From Mollusk to Swarms of Observers:

A Fully General, Observer-Based Operational Framework for General Relativity

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Abstract

A physically realizable, philosophically grounded, and pedagogically motivated construction of reference frames in general relativity is presented—one that fully captures the coordinate freedoms inherent in general covariance. Inspired by Einstein's original 'reference mollusk'—a physically intuitive but limited operationalization of a deformable coordinate system—this work presents a covariant, observer-based generalization that removes structural constraints such as spatial foliations. In this framework, a swarm of arbitrarily moving local observers assigns coordinates to nearby spacetime events using four independently and arbitrarily running clocks per observer. Freely programmable and unconstrained by synchronization or alignment, these clock readings can reproduce any mathematically admissible coordinate chart. At the same time, their evolving readings serve as operational labels for events, providing a fully local and physical basis for spacetime coordinate assignment. The resulting construction yields a smooth and physically transparent generalization of reference frames from special relativity to curved spacetimes—retaining their operational character without compromising general covariance. It further provides an intuitive bridge to standard practices in cosmology and numerical relativity, where coordinate freedom appears as gauge choice. Beyond offering a concrete realization of operationalism in general relativity, this framework also serves as a pedagogical model—bridging conceptual foundations with intuitive accessibility.

I. INTRODUCTION

A central insight of Einstein's general theory of relativity (GR) is that only events — physical coincidences, such as the intersection of worldlines — are directly observable. Einstein's 1913 "hole argument," and his subsequent resolution of it, made clear that general covariance demands a reassessment of the physical significance of coordinate systems. The essential conclusion is that it is spacetime coincidences—not the coordinate values themselves—that carry direct empirical meaning[1]. This reflects a view also emphasized by Norton, who argued that coordinate values, while devoid of intrinsic physical meaning, can acquire empirical relevance when tied to observable coincidences or invariant structures[2].

This insight has occasionally been misconstrued as suggesting that assigning operational meaning to coordinate values constitutes a conceptual mistake. Yet general covariance fully allows for the use of arbitrary coordinate systems, including those with physical implementations—so long as their interpretation rests on clear measurement procedures involving observation events, such as the (near) coincidence of a local observer with the event being recorded. Bridging the gap between mathematical coordinate freedom and operational implementability remains not only a longstanding pedagogical challenge but a philosophical one, requiring us to reconcile the abstract freedom of general covariance with the empirical role of measurement. The framework presented here addresses both concerns: it provides an explicit realization of coordinate systems as operational constructs and thereby reinforces the philosophical claim that coordinate values, when tied to observable events, need not be dismissed as physically meaningless.

Einstein himself recognized the need for operational grounding in the use of coordinate systems. In his 1917 popular exposition, he introduced a vivid metaphor: the "reference mollusk"—a deformable, mobile reference body intended to represent a general, non-inertial frame in curved spacetime[4]. This image was not conceived as a mathematical abstraction but as a physical construct—a malleable body populated by observers equipped with clocks, capable of assigning coordinates through local measurements. Despite its conceptual richness, the metaphor gained little traction in physics education and found only modest resonance beyond the discipline[5].

While Einstein's mollusk offers a powerful pedagogical metaphor—one that operationalizes deformable coordinate systems in curved spacetime—it remains structurally limited:

mollusk-adapted coordinates implicitly rely on a slicing into spatial hypersurfaces and comoving observers with fixed spatial labels. The present work extends this idea into a general, coordinate-free framework based on programmable local observers. We adopt a fully observer-based framework in which spacetime is populated by a freely moving swarm of local observers, each equipped with four independently and arbitrarily running clocks. These clocks—each running at an arbitrary rate and unconstrained by synchronization or foliation—serve to assign coordinate values to nearby events without appealing to abstract field configurations or spacetime slicing. It is important to understand that these clock readings do not correspond to physically independent measures of time. Rather, the clocks serve purely to produce numerical values—arbitrary scalar labels defined by observer-programmed algorithms which can assume the role of general coordinates. The resulting mollusk-inspired framework manifestly respects general covariance and offers a physically realizable and accessible interpretation of general coordinates—well-suited both for teaching and for clarifying the foundations of coordinate freedom in GR.

We emphasize an operational stance throughout the exposition of GR. Standard textbooks on general relativity typically begin by emphasizing that global inertial frames, as defined in special relativity, cannot be extended to curved spacetimes. This limitation motivates the introduction of curvilinear coordinates and the formal apparatus of pseudo-Riemannian geometry[6–8]. However, such treatments rarely offer explicit operational procedures for physically constructing these general coordinate systems. As a result, students are often introduced to the abstract formalism of general relativity without a fully developed physical picture that connects also curvilinear coordinates to easily measurable quantities. While fully realistic reference systems—such as those implemented in experimental physics or space missions—often involve technical complexities that make them unsuitable for introductory exposition, the proposed swarm of observers offers a compelling compromise. It is physically realizable in principle, fully general, and still conceptually transparent enough to illustrate the essential role of observer-based coordinate assignments within the broader framework of general covariance.

This dual-purpose framework is intended both as a philosophical analysis and as a pedagogical model. The aim is to bridge the conceptual clarity demanded by foundational discussions with the accessibility required for effective teaching. To this end, visualizations such as the "Einstein mollusk" and the freely floating observer swarm are deliberately presented in a style reminiscent of scientific popularization. This stylistic choice serves pedagogical clarity and conceptual accessibility: it serves to make the underlying ideas immediately graspable, while maintaining their scientific and conceptual rigor.

The presented approach aligns with longstanding philosophical traditions that emphasize the necessity of linking theoretical structures to concrete observational procedures. Thinkers such as Bridgman[9] and Reichenbach[10] underscored the importance of bridging the gap between formalism and measurement. More recently, Hetzroni and Read[11] have highlighted the interplay between pedagogical strategy and philosophical commitment. Building on this trajectory, recent work has sought to clarify the conceptual status of reference frames in general relativity by distinguishing purely mathematical coordinate systems from physically meaningful constructions. For instance, Bamonti[12] offers a taxonomy based on the dynamical properties of reference frames and their contribution to the gravitational field via stress-energy. His examples include dynamical reference frames based on physical fields—such as scalar fields obeying Klein–Gordon dynamics—that are explicitly coupled to the metric.

By contrast, the present framework introduces no such dynamical coupling: coordinate values are assigned through freely programmable readings of four arbitrary clocks carried by each local observer. These scalar quantities are not governed by field equations but function purely as operational labels, capable of instantiating any mathematically admissible chart, independent of the observers' physical state or interaction with the geometry. While formally reminiscent of scalar fields—each coordinate component being a smooth scalar function on spacetime—they are not dynamical fields in Bamonti's sense. Most importantly, they are *physically realizable*: they can be instantiated by actual clocks or smartphone-like devices, unlike hypothetical scalar fields, which have no empirical counterpart available for practical use. This distinction ensures both empirical accessibility and full coordinate freedom without introducing additional dynamical structure.

In particular, this approach resonates with earlier philosophical analyses of coordinate significance, such as Anderson's influential distinction between absolute and dynamical structures[3]. Yet whereas Anderson held that meaningful coordinates must be anchored in the dynamics of physical fields or matter, the present framework advances this perspective by allowing coordinate values to be freely assigned—independent of the observer's physical state or motion—so long as they are operationally tied to observable events. In this way, the proposed swarm of observers pushes the principle of general covariance to its operational limit, while preserving its empirical content.

II. SPECIAL RELATIVITY'S REFERENCE FRAMES

To prepare for general relativity, we begin by revisiting and slightly reframing the construction of physical reference frames in special relativity (SR). The foundational principle—shared by all relativistic theories—is that only local coincidences of events can be assigned physically unique meaning. This *locality principle* therefore also serves as the operational starting point for the formulation of reference frames.

Consequently, even in SR, an "observer" must be understood not as a pointlike abstraction but as a *swarm of local observers*, each equipped with measuring devices and capable of recording only those events that occur in their immediate vicinity. A spacetime diagram, then, is an operational representation of the totality of recorded measurements: the collective event data gathered by all these assistant observers who conceptually fill spacetime. Each local device assigns coordinates—its own clock reading and spatial position—to the events it directly registers.

