A New Formal Approach to Two-Dimensional Semantics: Building on Davies and Humberstone's Two-Dimensional Modal Logic

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Abstract: In "Two Conceptions of Necessity", Martin Davies and Lloyd Humberstone construct a two-dimensional modal logic to formalize Gareth Evans' distinction between superficial and deep modalities, thereby addressing Saul Kripke's notions of "contingent a priori propositions" and "necessary a posteriori propositions". However, Davies and Humberstone's two-dimensional modal logic fails to account for the necessity a posteriori of identity statements involving proper names, thus falling short of satisfying the explanatory demands of two-dimensional semantics. To overcome these limitations, this paper proposes a new formalization approach for two-dimensional semantics: replacing the doubly-indexed mechanism of possible worlds with variable semantic models, transforming the vertical axis in the 2D-matrix from a designated "actual world" to specific semantic models corresponding to distinct worlds—termed "world-models". Each possible world corresponds to a world-model that describes it, with the primary difference between world-models lying in the interpretation function's distinct valuations to individual constants. This formal framework not only more appropriately handles Kripkean identity statements involving proper names but also aligns more closely with David Chalmers' epistemic interpretation of two-dimensional semantics.

Keywords: two-dimensional semantics; two-dimensional modal logic; world-model; Martin Davies; Lloyd Humberstone

1. Introduction

In *Naming and Necessity*, Saul Kripke argues that linguistic expressions such as indexicals, demonstratives, proper names, and natural kind terms are rigid designators, referring to the same individual across all possible worlds. Statements containing rigid designators may lead to a separation between the cognitive dimension of meaning (i.e., a priori/a posteriori) and the modal dimension (i.e., necessity/contingency). For example (assuming the rigid designators in the following statements have non-empty extensions in all possible worlds):

(1) The Phosphorus is the Hesperus.
(2) The Phosphorus is the celestial body that shines brightly in the eastern sky at dawn.

Statement (1) is a "necessary a posteriori statemen". On the one hand, it expresses a necessary proposition about the self-identity of an individual (Venus); on the other hand, it's an a posteriori fact requiring astronomical observation. Statement (2) is a "contingent a priori statement". While it describes a contingent fact (Venus shining brightly in the eastern sky at dawn), the statement is a priori because it is derived purely from conceptual analysis of the term "Phosphorus".

The common approach to understanding the separation between cognitive and modal dimensions is to appeal to two-dimensional semantics. The term "dimension" here refers to the number of parameters relative to which an expression is assigned an extension[1]. Standard Kripkean semantics relativizes expressions to a single parameter (possible worlds), while two-dimensional semantics introduces a second parameter, forming a distinct axis alongside possible worlds. Depending on the nature of this additional parameter, different approaches to two-dimensional semantics emerge. For instance, Kaplan's "linguistic-contextual approach" treats context as the second parameter, while Chalmers' "epistemic approach" employs epistemic possibilities as the second parameter. Regardless of the approach, we can construct a two-dimensional matrix (2D matrix) following Stalnaker's framework:

Expression	w_0	w_1	w_2
i			
j			
k			

Table 1

In this matrix, the horizontal axis (w_0, w_1, w_2) represents different possible worlds, while the vertical axis (i, j, k) represents distinct second-dimensional parameters. Each coordinate (left blank in Table 1) denotes the extension of an expression relative to that coordinate. Typically, there is a one-to-one correspondence between horizontal and vertical coordinates (e.g., w_0 corresponds to i, w_1 to j), which may reflect perspectives where the former serves as the actual world or contexts incorporating the former. This correspondence depends on the specific two-dimensional approach. The matrix concretizes the separation between cognition and modality:

Necessity/Contingency: If a statement is true across all worlds in a row (blue region in Table 1) under a fixed vertical parameter (e.g., i), it is deemed necessary; if only partially true, contingent.

A Priority/A Posteriority: If a statement is true across all rows and their corresponding worlds (yellow region in Table 1) under varying vertical parameters, it is deemed a priori; if only partially true, a posteriori.

Following Chalmers' neutral terminology, the extensions under varying vertical parameters are called the 1-intension (yellow region), while those under fixed vertical parameters are called the 2-intension (blue region)[2].

[1] Schroeter (2021: §1.1.1).
[2] Chalmers (2004: 160).

