decoherent Hilbert subspaces and their recoherence have already been observed in the quantum teleportation experiment [33].

2. Mathematical Framework.

In this section, we present the fundamental concepts of BHSI: branching local Hilbert spaces, updating (engaging and disengaging) the observer's state, and the Born rule.

2.1. The Branching, Engaging, and Disengaging Operators

Assume the observer chooses to measure an observable $\hat{G}$. The following linear combination on the *G*-basis describes the initial quantum state ([2, p.29]):

$$|\Psi\rangle = \sum_{i=1}^{D} c_i \,|g_i\rangle, \quad \hat{G}\,|g_i\rangle = g_i\,|g_i\rangle, \quad \langle g_i \,|\, g_j\rangle = \delta_{i,j}, \quad \sum_{i=1}^{D} |c_i|^2 = 1, \quad \prod_{i=1}^{D} c_i \neq 0 \tag{1}$$

The initial Hilbert space is *D*-dimensional, corresponding to the *D* possible outcomes of the measurement, each with a non-zero probability. The *branching operator* $\hat{B}$ is a unitary operator that splits the *D*-dimensional Hilbert space $\mathcal{H}^D$ into *D* branches:

$$\hat{B} \equiv \sum_{k=1}^{D} |g_{B;k}\rangle\langle g_k|, \quad \hat{B}^{\dagger}\hat{B} = I, \quad \hat{B}\hat{B}^{\dagger} = I_B, \; \langle g_k \,|\, \Psi_B\rangle = \langle g_k \,|\, B^{\dagger} B \,|\, \Psi\rangle = \langle g_k \,|\, \Psi\rangle = c_k \tag{2}$$

$$\hat{B}(|\Psi\rangle \otimes |E\rangle_L) = |\Psi_B\rangle = \sum_{k=1}^{D} c_k\, |g_k\rangle\, |E_k\rangle_L = \sum_{k=1}^{D} c_k\, |g_{B;k}\rangle, \quad |g_{B;k}\rangle \equiv |g_k\rangle\, |E_k\rangle_L$$
$$\hat{B}\left(\mathcal{H}_S \otimes \mathcal{H}_L\right) = \bigoplus_{k=1}^{D} \mathcal{H}_{S,k}(\text{span c}_k\, |g_{B;k}\rangle), \quad |\langle g_k\, |\Psi\rangle|^2 = |\langle g_{B;k}\, |\Psi_B\rangle|^2 = |c_k|^2 \tag{3}$$

Note that the states $|g_{B,k}\rangle$ are locally decoherent, evolving in different branches; the surrounding environment $|E\rangle_L$ is involved to make them decoherent. Such subspaces are not merely a theoretical construct: *they already exist in teleportation* (see Section 5.2). The engaging and disengaging operator $\Sigma_\beta \equiv \Gamma_\beta \mathrm{T}_\beta \Lambda_\beta$ is a product of three unitary operators.[2] The first operator is the engaging operator Λ_β. It updates the observer's state from |ready⟩ in the environment $\mathcal{H}_E$ to |reads⟩ and entangles the observer's state with the β^{th} subspace. The operator product $\Lambda_\beta \hat{B}$ branches the Hilbert space and randomly engages the observer with the β^{th} subspace:

$$\Sigma_\beta \equiv \Gamma_\beta T_\beta \Lambda_\beta, \quad \Lambda_\beta : |\text{ready}\rangle_o \in \mathcal{H}_E \mapsto |\text{reads } g_\beta\rangle_o \in \mathcal{H}_{S,\beta} \tag{4}$$

$$\Lambda_\beta \hat{B}\text{: } \mathcal{H}_S \otimes \mathcal{H}_L \mapsto \mathcal{H}_B = \bigoplus_{k=1}^{D} \left\{\mathcal{H}_{S,k}(\text{span}\, c_k\, |g_{B,k}\rangle(|\text{ reads } g_\beta\rangle_O)^{\Delta(k,\beta)}\right\}, \quad \beta \in \{1.2.\cdots D\} \tag{5}$$

To simplify the expression, we have used the following notation:

$$\Delta(k,\beta) = \delta_{k,\beta} = \begin{cases} 1, \text{ if } k = \beta \\ 0, \text{ if } k \neq \beta \end{cases} \text{ (discreate case)}, \quad (|\text{reads}\rangle_O)^{\Delta(k,\beta)} = \begin{cases} |\text{reads}\rangle_O, \text{ if } k = \beta \\ \quad 1, \text{ if } k \neq \beta \end{cases} \tag{6}$$

After recording the outcome, operator T_β changes the observer's state to |ready⟩, then operator Γ_β disengages him from the branch, ensuring he is prepared for the next engagement.

$$T_\beta : |\text{ reads}\rangle_O \mapsto |\text{ ready}\rangle_O; \quad \Gamma_\beta T_\beta : \mathcal{H}_B \mapsto \mathcal{H}_f = \left\{\bigoplus_{k=1}^{D} \mathcal{H}_{S,k}(\text{span } c_k\, |g_{B,k}\rangle\right\} \otimes |\text{ ready}\rangle_O \tag{7}$$