Mathematically, these coordinates are typically denoted by x^μ with $\mu = 0, 1, 2, 3$, so that a spacetime event is labeled by $x = (x^0, x^1, x^2, x^3)$. The temporal coordinate is given by $x^0 \equiv c\bar{t}$, where c is the speed of light and $\bar{t}$ is the local clock reading. The spatial coordinates are written as $(x^1, x^2, x^3) \equiv \bar{\mathbf{x}}$.

Some introductory texts visually depict such a reference frame as a regular three-dimensional lattice, composed of idealized rigid rods and locally placed synchronized clocks[13,14]. These constructions are designed to illustrate the operational assumptions of inertial frames in SR—such as the Euclidean geometry of space, global clock synchronization, and the zero-distance idealization between the event and its recording observer.

An equivalent and more locality-focused depiction is shown in Fig. 1. Here, spacetime events are assigned coordinates by a distributed network of observers, each using their locally maintained synchronized clocks and pre-established positions relative to others. When needed, spatial alignment across the network can be re-calibrated by activating orthogonal laser pulses, which form a transient coordinate grid. This operational picture emphasizes that it is the observers—and not the coordinate mesh—that serve as the physical basis of the reference frame.

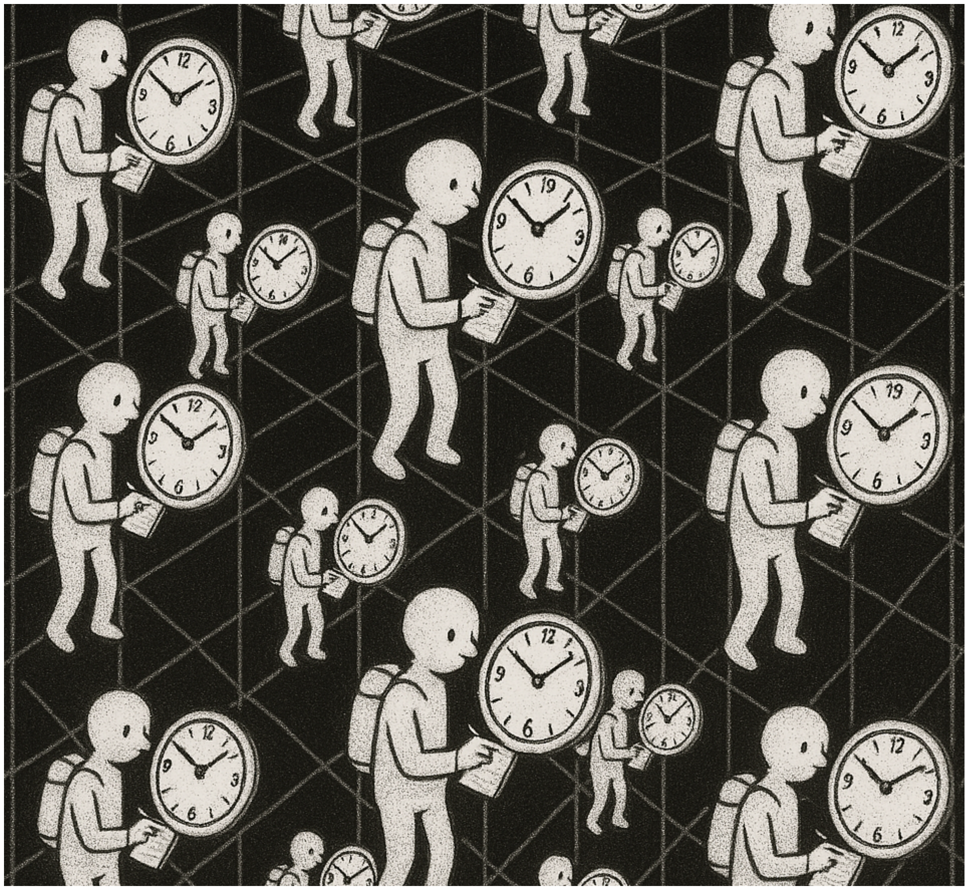

FIG. 1. A special relativistic reference frame depicted from an operational perspective.

Once established, a reference frame allows for the measurement of spatial distances, time intervals, and kinematic quantities by combining purely local observations of events, even when those events are separated in space and/or time. For example, measuring the length of a small rigid rod—whether at rest or moving uniformly relative to the frame—involves identifying two simultaneous but spatially separated events: one recorded by an observer located at one end of the rod, and the other by a second observer at the opposite end, both using their synchronized clocks to ensure simultaneity in the frame.

The configuration of free-floating observers shown in Fig. 1 naturally generalizes to multiple overlapping inertial reference frames, such as frame A (Alice) and frame B (Bob), covering the same region of spacetime. Each frame is constructed independently, using only its own network of observers, synchronized clocks, and internal measurement protocols.

As a result, any given spacetime event can be assigned two sets of coordinates—x and $\tilde{x}$—depending on which frame's observers perform the local assignment. Because the two networks are coextensive, observers in one frame can also access the measurements made by their local counterparts in the other. This operational comparison allows one to empirically determine a transformation function:

$$\tilde{x} = \Lambda_{A\to B}(x). \tag{1}$$

A striking empirical result is obtained: when two inertial frames A and B are constructed independently—each relying solely on its own local observers and synchronization protocol—the transformation $\Lambda_{A\to B}$ turns out not to be Galilean, as would be expected in Newtonian mechanics. Instead, it corresponds to a Lorentz transformation (up to translations), in accordance with the Poincaré symmetry that preserves the form of Maxwell's equations. This result is theoretically equivalent to the constancy of the speed of light as measured in any inertial reference frame — the principle originally postulated by Einstein and used to derive the Lorentz transformation. In modern terms, this invariance is understood as the *Poincaré invariance* of special relativity: the spacetime interval between any two events remains the same in all inertial frames.

Expressing the Poincaré transformation in component form in Einstein notation as

$$\tilde{x}^\mu = \Lambda^\mu{}_\nu x^\nu + o^\mu, \tag{2}$$

where the constant four-vector o^μ accounts for arbitrary (but physically irrelevant) offsets in spacetime origins between frames, and introducing the invariant Minkowski metric tensor

$$\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1), \tag{3}$$

the invariance of the infinitesimal spacetime interval ds^2 is expressed as

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu = \eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu. \tag{4}$$

These equations involve only the differences in spacetime coordinates between events, and as discussed above, such differences can be determined through purely local measurements within each reference frame, using synchronized clocks and known spatial positions. Consequently, the value of the invariant spacetime interval is directly computable from physically measurable quantities in either frame.

III. GENERAL RELATIVITY'S REFERENCE FRAMES

Introductory treatments of general relativity typically emphasize that globally inertial reference frames, as defined in special relativity, cannot be extended to generic curved spacetimes. This motivates the transition to curvilinear coordinates, the adoption of general covariance, and the use of the formalism of pseudo-Riemannian differential geometry. Yet, during this transition, the analogy between physically realizable reference systems and their mathematical counterparts often remains conceptually incomplete.

Only part of this analogy is typically developed. The equivalence principle establishes a local correspondence between a freely falling physical reference frame—realized, for instance, by an idealized observer equipped with rigid, mutually orthogonal measuring rods and synchronized clocks for temporal measurement—and the mathematical notion of a locally flat infinitesimal part of a curved manifold $\mathcal{M}$, and the associated tangent space $T_p\mathcal{M}$ at a point $p \in \mathcal{M}$. However, no corresponding operational framework is typically provided for constructing general finite coordinate charts on the manifold. The extension from infinitesimal inertial frames to arbitrary curvilinear coordinates is usually carried out purely in the abstract, without offering concrete procedures that could be implemented by individual observers using only local measurement processes. Pedagogical analogies—when offered—frequently appeal to cartographic projections, such as mapping Earth's surface, but these remain disconnected from the question of how general reference frames might be physically realized in spacetime. The step needed to complete the analogy—constructing arbitrary reference frames based solely on the data physically accessible to observers operating independently—is often omitted, or substituted with technically elaborate formalisms. As a result, the connection between arbitrary coordinate systems defined over finite regions of curved spacetimes and their operational observability remains underexplained and continues to call for more accessible and physically grounded introductions.