Not all approaches acknowledge the explanatory power of two-dimensional semantics regarding cognition and modality. Kaplan argues that two-dimensional matrices are only applicable to expressions containing indexicals ("I", "here", "now", etc.), while Stalnaker maintains that such matrices are merely tools for analyzing "assertions" in discourse and cannot be used to analyze *a priority*[1]. Chalmers, however, offers the most prominent defense of two-dimensional semantics' explanatory efficacy. In his 2004 paper "Epistemic Two-Dimensional Semantics", he argues that a two-dimensional matrix defined with appropriate epistemic terminology can accurately capture the cognitive and modal dimensions of meaning. To evaluate whether a two-dimensional approach meets the explanatory demands, Chalmers proposes the following Core Thesis:

> **Core Thesis:** For any statement S, S is *a priori* if and only if S has a necessarily true 1-intension.[2]

In short, an approach satisfies the Core Thesis if it aligns with Chalmers' demand for explanatory power in two-dimensional semantics. Although the Core Thesis is initially limited to statements, it can be extended to other linguistic expressions (e.g., proper names). Additionally, the Core Thesis imposes requirements only on the 1-intension (responsible for the cognitive dimension of meaning), as the 2-intension (responsible for the modal dimension) is already treated by standard Kripkean semantics in any approach.

This paper aims to propose a new two-dimensional semantic framework that satisfies the Core Thesis. Unlike Chalmers' epistemic approach, however, we adopt a formal approach based on quantified modal logic. The origins of this approach trace back to Martin Davies and Lloyd Humberstone's 1980 work "Two Notions of Necessity", in which the authors formalize two-dimensional semantics by constructing a two-dimensional modal logic with the "actually" operator **@** and the "fixedly" operator **F**[3]. However, their formal framework fails to satisfy the Core Thesis because its semantics cannot handle the a posteriori nature of identity statements involving proper names, such as (1). This paper begins by reconstructing Davies and Humberstone's original framework, then proposes a new formal approach to overcome its limitations, and finally demonstrates how this approach aligns with the goals of Chalmers' epistemic framework.

2. Davies and Humberstone's Two-Dimensional Modal Logic

The work of Davies and Humberstone builds on Gareth Evans' 1979 paper "Reference and Contingency", which introduces the concept of "descriptive names" and distinguishes between "superficial modality" and "deep modality". According to Evans, a descriptive name is a referential expression whose reference is fixed by a definite description. It functions like a proper name in denoting an object

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[1] See Kaplan (1989) & Stalnaker (2001).
[2] Chalmers (2004: 165).
[3] In Davies & Humberstone (1980), the authors used the symbols **A** and $\mathfrak{F}$, which I have updated in this paper to the more contemporary @(actually) and F(fixedly).

but retains the cognitive content of a definite description[1]. For example, "Phosphorus" in statement (2) can be viewed as a descriptive name fixed by the definite description "the celestial body shines brightly in the eastern sky at dawn" (though Kripke would argue that "Phosphorus" is a proper name). Necessary a posteriori propositions and contingent a priori propositions only involve statements containing descriptive names, where necessity/contingency corresponds to superficial modality, and a priority/a posteriority corresponds to deep modality. Superficial modality aligns with the standard Kripkean semantics, while deep modality concerns the realizability of possible worlds themselves. As Evans originally states:

> If a deeply contingent statement is true, there will be some state of affairs such that we can assert both that the statement would not have been true if that state of affairs had not obtained, and that that state of affairs might not have obtained.[2]

It is evident that superficial and deep modalities correspond to the 2-intension and 1-intension in the 2D-matrix, respectively. Inspired by Evans, Davies and Humberstone interpret the vertical axis of the 2D-matrix as a designated actual world, which belongs to the same set W of possible worlds as the horizontal axis. To lay the groundwork for subsequent technical discussions, we first define the semantic model for a simple quantified modal logic (QML)[3].

Definition 1 (QML Model): A QML model is a triple $\langle W, D, I \rangle$, where:

1. W is a non-empty set; ("possible worlds")
2. D is a non-empty set; ("domain")
3. I is a function such that: ("interpretation function")
   (a) If α is a constant then $I(\alpha) \in D$;
   (b) If Π^n is an n-place predicate then $I(\Pi^n)$ is a set of n+1-tuples $\langle u_1, \ldots, u_n, w \rangle$, where $u_1, \ldots, u_n$ are members of D, and $w \in W$.

Definition 2 (Assignment Function): g is an assignment function for the model $\langle W, D, I \rangle$ iff g is a function that assigns each variable to an object in D.