Let $U(t)$ be the time evolution operator of the system, which can be relativistic or not:

$$U(t)\,|\Psi(0)\rangle = |\Psi(t)\rangle, \quad |\Psi\rangle \equiv |\Psi(0)\rangle, c_k \equiv c_k(0), \quad U(t)\,|\Psi_B(0)\rangle = |\Psi_B(t)\rangle \tag{8}$$

The branching operator $\hat{B}$ commutes with the time evolution operator:

$$\hat{B}\hat{U}(t)\left\{|\Psi\rangle\right\} = \hat{B}\sum\nolimits_{k=1}^{D} c_k(t)\,|g_k\rangle = \sum\nolimits_{k=1}^{D} c_k(t)\,|g_{B;k}\rangle \tag{9}$$
$$= \hat{U}(t)\hat{B}\left\{|\Psi\rangle\right\} = \hat{U}(t)\left\{|\Psi_B\rangle\right\} = \sum\nolimits_{k=1}^{D} c_k(t)\,|g_{B;k}\rangle \tag{10}$$

Altogether, a measurement process can be described as a unitary transformation $\hat{M}_\beta$ (β is a random choice), which commutes with the time evolution operator $U(t)$:

$$\hat{M}_\beta \equiv \Sigma_\beta \hat{B} = \Gamma_\beta T_\beta \Lambda_\beta \hat{B}, \quad \hat{M}_\beta^\dagger \hat{M}_\beta = \hat{B}^\dagger \Sigma_\beta^\dagger \Sigma_\beta \hat{B} = I, \quad \hat{M}_\beta U(t) = U(t)\hat{M}_\beta, \quad \beta \in \{1,2,\cdots D\} \tag{11}$$

[2] They act like the unitary NOT gate, flipping between the observer's states [24, p.233].

$$\Gamma_\beta T_\beta \Lambda_\beta \hat{B}\{|\Psi\rangle \otimes |\text{ready}\rangle_O \,|E\rangle_L\} = \Gamma_\beta T_\beta \left\{\sum_{k=1}^{D} c_k \,|g_{B;k}\rangle \otimes (|\text{reads}\rangle_O)^{\Delta(k,\beta)}\right\} = |\Psi_B\rangle \otimes |\text{ready}\rangle_O \quad (12)$$

2.2. The Measurement Process and the Born Rule in BSHI

The initial Hilbert space is D-dimensional, as Eq. (1) describes. We discuss three cases.
Case 1: D = 1. The initial normalized state contains only one basis state.

$$|\Psi\rangle = |g_1\rangle \quad (13)$$

Since this reflects the observer's measurement basis, the observer consistently records g_1, with $P(g_1)$ =1, by unitarily branching, engaging, and disengaging. Only one branch exists, containing $|g_{B,1}\rangle$ after the measurement. There is no loss of information or gain of entropy.

Case 2: $D \geq 1$. Before the observation, the system (S), the local environment $|E\rangle_L$, and the state of the observer or the apparatus (O) are in the following pre-measurement state:

$$|\Psi_0\rangle = |\Psi\rangle \otimes |\text{ready}\rangle_O \,|E\rangle_L, \quad |\Psi\rangle = \sum_{k=1}^{D} c_k \,|g_k\rangle \quad (14)$$

According to Eq. (5), branching the system causes its local Hilbert space to split into D parallel subspaces, each spanning a basis state. The observer engages with one branch, which has an associated weight (chance) based on the initial state, thereby realizing the Born rule:

$$\mathcal{H}_S \otimes \mathcal{H}_L \to \bigoplus_{k=1}^{D} \mathcal{H}_{S,k}[\text{span } \mathrm{c}_k \,|g_{b,k}\rangle(|\text{ reads } g_\beta\rangle_o)^{\Delta(k,\beta)}], \quad P(g_\beta) = |c_\beta|^2 = |\langle g_\beta | \Psi\rangle_S|^2 \quad (15)$$

After the measurement, the observer disengages from the branched system state, as illustrated by Eq. (7):

$$|\Psi_f\rangle = |\Psi_B\rangle \otimes |\text{ready}\rangle_O = \left\{\sum_{k=1}^{M} c_k \,|g_{b;k}\rangle\right\} \otimes |\text{ready}\rangle_O \quad (16)$$

Case 3: D = 2. This is a specific example of Case 2: the initial state consists of only two basis states. We aim to use this case to compare step-by-step with the MWI. Assume that Bob is observing a qubit. Before the measurement, we have:

$$\text{MWI:} \quad |\Psi_0\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle)\,|B\rangle\,|E\rangle, \quad |\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (17)$$

$$\text{BHSI:} \quad |\Psi_0\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle)\,|\text{ready}\rangle_O \,|E\rangle_L, \quad |\alpha_0|^2 + |\alpha_1|^2 = 1 \quad (18)$$

The equation appears similar. The difference is the scope of the environment involved. After branching, their states have the following forms:

$$\text{MWI:} \quad |\Psi_f\rangle = \alpha_0 |0\rangle |B_0\rangle |E_0\rangle + \alpha_1 |1\rangle |B_1\rangle |E_1\rangle, \quad \langle B_0 | B_1\rangle \approx 0, \langle E_0 | E_1\rangle \approx 0 \quad (19)$$

$$\text{BHSI:} \quad |\Psi_B\rangle = \sum_{k=0}^{1} \alpha_k \,|k_B\rangle(|\text{ reads } k\rangle_O)^{\Delta(k,\lambda)}, \quad \lambda \in \{0,1\}, \quad P(\lambda) = |\alpha_\lambda|^2 \quad (20)$$

