The Epistemic Mismatch Problem: Scientific Progress and Knowledge of Approximate Truths

Finnur Dellsén & James Norton

Abstract

Many instances of scientific progress feature the development of theories that are not fully true, but merely approximately true to various extents. Since only fully true propositions can be known, this seems to rule out the view that scientific progress consists in the accumulation of knowledge. According to Bird's Cumulative Knowledge Account of progress, however, what becomes known in such instances is a (fully true) proposition expressing that the theory in question is approximately true to some extent. We present a general challenge for this idea–the Epistemic Mismatch Problem–and consider various strategies by which proponents of the Cumulative Knowledge Account might respond to it. We suggest, however, that the only plausible such strategies involve giving up on aspects of the Cumulative Knowledge Account that are central to why it has seemed plausible to begin with.

Keywords: scientific progress, epistemic account, knowledge accumulation, approximate truth

1 The Cumulative Knowledge Account and Approximate Truth

The burgeoning debate about the nature of scientific progress centres around what sort of cognitive achievement lies at the heart of progress of this kind. According to what Alexander Bird (2022, 38) dubs the *Epistemic Approach*, scientific progress should be understood in terms of epistemic notions such as knowledge or justification. By far the most influential incarnation of this approach is Bird's own *Cumulative Knowledge Account*, according to which:

> An episode in science makes progress precisely when it shows an accumulation of scientific knowledge in the relevant scientific community (Bird 2022, 39; see

also Bird 2007, 64).[1]

The Cumulative Knowledge Account is often contrasted with three other prominent accounts (see Dellsén, 2018; Shan, 2023), none of which is an incarnation of the Epistemic Approach.[2] These are the Verisimilitudinarian Account, on which science makes progress via the proposal of more truthlike theories (Popper, 1963; Niiniluoto, 2014); the Noetic Account, on which progress consists in putting people in a position to understand (Dellsén, 2016, 2021); and the Problem-Solving Account, on which progress occurs as scientific problems are solved or eliminated (Kuhn, 1970; Laudan, 1981).[3]

This paper argues that the Cumulative Knowledge Account faces serious difficulties accommodating the platitude that science often makes progress by developing new theories which, while false, are nonetheless more accurate than their predecessors. More precisely, consider:

> **The Approximation Platitude:** There is often scientific progress when a less accurate theory T_n is replaced with a more accurate theory T_{n+1}, even when T_n and T_{n+1} are both false.

For an illustrative example, let us look to scientific investigations into the fundamental constituents of matter. Progress was made with the advent of Dalton's early atomic theory–on which indivisible atoms comprise all matter; via the proposal of Thomson's plum-pudding model–on which atoms contain negatively-charged particles (electrons) that float around in a positively-charged substrate; when Rutherford proposed that an atom's positive change was contained within a comparatively small but dense nucleus; and indeed when Bohr proposed that the electrons orbit the nucleus at fixed energy levels. Although these theories are all false according to contemporary particle physics, the proposal of each was undoubtedly an instance of scientific progress.

Why might the Cumulative Knowledge Account be at odds with the Approximation Platitude? Well, in order for knowledge to be accumulated in the scientific community,

[1] Bird (2022, 39) refers to this thesis as '(CK) (cumulative knowledge)'. It is worth noting that Bird (2022, 41) also formulates an alternative and much more inclusive version of (CK), which he refers to as (CK'), on which one of several ways in which progress may be made is to bring the relevant scientific community "closer to adding to knowledge". However, (CK') is not the version of the Cumulative Knowledge Account that Bird (2022) goes on to defend in the subsequent discussion, nor is it a view that he has defended elsewhere to our knowledge. Indeed, Bird's primary concern with competing accounts is that they are too permissive regarding what counts as progress (Bird, 2022, 45-58,64-68), and (CK') is, at least on the face of it, considerably more permissive than any of the competing accounts to which he objects. We thus take (CK) to be the canonical statement of the account. With that said, in §4 we consider modifications to the Cumulative Knowledge Account that may be thought of as ways of spelling out what 'getting closer to adding to knowledge' might consist in (Bird himself leaves this phrase unanalysed and unremarked on).

[2] A recent account that does fall within the Epistemic Approach is Stegenga's (2024) justification-based account, to which we will return in footnote 24.

[3] As we note in the conclusion (§5), the problem we articulate and explore in this paper threatens none of these competing accounts, and for this reason we will set them aside for most of the paper.

scientists must in some sense come to believe, with warrant, propositions that are fully true.[4] After all, knowledge that P requires P to be fully true, not just approximately true. Dalton's atomic theory, for instance, isn't—and wasn't ever—known, since the theory is false (and *mutatis mutandis* for Thomson's, Rutherford's, and Bohr's models of the atom). In general, when a theory T_n is replaced by a more approximately true theory T_{n+1}, and neither theory is fully true, neither theory can be an object of knowledge. It is thus not immediately clear how knowledge could be accumulated in such cases.

In his defence of the Cumulative Knowledge Account, Bird (2007, 2022) addresses this problem head-on, arguing that its solution can be seen by looking more closely at the *content* of the knowledge that is often accumulated in instances of scientific progress. Roughly, the idea is that even when the relevant scientific theory is not fully true, there will nevertheless be a fully true proposition that can be the object of knowledge: the proposition expressing that the theory is *approximately true*.[5] This initial response requires some finessing, however, and Bird (2007, 78; see also 2022, 61) soon appeals to "a series of versions or precisifications of the approximation operator A: A_0, A_1, A_2, ..., where each later approximation in the series is more precise than its predecessors" in order to capture how progress is, in the cases in question, achieved by the successive proposal of *increasingly* approximately true theories. To explain, let $A_i(T)$ stand for the proposition that theory T is approximately true to degree i, allowing for a 'degree of approximate truth' to refer to a range rather than a point value.[6] Then we can say that progress from a less accurate T_n to a more accurate T_{n+1} occurs, on the Cumulative Knowledge Account, as scientists come to know a proposition of the form $A_i(T_{n+1})$—perhaps in addition to already knowing $A_j(T_n)$, where A_j is a less precise approximation operator than A_i. This, in essence, is how Bird's Cumulative Knowledge Account accommodates the Approximation Platitude.

There are various concerns one might have about the picture thus articulated. In our view the most critical issue is what we call the *Epistemic Mismatch Problem*: roughly, that the true propositions Bird identifies, warranted belief in which is supposed to constitute progress, are not plausibly propositions that are in fact believed with warrant by scientists. While this issue has been flagged by Niiniluoto (2014, 76), and discussed briefly by Rowbottom (2023, 18-19), it has not yet been systematically explored by either proponents or opponents of the Cumulative Knowledge Account. As we shall see, careful

[4]Following Plantinga (1993), we use 'warrant' here to refer to whatever it is that needs to be 'added' to a true belief to make it knowledge. Thus, depending on one's theory of what knowledge requires, warrant may be a matter of reliability, or safety, for example, or a combination of epistemic justification and an appropriate Gettier-condition.