Notations:

g_u^α is extensionally identical to assignment function g, except that $g_u^\alpha(\alpha) = u$;

$$[\alpha]_{M,g} = \begin{cases} I(\alpha) \text{ if } \alpha \text{ is a constant.} \\ g(\alpha) \text{ if } \alpha \text{ is a variable.} \end{cases}$$

This model follows metaphysical conventions by adopting the S5 frame of modal logic, disregarding accessibility relations between worlds, and assuming that all possible worlds share the same domain of individuals. Given a QML model, the next task is to define truth conditions for formulas. Unlike standard Kripkean semantics, Davies and Humberstone's two-dimensional modal logic employs a doubly-indexed mechanism of possible worlds, where each formula is evaluated relative to two

[1] Davies & Humberstone (1980: 7).
[2] Evans (1979: 185).
[3] All definitions presented in this section have been modernized and refined, differing from Davies and Humberstone's original formulations. For further details, see Sider (2010).

possible world parameters[1]. We stipulate $\rightarrow, \sim, \forall, \Box$ as primitive operators, while $\wedge, \vee, \exists, \Diamond$ can be derived from the primitives in the usual manner. The truth conditions for two-dimensional modal logic is defined as follows:

Definition 3 (Doubly-Indexed Truth Conditions): Let $M = \langle W, D, I \rangle$ and g be a QML model and assignment function respectively.

1. For any terms α and β, $M, v, w \Vdash_g \alpha = \beta \Leftrightarrow [\alpha]_{M,\mathrm{g}} = [\beta]_{M,\mathrm{g}}$;

2. For any n-place predicate, Π, and any terms $\alpha_1, \ldots, \alpha_n$,

$$M, v, w \Vdash_g \Pi\alpha_1, \ldots, \alpha_n \Leftrightarrow \langle [\alpha_1]_{M,\mathrm{g}}, \ldots, [\alpha_n]_{M,\mathrm{g}}, w \rangle \in I(\Pi);$$

3. For any wff ϕ and ψ, and variable, α,

$$M, v, w \Vdash_g \sim\phi \Leftrightarrow M, v, w \nVdash_g \phi,$$

$$M, v, w \Vdash_g \phi \rightarrow \psi \Leftrightarrow M, v, w \nVdash_g \phi \text{ or } M, v, w \Vdash_g \psi,$$

$$M, v, w \Vdash_g \forall\alpha\, \phi \Leftrightarrow \text{for each } u \in D, M, v, w \Vdash_{g_u^\alpha} \phi,$$

$$M, v, w \Vdash_g \Box\phi \Leftrightarrow \text{for each } w' \in W, M, v, w' \Vdash_g \phi.$$

It is evident that the doubly-indexed mechanism of possible worlds does not uniquely affect the standard logical operators in Definition 3. Davies and Humberstone introduced two new modal operators into the formal language: @ (actually) and **F** (fixedly). It is precisely these operators and their combinations that leverage the doubly-indexed mechanism:

Definition 3.3+:

$$M, v, w \Vdash_g @\phi \Leftrightarrow M, v, v \Vdash_g \phi,$$

$$M, v, w \Vdash_g \mathbf{F}\phi \Leftrightarrow \text{for each } v' \in W, M, v', w \Vdash_g \phi,$$

$$M, v, w \Vdash_g \mathbf{F}@\phi \Leftrightarrow \text{for each } v' \in W, M, v', v' \Vdash_g \phi.$$

Intuitively, the first parameter v in the doubly-indexed mechanism represents the designated actual world, while the second parameter w is the ordinary possible world. For any formula ϕ, $M, v, w \Vdash_g \phi$ can be interpreted as: "When v is the actual world, ϕ is true in w". This allows Davies and Humberstone to formally define Evans' distinction between superficial modality and deep modality:

[1] Prior to formally adopting the doubly-indexed mechanism, Davies and Humberstone employed variable designated models $\langle \mathrm{W}, \mathrm{w}^*, \mathrm{V} \rangle$—standard Kripke models augmented with a designated actual world w^*. The difference between designated models lies in the choice of w^*. Although variable designated models achieve the same effect as the doubly-indexed mechanism, Davies and Humberstone ultimately adopted the latter as their primary formalism for its technical convenience. For details, see Davies & Humberstone (1980: 4).

Superficial Modality:

1. ϕ is superficially necessary in M at $w \Leftrightarrow M, v, w \Vdash_g \Box\phi$ for all g.

2. ϕ is superficially contingent in M at $w \Leftrightarrow M, v, w \nVdash_g \Box\phi \wedge \Box{\sim}\phi$ for all g.

Deep Modality:

1. ϕ is deeply necessary in M at $w \Leftrightarrow M, w, w \Vdash_g \mathbf{F}@\phi$ for all g.