Or: $|\Psi_B\rangle = \alpha_0 |0_B\rangle (|\text{reads } 0\rangle_O)^{\Delta(0,\lambda)} + \alpha_1 |1_B\rangle (|\text{reads } 1\rangle_O)^{\Delta(1,\lambda)}, \quad \lambda \in \{0,1\}, \quad P(\lambda) = |\alpha_\lambda|^2$ (21)

The BHSI borrows the branching idea from the MWI. However, instead of updating the universal wave function in the global Hilbert space, the BHSI only updates the minimal environment with Bob's state in one of the local spaces (see Fig. 1). After the branching, in the MWI, each branch is a real world with a real Bob, which has no experimental evidence so far. In contrast, in the BHSI, Bob in the local Hilbert space is not a real person but rather the state of Bob as represented through the engaged part of his apparatus. The observer (or apparatus) becomes entangled with one branch and reads the corresponding outcome with a specific probability. Each branch contains only a single basis state, and it evolves unitarily, which has already been observed in teleportation (Section 5.2). Therefore, wave collapse in the CI is circumvented without the necessity of many worlds in the MWI.

After reading, Bob becomes disengaged, as described by Eq. (7). The final state contains two decoherent branches:

BHSI: $|\Psi_B\rangle = \alpha_0 |0_B\rangle + \alpha_1 |1_B\rangle \quad |\alpha_0|^2 + |\alpha_1|^2 = 1$ (22)

Assuming Bob reads 1 (λ = 1). During the entire measurement process, Bob experiences three stages (before, during, and after the measurement), as described by Eqs (11-12):

$|\Psi\rangle \otimes |\text{ready}\rangle_O |E\rangle_L \rightarrow \alpha_0 |0_B\rangle + \alpha_1 |1_B\rangle |\text{reads } 1\rangle_O \rightarrow |\Psi_B\rangle \otimes |\text{ready}\rangle_O$ (23)

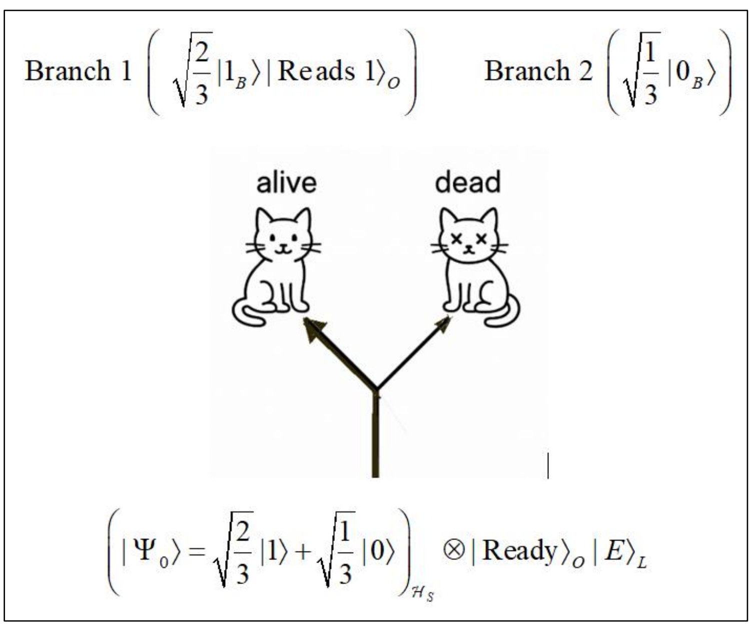

Fig. 1: The Branched Local Hilbert Subspaces

The branched local Hilbert spaces are eventually relocated into the environment at large by unitary transformations, complying with the No-Hiding Theorem (NHT, [23]):

$$U_E : |\Psi_B\rangle \otimes |E\rangle \to |E'\rangle \tag{24}$$

2.3. The Observer's Local View of the Measurement:

In quantum measurements or quantum computing, the observer must repeatedly measure the same initial states. Each time, he reads one possible outcome, with the probability predicted by the Born rule, which leads to the following density matrix [33, p.53]:

$$\rho = \sum_{k=1}^{D} |g_k\rangle |c_k|^2 \langle g_k|, \quad \sum_{k=1}^{D} |c_k|^2 = 1 \tag{25}$$

Locally, the observer sees that the initial pure state, Eq. (1), with zero von Neumann entropy [33, p.179], becomes a mixed state, and its von Neumann entropy is increased to:

$$S(\rho) = -\mathrm{Tr}(\rho \ln \rho) = -\sum_{k=1}^{D} \{|c_k|^2 \ln |c_k|^2\} > 0 \tag{26}$$

The observer concludes that his measurement is irreversible because the system's entropy increases and certain information is lost. However, in the entire Hilbert space encompassing all branches, there is no loss of information or gain in entropy. This is quite similar to the MWI, except that MWI consists of many independent, equally real worlds, while BHSI features numerous independent local Hilbert subspaces with predictable weights (probabilities).