[5]Note that Bird could instead have suggested that the fully true proposition that is known in such cases is a proposition expressing that the theory is *truthlike* to some degree. On the difference between truthlikeness and approximate truth, see, e.g., Niiniluoto (1999, 72-73; 2014, 74).

[6]In the former case, these ranges would presumably have to be non-overlapping intervals of approximations to the truth. Otherwise, the series of approximation operators may not be such that each is more precise than the last, in the way that Bird envisions.

examination of this problem reveals that there is no quick response or easy modification available, rendering the problem at least as serious as other concerns about the Cumulative Knowledge Account that have been discussed more thoroughly in the literature.[7]

To promptly expose what we take to be the core of the issue, we will grant that something can be said to address other pertinent concerns. In particular, we'll grant that the notion of approximate truth could be made sufficiently *precise* to serve its proposed purpose, e.g., such that one can discriminate between the approximate truth of T_n and T_{n+1}. We'll also grant that the notion of approximate truth could be made sufficiently *general* to cover all of the instances of increasingly accurate theories to which the Approximation Platitude refers. Finally, we will grant that that there is nothing about the concept of approximate truth that makes it impossible for scientists to have knowledge (or warrant, or true beliefs) about the extent to which theories are approximately true. So we will grant, for example, that if P is a proposition, and thus a possible object of knowledge, then so too is $A_i(P)$, for any i.

2 The Epistemic Mismatch Problem

Precisely how the kind of mismatch alluded to above constitutes a problem for the Cumulative Knowledge Account may be brought out by the following *Mismatch Argument*:

> **M1** In order for a not-fully-true new theory T_{n+1} to contribute to progress on the Cumulative Knowledge Account, scientists would have to believe, and be epistemically warranted in believing, a true proposition of the form $A_i(T_{n+1})$.
>
> **M2** Scientists hardly ever believe true propositions of the form $A_i(T_{n+1})$, and/or are hardly ever warranted in having such beliefs.
>
> **M3** So, not-fully-true new theories hardly ever contribute to progress on the Cumulative Knowledge Account.

We take it that this argument is valid, so let's consider, in turn, the plausibility of its premises, and the unacceptability of the conclusion.

The first premise, **M1**, seems to follow straightforwardly from Bird's clarification of his account in response to the Approximation Platitude. With that said, we will consider in §3 some ways in which a defender of the Cumulative Knowledge Account might try to modify the commitment enshrined in **M1** so as to avoid the problematic conclusion **M3**. To foreshadow, we will argue that each of these modifications either does not solve the underlying problem, or that the cure it provides turns out to be worse than the disease.

[7]For some of these other concerns about the Cumulative Knowledge Account, see, e.g., Rowbottom (2008, 2010); Cevolani and Tambolo (2013); Niiniluoto (2014); Dellsén (2016, 2022b).

To address the second premise, **M2**, consider what it would take, intellectually and epistemically, for scientists to have warranted, true beliefs of the requisite sort.[8] In particular, when progress occurs via an earlier theory T_n being replaced with a more approximately true theory T_{n+1}, the Cumulative Knowledge Account appears to require that scientists:

(a) are in possession of concepts of degrees of approximate truth that are precise enough to discriminate between increasingly approximately true theories, e.g., between T_n and T_{n+1};

(b) correctly estimate which of these concepts of degrees of approximate truth in fact apply to each theory with which progress is made, e.g., T_n and T_{n+1};

(c) subsequently devote part of their limited cognitive resources to forming beliefs on this basis, including storing these beliefs in their short- and long-term memory;

(d) do all this in a way that makes these beliefs epistemically warranted in the sense required for knowledge (e.g., with sufficient reliability).

We concede that it is conceivable that scientists occasionally satisfy (a)-(d). This may be so, for example, in those special cases of scientific progress in which theorising about some phenomenon explicitly involves proposing, within increasingly small margins of error, the value of some quantity (see, e.g., Bird's (2022, 59-61) discussion of increasingly precise estimations of the speed of light).[9] However, we submit that it is exceedingly implausible that they do so on a regular basis. These intellectual and epistemic requirements are

[8]Here and in what follows, we'll grant that knowledge that P may only require one to have a *dispositional belief*, rather than an *occurrent belief*, that P. Although accounts of this distinction differ, the standard story is one on which "[a] subject dispositionally believes P if a representation with the content P is stored in their memory or 'belief box' [...] When that representation is retrieved from memory for active deployment in reasoning or planning, the subject occurrently believes P" (Schwitzgebel, 2023). Importantly for our purposes, however, not all *dispositions to believe* are dispositional beliefs in this sense, for many things that an agent might be disposed to believe in the right circumstances (e.g., when presented with the appropriate evidence, led through the appropriate proof, provided with the appropriate concepts, etc.) will not in any reasonable sense be stored in their memory or 'belief box' (see also Audi, 1994). While it may be plausible that scientists are disposed to believe true propositions of the form $A_i(T_{n+1})$ in certain hypothetical circumstances, such as when they are presented with the concept of something being approximately true to degree i and asked whether this concept might apply to T_{n+1}, it is far less plausible that they are in possession of true dispositional beliefs of this form, i.e., that this type of belief is stored in their memory or 'belief box'. (Thanks to both Alexander Bird and Nick Hughes for highlighting the relevance of the distinction between occurrent and dispositional belief.)

[9]In this sort of case, there is a perfectly good sense in which scientists do have the required concepts of degrees of approximate truth, as per (a), and so provided that they apply these concepts correctly, as per (b), form and store the appropriate beliefs, as per (c), and do all this in an epistemically warranted manner, as per (d), scientists would indeed accumulate knowledge as their estimations of the value of this quantity become increasingly precise. It should be clear, however, that this is a very special sort of case, and not one from which we can generalise to all cases of scientific progress in which less accurate theories are replaced by more accurate ones. For example, the progress made by increasingly accurate theories of the atom (from Dalton, via Thomson, and Rutherford, to Bohr) does not simply involve estimating, within increasingly small margins of error, the value of some quantity.

simply far too demanding for it to be plausible that they are satisfied by all but the most sophisticated and reflective scientists in special circumstances. With that said, we will consider in §3 whether the requisite type of belief may be weakened in a way that would make (a)-(d) substantially easier to satisfy.[10]

Supposing, then, that the argument is sound, how problematic is its conclusion, **M3**? An immediate upshot of **M3** is that scientific progress would be surprisingly difficult to achieve via the development of new theories on the Cumulative Knowledge Account, corresponding to how difficult it is for scientists to satisfy (a)-(d) above. From this two further upshots follow. The first is that scientific progress would be considerably less frequent, and/or more modest, than it is generally assumed to be, occurring almost exclusively when scientists hit upon fully true theories which could subsequently become known, or when scientists accumulate piecemeal knowledge that falls well short of a theory (e.g., knowledge of the results of an experiment).[11] In addition to this surprisingly pessimistic upshot about the prevalence of scientific progress, we may add a revisionist upshot about scientific practice. For if scientific progress is largely or even primarily impeded by scientists not satisfying requirements (a)-(d) above, then it would follow that progress-seeking scientists should, at least to much a greater extent than they currently do, strive to overcome these hurdles. This would involve spending their limited time and resources on, for instance, acquiring the requisite concepts of different degrees of approximate truth; developing the skills required for accurately estimating the degrees of approximate truth for each new theory; and storing the relevant beliefs about the degrees to which theories are approximately true in whatever way is required for the corresponding knowledge to be accumulated in the scientific community. We take it that these are undesirable and perhaps unacceptable upshots of the Cumulative Knowledge Account, so we turn now to considering how **M3** might be avoided.