2. ϕ is deeply contingent in M at $w \Leftrightarrow M, v, w \nVdash_g \mathbf{F}@\phi \wedge \mathbf{F}@{\sim}\phi$ for all g.

Furthermore, descriptive names can be reformulated using the @ operator as "the actual G"[1]. For example, "Phosphorus" can be rewritten as "the celestial body that actually shines brightly in the eastern sky at dawn". Consequently, statements (1) and (2) can be reformulated as:

(1') The celestial body that actually shines brightly in the eastern sky at dawn is the celestial body that actually shines brightly in the western sky at night.
(2') The celestial body that actually shines brightly in the eastern sky at dawn is the celestial body that shines brightly in the eastern sky at dawn.

Following Russell's theory of descriptions, these statements can be further formalized as:

(1*) $\exists \mathrm{x}(@G\mathrm{x} \wedge \forall \mathrm{y}(@G\mathrm{y} \rightarrow \mathrm{x} = \mathrm{y}) \wedge \exists \mathrm{z}(@H\mathrm{z} \wedge \forall \mathrm{y}(@H\mathrm{y} \rightarrow \mathrm{z} = \mathrm{y}) \wedge \mathrm{z} = \mathrm{x}))$

(2*) $\exists \mathrm{x}(@G\mathrm{x} \wedge \forall \mathrm{y}(@G\mathrm{y} \rightarrow \mathrm{x} = \mathrm{y}) \wedge G\mathrm{x})$

Here, predicate G denotes "shining brightly in the eastern sky at dawn", and predicate H denotes "shining brightly in the western sky at night". Consider a model M where the set of worlds W contains only two possible worlds: w_0 (the actual world) and w_1 (a world where Mars shines brightly in the eastern sky at dawn, with other details resembling the actual world). Using the formal semantics above, we can construct the following 2D matrices:

(1*)	w_0	w_1
w_0	1	1
w_1	0	0

Table 2.1

(2*)	w_0	w_1
w_0	1	0
w_1	0	1

Table 2.2

To interpret these matrices: The value of formula ϕ at coordinate (w_i, w_j) is 1 iff $M, v, w \Vdash_g \phi$, where g is an arbitrary assignment. These matrices appear to satisfy

[1] Davies & Humberstone (1980: 11).

the explanatory requirements of two-dimensional semantics. According to Tables 2.1 and 2.2, the 1-intension of the necessary a posteriori proposition (1*) is contingently true, while that of the contingent a priori proposition (2*) is necessarily true. This suggests that the framework partially satisfies the Core Thesis. However, to fully satisfy the Core Thesis, the formalization must meet two assumptions: First, necessary a posteriori and contingent a priori propositions only involve statements containing descriptive names. Second, all descriptive names can be reformulated using the @ operator as "the actual G". If either assumption fails, the "right-to-left" direction of the Core Thesis collapses.

Davies and Humberstone concede that at least the first assumption is untenable. Most of Kripke's necessary a posteriori propositions are identity statements involving proper names, but not all proper names can be treated as descriptive names[1]. For instance, if "Phosphorus" is strictly interpreted as a proper name (à la Kripke) rather than a descriptive name, statement (1) cannot be formalized as (1*), and its 1-intension cannot be represented by Table 2.1. Even if "Phosphorus" is a descriptive name, numerous proper names in ordinary language (e.g., "Davies" or "Humberstone") resist such treatment. For necessary a posteriori identity statements like "Davies is Humberstone", two-dimensional modal logic can only formalize them as $a = b$, where a and b are individual constants. Since such propositions contain no @ or $\mathbf{F}$ operators, the doubly-indexed mechanism becomes inert, and their 1-intensions are trivially necessarily true[2]. Consequently, "Davies is Humberstone" under this framework constitutes a counterexample to the Core Thesis: it is a posteriori yet has a necessarily true 1-intension.

3. The World-Model Approach to Two-Dimensional Semantics

Given the limitations of two-dimensional modal logic in handling identity statements involving proper names, this paper proposes a more explanatorily robust formal framework, termed the world-model approach. While rooted in quantified modal logic, this approach replaces the doubly-indexed mechanism of possible worlds with variable semantic models, transforming the vertical axis of the 2D-matrix from designated actual worlds to specific semantic models corresponding to distinct worlds—referred to as world-models. This section is divided into two subsections: the first introduces the technical details of the approach, and the second explores its philosophical implications and compliance with the Core Thesis.

3.1 Technical Content of the World-Model Approach

The world-model approach retains the QML model and assignment function from Definitions 1 and 2 but adopts a revised truth condition. Specifically, it employs a traditional single-indexed mechanism, effectively eliminating the parameter v from Definition 3 and excluding the @ and $\mathbf{F}$ operators:

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[1] Davies＆Humberstone (1980: 11-12).