Even as thorough a textbook as *Gravitation*[15] by Misner, Thorne, and Wheeler can exemplify this tendency. It begins its treatment of coordinates (pp. 5-10) with a vivid physical description of events as intersections of worldlines and physical interactions. The introduction of coordinates is explicitly delayed until events have been described physically. But it ultimately also does not fully preserve the operational foundation of reference frames. No physical system of local observers, clocks, or measuring devices is constructed when making the leap to event coordinates. Coordinates are not related to physical operations, but are introduced as abstract ordering devices applied *post hoc* to an idealized event structure:

> *Nothing is more distressing on first contact with the idea of "curved spacetime" than the fear that every simple means of measurement has lost its power in this unfamiliar context. [...] No numbers. No coordinate system. No coordinates. [...] To order events, introduce coordinates! [...] Coordinates are four indexed numbers per event in spacetime. [...] In christening events with coordinates, one demands smoothness but foregoes every thought of mensuration.*

For some students, such remarks may be sufficient to address the acknowledged "fear that every simple means of measurement has lost its power." However, given that Einstein's mollusk (and in particular its generalization, introduced below) offers a direct and conceptually smooth transition from the physical reference frames of special relativity to their general-relativistic counterparts, entirely omitting it at this stage represents a lost teaching opportunity. The leap from physical "events" to the abstract notion of purely mathematical coordinates bypasses an important operational construction.

In what follows, we demonstrate how the extension from SR to GR reference frames can proceed continuously and transparently, requiring only a few well-motivated generalizations.

A. The Einstein Mollusk

As previously mentioned, the first popular exposition of general relativity[4] was written by Einstein himself, who did not shy away from introducing the vivid operational image of a physical "reference mollusk":

> *What does it mean to assign to an event the particular co-ordinates x_1, x_2, x_3, x_4, if in themselves these co-ordinates have no significance? More careful consideration shows, however, that this anxiety is unfounded [...] For this reason non-rigid reference-bodies are used, which are as a whole not only moving in any way whatsoever, but which also suffer alterations in form ad lib. during their motion. Clocks, for which the law of motion is of any kind, however irregular, serve for the definition of time. We have to imagine each of these clocks fixed at a point on the non-rigid reference-body. These clocks satisfy only the one condition, that*

the "readings" which are observed simultaneously on adjacent clocks (in space) differ from each other by an indefinitely small amount. This non-rigid reference-body, which might appropriately be termed a "reference-mollusk", is in the main equivalent to a Gaussian four-dimensional co-ordinate system chosen arbitrarily. That which gives the "mollusk" a certain comprehensibility as compared with the Gauss co-ordinate system is the (really unjustified) formal retention of the separate existence of the space co-ordinates as opposed to the time co-ordinate. Every point on the mollusk is treated as a space-point, and every material point which is at rest relatively to it as at rest, so long as the mollusk is considered as reference-body. The general principle of relativity requires that all these mollusks can be used as reference-bodies with equal right and equal success in the formulation of the general laws of nature; the laws themselves must be quite independent of the choice of mollusk.

A visual representation of this idea is shown in Fig. 2, which generalizes the spatial lattice of special relativity to a fully flexible reference structure. Local observers, each equipped with a clock that may run non-uniformly, are distributed across spatial coordinates that are allowed to warp arbitrarily. These coordinates define a continuous network of physically distinguishable events without relying on global synchronization or rigid rulers. The surface shown in the figure represents a two-dimensional spatial slice. Multiple such surfaces are imagined to be stacked spatially to fill a three-dimensional region, like the layers of an onion or the nested shells of a matryoshka doll. The mollusk as a whole may evolve in time, with the shape of each surface changing and clocks ticking at locally arbitrary rates — it is, metaphorically speaking, "alive and kicking."

It is worth emphasizing how deeply Einstein's explanation remains grounded in physical entities. In his account, events — even taken collectively — do not automatically correspond to coordinate values. Rather, coordinates indicate the position of an event relative to *some other physical entity* acting as a reference frame. This relational view may appear trivial at first glance, but it is easily obscured when no attempt is made to operationalize a general physical reference frame for general relativistic coordinates.

Since we are interested in physically realizable reference frames, and because the entire spacetime manifold $\mathcal{M}$ can be covered by a countable collection of overlapping open sub-

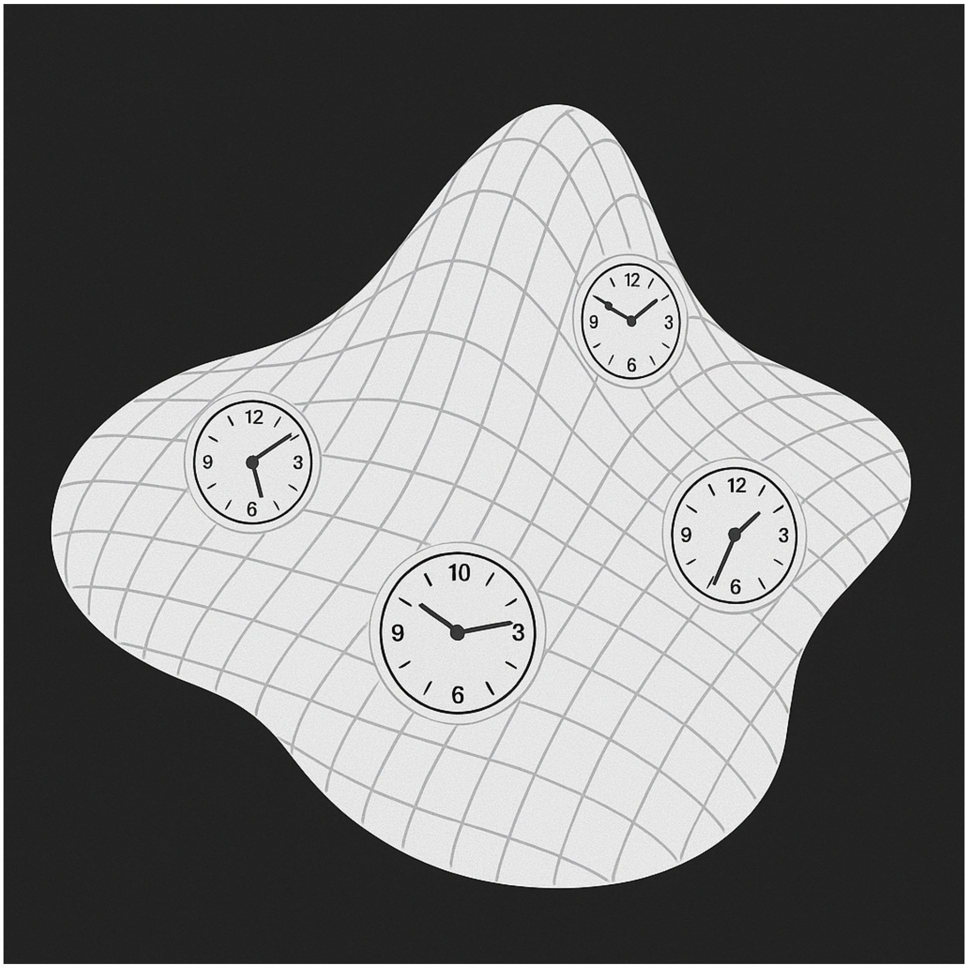

FIG. 2. One spatial surface of an Einstein mollusk, representing a local physical reference frame. Observers with arbitrarily varying clocks are positioned along nonlinearly deformed spatial coordinates. The surface is one of many spatial layers that, together, define a fully deformable reference structure. The entire mollusk may change its shape over time.

sets $U \subset \mathcal{M}$, we may approximately represent each of those regions U in which spacetime is probed using a mollusk by a distinct physical Einstein mollusk — that is, a finite, deformable reference frame constructed from comoving observers equipped with local clocks and measuring devices[25]. If mollusks are defined on multiple overlapping regions, their respective coordinate labels must be defined so as to ensure mutual consistency via smooth transition functions on the intersections of their domains.