3 Cumulative Knowledge without Epistemic Mismatch?

3.1 Knowing that Later Theories Are More Approximately True

Consider first a response that rejects **M1** by insisting that properly understood, the Cumulative Knowledge Account does not require that scientists believe, or are epistemically

[10]It is worth noting that **M2**, is, in principle, subject to empirical investigation. However, a direct empirical investigation of **M2** would require, first of all, that we determine to what degree a given theory T_{n+1} is approximately true (i.e., which approximation operator A_x makes $A_x(T_{n+1})$ fully true). It would also require some means of empirically detecting whether scientists' beliefs with respect to this theory have exactly this content, e.g., $A_x(T_{n+1})$ as opposed to $A_{x-1}(T_{n+1})$ or $A_{x+1}(T_{n+1})$. And finally, it would require that we could determine, empirically, whether scientists are in fact warranted in having beliefs with these contents. It is because of these obstacles to a direct empirical investigation of **M2**, that we have adopted a slightly different tack in evaluating its plausibility.

[11]This would certainly seem antithetical to Bird's own views, since he explicitly states that, as a scientific realist, he is at the very least committed to the "minimal realist claim" that "science has always progressed: the history of science is marked by the accumulation of knowledge" (Bird, 2007, 79).

warranted in believing, a proposition of the form $A_i(T_{n+1})$. Instead, in cases like those described by the Approximation Platitude, the progress made by moving from T_n to T_{n+1} comes via scientists coming to know (and thus believe, with warrant) that T_{n+1} is *more approximately true* than T_n. Semi-formally, the knowledge that is accumulated would then be of propositions of the form $T_{n+1} >_A T_n$.[12] Even if it is true, as per **M2**, that scientists rarely know propositions of the form $A_i(T_{n+1})$, they may nevertheless know propositions of the form $T_{n+1} >_A T_n$. Indeed, a proposition of this form can be known without knowing to what degree either T_{n+1} or T_n is approximately true.

However, conceiving of scientific progress in this way is not without its drawbacks. For one, if the knowledge that is gained in progressive episodes is comparative in this way, its content is disconnected from the propositions that seem most central to scientific practice. Propositions of the form $T_{n+1} >_A T_n$ are decidedly not the propositions that are normally appealed to in explanations, predictions, model-building, theory-testing, experimentation, and so forth for almost any scientific activity in which theories are actually put to use. Similarly, science communication and science advice to policymakers rarely focuses on comparative claims of this sort, expressing that a given theory is more approximately true than its predecessor. After all, a comparative claim of the form $T_{n+1} >_A T_n$ can easily be true even when T_{n+1}'s approximate truth is extremely low (provided that T_n's approximate truth is even lower). Indeed, a claim of the form $T_{n+1} >_A T_n$, by itself, tells us almost nothing about the approximate truth (or lack thereof) of either T_{n+1} or T_n. There is therefore a large logical gap between believing $T_{n+1} >_A T_n$, on the one hand, and on the other believing that T_{n+1} is even moderately approximately true, let alone sufficiently approximately true to be relied upon for the various purposes that scientific theories are in fact relied upon. This response thus generates a new sort of mismatch problem: in order to make progress, scientists must only believe comparative claims of the form $T_{n+1} >_A T_n$, but in order to practice and communicate science they must adopt some non-comparative attitude towards T_{n+1} itself.

To see a further concern, consider that according to the Cumulative Knowledge Account, if knowledge is lost—because it has been forgotten, say—there would be scientific regress (unless, one might add, there are corresponding gains in knowledge in the meantime). Bird embraces this implication, although he adds that "such occurrences are rare for knowledge in general and even rarer for scientific knowledge" (Bird, 2007, 79). However, note that in order for knowledge of the form $T_{n+1} >_A T_n$ to be retained, scientists would have to continuously consider the earlier theory T_n in addition to considering T_{n+1};

[12]Note that no proposition of this form can come to be believed or known when the first theory of some phenomenon, T_1, is first developed. In this special case, perhaps progress comes on the cheap: moving from no theory at all to T_1 counts as progress regardless of how approximately true T_1 is. Exactly which proposition should be said to become known in this special case is not entirely clear, although one proposal would be that scientists come to know the proposition that T_1 has *some* degree of approximate truth.

otherwise they cannot continue to believe (and thus know) $T_{n+1} >_A T_n$. Indeed, if there is to be accumulation of such knowledge over time, and no losses (with corresponding regress), scientists would have to continue to believe $T_n >_A T_{n-1}$, $T_{n-1} >_A T_{n-2}$, and so forth all the way to $T_2 >_A T_1$. Not only does this seem too demanding a mental load for scientists to maintain on a regular basis; it simply seems implausible that progress would depend on their capacity to maintain it.

3.2 Knowing that Theories Are At Least This Approximately True

Another way to reject **M1** is to argue that, when our theories become increasingly approximately true, scientists need not believe that the latest theory is approximately true to some degree i in order for there to be progress on the Cumulative Knowledge Account; rather, they need only believe that the theory in question is approximately true to *at least* degree i. In other words, scientists need not believe, and be warranted in believing, a proposition of the form $A_i(T_{n+1})$; they must instead only believe, and be warranted in believing, a proposition of the form $A_{\geq i}(T_{n+1})$, which is true whenever T_{n+1} is in fact approximately true to any degree equal to, or higher than, i. Notably, $A_{\geq i}(T_{n+1})$ is a less demanding belief than $A_i(T_{n+1})$ for all but the highest possible value of i.

This response, by itself,[13] is hardly an improvement over Bird's original strategy for accounting for the Approximation Platitude. To see why, note first that in order for an accumulation of warranted beliefs of the form $A_{\geq i}(T_{n+1})$ to constitute scientific progress, the relevant approximation operators $A_{>1}(\cdot)$, $A_{>2}(\cdot)$, ..., would still have to be discriminating enough to make it true that, for any pair of theories T_n and T_{n+1} where the latter is more accurate than the former, there is some i such that $A_{\geq i}(T_{n+1})$ is true and $A_{\geq i}(T_n)$ is false. This is required in order to capture the sense in which the successor theory T_{n+1} is an improvement over its predecessor T_n. For if scientists lacked sufficiently discriminating approximation operators—i.e., if their approximation operators $A_{>1}(\cdot)$, $A_{>2}(\cdot)$, ... were such that, for all i, $A_{\geq i}(T_{n+1})$ and $A_{\geq i}(T_n)$ are either both true or both false—then progress would also occur, to the very same extent, in a scenario in which T_n is replaced by an alternative subsequent theory T_n^* which is exactly as (or, possibly, even slightly less) approximately true as T_n itself.[14] After all, in both cases scientists' *knowledge* with respect to the theories in question, viz. T_{n+1} and T_n^*, would be that they are approximately true to at least degree i: $A_{\geq i}(T_{n+1})$ and $A_{\geq i}(T_n^*)$. There would thus be no way in which, on the Cumulative Knowledge Account, the development of T_{n+1} (which is more approximately true than T_n) would contribute more to progress than T_n^* (which isn't more approximately true than T_n). In short, the upshot is that the Cumulative

[13] In §3.3, we consider whether this response may be combined with another response to successfully address the Epistemic Mismatch Problem.