[2] Sider (2010: 333).

Definition 3* (Single-Indexed Truth Condition): Let $M = \langle W, D, I \rangle$ and g be a QML model and assignment function respectively.

1. For any terms α and β, $M, w \vDash_g \alpha = \beta \Leftrightarrow [\alpha]_{M,\mathrm{g}} = [\beta]_{M,\mathrm{g}}$;

2. For any *n*-place predicate, Π, and any terms $\alpha_1, \ldots, \alpha_n$,

$$M, w \vDash_g \Pi\alpha_1, \ldots, \alpha_n \Leftrightarrow \langle [\alpha_1]_{M,\mathrm{g}}, \ldots, [\alpha_n]_{M,\mathrm{g}}, w \rangle \in I(\Pi);$$

3. For any wff ϕ and ψ, and variable, α,

$$M, w \vDash_g \sim\phi \Leftrightarrow M, w \nvDash_g \phi,$$

$$M, w \vDash_g \phi \to \psi \Leftrightarrow M, w \nvDash_g \phi \text{ or } M, w \vDash_g \psi,$$

$$M, w \vDash_g \forall\alpha\, \phi \Leftrightarrow \text{for each } u \in D, M, w \vDash_{g^{\alpha}_{u}} \phi,$$

$$M, w \vDash_g \Box\phi \Leftrightarrow \text{for each } w' \in W, M, w' \vDash_g \phi.$$

By eliminating the double-indexing, the vertical axis of the 2D-matrix no longer represents designated actual worlds but instead world-models. Each possible world w_n corresponds to a unique world-model M_n, which is a QML model describing w_n. We define an accessibility relation between world-models:

Definition 4 (Inter-Model Accessibility Relation):

$$M_i \approx M_j \Leftrightarrow W_i = W_j \text{ and } D_i = D_j, \text{ where } W_i, D_i \in M_i \text{ and } W_j, D_j \in M_j.$$

The relation $\approx$ is an equivalence relation, partitioning models into equivalence classes. Members of the same class differ only in the interpretation function I_n's valuation to individual constants. While world-models in the same 2D-matrix are typically accessible to each other, we allow the inclusion of inaccessible models (i.e., models with distinct domains). The next subsection will demonstrate how this flexibility aids in satisfying the Core Thesis.

The world-model approach does not require proper names to be paraphrased as @-operator-based descriptive names. Instead, they are treated as individual constants, compatible with Kripkean rigid designators. Analytic propositions like (2) need not be formalized as complex formulas like (2*) but can be expressed directly as atomic formulas $\Pi\alpha_1, \ldots, \alpha_n$ (augmented with tools like λ-calculus if necessary). Under this framework, (1) and (2) are formalized as:

(1**) $\mathrm{a} = \mathrm{b}$

(2**) $G\mathrm{a}$

Recalling the two worlds w_0 and w_1 from Section 2, we define their corresponding world-models M_0 and M_1:

$$M_0 \begin{cases} W_0 = \{w_0, w_1, \ldots\} \\ D_0 = \{\text{Venus}, \text{Mars}, \ldots\} \\ I_0(\mathrm{a}) = I_0(\mathrm{b}) = \text{Venus} \\ I_0(G) = \{\langle \text{Venus}, \mathrm{w}_0\rangle, \langle \text{Mars}, \mathrm{w}_1\rangle, \ldots\} \end{cases} \quad M_1 \begin{cases} W_1 = \{w_0, w_1, \ldots\} \\ D_1 = \{\text{Venus}, \text{Mars}, \ldots\} \\ I_1(\mathrm{a}) = \text{Mars}, I_1(\mathrm{b}) = \text{Venus} \\ I_1(G) = \{\langle \text{Venus}, \mathrm{w}_0\rangle, \langle \text{Mars}, \mathrm{w}_1\rangle, \ldots\} \end{cases}$$

Based on models $M_0 \approx M_1$, we construct the following 2D matrices:

$\mathrm{a} = \mathrm{b}$	w_0	w_1
M_0	1	1
M_1	0	0

Table 3.1

$G\mathrm{a}$	w_0	w_1
M_0	1	0
M_1	0	1

Table 3.2

The value of formula ϕ at coordinate (w_i, M_j) is 1 iff $M_j, w_i \vDash_g \phi$, where g is an arbitrary assignment. Tables 3.1 and 3.2 demonstrate that the world-model approach surpasses two-dimensional modal logic in satisfying the Core Thesis. It accurately captures the necessary a posteriori nature of (1) and the contingent a priori nature of (2) while remaining faithful to Kripke's original intent—no ad hoc treatment of proper names is required.