A mollusk-adapted coordinate system within U is then given by the coordinate functions

$$(x^0, x^1, x^2, x^3) := (\lambda(\tau, x^1, x^2, x^3), x^1, x^2, x^3), \tag{5}$$

where each observer remains at fixed spatial coordinates x^i, with $i = 1, 2, 3$, relative to the arbitrarily moving mollusk, while the local clock shows λ, which increases monotonically with the observer's proper time τ. These functions define a smooth local coordinate chart on spacetime, i.e., a local diffeomorphism from $\mathbb{R}^4$ to U. However, it is important to distinguish between the mathematical existence of arbitrary local charts—guaranteed by the manifold structure of spacetime—and the construction of such charts via physical procedures. Mollusk-adapted coordinates form a subclass of all admissible charts, characterized by their operational origin: they arise from measurements performed by comoving observers, each recording their proper time (or a monotonic function thereof) and fixed spatial label.

It is crucial to recognize that a mollusk is not merely a physical realization of a coordinate chart, but a realization that is structurally constrained by its mode of implementation. A mollusk-adapted chart is more than a smooth diffeomorphism from $\mathbb{R}^4$ to a spacetime region U: it is a chart with built-in metric constraints. Specifically, the coordinate direction $x^0 = \lambda(\tau, x^i)$ must be timelike, while the spatial directions x^i must be spacelike within the mollusk's domain. This restriction is inherited from the mollusk's realization by a congruence of comoving timelike observers, each with a single clock and fixed spatial label.

We may therefore ask a sharper question: can the mollusk instantiate *any* admissible mathematical chart? Importantly, we are not asking whether a mollusk-adapted chart can be transformed *into* any other chart via coordinate change; rather, we ask whether it can *directly represent* arbitrary charts through its physical construction. The answer is negative: the mollusk cannot instantiate coordinate systems that require, for example, null directions, as found in double-null charts or Eddington–Finkelstein coordinates. In such cases, there is no way to associate the required coordinate structure with the mollusk's single timelike direction and its fixed spatial labeling (see Appendix A).

This limitation reveals a deeper insight: physically implemented coordinate charts typically carry not only a smooth labeling of events, but also structural constraints inherited from the physical systems that realize them.

B. The reference swarm of unconstrained observers

To overcome this, and to realize *any* chart—regardless of its causal or foliation structure—we require a physical system in which the coordinate labels are fully decoupled from the geometry of the observers who carry them. This is precisely what the generalized observer swarm provides: each observer carries four independently programmable clock readings, which serve purely as numerical labels. These can be chosen to reflect any smooth chart on spacetime, without implying anything about the causal or geometric structure of the observer's worldline. The role of the observers' timelike worldlines is merely to ensure that spacetime is locally covered by measurement devices; once this coverage is achieved, the freely programmable clocks can assign arbitrary coordinate values to any point in spacetime, independent of the observers' physical motion or causal structure. In this way, the observer swarm furnishes a physically realizable chart that is free from geometric constraints—a direct operational realization of the full diffeomorphism freedom of general relativity.

We may arrive at the same insight from a slightly different point of view: we may build on our earlier shift in special relativity from a rigid lattice to a distributed network of local observers (as depicted in Fig. 1), and complete this progression by shifting attention away from coordinate labels derived from spatial slices. Instead, we regard the observers themselves—and in particular the information they locally assign—as the primary carriers of coordinate structure. Specifically, we suggest eliminating any ties to fixed spatial coordinate values by implementing a general physical reference frame in general relativity as an *arbitrarily moving swarm of local observers*, each equipped with *four* independent and arbitrarily running clocks (replacing any constant readings or fixed spatial positions). The readings of these clocks provide unique and smooth but otherwise arbitrary numerical labels to be associated with events observed in the neighborhood of the observer, and together they define a fully general local coordinate system — not restricted by any geometric structure such as orthogonality, synchronization, or foliation. For concreteness and clarity we may imagine that the four clock readings are shown on a single device — for instance, a smartphone screen carried by each observer — as depicted in Fig. 3.

The proposed generalization thus yields an operational realization of a reference frame that fully respects general covariance. It introduces no additional structure beyond the smooth invertibility of the observer-assigned labels and makes no assumptions about slicing, symmetries, or background geometry. Indeed, it remains agnostic even to spacetime dimensionality (if more or fewer than four numbers are displayed).

Moreover, since the numerical displays of the observers are entirely unconstrained — evolving arbitrarily in both space and time — the framework naturally accommodates the

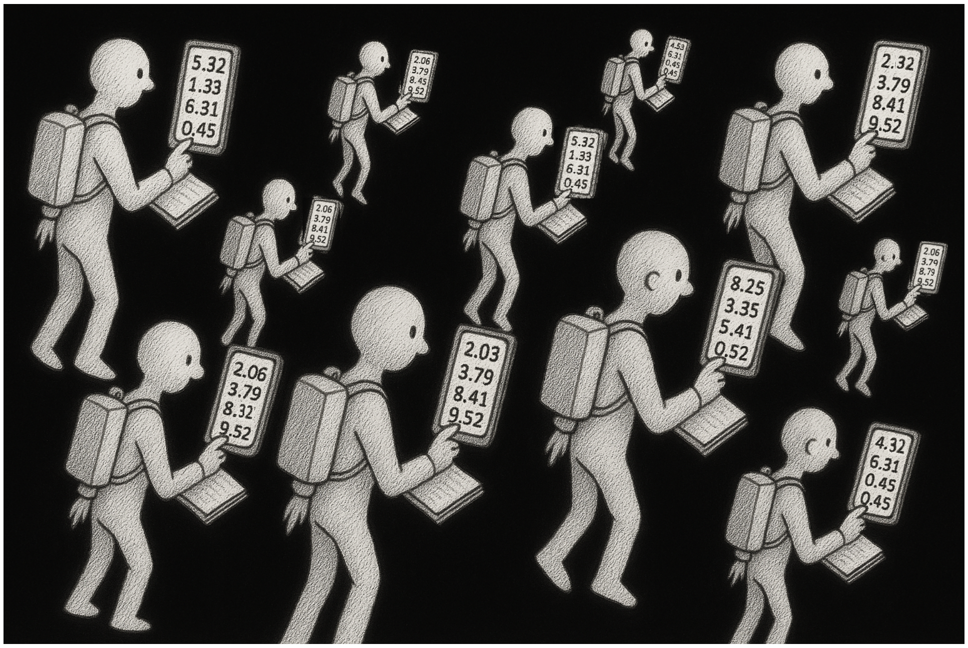

FIG. 3. A swarm, flock or cloud of unconstrained observers fills spacetime without relying on any foliation. Each observer locally records observed events in a notebook and uses four arbitrarily evolving numerical labels to index them — for instance, as displayed on a single device. These values define general local coordinates in a fully covariant, slicing-independent framework.

mathematical structure of an arbitrary atlas on a smooth manifold. The local continuity of the four numbers ensures that overlapping neighborhoods can collectively form a smooth cover of spacetime in which chart transitions are realized algorithmically through the consistent labeling of events in intersecting regions.

In more detail, let $\mathcal{M}$ be a smooth 4-dimensional Lorentzian manifold, and let $U \subset \mathcal{M}$ be an open neighborhood in which we implement our physical reference frame—again assumed to be sufficiently small to neglect self-gravitational effects and exclude collapse. We define a local congruence of timelike worldlines by a smooth, future-directed, unit timelike vector field $u^\mu(x)$. The integral curves of u^μ then define the worldlines of the observers that constitute the swarm. Let $\omega_p(\tau)$ denote the worldline through point $p \in U$, parametrized by a scalar parameter τ denoting proper time of the observer as measured by a (fifth) clock which is no longer arbitrary (e.g. an atomic clock). Associated with this congruence we introduce four functions $X^\mu(x)$ interpreted as the four numerical values assigned to events by local observers, i.e. the four arbitrary clock readings or arbitrary, algorithmically defined numbers displayed on a screen. Because these arbitrary clock readings $X^\mu(x)$, with $\mu = 0,1,2,3$, are local to the observers, they are naturally transported along the worldlines of the congruence.

$$\frac{d}{d\tau} X^\mu(\omega_p(\tau)) = u^\nu \nabla_\nu X^\mu = \mathcal{L}_u X^\mu, \tag{6}$$

where $\mathcal{L}_u X^\mu$ is the Lie derivative along the vector field u^ν. This setup is analogous to the situation in fluid mechanics: the observer worldlines correspond to the Lagrangian viewpoint, while the quadruples $X^\mu(x)$ reflect an Eulerian perspective.