[14] To see why the parenthetical claim holds, note that as long as T_n's approximate truth is slightly above the threshold for the relevant approximation operator $A_{\geq i}(\cdot)$, T_n^* may be slightly less approximately true than T_n even if $A_{\geq i}(T_n)$ and $A_{\geq i}(T_n^*)$ are both true.

Knowledge Account would imply that we would also have progress, to precisely the same extent, in cases of no increasing approximation to the truth whatsoever, such as when T_n is replaced with the equally (or slightly less) approximately true T_n^*.[15]

In order to correctly identify instances of scientific progress, then, the approximation operators at play in the proposed beliefs of the form $A_{\geq i}(T_{n+1})$ would have to be just as discriminating as those envisaged in Bird's original strategy. It thus remains exceedingly implausible that scientists have beliefs, let alone warranted true beliefs, of the requisite type, viz. expressing fully true propositions of the form $A_{\geq i}(T_{n+1})$. After all, this would once again require that scientists are in possession of the relevant range of concepts of degrees of approximate truth (albeit of the form $A_{\geq i}(\cdot)$ rather than $A_i(\cdot)$); that scientists correctly estimate which such concepts apply to a given theory; devote part of the cognitive resources on forming and storing the relevant beliefs; and that they do all this in a way that makes their beliefs warranted. In short, the requirements (a)-(d) discussed in §2 apply even with the account reimagined in the way envisaged here, *mutatis mutandis*, and it is not significantly more plausible that they are satisfied now than it was that they were satisfied before.

3.3 Knowing Approximate Truths Through Outright Belief in Theories

A third response to the Epistemic Mismatch Problem builds on the previous response but also rejects **M2**.[16] To introduce this response, note that there is a sense in which belief in the proposition T_{n+1} includes a belief in a proposition of the form $A_{\geq i}(T_{n+1})$. After all, the proposition that a theory is (fully) true directly entails the proposition that the theory in question is approximately true to a degree greater than or equal to i, for any i. In other words, T_{n+1} entails $A_{\geq i}(T_{n+1})$, for any i. One might take this to mean that scientists possessing outright belief in the truth of a theory also thereby believe any proposition stating that the theory is true to at least some particular degree. If so, then if scientists believe theories to be true at all, they *ipso facto* have the requisite beliefs about the theories being at least approximately true to the degree they are in fact approximately true. In this way, it may seem, the support offered for **M2** in §2 is undermined.[17] For, according to the present line of thought, the supposedly-rare belief in $A_{\geq i}(T_{n+1})$ comes on the cheap for any agent who believes the original theory T_{n+1} itself.[18]

Our first concern with this third response is that if this is not to be an entirely *ad*

[15]This same point, as it applies to Bird's original strategy, is made by Niiniluoto (2014, 77).

[16]This line of response is mentioned briefly by Rowbottom (2023, 18), who also suggests a rejoinder along the lines of the one we develop below (see footnote 19).

[17]More precisely, what would be undermined is support for the version of **M2** in which '$A_i(T_{n+1})$' has been replaced by '$A_{\geq i}(T_{n+1})$', in line with the response considered in §3.2.

[18]To see why this response builds on the previous response, note that while T_{n+1} entails $A_{\geq i}(T_{n+1})$, it does not also entail $A_i(T_{n+1})$, for any i except for the maximum value of i. There is thus no initial plausibility to the thought that believing T_{n+1} somehow carries with it a true belief of the form $A_i(T_{n+1})$ as well.

hoc move, it would have to assume a theory of belief on which it is closed under all of the relevant approximation operators $A_{\geq i}(\cdot)$. Put differently, the theory—if it is to avoid being entirely *ad hoc*—would have to imply that, for any proposition P that one believes, one also believes that P is approximately true to at least degree i, for any i. While this would indeed ensure that scientists in fact believe the relevant true proposition of the form $A_{\geq i}(T_{n+1})$, it also ensures that scientists believe a plethora of other propositions about the minimum degree to which T_{n+1} is approximately true. Suppose, then, that T_{n+1} is approximately true to degree i. According to the view under consideration, scientists will believe (i) some propositions according to which T_{n+1} is at least approximately true to degrees *lower* than i, and (ii) some propositions according to which T_{n+1} is at least approximately true to degrees *higher* than i. The beliefs referred to in (i) will, while true, be 'junk beliefs', in that they are strictly less informative implications other propositions one believes (Harman, 1986). The current response thus implies, problematically, that scientists must have a large number of such junk beliefs in order to make progress. Even more problematically, the beliefs referred to in (ii) will be *false* junk beliefs – since they attribute to the theory a degree of approximate truth greater than some threshold the theory does not meet. Thus the theory of belief behind the current response not only attributes to progress-making scientists a whole range of junk beliefs, but also many such beliefs that are simply false.

A second concern with the current response is that if a true proposition $A_{\geq i}(T_{n+1})$ is believed only in virtue of believing every approximate-truth-involving entailment of T_{n+1}, then $A_{\geq i}(T_{n+1})$ would seem not to be known after all due to the remaining requirements for knowledge, beyond truth and belief, not being satisfied in such a case.[19] To see this, consider that in the imagined case T_{n+1} is not fully true, and thus false. Now, depending on how the case is spelled out further, outright belief that T_{n+1} is true may or may not be epistemically justified for the scientists in question. Suppose first that it is not. In that case, belief in the true proposition $A_{\geq i}(T_{n+1})$ would clearly also not be justified, since scientists' justification for $A_{\geq i}(T_{n+1})$ would be derived from their (inadequate) justification for T_{n+1}. Suppose, then, that the scientists' belief in T_{n+1} *is* justified. In this case, $A_{\geq i}(T_{n+1})$ is being inferred from a false yet justified premise, viz. T_{n+1}. The proposition $A_{\geq i}(T_{n+1})$ would thus exemplify the original and arguably least controversial type of Gettier-case, viz. the type of case in which the agent infers a true conclusion from a justified but false premise (Gettier, 1963). In sum, then, regardless of whether the belief in T_{n+1} simpliciter on which the belief in $A_{\geq i}(T_{n+1})$ is based is itself justified or not, $A_{\geq i}(T_{n+1})$ would either not be justified, or be justified in precisely the way that creates paradigmatic Gettier-cases; either way, $A_{\geq i}(T_{n+1})$ would not be known.