The only caveat is that this approach may presuppose the following assumption: For every proper name α, there exists a unique unary predicate Π (potentially generated by the λ-calculus) such that the 1-intension of $\Pi\alpha$ is necessarily true. Intuitively, Π captures α's cognitive content or quasi-Fregean sense—what we may term α's "sense predicate". This assumption aligns with descriptivist theories of meaning, wherein the meaning of α is determined by a lengthy descriptive cluster, and Π is the complex predicate derived from this cluster via λ-calculus.

Another advantage of the world-model approach over two-dimensional modal logic lies in its ability to extend the applicability of 2D-matrices from sentences to other types of linguistic expressions, such as proper names. While two-dimensional modal logic must rely on Russell's theory of descriptions to formalize statements containing descriptive names—thereby failing to directly represent the reference of descriptive names across matrix coordinates without additional technical apparatus — the world-model approach avoids this limitation. Taking the individual constant a (denoting "Phosphorus") as an example, we can effortlessly construct the following 2D-matrix:

a	w_0	w_1
M_0	Venus	Venus
M_1	Mars	Mars

Table 4

The value of individual constant a at coordinate (w_i, M_j) corresponds to $I_j(\mathrm{a})$, where $I_j \in M_j$. As shown in Table 4, "Phosphorus" has a contingent 1-intension and a necessary 2-intension, making it a strict designator that is *a posteriori*[1]. In contrast, it is difficult to construct a similar 2D-matrix for descriptive names within two-dimensional modal logic, since descriptive names cannot be directly formalized—only statements containing them can. This advantage is not entirely trivial. Taking Table 4 as an example, because aa has a contingent 1-intension, we can immediately infer: For any unary predicate Π and term β, if Π is not a's sense predicate, then neither Πa nor a $= \beta$ has a necessarily true 1-intension. Such inferences significantly streamline the evaluation of 1-intensions for statements involving rigid designators—a feat achievable only by directly constructing accurate 2D-matrices for individual constants.

3.2 Philosophical Implications of the World-Model Approach

Having clarified the technical details of the world-model approach, we now re-examine it from a more philosophical perspective. This subsection focuses on two questions: First, does the world-model approach align with Kripke's original understanding of necessary a posteriori propositions and contingent a priori propositions? Second, does the world-model approach satisfy Chalmers' Core Thesis?

We begin with Kripke's analysis of necessary a posteriori and contingent a priori propositions. Kripke argues that while a necessary a posteriori proposition is metaphysically necessary, it exhibits an "apparent contingency". For example, consider the necessary a posteriori proposition "This table is made of wood". We might entertain the thought that "This table could have been made of ice". However, this is impossible because (at least according to Kripke) being made of wood is an essential property that determines the table's transworld identity. Kripke explains this as follows:

> Any necessary truth, whether a priori or a posteriori, could not have turned out otherwise. In the case of some necessary a posteriori truths, however, we can say that under appropriate qualitatively identical evidential situations, an appropriate corresponding qualitative statement might have been false.[2]

In the case of the aforementioned table, when we consider the apparent contingency expressed in the statement "this table could have been made of ice", Kripke argues that what we are actually considering is a scenario where there exists a table that is qualitatively identical to the given table, but which is indeed made of ice. This naturally raises the question: how should we determine the context of qualitative identity? Kripke seems to imply that this can be delineated through non-rigid designators that fix the reference of rigid designators, a point particularly evident in cases of identity statements involving proper names:

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[1] Here analogously to a posteriori statements, meaning its reference is sensitive to model variations.

[2] Kripke (1980: 142).

Let 'R_1' and 'R_2' be the two rigid designators which flank the identity sign. Then '$R_1 = R_2$' is necessary if true. The references of 'R_1' and 'R_2', respectively, may well be fixed by non-rigid designators 'D_1' and 'D_2', in the Hesperus-Phosphorus case these have the form 'the heavenly body in such-and-such position in the sky in the evening (morning)'. Then although '$R_1 = R_2$' is necessary, '$D_1 = D_2$' may well be contingent, and this is often what leads to the erroneous view that '$R_1 = R_2$' might have turned out otherwise.[1]

In short, the illusion of contingency arises from conflating a necessary proposition with a corresponding qualitative statement that differs from it and could be false. A similar logic applies to contingent a priori propositions: If the reference of a rigid designator R is fixed by a non-rigid designator D, one can know a priori (assuming R exists) that propositions like "R is D" are true. Statement (2) is a paradigmatic example—though (2) expresses a contingent proposition, there exists a "corresponding qualitative statement" that expresses a necessary proposition.