If we model the swarm of observers as a dust fluid (albeit with each "dust" particle being equipped with a recording device and four arbitrarily running clocks), their energy-momentum tensor in general coordinates can be approximated as

$$T^{\mu\nu} = \rho\, u^\mu u^\nu, \tag{7}$$

where $\rho(x)$ is the mass-energy density of the observer swarm, and $u^\mu(x)$ is their four-velocity field. To ensure that the reference frame itself does not distort the results (by acting as a source), we must conceptually take the limit $\rho \to 0$. Operationally, this can be interpreted as performing measurements with increasingly dilute distributions of test observers, and extrapolating the results to the limit of vanishing density. In this way, the reference system becomes asymptotically passive, serving only to reveal the intrinsic structure of spacetime without back-reaction.

For the following, the Eulerian description $X^\mu(x)$ is preferred because the mapping

$$X : x \mapsto \big(X^0(x), X^1(x), X^2(x), X^3(x)\big) \tag{8}$$

can define a physically realized coordinate chart on the region $U \subset \mathcal{M}$, mapping events to coordinate tuples in $\mathbb{R}^4$, provided the Jacobian $\partial X^\mu / \partial x^\nu$ is invertible. We intentionally use Greek superscripts for both the abstract manifold coordinates x^μ and the arbitrary observer-assigned functions X^μ, to highlight their respective roles as abstract and observable coordinates. The use of a capital X emphasizes the operational origin — a reference frame realized physically by measurement procedures. This distinction, while useful here, may be safely omitted in what follows, since any mathematical coordinate chart x defined on a region $U \subset \mathcal{M}$—for instance, one used in solving the Einstein field equations—can now be trivially realized on the observers' screens by choosing the observable coordinates $X^\mu(x)$ to coincide with the abstract coordinates:

$$X^\mu(x) := x^\mu.$$

This construction reflects a fundamental feature of the observer swarm: the coordinate values carried by each observer need not correspond to physically intrinsic quantities like proper time or spatial position. Rather, these values can be freely assigned and algorithmically updated, allowing the observers to carry abstract coordinate labels that encode structures not tied to their own physical state. In particular, this freedom allows the representation of coordinate systems not adapted to the underlying timelike congruence, including those based on null foliations, or even formal coordinate singularities such as those encountered at event horizons. The observers are thus best understood as carriers of coordinate values—programmable labels that reflect the chosen chart, not the local dynamics of the observers themselves. More generally, a diffeomorphism between two coordinate systems, $x \mapsto \tilde{x}(x)$, corresponds in the operational setting to a transformation between two physically realized coordinate systems:

$$\tilde{X}^\mu(X) = \tilde{x}^\mu\big(x(X)\big), \tag{9}$$

where $x(X)$ is the inverse of the physically realized chart $X : x \mapsto X(x)$, and $\tilde{X}^\mu$ denotes the new observer-assigned coordinates. This shows that diffeomorphisms appear as transformations between observable coordinate assignments, and that any mathematical chart or coordinate transformation can be physically realized by a suitably constructed swarm of observers and their associated quadruples of clock readings.

Once the observer-assigned labels X^μ are identified with a chosen mathematical chart x^μ, it therefore becomes convenient—though conceptually distinct—to use the same notation. Hence, both the abstract and the physically realized coordinates may be denoted by lower case notation, i.e. x^μ and $\tilde{x}^\mu(x)$, as they share the same mathematical properties and differ only in their operational interpretation.

Operationally, coordinate transformations can be realized in different equivalent ways. For example, we may again introduce two interspersed swarms of observers — defining two independent reference frames, A (for Alice) and B (for Bob) — each covering the same region U of spacetime. Alternatively, we may choose to assign multiple distinct sets of four numerical values to each observer within a single swarm. In this case, each observer carries several coordinate quadruples simultaneously, which may be displayed on two screens per observer (or a shared screen for both coordinate systems). In all of these scenarios, a general coordinate transformation between frames is implemented physically by having each observer record both their own set of coordinate labels x and those of the nearest observer from the other frame $\tilde{x}$. The transformation $\tilde{x}^\mu = \tilde{x}^\mu(x)$ in all of U is then inferred from all local comparisons.

Accordingly, an infinitesimal coordinate displacement $d\tilde{x}$ in the reference frame A of the Alice observers is related to the corresponding difference dx in the reference frame B of the Bob observers via the Jacobian:

$$d\tilde{x}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}\, dx^\nu. \tag{10}$$

Again all quantities appearing in Eq. (10) correspond to observable differences: both $d\tilde{x}^\mu$ and dx^ν represent measurable differences between the coordinates of two nearby events, as recorded in two distinct reference frames by the respective swarms of Alice and Bob observers. The partial derivatives in the Jacobian can be obtained directly from such measured values using basic linear algebra, or by numerically differentiating the transformation relation given in Eq. (9). All terms thus admit a concrete operational interpretation — each reflects an actual measurement process followed by computational analysis.

Although physical laws must be expressed in locally covariant form to ensure consistency under coordinate transformations, non-covariant expressions—such as coordinate values or Christoffel symbols—retain operational significance *within a fixed reference frame*. Their empirical relevance derives from the fact that they correspond to directly measurable quantities, once the reference frame is specified. For example, coordinate values correspond to actual numbers displayed on screens in Fig. 3, reflecting measurable quantities within a physically realized reference frame. These values can be freely chosen, but once assigned, they constitute real outputs of local measurements.

Such distinctions are easily overlooked when physical constructions of reference frames are not explicitly considered. Yet they are both pedagogically and conceptually important. For example, in Rovelli's terms[18], coordinate values and non-tensorial quantities like the Christoffel symbols qualify as *partial observables*—quantities that are directly measurable, even though they are not invariant under diffeomorphisms.

All operations required to verify Eqs. (9)–(10) can be carried out locally by the observers in the frame. Any quantity computed from coordinates—whether tensorial or not—can thus be viewed as the result of data processing based on numerical results obtained from physical operations. Properties like tensoriality can, in principle, be operationally tested.

C. Measuring the Metric in GR

In special relativity (SR), the flatness of spacetime[26] is reflected in the ability to choose globally rectilinear coordinates in which the metric tensor assumes its canonical Minkowski form η. Its global constancy ensures that coordinate differences dx^μ correspond directly to physically meaningful intervals, as shown in Eq. (4). In curvilinear coordinates, coordinate differences dx^μ alone are not physically meaningful. To obtain, for example, proper distances or durations, one must also know the locally varying metric tensor $g_{\mu\nu}(x)$, which allows to extend beyond SR by encoding spacetime curvature. This section shows how this additional structure can be gently introduced using the operational framework developed so far.