3. Feature Comparison of CI, MWI, BM, and BHSI

Feature	Copenhagen (CI)	Many-Worlds (MWI)	Bohmian Mechanics (BM)	BHSI
1. Wave Collapse? Unitarity?	Yes. Non-unitary	No. Fully unitary by splitting the global Hilbert space	No. Fully unitary (wavefunction guides particles)	No. Fully unitary by splitting the local Hilbert space
2. Ontology: Number of Worlds and "Me"	A single world, a single "Me."	Many real worlds, each with a "Me."	A single world, a single "Me."	A single world, a single "Me."
3. Probability: The Born Rule	Fundamental postulate (no deeper explanation)	Emergent from decision theory? (self-locating uncertainty?)	Explained by the equilibrium distributions of hidden variables	Interpreted as the weights of local Hilbert branches.
4. The Role of the Observer	Passive, external to the system, and causes collapse	Branching, then following one world, and all worlds are real.	Passive (particles have definite positions at all times)	Branching, engaging, then disengaging from one Hilbert branch.
5. Determinism	Indeterministic (collapse	Deterministic (but observers experience	Deterministic (hidden variables	Deterministic (but observers

	introduces randomness)	subjective randomness)	define definite trajectories)	experience local randomness)
6. Information Loss	Yes (collapse destroys superpositions permanently)	No (information persists in different worlds)	No (global wave function guides particles deterministically)	No (information persists in different Hilbert subspaces)
7. Can Branches Recombine?	N/A (only one world exists)	No (recoherence leads to identity crises)	N/A (only one world exists)	Yes? In theory, it is possible.
8. Locality of Physical Laws	Local (except for nonlocal collapse)	Local (no signal between branches)	Nonlocal (built-in by the global wave function)	Local (no faster-than-light action)

Table 1. Comparison of the Four Interpretations of Measurements

4. Comparison of BHSI with CI, MWI, and Other Interpretations

Using several real or thought experiments, the first subsection compares BHSI with MWI and CI. Then, BHSI is compared to QBism, Relative QM, and Model Interpretations.

4.1. Comparing BHSI with MWI and CI by Examples

The BHSI is proposed as a "cost-effective" version of the MWI to avoid the collapse issue in the CI without the ontological excess of MWI. This section uses several examples to illustrate the similarities and differences between the three interpretations.

Example 4.1.1. *The Double-Slit Experiment*: It is the most popular experiment to explain the particle-wave duality of QM [15-16], including photons, electrons, and large C_{60} molecules [17]. When a particle hits the screen, the local Hilbert space in BHSI splits into uncountable infinite branches (in theory), and the observer reads it at one position *x*.

$$|\Psi_B\rangle = \int dx'|x'\rangle\langle x'|\Psi_B\rangle[|\text{reads } x\rangle_O]^{\Delta(x,x')}, \quad \Delta(x,x') = \begin{cases} 1, & \text{if } x = x' \\ 0, & \text{if } x \neq x' \end{cases} \text{ (continuous case)} \qquad (27)$$

$$|\langle x|\Psi_B\rangle|^2 = |\Psi(x)|^2 = |\Psi_I(x) + \Psi_{II}(x)|^2 \qquad (28)$$

Because of the limitations of the experimental equipment, the integral in Eq. (27-28) should be replaced by a discrete summation over tiny pieces Δ_k:

$$|\Psi_B\rangle = \sum_{k'} \Delta_{k'}|x_{k'}\rangle\langle x_{k'}|\Psi_B\rangle[|\text{reads } x_k\rangle_O]^{\Delta(k,k')}, \quad P(\Delta_k) = |\Psi_{\mathrm{I}}(x_k) + \Psi_{\mathrm{II}}(x_k)|^2\ \Delta_k \qquad (29)$$

The BHSI and MWI rely on branching to maintain unitarity and interference without total information loss. In the BHSI, the observer disengages with the system after reading, and the interference or probability distribution (the Born rule) can be assigned naturally; however, in the MWI, the environment coherent with each piece Δ_k is a whole world with a real observer. In a typical double-slit experiment, tens of thousands of photons hit the screen, and each photon updates thousands of branches. Because of the ontological issue, there is no convincing interpretation of probability in MWI yet: Many minds? Indexicalism? Decision theory? A rational bet on a particular result? Or Envariance [8,9]?

The CI can explain the interference by simply assuming the Born rule. Still, each particle's hit causes a wave collapse (FTL action), breaking unitarity and causing information loss.

Example 4.1.2. *The Bell Tests of Entanglement*: Applying the Born rule, all three interpretations can explain the violation of the Bell inequality [18-19] without spooky actions at a distance between the paired particles or the two observers. However, the costs are different. In CI, the measurements by Alice and Bob cause two wave collapses (FTL actions), leading to information loss. MWI and BHSI have no collapse and no total information loss. But, MWI ends with four Hilbert branches of worlds per photon pair, each containing Alice and Bob, while BHSI ends with four local Hilbert branches without multiple Alice and Bob.