Is the world-model approach consistent with Kripke's ideas? The answer is affirmative, as Kripke's original insights can be equivalently expressed through the technical tools of the world-model approach. Crucially, Kripke's non-rigid designators (used to fix the reference of rigid designators) correspond to sense predicates in this framework. Just as every necessary a posteriori proposition $R_1=R_2$ has a corresponding contingent qualitative statement $D_1=D_2$, and every contingent a priori proposition "R is D" has a corresponding necessary qualitative statement, we can use sense predicates to construct such qualitative counterparts for all necessary a posteriori and contingent a priori propositions. For instance, we formalize the counterparts of (1**)and (2**) as follows:

(1#) $\exists x(Gx \wedge Hx)$

(2#) $\exists x(Gx \leftrightarrow Gx)$

Here, G and H are sense predicates for a and b, respectively, ensuring Ga and Hb have necessarily true 1-intensions. While (1**) and (2**) are necessary and contingent, their counterparts (1#) and (2#) invert these modal profiles, as shown in Tables 5.1 and 5.2:

(1#)	w_0	w_1
M_0	1	0
M_1	1	0

Table 5.1

(2#)	w_0	w_1
M_0	1	1
M_1	1	1

Table 5.2

Next, we examine whether the world-model approach satisfies Chalmers' Core Thesis. To do this, we must first analyze Chalmers' critique of other two-dimensional semantic approaches. According to him, beyond formal approaches like two-dimensional modal logic, two-dimensional semantics can be broadly divided into

[1] Kripke (1980: 143-144).

two categories: contextual approaches and epistemic approaches[1]. Contextual approaches are further subdivided into: Orthographic contextualism (e.g., Stalnaker's "diagonal propositions"), Linguistic contextualism (e.g., Kaplan's "character"), Semantic contextualism, Cognitive contextualism, etc. All contextual approaches interpret the vertical axis of the 2D-matrix as a generalized context, differing in how they define the type to which an expression token belongs. For example: Two tokens belong to the same orthographic type if they share identical spelling. Two tokens belong to the same linguistic type if they are instances of the same expression in a language. However, no contextual approach satisfies the Core Thesis, as they presuppose the existence of expression tokens (regardless of their type)[2]. Specifically, in any contextual framework, a posteriori propositions like "A sentence token exists", "A language exists", or "I am uttering now" have trivially necessary 1-intensions, thereby violating the "right to left" direction of the Core Thesis. The root issue is that the necessity of 1-intensions in contextual approaches merely reflects "truth whenever uttered," which does not equate to a priority[3]. Chalmers argues that only by defining the 2D-matrix through primitive epistemological notions can 1-intensions accurately capture a priority—hence the need for an epistemological approach.

Chalmers' epistemological approach is rigorously defined, but we summarize its key ideas here. This approach interprets the vertical axis as distinct epistemic possibilities or "scenarios". Each scenario corresponds to a canonical description—a complete specification using neutral qualitative terms and indexicals anchored to the scenario's center. A scenario W verifies a statement S iff $D \rightarrow S$ is epistemically necessary, where D is W's canonical description. A claim is epistemically possible iff it is not ruled out a priori. Chalmers enshrines this in the Plentitude Principle:

> **Plentitude Principle:** For any statement S, S is epistemically possible if and only if there exists a scenario that verifies S.[4]

The relationship between scenarios and epistemic possibilities parallels that between possible worlds and metaphysical possibilities: Just as every metaphysical possibility corresponds to a possible world, the Plentitude Principle guarantees sufficient scenarios to verify every epistemic possibility. If epistemic necessity is equated with a priority, the Plentitude Principle becomes equivalent to the Core Thesis[5].