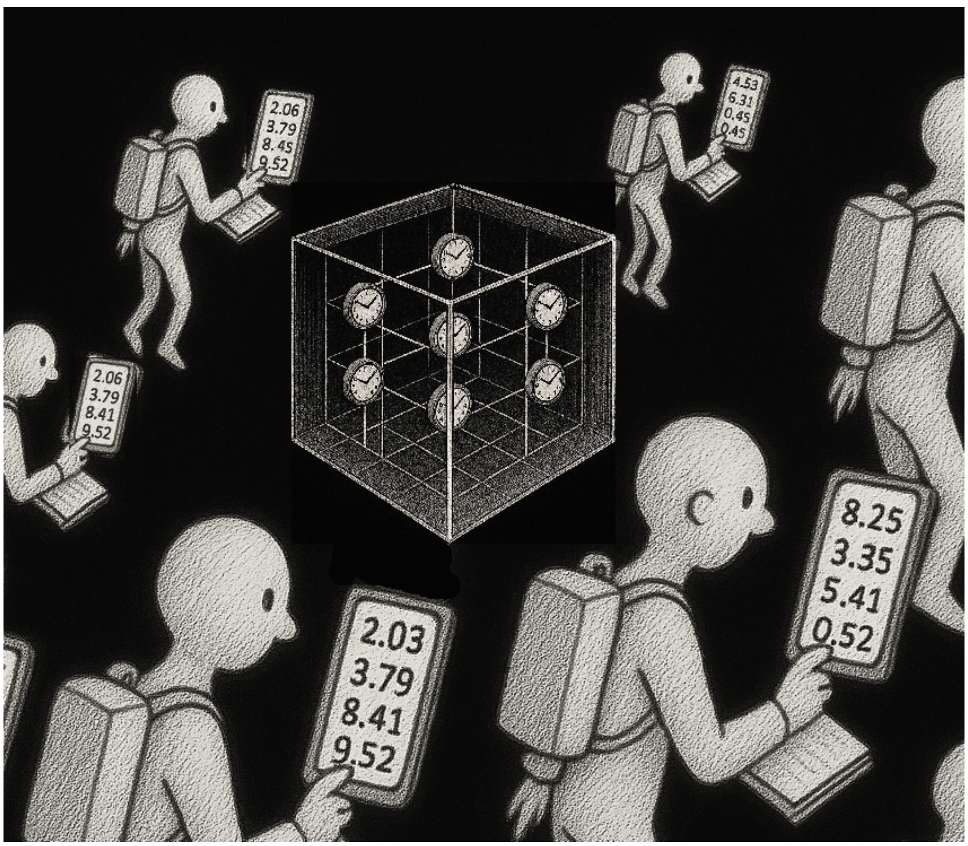

FIG. 4. A GR reference frame consists of a swarm of unconstrained observers traveling arbitrarily in spacetime, measuring whatever happens in their immediate vicinity. A second reference frame, in the form of an Einstein elevator, covers an infinitesimal region in space and time in which the laws of SR hold and can be expressed in the elevator's special relativistic coordinate system.

Einstein was led to general relativity (GR) by a second stroke of genius: he recognized the true reason why all bodies fall with the same acceleration in Galileo's famous Leaning Tower of Pisa experiment. Gravitational effects can be locally eliminated by using the coordinate values of a freely falling reference frame. More figuratively, the interior of an infinitesimally small, freely falling elevator constitutes a local inertial frame in which the laws of special relativity (SR) hold—without curvature or gravitational effects—for a very short duration and within a very small spatial region. This is the content of the equivalence principle.

This empirical insight can be applied to the analysis of any event p with coordinate values x^μ in an arbitrary GR reference frame. In the infinitesimal neighborhood of p, one can always construct a freely falling Einstein elevator passing through the event, as illustrated in Fig. 4. Since SR holds locally inside this tiny, transient frame, an orthonormal local SR coordinate system $\tilde{x}$ can also be constructed—a latticework as in Fig. 1—but now centered on p and valid only within an infinitesimally small region of spacetime.

For ease of visualization, we assume that the Einstein elevator has no physical walls, allowing the observers from both reference frames—the general relativistic (GR) frame and the local inertial special relativistic (SR) frame—to be interspersed throughout the same region, including inside the elevator. Observers from both systems can thus record the same events occurring within the elevator. The GR observers assign four coordinate values x^μ, while the SR observers use locally valid coordinates $\tilde{x}^\mu$. These coordinate systems are related by a smooth transformation $\tilde{x}^\mu = \tilde{x}^\mu(x)$, which is valid only in a small neighborhood around the event p, but is otherwise no different from a conventional coordinate transformation.

Operationally, this transformation can be physically realized in the usual manner: each observer in one reference frame simply records the coordinate values displayed on the device of their neighboring counterpart in the other frame. Since the $\tilde{x}^\mu$ coordinates belong to a local SR reference frame, spacetime intervals expressed in these coordinates have immediate physical significance; in particular, the invariant expression of the interval $ds^2 = \eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu$ remains valid throughout the short lifetime and spatial extent of the Einstein elevator.

By applying the coordinate transformation $\tilde{x}^\mu = \tilde{x}^\mu(x)$ and using Eq. (10), the spacetime interval can be rewritten in terms of the general coordinates x^μ, which alone do not carry immediate physical meaning:

$$ds^2 = \eta_{\mu\nu} \frac{\partial \tilde{x}^\mu}{\partial x^\alpha} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} dx^\alpha dx^\beta \equiv g_{\alpha\beta}(x)\, dx^\alpha dx^\beta, \tag{11}$$

where we have defined the general relativistic metric tensor by

$$g_{\alpha\beta}(x) = \eta_{\mu\nu} \frac{\partial \tilde{x}^\mu}{\partial x^\alpha} \frac{\partial \tilde{x}^\nu}{\partial x^\beta}. \tag{12}$$

Once the functional form of the coordinate transformation $\tilde{x}^\mu(x)$ is determined—through local comparison of observer readings—its derivatives yield the metric components via Eq. (12), completing the reconstruction from local SR measurements.

Alternatively, one may reconstruct the components of $g_{\mu\nu}(x)$ directly from physically measured infinitesimal displacements $d\tilde{x}^\mu$ in the elevator frame and the corresponding coordinate differences dx^μ in the general frame, using the defining relation $g_{\alpha\beta}dx^\alpha dx^\beta \equiv \eta_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu$ and standard linear algebra.

If we switch to a different GR reference frame with coordinates x'^μ, and construct a corresponding local SR frame $\tilde{x}'^\mu$, we again define the metric via local measurements as

$$ds^2 = \eta_{\alpha\beta}\, d\tilde{x}'^\alpha d\tilde{x}'^\beta = g'_{\mu\nu}(x')\, dx'^\mu dx'^\nu. \tag{13}$$

Since both the original and the new GR frames use the same underlying local SR measurements (up to Lorentz or Poincaré transformations, which preserve ds^2), and since both expressions yield the same invariant interval, we have

$$g_{\mu\nu}(x)\, dx^\mu dx^\nu = g'_{\alpha\beta}(x')\, dx'^\alpha dx'^\beta. \tag{14}$$

But the coordinate displacements are related via the Jacobian in eq. (10).

Substituting into the left-hand side of eq. (14), we get (since the infinitesimal displacements were arbitrary):

$$g'_{\alpha\beta}(x') = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} g_{\mu\nu}(x), \tag{15}$$

which is the transformation law of a (0,2)-tensor. Thus, the metric's tensorial character follows from the invariance of ds^2 which is operationally defined through local measurements in overlapping reference frames.

Since the local GR metric $g_{\alpha\beta}(x)$ is operationally defined via a transformation from a local SR coordinate system, it can also be used to analyze causal relationships in the GR coordinates. For any event p with assigned coordinates x^μ in a general reference frame, one can construct the local light cone by first identifying the standard SR light cone at p in the local inertial frame, defined by the null condition

$$ds^2 = \eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu = 0, \tag{16}$$

and then applying the local coordinate transformation to express these directions in terms of the GR coordinates. This yields the GR light cone at x, defined by

$$ds^2 = g_{\alpha\beta}(x)\, dx^\alpha dx^\beta = 0, \tag{17}$$

whose causal structure thus arises directly from the mapping of the local SR structure into the general frame.

This provides a powerful practical insight, especially in spacetimes with potentially misleading coordinate labels (such as the Schwarzschild interior). It allows one to determine unambiguously which coordinate displacements dx^α are timelike, spacelike, or null by evaluating the associated invariant:

$$ds^2 \begin{cases} < 0 & \text{timelike,} \\ = 0 & \text{null,} \\ > 0 & \text{spacelike.} \end{cases} \tag{18}$$

Regardless of the coordinate names or conventions, the causal character of any direction can thus be determined locally from the metric.

Slightly more advanced approaches may start from the idea that local light cone structures could anyway be determined directly through the observation of light rays, without explicitly constructing any local inertial system. This provides an independent observational route to determining the local metric structure[16]. Since null displacements satisfy eq. (17), knowledge of enough linearly independent null directions dx^α, obtained for instance by tracing the trajectories of different light rays through the same event p having coordinates x, determines the metric $g_{\alpha\beta}(x)$ up to a conformal factor. The remaining scale ambiguity can be fixed, for example, via a (fifth) clock which is no longer arbitrary but rather has to record proper time along the timelike worldline of the observer passing through p. While this approach may be more directly practical, it tends to obscure the conceptual link to the equivalence principle and the analogy with locally flat coordinate patches in differential geometry.