MWI: Alice and Bob update four worlds per photon pair, each containing an Alice and a Bob:

$$\mathrm{A}_1: |0\rangle_a\,|\mathrm{Alice}_{0,A}\rangle\,|\mathrm{Bob}_{0,A}\rangle\,|\mathrm{E}_{0,A}\rangle, \qquad \mathrm{A}_2: |1\rangle_a\,|\mathrm{Alice}_{1,\mathrm{A}}\rangle\,|\mathrm{Bob}_{1,\mathrm{A}}\rangle\,|\mathrm{E}_{1,\mathrm{A}}\rangle \tag{30}$$

$$\mathrm{B}_1: |0\rangle_b\,|\mathrm{Alice}_{0,B}\rangle\,|\mathrm{Bob}_{0,B}\rangle\,|\mathrm{E}_{0,B}\rangle, \qquad \mathrm{B}_2: |1\rangle_b\,|\mathrm{AliceE}_{1,\mathrm{B}}\rangle\mathrm{BobE}_{1,\mathrm{B}}\rangle\,|\mathrm{E}_{1,\mathrm{B}}\rangle \tag{31}$$

BHSI: Alice and Bob update four branches per photon pair in their local Hilbert space,

$$\mathrm{A}_1: |0_B\rangle_a(|\text{reads } 0\rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle_a, \qquad \mathrm{A}_2: |1_B\rangle_a(|\text{reads } 1\rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle_a, \quad \alpha \in \{0,1\} \tag{32}$$

$$\mathrm{B}_1: |0_B\rangle_b(|\text{reads } 0\rangle_O)^{\Delta(\beta,0)} \rightarrow |0_B\rangle_b, \qquad \mathrm{B}_2: |1_B\rangle_b(|\text{reads } 1\rangle_O)^{\Delta(\beta,1)} \rightarrow |1_B\rangle_b, \quad \beta \in \{0,1\} \tag{33}$$

Typically, millions of photon pairs are measured by Alice and Bob in a Bell test.

Example 4.1.3. *Wigner's Friend Thought Experiment* [5, 19-20] is a compelling example involving mixed observers. Setup: The Friend (F) observes a qubit state: $(|0\rangle + |1\rangle)/\sqrt{2}$ in a Lab; simultaneously, Wigner (W), outside, observes F and the qubit. What occurs?

CI: F collapses the qubit, and W sees what F sees. One collapse. Why? F is the preferred observer (he measures the qubit), and F is a classical object that cannot entangle with a qubit.

MWI: F updates two worlds in the global Hilbert space, each containing an F and a W:

$$\mathrm{H}_1: |0\rangle\,|\mathrm{F}_0\rangle\,|\mathrm{W}_0\rangle\,|\mathrm{E}_0\rangle, \qquad \mathrm{H}_2: |1\rangle\,|\mathrm{F}_1\rangle\,|\mathrm{W}_1\rangle\,|\mathrm{E}_1\rangle \tag{34}$$

At the same time, W also updates two worlds, each containing an F and a W, too:

$$\mathrm{H}_3: |0\rangle\,|\mathrm{F}_0\rangle\,|\mathrm{W}_0\rangle\,|\mathrm{E}_0\rangle, \qquad \mathrm{H}_4: |1\rangle\,|\mathrm{F}_1\rangle\,|\mathrm{W}_1\rangle\,|\mathrm{E}_1\rangle \tag{35}$$

There is no collapse, no preferred observer, and F can be entangled with a qubit. Moreover, we can set $H_1 = H_3$ and $H_2 = H_4$, because H_1 & H_3 (H_2 & H_4) are physically indistinguishable, leading to one branching, two worlds. No matter whether it is two or four worlds, there is no identity conflict. If F and W shake hands, they must see the same result and in the same world.

BHSI: Friend updates two decoherent local branches, engages one, and then disengages:

$$\mathrm{H}_1 : |0_B\rangle(|\text{reads } 0\rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle, \qquad \mathrm{H}_2 : |1_B\rangle(|\text{reads } 1\rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle, \quad \alpha \in \{0,1\} \tag{36}$$

Because Friend measures the qubit, his branching is dominant; the local Hilbert subspaces must be updated synchronously with his, so Wigner's two branches should synchronize with Friend's:

$$\mathrm{H}_3 : |0_B\rangle(|\text{F reads } 0\rangle\,|\text{reads } 0\rangle_O)^{\Delta(\alpha,0)} \rightarrow |0_B\rangle, \quad \mathrm{H}_4 : |1_B\rangle(|\text{F reads } 1\rangle\,|\text{reads } 1\rangle_O)^{\Delta(\alpha,1)} \rightarrow |1_B\rangle \tag{37}$$

Wigner will see an outcome of 0/1 if his friend engages with H_1/H_2. Like the MWI, the process is unitary, with no information loss or collapse; the friend's state can be entangled with a qubit, with no preferred observer but a causally dominant branching. Similar to the CI, with only one world, one Wigner, and one Friend, they see the same result and can always shake hands. Suppose Alice and Bob are outside, watching Wigner or his friend simultaneously and shaking hands afterward. What happens in MWI and BHSI? We will use this scenario in our next section.

Example 4.1.4. *The Black hole information paradox*: Hawking's semi-classical calculations suggest that black hole evaporation via Hawking radiation is thermal and random [22]. If so, it destroys information about the infalling matter, violating unitarity. MWI and BHSI have different branching structures (global vs. local) for modeling Hawking radiation, both of which are consistent with the No-Hiding theory (NHT, [23]). However, the Hawking radiation in the CI causes collapses and information loss, violating the NHT.

