Comment on “Energy level shift of quantum systems via the scalar electric Aharonov-Bohm effect”

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Abstract

Recently Chiao and his collaborators proposed a novel scalar electric Aharonov-Bohm (AB) effect [Phys. Rev. A 107, 042209 (2023)]. They claimed that a quantum system inside a Faraday cage with a time varying but spatially uniform scalar potential acquires an AB phase, resulting in observable energy level shifts. This comment argues that their analysis is flawed: a spatially uniform scalar potential inside the cage, despite external variations, can be gauged away without altering gauge-invariant observables, such as energy differences, thus invalidating their claim. A possible explanation of this seemingly puzzling result is also given.

1 Introduction

The Aharonov-Bohm (AB) effect [1] demonstrates that electromagnetic potentials influence quantum systems even in field-free regions, traditionally via a phase shift in interference patterns. The magnetic AB effect involves a vector potential $\mathbf{A}$ around enclosed flux, while the scalar electric AB effect requires a potential difference between interferometer paths. Chiao et al. [2] propose a variant where a quantum system (e.g., rubidium atoms) inside a Faraday cage with a time-varying, spatially uniform scalar potential $V(t) = V_0 \cos(\Omega t)$ exhibits energy level shifts, detectable via spectroscopy, rather than fringe shifts. Given subsequent studies based on this proposal [3-6], its validity warrants scrutiny. In this comment, I argue that a spatially uniform $V(t)$ inside the cage, screened from external fields, cannot produce observable energy shifts due to gauge invariance, and thus Chiao et al's proposal is not a version of the electric AB effect. I also give a possible explanation of this seemingly puzzling result.

2 Chiao et al.'s Proposal and Analysis

In Chiao et al's proposal [2], the basic set-up consists of a Faraday cage with a time varying voltage on its surface. Inside the Faraday cage, the **E**-field is zero, and there is only a time-varying, spatially uniform scalar potential $V(t) = V_0 \cos(\Omega t)$, where $\frac{\Omega}{2\pi}$ is the frequency and V_0 is the amplitude. The quantum system used to register the effect of this $V(t)$ is a gas of hydrogen-like atoms inside the Faraday cage such as rubidium gas. According to Chiao et al's analysis, the time varying, spatial uniform potential, $V(t)$, will split the energy levels of the quantum system into a series of energy levels, and the observable energy level shift can be used to probe the scalar electric AB effect. Chiao et al's derivation is as follows.

The Faraday cage has a time-varying voltage $V(t) = V_0 \cos(\Omega t)$ on its surface (radius r_0), yielding inside:

$$V(t) = V_0 \cos(\Omega t), \quad \mathbf{A} = 0 \quad \text{for} \quad r < r_0, \tag{1}$$

with $\mathbf{E} = -\nabla V = 0$ due to screening, and outside:

$$V(r,t) = V_0 \frac{r_0}{r} \cos(\Omega t), \quad \mathbf{A} = 0 \quad \text{for} \quad r > r_0, \tag{2}$$

where $\mathbf{E} = -\nabla V \neq 0$. The Hamiltonian inside is:

$$H = H_0 + eV(t), \tag{3}$$

with:

$$H_0 \Psi_i(\mathbf{r}) = E_i \Psi_i(\mathbf{r}). \tag{4}$$

Solving the time-dependent Schrödinger equation, they obtain:

$$\psi_i(\mathbf{r},t) = \Psi_i(\mathbf{r}) \sum_{n=-\infty}^{\infty} (-1)^n J_n(\alpha) \exp\left(-\frac{i(E_i - n\hbar\Omega)t}{\hbar}\right), \tag{5}$$

where n is an integer, $J_n(\alpha)$ are the Bessel functions, and $\alpha = \frac{eV_0}{\hbar\Omega}$, suggesting energy sidebands $E_i^{(n)} = E_i \pm n\hbar\Omega$, with dominant shifts $E_i \pm eV_0$ for large α.

3 Critique of the Analysis

3.1 Gauge Invariance and Energy Observables

Consider two gauges for the internal system:

- **Gauge 1 (unprimed, as in [2]):** $V(t) = V_0 \cos(\Omega t)$, $\mathbf{A} = 0$, with:
$$H = H_0 + eV(t). \tag{6}$$
The wave function is:
$$\psi_i = \Psi_i \exp\left(-\frac{i}{\hbar}E_i t - i\varphi(t)\right), \tag{7}$$
where:
$$\varphi(t) = \frac{e}{\hbar}\int V(t)dt = \alpha \sin(\Omega t). \tag{8}$$
- **Gauge 2 (primed):** Apply $\lambda(t) = \frac{V_0}{\Omega}\sin(\Omega t)$, so:
$$V' = V - \partial_t \lambda = 0, \quad \mathbf{A}' = \mathbf{A} + \nabla\lambda = 0, \tag{9}$$
(since $\nabla\lambda = 0$ inside). Then:
$$H' = H_0, \tag{10}$$
and:
$$\psi_i' = \exp\left(i\frac{e}{\hbar}\lambda\right)\psi_i = \Psi_i \exp(-iE_i t/\hbar). \tag{11}$$

This is consistent with the initial presupposition (4).