Regardless of which operational route is chosen—local SR-based construction or direct causal probing—the same procedure can be repeated (at least conceptually) at each event

$p, q, r, \ldots$ in the reference frame. In this way, one reconstructs the full spacetime-dependent metric tensor $g_{\alpha\beta}(x)$ over the domain of interest.

Once the metric tensor is known throughout spacetime, all geometrical quantities—such as the connection and curvature tensors—become, at least in principle, derivable from observable data. The foundational structure of general relativity is therefore rooted in conceptually transparent measurement procedures, based entirely on coordinate readings and local observations[19].

For completeness, we note that the physical framework developed so far already suffices to describe non-gravitational physical processes in GR coordinate frames. The procedure is straightforward: such processes are first analyzed in the local inertial SR frame, where the known laws of special relativity apply for short durations and over small spatial regions. These results are then translated into GR coordinates using the local coordinate transformation and the metric tensor defined in Eq. (12), as needed.

In this way, locally force-free motion becomes geodesic motion in spacetime, ordinary derivatives in SR translate to covariant derivatives in GR coordinates, and so forth. Moreover, the notion of a local inertial frame can be formalized *a posteriori* through the construction of Riemann normal coordinates $\tilde{x}$ centered at a point p, in which the metric reduces to the Minkowski form $\eta_{\mu\nu}$ and its first derivatives vanish[20]. Related constructions, such as Fermi–Walker coordinates, show that even an accelerated observer—such as one aboard a rocket or propelled by a jetpack—possesses an observer-specific momentary rest frame in which the metric locally approximates Minkowski space. This approximation, however, holds only at a single point, as the first derivatives of the metric generally do not vanish. Nevertheless, this completes the conceptual circle: the observer swarm itself is composed of local observers, each of whom experiences a momentarily Minkowski-like, though possibly accelerated, description of spacetime[27].

It is worth emphasizing, however, that we have so far only scratched the surface of the mathematical structure of differential geometry. The reason further mathematical development becomes useful is not abstract preference, but physical necessity: the local flatness implied by the equivalence principle is conceptually analogous to the flatness of small patches on curved surfaces. In this sense, physics has led us naturally to the tools of differential geometry, not the other way around. This conceptual flow can be obscured in treatments that begin with a few physical images (such as local SR latticeworks) but then make a rapid leap to abstract mathematics (e.g., tangent spaces to curved manifolds), without developing the underlying physical motivation.

Moreover, since we were forced to engage in local metric reconstruction precisely because gravity prohibits the existence of a global SR frame, it should come as no surprise that local variations in the metric tensor $g_{\alpha\beta}(x)$ will also play a central role in the gravitational field equations themselves. It is instructive to contrast this with the case of curvilinear coordinates in special relativity, which also yield a locally varying metric, but one with a vanishing Riemann curvature tensor[20]. Hence, also the operational framework developed here makes it clear from the outset that any genuine generalization of special relativity to include gravitation must involve field equations that yield nonzero spacetime curvature.

D. Caveats on the use of observer-based coordinate systems

Before concluding, it is worth noting several important limitations of the physical picture developed here, in which arbitrarily moving, timelike observers serve as carriers of a general reference frame via their four coordinate readings.

Although a smooth timelike congruence—a smoothly varying family of non-intersecting timelike worldlines—can always be constructed locally, such congruences may not extend globally across spacetime. Geometric features such as horizons, topological obstructions, or the development of caustics (where worldlines intersect or focus) can prevent any single congruence from covering an entire region smoothly. This limitation arises independently of whether observers follow geodesics or are accelerated: even idealized observers with unlimited maneuverability cannot avoid convergence in regions where spacetime geometry enforces it. For instance, no choice of acceleration or initial conditions can prevent timelike worldlines from intersecting inside the event horizon of a Schwarzschild black hole, where they are geometrically compelled to converge.

The limit $\rho \to 0$ in the dust energy-momentum tensor $T^{\mu\nu} = \rho\, u^\mu u^\nu$ ensures that the observer swarm exerts no gravitational influence. When combined with a restriction to suitably small local patches $U \subset \mathcal{M}$, this limit can be safely taken: for small but finite ρ, the swarm does not dynamically collapse onto itself within such regions. Moreover, for sufficiently small U, one can also avoid caustic formation due to the curvature of the underlying spacetime. For larger regions U, however, the development of caustics remains possible as a purely geometric effect, independently of the observers' dynamics or backreaction, as exemplified by the interior of a Schwarzschild black hole.

Importantly, however, these limitations are not unique to the observer swarm: they affect *all* physically realized coordinate systems. In each case, the physical influence of the reference structure may be considered negligible by taking appropriate limits, yet the underlying spacetime curvature can still prevent the existence of a global reference frame. Nevertheless, spacetime can be probed within suitably chosen finite regions U at any regular point of the manifold $\mathcal{M}$, provided that topological defects and singularities are excluded. In this sense, the construction of physical reference frames permits unrestricted local exploration of spacetime structure throughout the manifold.

In addition, while going beyond the classical framework of this paper, quantum field-theoretic considerations may impose further constraints. Even in flat spacetime, accelerated observers detect particle-like excitations in the vacuum, as exemplified by the Unruh effect. In curved spacetimes, even inertial (geodesic) observers may register such detections, depending on the global properties of the spacetime and the choice of vacuum state[21]. This raises the subtle issue that particle content—and thus part of what an observer records as physical—can depend on the observer's trajectory.

The observer swarm is intended to faithfully register all physically occurring phenomena, from classical events to quantum effects such as Hawking radiation. However, it is not always theoretically possible to cleanly separate observer-induced effects (such as Unruh radiation) from those attributable to the ambient spacetime structure alone. All such phenomena are real in the operational sense that they affect detectors and can be measured. Nevertheless, their correct interpretation may require contextual knowledge of the spacetime—including its global structure and vacuum specification—as well as corresponding artifact reduction to distinguish motion-induced signals from external physical processes.

Thus, while the picture of an arbitrarily moving swarm of observers with freely programmable coordinate values is conceptually powerful and operationally flexible, its application must be accompanied by care. Additional assumptions—such as the restriction to suitably small local patches $U \subset \mathcal{M}$, or the ability to distinguish observer-dependent detector artifacts from physical phenomena attributable to the underlying spacetime structure—may be required to ensure completeness and consistency in more general settings.

Again, however, these considerations are not specific to the observer swarm: they apply equally to all physically realized reference frames.

CONCLUSION

The foundational concepts of reference frames, general curvilinear coordinates, and the metric tensor have been systematically grounded in explicit, physically realizable operations performed by local observers. This yields a fully general yet conceptually transparent operational foundation for general relativity, one that avoids reliance on abstract geometric structures introduced *a priori*.

The presented framework elucidates how fundamental concepts of relativity—such as coordinate assignments, spacetime intervals, and the metric tensor—can be regarded as observable in principle, regardless of their transformation properties. As such, it may serve as useful complementary reading to ongoing debates about the physical interpretation of observables in both classical and quantum gravity[18,23,24].

The construction also modernizes and generalizes traditional pedagogical models—such as the spacetime lattice of special relativity and Einstein's "reference mollusk" in general relativity—into a manifestly diffeomorphism-invariant operational scheme. A swarm of arbitrarily moving observers, each equipped with four freely evolving numerical labels (obtained from four arbitrarily running local clocks, or smartphone-like displays showing four algorithmically changing numbers), suffices to define a general coordinate system without conceptual or mathematical overhead. Aside from smoothness, no structural assumptions such as $3+1$ foliations are required.

The framework developed here may also complement mathematically oriented presentations of general relativity by offering a fully general yet physically grounded operational starting point. The observers record events and function as carriers of completely unconstrained coordinate labels, enabling the representation of arbitrary charts—including those based on null foliations or singular coordinates—thereby reinforcing the full coordinate generality of the physical construction.

By beginning with elementary yet fully general procedures, the present approach directly addresses a pedagogical concern first articulated by Einstein[4] and later echoed in *Gravitation* by Misner, Thorne, and Wheeler[15]: the fear that general relativity lacks a conceptually simple account of how spacetime concepts can be endowed with operational meaning.