Example 4.1.5. The delayed choice quantum eraser experiments [24-25]: In the MWI and BHSI, all Hilbert branches were already recorded when signal photons (pair a) hit the screen. The observer later chooses which branches based on the path of the idle photons (pair b), i.e., which-way information (w) is kept (w = 1, no interference) or erased (w = 0, seeing interference). There is no collapse or retrocausality, but many worlds in MWI compared to many local subspaces in BHSI. In the CI, reality is only determined when the measurement is fully completed, so retrocausality does not occur either. By the way, does the experiment imply retrocausality? No. The state of the entangled signal photon pair a and the idle photon pair b can be written as:

$$|\Psi\rangle = (1/\sqrt{2})\sum_{w=0}^{1} |w\rangle_a\,|w\rangle_b = (1/\sqrt{2})\big(|0\rangle_a\,|0\rangle_b + |1\rangle_a\,|1\rangle_b\big) \tag{38}$$

When photon pairs (a) strike the screen, they leave objective records for $w_a = 0$ and $w_a = 1$. Crucially, the interference ($w_a = 0$) can be recovered only if the timing-matched photon pairs (b) are later chosen to take the path for $w_b = 0$, occurring with probability $|\langle 00|\Psi\rangle|^2 = ½$. This mirrors a Bell test: Alice's ability to interpret her data depends on receiving Bob's correlated records—regardless of their spatial separation from the source—but requires no retrocausality, only pre-established entanglement.

4.2: **Comparing BHSI with QBism, Relational QM, and Modal Interpretation**

BHSI vs. QBism: Objective Branching vs. Subjective Belief.
Both BHSI and QBism reject the need for wavefunction collapse or parallel worlds, but they diverge radically in ontology. QBism treats quantum states as agent-centered beliefs, framing probabilities as subjective Bayesian updates based on personal experience [26]. In contrast, BHSI posits an agent-independent Hilbert space structure, where branching subspaces represent objective measurement records—physical correlates of decoherence as described in Eq. (3). Whereas QBism addresses the measurement problem through observer psychology ("What does the agent expect?"), BHSI resolves it through geometric and causal mechanisms: subspace decomposition governed entirely by unitary dynamics, independent of observers. In Example 4.1.3, the lab equipment determines the branching outcome; Wigner's later observation—and those of Alice and Bob—must align with this objectively existing branch, regardless of personal beliefs or expectations.

BHSI vs. Relational QM (RQM): Causal Structure vs. Observer-Dependent "Reality".
Like RQM, BHSI rejects the notion of absolute quantum states but categorically denies RQM's claim that observables are intrinsically observer-relative [27]. BHSI's branches emerge deterministically through unitary dynamics, encoding measurement outcomes as objective subspace decompositions—no observer required. RQM, by contrast, risks conceptual instability: if facts are always relative, what anchors reality in the absence of observers? This opens the door to solipsistic implications ("Did the measurement happen if no one observed it?") and communication paradoxes (see below). BHSI avoids such pitfalls by grounding reality in causally coherent branching: decoherence-induced subspaces exist physically, whether or not anyone observes them.

A critical challenge for RQM emerges in the extended Wigner's Friend scenario (Example 4.1.3), where Alice and Bob each observe distinct parts of the system and later compare notes. In RQM, this handshake entails a post-hoc reconciliation of "facts"—but who updates their state first? Alice, who saw the Friend? Bob, who saw Wigner? This negotiation of realities is not only conceptually murky but potentially violates causal structure. BHSI sidesteps the dilemma: all observers align with the dominant causal branch, defined by the objective decoherence history of the system (e.g., the lab equipment)—no contradictions, no subjective redefinitions—just the unitary dynamics of quantum mechanics doing its work.

BHSI vs. Modal Interpretations: Actual Branches vs "Possible" Properties
Like modal interpretations, BHSI decomposes Hilbert space into subsystems within a single world—but it diverges crucially by treating all branches as equally actual, not merely "possible" [28–29]. Modal frameworks typically depend on preferred factorizations (e.g., system–apparatus splits) to define provisional properties. In contrast, BHSI's branching is dynamically determined: it is governed by the unitary operator in Eq. (3) and produces random but objectively real outcomes, as formalized in Eqs. (11–12).

The core conflict lies in the timing and status of actualization. Modal interpretations delay actuality until a measurement context emerges (e.g., "Spin ↑ is possible until observed"), leaving open when or how possibilities become facts. BHSI, by contrast, treats branching as intrinsic to unitary evolution—decoherence alone records outcomes, and no contextual trigger is needed.

This avoids modal approaches' ad hoc choices of factorization and their ambiguity about when possibilities become facts. In example 4.1.3, the lab equipment's interaction with the qubit definitively branches the state; all observers (Wigner, his friend, Alice, and Bob) follow this shared, causally determined history—there is no "maybe."

In all the above cases, BHSI dispenses with ad hoc rules, psychological constructs, or arbitrary factorizations, relying solely on the geometry and dynamics of Hilbert space to resolve the measurement problem. As such, BHSI presents a compelling candidate for a realist, no-collapse interpretation firmly grounded in standard quantum mechanics.