[19] A version of this concern is mentioned briefly by Rowbottom (2023, 18), who claims that "[o]ne cannot come to know A(T) simply by justifiably believing the false claim that T and deriving A(T)." Here we develop and argue for Rowbottom's claim, and show in more detail how it closes off the current line of response to the Epistemic Mismatch Problem.

3.4 Knowledge Accumulated by Science as a Collective Entity

A fourth and final response we will consider is to look more closely at what it would be for *scientists* to believe, and be epistemically warranted in believing, a true proposition of the form $A_i(T_{n+1})$. While this is most straightforwardly read as referring to what individual scientists believe, another option is to interpret it as referring to what *science*, understood as a collective entity in its own right, believes. In particular, the proponent of the Cumulative Knowledge Account might concede that individual scientists rarely believe with warrant true propositions of the form $A_i(T_{n+1})$ (or $A_{\geq i}(T_{n+1})$; see §3.2), but insist that a collective entity can know propositions which are not known by any members of the collective.[20] Thus, *science*$_c$, understood as a collective agent in its own right, may have these beliefs after all, and be warranted in having them. If we add to this that scientific progress can, at least in some cases, consist not in the accumulation of individual scientists' knowledge, but also of the accumulation of science$_c$'s collective knowledge, then we may seem have accumulation of knowledge after all in cases of increasingly approximately true theories.

The main problem facing this response stems from the fact that even if one grants that science$_c$ may have knowledge that no individual scientist has, this in no way implies that science$_c$'s beliefs have exactly the right propositional content in order for its knowledge to accumulate as T_n is replaced by T_{n+1}, for instance. All it really implies is that there is one additional subject (as distinguished from any individual scientist) who is capable of possessing knowledge, viz. science$_c$, and whose accumulation of knowledge may make for scientific progress. But unless and until we have been given reasons for thinking it is significantly more plausible, on balance, that science$_c$ possesses exactly the right sort of beliefs in fully true propositions of the form $A_i(T_{n+1})$ (or $A_{\geq i}(T_{n+1})$), and moreover is warranted in having those beliefs, there is no compelling reason for thinking that adding this particular subject to the list of potential accumulators of scientific knowledge would help with the Epistemic Mismatch Problem.

So *is* it significantly plausible on balance that science$_c$, the collective agent, has these sorts of true beliefs, and is moreover warranted in having them? It is hard to see why this would be the case. Consider again the requirements for having such true warranted beliefs that we suggested it would be implausible to suppose that individual scientists regularly satisfy (see (a)-(d) in §2). As far as we are aware, there are no reasons for thinking that collective agents, such as science$_c$, possess precise concepts of degrees of approximate truth, as per (a); it would be surprising indeed if science$_c$ was consistently able to hit upon those beliefs about the degree to which scientific theories are approximately true

[20]Indeed, Bird himself has independently argued that science is such a collective entity, itself the subject of knowledge ascriptions (Bird, 2019, 2022, ch.2); and moreover that a collective entity can know propositions which are not known by any members of the collective Bird (2010, 2022, ch.4). So this type of response would not be *ad hoc* from Bird's own point of view.

that happened to be (fully) true, as per (b); it does not seem as if science$_c$ regularly forms and stores beliefs about the extent to which scientific theories are approximately true, as per (c);[21] finally, even if science$_c$ somehow succeeded in hitting upon precisely those beliefs of the form $A_i(T_{n+1})$ that are fully true, it is far from clear how science$_c$ could be warranted in having these beliefs. Even supposing that science$_c$ draws upon all the publicly available evidence collected in science, it is unlikely that the result is anything approaching warrant for believing a particular proposition about the extent to which current theories are approximately true, e.g., $A_i(T_{n+1})$ as opposed to $A_j(T_{n+1})$ where i and j are close but not identical.

4 Alternative Accounts within the Epistemic Approach?

Having considered several potential responses to the Epistemic Mismatch Problem available to the Cumulative Knowledge Account, we're yet to find one that addresses the problem in a plausible way. We therefore move on to consider some alternative incarnations of what Bird calls the Epistemic Approach—on which scientific progress should be understood in terms of knowledge or justification or related concepts—that can correctly identify the progress that comes via increases in the approximate truth of successive theories. While there are presumably many ways of analysing scientific progress that belong within the Epistemic Approach, we shall sketch and evaluate two accounts that we take to be quite close cousins of the Cumulative Knowledge Account (i.e., two epistemic accounts of progress, in a broad sense). We call them the *Warrantedness Account* (§4.1) and the *Approxistemic Account* (§4.2). While these accounts fare rather better than the Cumulative Knowledge Account when it comes to the Epistemic Mismatch Problem, they are somewhat at odds with the role that many have thought knowledge ought to play in our scientific and philosophical theorising (§4.3).

4.1 The Warrantedness Account

By contrast to the Cumulative Knowledge Account, the Warrantedness Account does away with the requirement that in order for scientific progress to occur scientists must have any particular *beliefs* regarding the theories proposed in progressive episodes, including beliefs regarding these theories' degrees of approximate truth. With this requirement removed, there are no issues arising from a mismatch between the propositions that are true and those that can realistically be taken to be believed with warrant by scientists.

[21] This would presumably require the content of said beliefs to published or otherwise made publicly available, as per the cases discussed by Bird (2010; 2022, ch.4), but we are not aware of even a single instance of a scientific theory being explicitly declared to be approximately true to a specific degree i.(As Niiniluoto (2014, 76) pithily puts it, "we don't find A(NEWTON) in the *Principia*.") Note that, as argued in §2, it would not be enough for science$_c$ to believe that T_{n+1} is approximately true *simpliciter*, as in the content of utterances such as 'T_{n+1} is roughly true'.

However—in line with the Cumulative Knowledge Account's focus on warrant—the Warrantedness Account holds that scientific progress occurs precisely when (and to the extent that) there is accumulation of true claims for which the scientific community has the type and level of epistemic warrant required for knowledge.[22][23]

So, for instance, if scientific research generates sufficient evidence to warrant belief in T_{n+1}, and T_{n+1} is true, then this will amount to progress regardless of whether scientists propose, let alone believe, T_{n+1}. Moreover—and this is where the Warrantedness Account does better with the Approximation Platitude than does the Cumulative Knowledge Account—if scientific research generates sufficient evidence to warrant belief in $A_i(T_{n+1})$ (or $A_{\geq i}(T_{n+1})$), and that proposition is true, then this will amount to progress regardless of whether anyone believes these propositions. Progress would thus be independent of what scientists actually believe; changes in scientists' epistemic warrant alone would determine whether, and the extent to which, there is progress in a given episode.[24]

However, even with the mismatch between what is believed with warrant versus what is true circumvented in this way, we are not convinced that those who are drawn towards the Epistemic Approach will consider this view to be an attractive incarnation of the approach. This is so for three reasons. First, it remains questionable that scientists typically have epistemic warrant for propositions like $A_i(T_{n+1})$ (or $A_{\geq i}(T_{n+1})$). In particular, one might worry that, in a typical case of progress via increasingly approximately true theories, the evidence gathered in support of a false yet frontrunning theory T_{n+1} simply provides some sort of modest warrant for T_{n+1}, rather than providing epistemic warrant (of the level required for knowledge) for a rather contrived proposition of the form $A_i(T_{n+1})$ (or $A_{\geq i}(T_{n+1})$).