Indeed, by formulating the construction in terms so elementary that they may appear obvious in retrospect—a perception that is itself part of its pedagogical value—the proposal serves not merely a teaching function, but also stands as a full proof-of-concept realization of the operationalist philosophical stance within general relativity.

Appendix A: Mollusk Limitations compared to Eddington–Finkelstein coordinates

This appendix provides a concrete example of why mollusk-adapted coordinates cannot instantiate every mathematically admissible chart, even locally. We focus on Eddington–Finkelstein (EF) coordinates, which are based on a null foliation, and show that such a chart cannot be physically realized by a mollusk composed of comoving timelike observers with fixed spatial coordinates and a single arbitrary clock.

The Einstein mollusk is often presented as a general coordinate system that deforms freely in space and time, implemented by a continuous distribution of comoving observers. However, we must pay close attention to what this physical realization implies. A mollusk is not merely a smooth physical instantiation of a chart that assigns numbers to spacetime points. Rather, it is a chart realized by a physical congruence of timelike worldlines, each labeled by fixed spatial coordinates and equipped with a single clock that defines the time coordinate. This construction imposes additional geometric constraints that go beyond the mathematical smoothness of charts.

In particular, the coordinate system defined by a mollusk-adapted frame takes the form:

$$(x^0, x^1, x^2, x^3) := (\lambda(\tau, x^1, x^2, x^3), x^1, x^2, x^3),$$

where λ increases monotonically with proper time τ, and the observers are located at fixed values of (x^1, x^2, x^3). The coordinate vector corresponding to x^0 is aligned with the direction of increasing proper time along the observers' worldlines and must therefore be timelike. Conversely, the spatial coordinates x^i remain constant along each observer's path and are thus orthogonal to it, implying they must be spacelike. These are physical constraints imposed by the implementation of the mollusk through timelike observers.

This constraint can be expressed directly at the level of the metric tensor $g_{\mu\nu}(x)$ in mollusk-adapted coordinates. Since the coordinate $x^0 = \lambda(\tau, x^i)$ increases monotonically with proper time and the observers remain at fixed x^i, the metric must satisfy:

$$g_{00}(x) < 0 \quad \text{(timelike direction)} \tag{A1}$$

$$g_{0i}(x) = 0 \quad \text{for all } i = 1, 2, 3 \quad \text{(no cross-terms from fixed spatial labels)} \tag{A2}$$

Now consider the Schwarzschild metric in standard coordinates t, r, θ, ϕ:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{A3}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$. This chart and its associated metric satisfy the mollusk conditions outside the horizon ($r > 2M$): the mollusk can be used as a physical reference frame in this situation.

But in ingoing EF coordinates:

$$v = t + r^* = t + r + 2M \ln \left| \frac{r}{2M} - 1 \right|, \tag{A4}$$

we have

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dv\, dr + r^2 d\Omega^2, \tag{A5}$$

which violates mollusk constraints: $g_{vr} \neq 0$ and g_{vv} becomes null on the horizon.

Thus, EF coordinates cannot be instantiated by mollusk observers. EF charts require a null direction, while mollusks require a timelike one. This reflects a structural incompatibility due to the way the mollusk coordinate system is physically realized. It is therefore not sufficient that the mollusk can model the local geometric situation in some coordinate system; to be truly covariant in the operational sense, it would need to reflect the complete freedom of coordinate choice—which it cannot.

This highlights that, while mathematically any smooth coordinate chart is admissible, only a subset can typically be physically realized by mollusk observers who are by definition subject to built-in causal and metric constraints.

By contrast, the observer swarm can assign arbitrary numerical values to cover such charts, since the labels are decoupled from the observers' physical motion.

The mollusk realizes only a constrained subset of charts.

The observer swarm realizes them all.

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[1] M. Giovanelli, "Nothing but coincidences: the point-coincidence and Einstein's struggle with the meaning of coordinates in physics", *European Journal for Philosophy of Science*, vol. 11, no. 2, 45, 2021.

[2] J. D. Norton, "Coordinates and covariance: Einstein's views on the foundations of general relativity", *Foundations of Physics*, vol. 19, pp. 1215–1263, 1989.

[3] J. L. Anderson, *Principles of Relativity Physics*, Academic Press, New York, 1967.

[4] A. Einstein, *Über die spezielle und die allgemeine Relativitätstheorie (Gemeinverständlich)* (Vieweg & Sohn, Braunschweig, 1917); English translation: *Relativity: The Special and the General Theory — A Popular Exposition* (Methuen & Co. Ltd., London, 1920).

[5] P. Villa, "Mollusk-Writers: Spacetime Revolutions in a Literary Shell", *Journal of Modern Literature*, vol. 43, no. 2, pp. 21–40, 2020.

[6] J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity*, Addison-Wesley, 2003.

[7] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity*, Addison-Wesley, 2004.

[8] B. F. Schutz, *A First Course in General Relativity*, 2nd ed., Cambridge University Press, 2009.

[9] P. W. Bridgman, *The Logic of Modern Physics*, Macmillan, 1927.

[10] H. Reichenbach, *The Philosophy of Space and Time*, Dover Publications, 1958.

[11] G. Hetzroni and J. Read, "How to Teach General Relativity", *PhilSci-Archive; Archive for Preprints in Philosophy of Science*, 2023.

[12] N. Bamonti, "What Is a Reference Frame in General Relativity?", *arXiv preprint*, 2024. arXiv:2307.09338

[13] E. F. Taylor and J. A. Wheeler, *Spacetime Physics: Introduction to Special Relativity*, 2nd ed., W. H. Freeman, 1992, See especially Fig. 2-6.

[14] P. A. Tipler and R. A. Llewellyn, *Modern Physics*, 5th ed., W. H. Freeman, 2008. See Fig. 1-13.

[15] Charles W. Misner, Kip S. Thorne, and John A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).

[16] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, 1973.

[17] R. M. Wald, *General Relativity* (University of Chicago Press, 1984), Sec. 10.2.

[18] C. Rovelli, "Partial observables", Phys. Rev. D **65**, 124013 (2002).

[19] R. Geroch, *General Relativity from A to B*, University of Chicago Press, 1978.

[20] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley & Sons, 1972. s

[21] R. M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press, 1994.

[22] S. Takagi, "Vacuum Noise and Stress Induced by Uniform Acceleration: Hawking-Unruh Effect in Rindler Manifold of Arbitrary Dimension", *Progress of Theoretical Physics Supplement*, 88:1–142, 1986.

[23] C. Rovelli, "What is Observable in Classical and Quantum Gravity?", *Classical and Quantum Gravity*, vol. 8, no. 2, pp. 297–316, 1991.

[24] E. Castro-Ruiz, et al., "Quantum clocks and the temporal localisability of events in the presence of gravitating quantum systems", *Nature Communications* **11**, 2672 (2020).

[25] To remain consistent with the geometric and dynamical constraints of general relativity, each mollusk must be of sufficiently low density to avoid backreactions on spacetime geometry and restricted to a region U small enough to neglect its own self-gravitation and to exclude the formation of collapsing matter.

[26] Flatness corresponds to the vanishing of the Riemann curvature tensor[20]. In such cases, the metric can always be brought locally — and globally if the manifold is topologically trivial — into the canonical Minkowski form η via a coordinate transformation. Thus, any coordinate system defined in a flat spacetime, even if curved, describes special relativity when properly interpreted. For pedagogical reasons, we assume rectilinear coordinates and the canonical form η for SR in freely falling local inertial frames, cf. also Fig. 4, such that the correspondence between coordinate differences and physical intervals becomes most transparent.

[27] While going beyond the classical setting of this work, a brief remark is in order regarding a potential tension between the existence of Fermi–Walker coordinates—which render the metric locally Minkowskian along the worldline of an accelerated observer—and the Unruh effect, whereby such an observer detects a thermal bath of particles in the Minkowski vacuum. This

apparent contradiction is resolved by noting that Fermi–Walker coordinates describe local spacetime geometry, while the Unruh effect reflects the observer's non-inertial coupling to quantum field modes with nonlocal support. Accordingly, local flatness does not preclude nontrivial detector response[22]. For a discussion on how this affects assumptions underlying the observer swarm see also the caveats in the main text.