It can be seen that the quantum system has different energy spectra in different gauges, and the energy eigenvalues in each gauge are not gauge invariant. This is a general feature of a time-dependent Hamiltonian with minimal coupling (see [7, p.326]). Now the gauge-invariant energy is:
$$-\left(\partial_t S + eV\right) = -\left(\partial_t S' + eV'\right), \tag{12}$$
where $S = \hbar \times$ phase. In Gauge 1, $S = -E_i t - e\int V(t)dt$, so:
$$-\left(\partial_t S + eV\right) = E_i + eV(t) - eV(t) = E_i. \tag{13}$$
In Gauge 2, $S' = -E_i t$, so:
$$-\left(\partial_t S' + eV'\right) = E_i. \tag{14}$$
Thus, the observable energy remains E_i, and transition energies $E_i - E_j$ are unchanged.

Note that my definition of E_i is not an assertion of static eigenvalues but the gauge-invariant energy eigenvalue for an eigenstate of H_0. It reflects the minimal coupling rule, where $-\left(\partial_t S + eV\right)$ is the observable energy, invariant under gauge transformations. By contrast, the sidebands $E_i^{(n)} = E_i \pm n\hbar\Omega$ obtained by Chiao et al are gauge-dependent, not gauge-invariant shifts of the quantum system's spectrum. Chiao et al propose absorption spectroscopy or EIT, requiring a probe (e.g., a laser) with interaction Hamiltonian H_{int}. If $H_{\text{int}} \ll H_0$, the measurement minimally disturbs the system, then the measured spectrum will reflect H_0's gauge-invariant energy eigenvalues, with no shifts. Stronger H_{int} will alter the system, producing shifts from the interaction Hamiltonian, still not from $V(t)$, unlike the AC Stark effect's field-driven shifts [2].

3.2 Comparison to the AB Effect

We can compare Chiao et al's setup with the magnetic AB effect, where a ring of radius R enclosing flux Φ shifts energies to:

$$E_n = \frac{\hbar^2}{2mR^2}\left(n - \frac{e\Phi}{2\pi\hbar}\right)^2, \tag{15}$$

despite $\mathbf{B} = 0$ [8]. This shift arises from a gauge-invariant phase:

$$\frac{e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{l} = \frac{e\Phi}{\hbar}, \tag{16}$$

tied to nonlocal topology. In contrast, in Chiao et al's setup, $V(t) = V_0\cos(\Omega t)$ is spatially uniform inside ($\nabla V = 0$), with the phase $\varphi(t) = \frac{e}{\hbar}\int V(t)dt$ being local and gauge-dependent, lacking spatial nonlocality. The external $\mathbf{E}$ and $V(r,t)$ are screened by the Faraday cage, irrelevant to the internal system. Unlike the magnetic AB effect, $V(t)$'s spatial uniformity allows $\lambda(t) = -\frac{V_0}{\Omega}\sin(\Omega t)$ to nullify it, leaving no observable effect.

4 Resolution and Broader Context

There remains a puzzle in Chiao et al's setup: when a time varying voltage is added to the surface of the Faraday cage, there will be a time-varying scalar potential $V(t)$ inside the cage, although the E-field will be zero there. Then, it is natural to expect that something inside the Faraday cage must be changed when a time varying voltage is added to the cage. This might be the main reason why Chiao et al think that the quantum system inside the cage such as its energy levels should be affected by the added voltage. But if the physical reality is required to be gauge invariant, then nothing physical inside the Faraday cage is changed by the added voltage, as the above analysis demonstrates.

A possible way to resolve this puzzle is to assume that there is one true gauge in which the potentials accurately represent the state of reality, although it cannot be measured due to gauge invariance [9]. When a time varying voltage is not added to the Faraday cage, the true gauge potentials inside the cage are $V(t) = 0$ and $\mathbf{A} = 0$. While when a time varying voltage is added to the Faraday cage, the true gauge potentials inside the cage are $V'(t) = V_0\cos(\Omega t)$ and $\mathbf{A}' = 0$. Then, the adding of the time varying voltage indeed results in the change of the physical state inside the cage, which is represented by the potentials in the true gauge. However, due to the minimal coupling rule and the local gauge invariance of laws of motion, neither the potentials nor the wave function alone is gauge invariant and measurable, and only certain combining properties of the wave function and the potentials (besides the probability density) are gauge invariant and can be measured, such as $\nabla S - e\mathbf{A}$ and $\partial_t S + eV$.

5 Conclusion

To sum up, I have argued that a spatially uniform scalar potential inside a Faraday cage, despite external variations, cannot result in observable energy level shift of the quantum systems inside the cage, and thus Chiao et al's proposal of a novel version of the electric AB effect is not valid. This critique extends to gravitational analogs [3], where similar gauge properties apply.

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