The central question now becomes: Can the world-model approach avoid the aforementioned shortcomings of contextual approaches while satisfying the epistemic approach's requirement of the Plentitude Principle? For one thing, the world-model approach is not equivalent to any specific variant of the contextual approaches, as it does not require the actual existence of expression tokens characterized by the 2D matrix. Thus, it can entirely circumvent the deficiencies inherent to contextualism. In fact, Chalmers himself recognizes that formal frameworks akin to two-dimensional

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[1] Chalmers (2004: 166).
[2] Chalmers (2004: 174).
[3] Chalmers (2004: 174).
[4] Chalmers (2004: 184).
[5] Chalmers (2004: 184).

modal logic cannot be neatly categorized under either contextual or epistemic approach, but instead constitute a unique "third path": "It is clearly not a contextual approach: sentence tokens present in counterfactual worlds play no special role here. And it seems not to be an epistemic approach: epistemic notions play no role in defining the key concepts"[1]. Like two-dimensional modal logic, the world-model approach is a formal framework capable of overcoming the limitations of the former. Chalmers himself acknowledges that one could construct a new formal approach distinct from two-dimensional modal logic, potentially bringing it closer to the epistemic approach:

> One might hold that utterances of indexicals and ordinary proper names involve the rigidification of some sort of Fregean content, even if these expressions are not equivalent in logical form to corresponding A-involving descriptions (i.e., definite descriptions expressed using the **F** and @ operators). If so, one could use this behavior todefine a broader sort of two-dimensional evaluation of sentences that does not turn entirely on the presence of **F** and operators. If one generalized Davies and Humberstone's framework in this way, the resulting framework would more closely resemble the epistemic framework that I have outlined.[2]

The world-model approach proposed in this paper can be seen as a partial realization of the ideas outlined in the above quotations. However, unlike an extension of Davies and Humberstone's framework, it independently introduces a novel formalism that completely replaces the doubly-indexed mechanism of possible worlds with variable semantic models.

For another, the world-model approach satisfies the Plentitude Principle. According to Chalmers, compliance with the Plentitude Principle hinges on how scenarios are defined. He proposes two methods for defining scenarios[3]: First, treating scenarios as centered possible worlds—possible worlds augmented with indexical information (e.g., a designated "here" and "now"). Second, treating scenarios as equivalence classes of epistemically complete sentences in an ideal language L, where L contains infinitely many sentences and terms expressing all possible concepts, with its expressions being epistemically invariant. A sentence D in L is epistemically complete if and only if D is epistemically possible, and there exists no sentence S such that both $\mathrm{D} \wedge \mathrm{S}$ and $\mathrm{D} \wedge \sim\mathrm{S}$ are epistemically possible. Chalmers argues that the second definition uncontroversially satisfies the Plentitude Principle because it is fully grounded in primitive epistemic terms. If we accept that every centered possible world corresponds to an epistemically complete sentence, the first definition can also satisfy the principle.

For the world-model approach, scenarios are world-models—specifically, the QML models defined in Definition 1. The approach's compliance with the Plentitude Principle manifests in three key aspects: (1) World-models formalize "canonical descriptions" through the model theory of quantified modal logic: The domain D and interpretation function I correspond to neutral qualitative terms. Distinct I_n to

[1] Chalmers (2006: 126).
[2] Chalmers (2006: 127), the content in parentheses was added by the translator.
[3] See Chalmers (2004: 185-190).

individual constants across world-models correspond to indexicals anchored to the center w_n. (2) The formal language of quantified modal logic meets the requirements of the ideal language L: It contains infinitely many well-formed formulas and individual constants. For each world-model, we can identify a set of formulas true in its corresponding world; their conjunction constitutes an epistemically complete sentence. (3) The world-model approach permits inaccessible models (models with distinct domains) to coexist in the same 2D-matrix, ensuring that every scenario can serve as a vertical coordinate of the matrix—provided the claim it represents is not ruled out a priori.

Returning to the satisfaction of the Core Thesis, we have demonstrated that the world-model approach not only avoids the deficiencies of two-dimensional modal logic and contextual approaches in satisfying the Core Thesis, but also preserves the advantages of the epistemological approach to the greatest extent. These two points jointly establish that the world-model approach satisfies the Core Thesis.

4. Conclusion

By replacing the two-dimensional modal logic's doubly-indexed mechanism with a variable semantic model, this paper proposes a new formal approach to two-dimensional semantics: the world-model approach. This framework offers technical advantages in handling the a posteriori necessity of identity statements involving proper names, while philosophically aligning with Kripke's and Chalmers' conceptions of cognition and modality.

Notably, although the present understanding of world models is grounded entirely in the model theory of quantified modal logic, this does not preclude the use of alternative semantic models distinct from QML as world-models. The philosophical function of world-models—describing their corresponding possible worlds—can be achieved through diverse technical methods, with QML models being merely one option. The paper's core contribution lies in offering a reinterpretation of the two-dimensional matrix: its vertical axis should be understood as a world model rather than traditional contexts, centered possible worlds, or epistemically complete sentences, while remaining compatible with varying technical interpretations of world-models. Constructing a semantic model with greater expressive power than QML models would constitute a valuable extension of the world-model approach.

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