5. Branching, Possible Debranching, and Teleportation: MWI vs BHSI

5.1. The environment involved in MWI and BHSI

In MWI, each branch is a whole, independent, and real world. Within a world, "objects" have definite macroscopic states by fiat [Eq. (1), 8]:

$$|\Psi_{\text{WORLD}}\rangle = |\Psi_{\text{OBJ. 1}}\rangle |\Psi_{\text{OBJ. 2}}\rangle \cdots |\Psi_{\text{OBJ. }N}\rangle |\Phi\rangle \tag{39}$$

The product state is only for relevant variables for the macroscopic description of the objects. There might be some entanglement between weakly coupled variables, which should belong to $|\Phi\rangle$. The universe is expressed as a superposition of all existing worlds:

$$|\Psi_{\text{UNIVERSE}}\rangle = \sum_i^M \alpha_i |\Psi_{\text{WORLD }i}\rangle, \qquad \sum_i^M \alpha_i = 1 \tag{40}$$

Determining how to configure a world with about 10^{80} particles is hard (preferred basis?), and nobody knows how big the total M is, except that it is exponentially growing (just one double-slit experiment will add millions of branches. As described in Case 3, Section 2.2, when measuring a qubit, one of the branches in Eq. (40) (where the observer lives) is entangled with the two-qubit states described in Eqs. (17) and (19), resulting in two independent worlds, each having a Bob. Although mathematically possible, recohering the two branches in Eq. (19) or any two in Eq. (40) is ontologically forbidden (it causes identity crises) and practically impossible on the scale of worlds.

Contrary to MWI, the branches in the BHSI are local Hilbert subspaces, and each observation triggers a branching in its own local Hilbert space. There is no need for a preferred basis: the basis chosen by the observer in Eq. (1) is the basis for branching. Based on the quantum decoherence theory [26-28], the branching operator in Eq. (3) can be understood as follows:

$$\hat{B}: \left(\sum_{k=1}^{N} c_k |g_k\rangle \right) \otimes |E\rangle_L \rightarrow \sum_{k=1}^{N} c_k |g_k\rangle |E_k\rangle_L \equiv \sum_{k=1}^{N} c_k |g_{B;k}\rangle, \quad {}_L\langle E_i | E_k\rangle_L \approx \delta_{i,k} \tag{41}$$

Here, $|E\rangle_L$ represents the minimal local environment, which directly interacts with the quantum system and contains about 10 ~100 particles. The nature of branching is the same for MWI and BHSI. The difference lies in the size of their respective environments: a whole world versus the local environment (maximal versus minimal, or 10^{80} versus 10^2). Therefore, controlled recoherence in BHSI is mathematically, ontologically permissible, and practically conceivable. In theory, one may construct a debranching operator for the recoherence of decohered branches:

$$B^{\dagger}(\alpha_1 |\psi_{B,1}\rangle + \alpha_2 |\psi_{B,2}\rangle) = B^{\dagger}(\alpha_1 |\psi_1\rangle |E_1\rangle_L + \alpha_2 |\psi_2\rangle |E_2\rangle_L) = (\alpha_1 |\psi_1\rangle + \alpha_2 |\psi_2\rangle) \otimes |E\rangle_L \quad (42)$$

This suggests a potential test to differentiate between MWI and BHSI. Experiments such as delayed choice and quantum eraser [24-25], quantum error correction [34], or trapped ions entangled with photons [35] could be utilized for this purpose. Although the possibility of practically debranching measurement-branched local Hilbert spaces before being relocated within the environment remains an open question, the quantum teleportation experiment demonstrates a locally controlled decoherence-recoherence process in BHSI.

5.2. Teleportation: decoherent subspaces and the recoherence process in BHSI

Assume that Alice has a pair of photons *C* and *D*, entangled in the Bell state $|B_1\rangle_{CD} = |\Phi^+\rangle_{CD}$, and she also has a photon *B* rotated to the following state:

$$|\psi\rangle_B = \alpha\, |0\rangle_B + \beta\, |1\rangle_B\,,\ |\alpha|^2 + |\beta|^2 = 1 \quad (43)$$

Photon *D* will teleport this state. Before swapping, Alice has two photons (*C* and *B*), while Bob will receive one photon (D). The state of the three photons is given by a separable pure state in the 8-dimensional product Hilbert space:

$$|\Psi\rangle = |\Phi^+\rangle_{CD} \otimes |\psi\rangle_B = \frac{1}{\sqrt{2}}\left(|0\rangle_C \otimes |0\rangle_D + |1\rangle_C \otimes |1\rangle_D\right) \otimes \left(\alpha\, |0\rangle_B + \beta\, |1\rangle_B\right) \quad (44)$$