Second, once we give up on the idea that progress consists in the accumulation of

[22]Since the warrant in question would not be attached to the beliefs of any particular agent, but rather to the propositions themselves (which may or may not be believed), the sort of warrant we have in mind would be *propositional*, as opposed to *doxastic*, epistemic warrant (see, e.g., Silva and Oliveira, 2024). The proximity of this account to the Cumulative Knowledge account can be brought out by noting that when scientists have warrant for propositions which they do not believe, there is a sense in which they are *in a position* to know these propositions.

[23]Thanks here to Maria Lasonen-Aarnio and Daniel Stoljar, who both independently made suggestions along these lines.

[24]In a recent paper, Stegenga (2024), defends a justification-based account of scientific progress according to which sciences progresses just in case there is a change in scientific justification. Since justification and warrant are closely related concepts, the Warrantedness account might seem almost identical to Stegenga's account. However, a closer look reveals that the two accounts differ in several important respects, including the following two. First, Stegenga's account makes no reference to truth, approximate truth, or indeed any other veritistic requirement on scientific progress; the Warrantedness Account, by contrast, conceives of progress in terms of *true* claims coming to be warranted in the way required for knowledge (where many of these true claims will be about what's approximately true to various degrees). Second, unlike the Warrantedness Account, Stegenga's account does not imply that progress comes when theories are justified to any specific degree—certainly not to the degree required for knowledge. Indeed, one surprising way of making progress on Stegenga's account is through *decreases* in justification; progress can come via an already unjustified theory coming to be *even less* justified. (For a critical discussion of Stegenga's account, see Dellsén and Norton, 2025.)

knowledge, it becomes somewhat unclear why we should retain the commitment that progress can only occur when the level of epistemic warrant for truths reaches that which is required for knowledge. Why not, for instance, develop a version of the Epistemic Approach according to which scientific progress consists in *increasing* warrant for truths, regardless of whether the level of warrant reaches that which is required for knowledge?[25] To concede that there is progress in such cases as well, i.e., that there can be progress regardless of whether any particular fully true proposition becomes sufficiently warranted to be known if believed, is to move even further away from the Cumulative Knowledge Account.

Third, we think that in giving center stage to a notion other than knowledge, the Warrantedness Account cannot be motivated by the kinds of reasoning that lead many towards the Cumulative Knowledge Account. Since this concern applies equally to the other account we consider in this section, we hold off on discussing it until that account has also been spelled out (see §4.3).

4.2 The Approxistemic Account

In slogan form, what we are calling the Approxistemic Account amounts to replacing the idea that progress occurs as scientists accumulate knowledge that theories are approximately true to an increasing degree, with the idea that progress occurs as it becomes increasingly approximately true that scientists know these theories.[26] Put differently, *knowledge of approximate truth* is replaced by *approximate knowledge*.[27] To spell this out, note that we could replace the implicit requirement of the Cumulative Knowledge Account that the warranted belief had by scientists must exactly match what's true, with the requirement that there is some kind of 'approximate match' between that which is warrantedly believed and that which is true. We'll build up to what we consider the most plausible way to expand on this idea by considering three increasingly sophisticated versions of the Approxistemic Account.

A first pass at an Approxistemic Account would have it that scientific progress occurs precisely when (and to the extent that) there is accumulation of approximately true theories which are believed with epistemic warrant by the scientific community. Note that the theories in question are not *believed to be approximately true* by the relevant scientists. Rather, they are believed outright, and while they are often false, their approximate truth

[25] In this vein, Dellsén and Norton (2025) formulate and motivate (but do not endorse) a view according to which scientific progress consists in *increasing* warrant for true answers to questions. Note that this account does not require warrant for propositions about the approximate truth of theories at all, and does not require epistemic warrant to meet the threshold required for knowledge; any increase in warrant for the true answers constitutes progress.

[26] Thanks to Jaakko Hirvelä for this way of putting it.

[27] As we noted in footnote 1, this may be what Bird (2022, 41) has in mind when he briefly presents a more inclusive version of the Cumulative Knowledge Account which allows that progress can be made by the scientific community getting "closer to adding to knowledge".

is enough for there to be progress on this version of the Approxistemic Account.

This first pass at an Approxistemic Account is clearly inadequate, however, since—as we've seen—a single notion of approximate truth is insufficiently discriminating (see §2). In particular, this account cannot explain how more progress is made when a very highly approximately true theory is believed with warrant than when the same is true of a theory that only barely passes the threshold for being approximately true in this binary sense. Furthermore, even when neither of two theories is approximately true in this binary sense, there may well be progress from one to the other provided that the later theory is more approximately true than the earlier theory (and the two theories are believed with warrant).

This leads to a second pass at an Approxistemic Account. This is the view that scientific progress occurs precisely when (and to the extent that) there is an increase in the degree to which the theories that are believed with epistemic warrant by the scientific community are approximately true. On this picture, there is an ever-changing set of theories that are believed, with warrant, by scientists; and scientific progress occurs precisely when the degree of approximate truth of these theories increases.[28]

Problematically, however, many of the theories in virtue of which progress is made according to the Approximation Platitude will be obviously false (i.e., obviously not fully true), even if they have a some, potentially a very high, degree of approximate truth. For example, Bohr was well aware that his orbit model of the atom was not, indeed could not be, fully true in the form in which he presented it. Indeed, Bohr even commented at the time that it was "perhaps no serious drawback" that "the deficiencies of the atomic model we are considering stand out so plainly" (quoted in Kragh, 2012, 68). In cases such as these, where a theory's lack of full truth is obvious, scientists will not generally be warranted in believing these theories. Thus, the theories in question are not included in the ever-changing set of propositions that are believed with warrant by scientists, so

[28]This account could be supplemented with a forgiving definition of 'knowledge' on which agents can know that P when P is false but sufficiently approximately true (see Niiniluoto, 1999, 84 and Decock et al., 2014 for suggestions along these lines). In that case, some of the theories in virtue of which progress is made at a given time would count as known by those scientists, viz. those theories that are not only believed with warrant, but also sufficiently approximately true. However, note that there will also be cases described by the Approximation Platitude in which the successor theory will *not* have a degree of approximate truth sufficient for it to count as known by any reasonable standard, even if those theories were believed with warrant. (The Bohr model discussed immediately below is a case in point.) So this amendment to the Approxistemic Account does not secure the result that all progress comes via knowledge, since there will also be progress in many cases in which no knowledge is gained even on this more forgiving definition of 'knowledge'. Besides, the envisioned (re)definition of 'knowledge' to incorporate false but sufficiently approximately true theories as possible objects of knowledge would very much go against the mainstream of philosophical thinking about knowledge—including most incarnations of the knowledge-first program that provides an important motivation for the Epistemic Approach (see §4.3 below)—so this amendment to the Approxistemic Account would risk alienating many philosophers who might otherwise be sympathetic to it. For these reasons, we do not think this amendment to the Approxistemic Account is ultimately one that proponents of the Epistemic Account should want to make. (Thanks to an anonymous referee for calling our attention to this possible amendment.)

their high degree of approximate truth cannot serve to increase the degree to which that set is approximately true.