Then, photons *C* and *B* are entangled to the four Bell states (B_k) by unitary swapping U_A, forcing photon *D* to carry correspondingly rotated states from photon *B* [33. P.165]:

$$U_A: \ |\Psi\rangle = \frac{1}{2}\Big[|\Phi^+\rangle_{CB} \otimes (\alpha\, |0\rangle_D + \beta\, |1\rangle_D) + |\Phi^-\rangle_{CB} \otimes (\alpha\, |0\rangle_D - \beta\, |1\rangle_D)$$

$$+ |\Psi^+\rangle_{CB} \otimes (\beta\, |0\rangle_D + \alpha\, |1\rangle_D) + |\Psi^-\rangle_{CB} \otimes (\beta\, |0\rangle_D - \alpha\, |1\rangle_D)\Big] \equiv \sum_{k=1}^{4} |B_k\rangle_{CB}\, |\psi_k\rangle_D \quad (45)$$

We can rewrite Eq. (45) as the decoherent state in Eq. (43) of photon *B*, realized in photon *D*:

$$U_A: \ |\Psi\rangle = \alpha\, |0\rangle_D \otimes |E_0\rangle + \beta\, |1\rangle_D \otimes |E_1\rangle, \ \langle E_0 | E_1\rangle = 0 \quad (46)$$

$$|E_0\rangle \equiv (1/2)(|\Phi^+\rangle_{CB} + |\Phi^-\rangle_{CB}) + (\beta/2\alpha)\cdot(|\Psi^+\rangle_{CB} + |\Psi^+\rangle_{CB}) \quad (47)$$

$$|E_1\rangle \equiv (\alpha/2\beta)\cdot(|\Phi^+\rangle_{CB} - |\Phi^-\rangle_{CB}) + (1/2)(|\Psi^+\rangle_{CB} - |\Psi^+\rangle_{CB}) \quad (48)$$

Eq. (46) shows photon *D* is *locally decoherent with minimal environmental involvement* (an entangled photon pair *CB*). Now, Alice chooses Bell states as her measurement basis, splitting the local system into four branches and forcing photon *D* to take one of the four possible states as described in Eq. (45). After receiving the record from Alice about which Bell state (B_i) she observes, Bob rotates photon *D* accordingly using a unitary transformation U_i, allowing him to fully recover the original state of photon *B* in the teleported photon *D*. Therefore, the operations on photon *D* can also be viewed as a locally controlled decoherence-recoherence process:

$$\alpha\,|\,0\rangle_B + \beta\,|1\rangle_B \xrightarrow{U_A} \alpha\,|\,0\rangle_D\,|\,E_0\rangle_{CB} + \beta\,|1\rangle_D\,|\,E_1\rangle_{CB} \xrightarrow{\hat{M}_i} |\,\psi_i\rangle_D \xrightarrow{U_i} \alpha\,|\,0\rangle_D + \beta\,|1\rangle_D \quad (49)$$

Although this teleportation process realizes a decoherence–recoherence operation, it is not a true debranching process as described in Eq. (42): Bob's rotation U_i does not merge the four branches of photons *BC* created by Alice's measurement; however, Eqs. (46–49) demonstrate that, for the single qubit (photon *D*), the *locally decoherent Hilbert subspaces* are *observable in teleportation*, not merely as a theoretical construct, and locally decoherent subspaces can be recohered via unitary transformations ($U_i\hat{M}_i$). Critically, this interpretation is only viable within BHSI because Eq. (46) does not match the decoherent worlds in MWI, where each branch represents a distinct world with its own Alice and Bob. Moreover, any mismatch in messaging (e.g., the wrong message or a missed one) leads to ontological ambiguity in MWI: Bob cannot determine *which* Alice he inhabits a world with, since the four branches created by Alice's measurement are causally disconnected. BHSI avoids this ambiguity entirely—Alice and Bob inhabit a single world, and their causal communication is subspace-specific, grounded in objective, measurable decoherence records.

6. Conclusion and Discussion

The Branched Hilbert Subspace Interpretation (BHSI) achieves a unique synthesis of quantum foundations, retaining the unitarity, determinism, and collapse-free evolution of the Many-Worlds Interpretation (MWI) while avoiding its ontological excess. Like the Copenhagen Interpretation (CI), BHSI preserves a single world and observer, yet eliminates CI's ad hoc collapse postulate. The teleportation protocol, Eqs. (46–49), exemplifies BHSI's physicality: it realizes a unitary-controlled decoherence-to-recoherence transition, impossible in MWI's eternally diverging branches. Crucially, BHSI resolves tensions inherent to both frameworks:

- Ontological minimalism: No parallel worlds or preferred basis.
- Unitary preservation: No information loss or nonlocal structures.
- Causal primacy: Branching is defined by decoherence dynamics, not observer choices.
- Born rule emergence: Probabilities arise from branch weights, not axiomatically.
- Testability: Predicts standard quantum results with fewer metaphysical commitments.
- Experimental grounding: Decoherent Hilbert subspaces are observable, as shown in teleportation experiments.

While BHSI requires further refinement, particularly in scaling to complex systems, it provides a causally consistent, single-world framework that bridges the CI-MWI divide. For those skeptical of collapse *and* parallel universes, BHSI offers a third way, rooted in Hilbert space geometry and empirically accessible branching.

Abbreviations

BHSI	Branched Hilbert Subspace Interpretation
BM	Bohmian Mechanics
CI	Copenhagen Interpretation
FTL	Faster Than Light
MWI	Many-Worlds Interpretation
NHT	No-Hiding Theorem
RQM	Relational Quantum Mechanics
QM	Quantum Mechanics

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