This leads us to a final, third pass at the Approxistemic Account. On this version of the account, scientific progress occurs precisely when, and to the extent that, there is an increase *either* in the extent to which the theories that are believed by scientists are approximately true, *or* in the extent to which they are warranted. This version of the account effectively splits the notion of progress into two dimensions, based on two different ways in which the theories believed by the scientific community can improve from an epistemic point of view. On the one hand, those theories might come to be approximately true to a higher degree; on the other hand, scientists might become more warranted in believing those theories. In both cases, however, what scientists believe is simply the scientific theories in question, rather than some proposition expressing the extent to which they are approximately true.[29]

This third pass of the Approxistemic Account, we suggest, might be the closest cousin of the Cumulative Knowledge Account which accommodates the Approximation Platitude without falling foul of the Epistemic Mismatch Problem, or generating obvious new concerns like those we raised for the Warrantedness Account. Nevertheless, we think that this account faces some serious motivational challenges.

4.3 Motivational Challenges

Both the Warrantedness Account and the Approxistemic Account are incarnations of the Epistemic Approach which circumvent the Epistemic Mismatch Problem. They do so by denying that scientists must *know* propositions about the approximate truth of scientific theories. However, since these accounts analyse the notion of scientific progress in terms of notions other than knowledge—albeit in terms of the closely related notions of warrant, belief and approximate truth—we think that these views cannot be motivated by the same sorts of considerations that have been adduced in favor of the Cumulative Knowledge Account. In particular, there are three sorts of motivations for that account which don't seem to carry over to the Warrantedness and Approxistemic Accounts.

[29]This account could clearly be developed further in various ways. For example, one might want to weigh improvements across each dimension differently, e.g., such that improvements in the extent to which believed theories are approximately true count for twice as much as improvements in the extent to which they are warranted. Alternatively, one might give lexical priority to improvements across one of the two dimensions, e.g., such that no amount of improvement in the approximate truth of believed theories can outweigh an improvement in the extent to which they are warranted. Another way to develop the account would be to flesh out, or even replace, the notion of *belief* appealed to in the account. In particular, since the Approxistemic Account does not require scientists to have knowledge, the notion of belief (or something belief-like) involved in the account could be weakened to something considerably more modest, such as *endorsement* (Fleisher, 2018), *acceptance* (Cohen, 1992), or a moderate *credence* (Jeffrey, 1970). Since we do not endorse or defend the Approxistemic Account, we leave it to others to develop the account in these, and other, respects, and to motivate their preferred precisification of the account.

The first such motivation stems from broadly held views about the connections between science and knowledge. Beyond straightforward etymological connections between 'science' and 'knowledge' (see Bird, 2022, 18-19), part of what makes the Cumulative Knowledge Account attractive is its ability to capture the commonplace idea that science progresses when scientists come to have more knowledge. Indeed, some proponents of the Cumulative Knowledge Account make much of the fact that scientists themselves often seem to conceive of what they are trying to achieve in terms of gaining knowledge (see, e.g., Bird 2022, 37-38 drawing upon Mizrahi 2013; see also Mizrahi 2021). If progress cannot in fact be directly understood in terms of knowledge, but rather must be understood in terms of some nearby notions such as warrantedness, approximate truth, and/or degrees thereof, these considerations no longer carry much weight in motivating the Epistemic Approach.

A second motivation for the Cumulative Knowledge Account stems from its association with the influential knowledge-first program in epistemology (see, e.g., Bird, 2010, 2024, 2022, viii). Williamson (2000) influentially argues for the view that knowledge is conceptually primitive and explanatorily fundamental, i.e., that the concept of knowledge cannot be analysed in terms of, or otherwise reduced to, other broadly-speaking epistemic notions such as warrant, justification, truth, and belief. A closely related idea is that knowledge—despite or perhaps because it is primitive and fundamental in this way—is central to understanding a wide range of other notions that are (broadly speaking) epistemic, including assertion, action, inquiry, and—importantly for our purposes—*progress* (see, e.g., Logins and Vollet, 2024). Each attempt to understand some notion in terms of knowledge is then taken to be supported, in a holistic way, by the general program's success in accounting for other notions in terms of knowledge. However, putative accounts of progress that merely appeal to related epistemic notions—such as the Warrantedness and Approxistemic Accounts—cannot obviously draw upon the holistic support granted by the general success of the knowledge-first program.

A third, related, motivation for the Cumulative Knowledge Account stems from influential arguments that knowledge is central to the constitutive norms governing various sorts of behaviour, especially *assertion* and *action* (see, e.g., Williamson 1996, 2017).[30] Roughly, the idea is that assertion and action are constitutively governed by norms according to which one should only assert that P, or act on P, if one knows that P. In particular, it is forcefully argued that nothing short of knowledge suffices for these purposes—no degree of warrant, for example, or of approximate truth, suffice by themselves. These norms clearly provide motivation for the Cumulative Knowledge Account since they imply that, according to that account, scientific progress puts scientists in a

[30]To be clear, while these arguments are often put forward by proponents of the knowledge-first program, they can also be endorsed by those otherwise unconvinced that knowledge is conceptually primitive and explanatorily fundamental (Gerken and Petersen, 2018, 687).

position to comply with the norms governing assertion and action as they communicate and act on scientific developments. By contrast, given these norms it would be strange indeed if, as per the Warrantedness and Approxistemic Accounts, there could be scientific progress without scientists accumulating knowledge. After all, the resulting picture would be one which allows for scenarios in which there is much scientific progress, and yet scientists could not legitimately assert or act upon their theories.

In summary, then, at least three powerful considerations in favour of the Cumulative Knowledge Account do not also serve to motivate the Warrantedness and Approxistemic Accounts. As such, while these accounts fare rather better than the Cumulative Knowledge Account when it comes to the Epistemic Mismatch Problem, it is unclear to what extent those who were predisposed towards that account specifically, or indeed the Epistemic Approach more broadly, will have an appetite for these alternative accounts.

5 Conclusion

Taking a step back, where does our consideration of the Epistemic Mismatch Problem leave the Epistemic Approach, and our theorising about scientific progress?

Firstly, note that it does not seem that any of the other prominent accounts of scientific progress mentioned in §1 face analogues of the Epistemic Mismatch Problem. The Verisimilitudinarian Account (Popper, 1963; Niiniluoto, 2014), for example, requires only that the theories that are proposed or accepted in science become more truthlike, with no further requirement that scientists have true and warranted beliefs about their degree of truthlikeness. Likewise, the Noetic Account (Dellsén, 2016, 2021; see also Dellsén, 2022a) requires only that science generates publicly available information which puts people in a position to understand, and explicitly does away with the requirement that scientists must have any particular beliefs in order for the discipline to progress. Finally, the Problem-Solving Account (Kuhn, 1970; Laudan, 1981) requires only that scientific problems are solved or eliminated by the lights of scientists themselves, explicitly denying that these solutions must be true in order for progress to be made. In general terms, no analogue of the mismatch problem arises for these accounts because none requires that, in order for progress to occur, the content of scientists' propositional attitudes—e.g., with respect to a theory—must *exactly match* the content of a fully true proposition.

The Epistemic Mismatch Problem, then, appears to be particular to the Cumulative Knowledge Account. Moreover, it is telling that the versions of the Epistemic Approach that do not suffer from the Epistemic Mismatch Problem appear to be precisely those versions which sever the direct connection between progress and knowledge. As we suggested in §4.3, however, we suspect that those who are drawn to the Epistemic Approach will not be well-disposed towards accounts that allow for scientific progress without the accumulation of knowledge. This leaves us with a question to which we do not pre-

tend to have a definite answer: is there some other way, for those who are motivated to analyse scientific progress in terms of the accumulation of knowledge, to account for the Approximation Platitude while avoiding the Epistemic Mismatch Problem?

Acknowledgements

We are very grateful for insightful comments from two anonymous reviewers for this journal, members of the Philosophical Progress Lab, and the audience at the Nordic Network in Epistemology Workshop II in Lillehammer. Last but not least, we'd like to specifically thank Alexander Bird for graciously providing us with helpful feedback on an earlier draft of this paper. This work was generously supported by the Icelandic Research Fund (Grant 228526–051) and the Australian Research Council (Grant DE230101136).

References

Audi, R. (1994). Dispositional beliefs and dispositions to believe. *Noûs 28*(4), 419–34.

Bird, A. (2007). What is scientific progress? *Noûs 41*(1), 64–89.

Bird, A. (2010). Social knowing: The social sense of 'scientific knowledge'. *Philosophical Perspectives 24*, 23–56.

Bird, A. (2019). The aim of belief and the aim of science. *Theoria: An International Journal for Theory, History and Foundations of Science 34*, 171–193.

Bird, A. (2022). *Knowing Science*. Oxford: Oxford University Press.

Bird, A. (2024, 08). 221Knowledge-First Philosophy of Science. In *Putting Knowledge to Work: New Directions for Knowledge-First Epistemology*. Oxford University Press.

Cevolani, G. and L. Tambolo (2013). Progress as approximation to the truth: A defence of the verisimilitudinarian approach. *Erkenntnis 78*, 921–935.

Cohen, L. J. (1992). *An Essay on Belief and Acceptance*. New York: Clarendon Press.

Decock, L., I. Douven, C. Kelp, and S. Wenmackers (2014). Knowledge and approximate knowledge. *Erkenntnis 79*(6), 1129–1150.

Dellsén, F. (2016). Scientific progress: Knowledge versus understanding. *Studies in History and Philosophy of Science Part A 56*, 72–83.

Dellsén, F. (2018). Scientific progress: Four accounts. *Philosophy Compass 13*(11), e12525.

Dellsén, F. (2021). Understanding scientific progress: The noetic account. *Synthese 199*, 11249–11278.

Dellsén, F. (2022a). Scientific progress: By-whom or for-whom? *Studies in History and Philosophy of Science Part A 97*(C), 20–28.

Dellsén, F. (2022b). Scientific progress without justification. In I. Lawler, K. Khalifa, and E. Shech (Eds.), *Scientific Understanding and Representation: Modeling in the Physical Sciences*. Routledge.

Dellsén, F. and J. Norton (2025). Dejustifying scientific progress. *Philosophy of Science 92*(2), 488–495.

Fleisher, W. (2018). Rational endorsement. *Philosophical Studies 175*(10), 2649–2675.

Gerken, M. and E. N. Petersen (2018). Epistemic norms of assertion and action. In S. C. Goldberg (Ed.), *The Oxford Handbook of Assertion*. Oxford University Press.

Gettier, E. (1963). Is justified true belief knowledge? *Analysis 23*(6), 121–123.

Harman, G. (1986). *Change in View: Principles of Reasoning*. Cambridge, MA, USA: MIT Press.

Jeffrey, R. (1970). Dracula meets wolfman: Acceptance vs. partial belief. In M. Swain (Ed.), *Induction, acceptance, and rational belief*, pp. 157–185. Reidel.

Kragh, H. (2012). *Niels Bohr and the Quantum Atom: The Bohr Model of Atomic Structure 1913-1925*. Oxford: Oxford University Press.

Kuhn, T. (1970). *The Structure of Scientific Revolutions*. Chicago: Chicago University Press.

Laudan, L. (1981). A confutation of convergent realism. *Philosophy of Science 48*(1), 19–49.

Logins, A. and J. H. Vollet (Eds.) (2024). *Putting Knowledge to Work: New Directions for Knowledge-First Epistemology*. Oxford: Oxford University Press.

Mizrahi, M. (2013). What is scientific progress? lessons from scientific practice. *Journal for General Philosophy of Science / Zeitschrift für Allgemeine Wissenschaftstheorie 44*(2), 375–390.

Mizrahi, M. (2021). Conceptions of scientific progress in scientific practice: An empirical study. *Synthese 199*(1-2), 2375–2394.

Niiniluoto, I. (1999). *Critical Scientific Realism*. Oxford: Clarendon Press.

Niiniluoto, I. (2014). Scientific progress as increasing verisimilitude. *Studies in History and Philosophy of Science Part A 46*, 73–77.

Plantinga, A. (1993). *Warrant: The Current Debate*. New York, US: Oxford University Press.

Popper, K. R. (1963). *Conjectures and Refutations: The Growth of Scientific Knowledge*. London: Hutchinson.

Rowbottom, D. P. (2008). N-rays and the semantic view of progress. *Studies in History and Philosophy of Science 39*, 277–278.

Rowbottom, D. P. (2010). What scientific progress is not: Against bird''s epistemic view. *International Studies in the Philosophy of Science 24*(3), 241–255.

Rowbottom, D. P. (2023). *Scientific Progress*. Cambridge University Press.

Schwitzgebel, E. (2023). Belief. In *Stanford Encyclopedia of Philosophy*.

Shan, Y. (Ed.) (2023). *New Perspectives on Scientific Progress*. New York and London: Routledge.

Silva, P. and L. R. G. Oliveira (2024). Propositional justification and doxastic justification. In M. Lasonen-Aarnio and C. Littlejohn (Eds.), *The Routledge Handbook of the Philosophy of Evidence*. Routledge.

Stegenga, J. (2024). Justifying scientific progress. *Philosophy of Science 91*(3), 543–560.

Williamson, T. (1996). Knowing and asserting. *Philosophical Review 105*(4), 489.

Williamson, T. (2000). *Knowledge and its Limits*. New York: Oxford University Press.

Williamson, T. (2017). Acting on knowledge. In J. A. Carter, E. C. Gordon, and B. W. Jarvis (Eds.), *Knowledge First: Approaches in Epistemology and Mind*, pp. 163–181. Oxford University Press.