Clarifying the foundations of Teleparallel Gravity: translational gauge freedom *vs.* local Lorentz invariance

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Abstract

Teleparallel Gravity (TPG) is an alternative, but empirically equivalent, spacetime theory to General Relativity. Rather than as a manifestation of spacetime curvature, TPG conceptualises gravitational degrees of freedom as a manifestation of spacetime torsion. In its modern formulation (as presented e.g. in the book-length study by Aldrovandi and Pereira (2013)), TPG also and expressly purports to be both a gauge theory of translations (G), as well as locally Lorentz-invariant (L). However, the reasoning which these authors invoke in order to implement (L) and (G) is often involved; indeed its mathematical coherence seems on occasion to be questionable. As such, clarification of the reasoning upon which TPG proponents rely in constructing the theory is sorely needed. The present paper will address this need. More broadly, we aim at achieving three interrelated tasks: (i) to shed light on TPG's aspirations of maintaining (G) and (L) at the same time, (ii) to illuminate TPG's conceptual and interpretative structure, and (iii) to offer a succinct methodological assessment of TPG as a theory *per se*.

keywords: *spacetime, geometrisation, gravity, gauge theories, underdetermination*

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1. Introduction

According to received wisdom—which nowadays permeates even popular culture—Einstein taught us that gravity is a manifestation of the curvature of spacetime geometry. Taken to encapsulate the gist of General Relativity (GR), Wheeler's slogan that "spacetime tells matter how to move; matter tells spacetime how to curve" (Misner et al. 1973, p. 5) has been properly canonised.

Intriguingly, though, an idea dating back to Einstein (1928) himself (see, e.g., Sauer (2006)) squarely challenges this orthodoxy. A geometric alternative to GR, equally consistent with observations, Teleparallel Gravity (TPG) represents gravity not as a manifestation of spacetime curvature, but of a different geometric structure: spacetime *torsion*.[1] Following Møller (1961), broader awareness of TPG soon grew within the (gravitational) physics community. Since the work of Lyre and Eynck (2003) and, more recently, Knox (2011), TPG has also started to captivate philosophers.[2]

Vis-à-vis an observationally viable competitor to GR, physicists and philosophers of science will immediately wonder (cf. Stanford (2023)): how to evaluate TPG's merits as a physical theory, over and above its empirical adequacy? The present paper will proffer a systematic answer: we'll methodologically assess the reasoning that TPGists deploy in TPG's construction, as well as the resulting theory itself. As a spin-off, our analysis will clarify several conceptual-interpretative questions that TPG raises (and that have spawned ample misunderstanding). In particular:

- What are TPG's fundamental physical variables and their interpretation?
- To what spacetime setting is TPG committed?
- Does TPG geometrise gravity, or should we rather regard it as a force theory?

Throughout, we'll focus on TPG's modern-day version, expounded most explicitly and comprehensively by Aldrovandi and Pereira (2013). With its main champions being predominantly based in Brazil, we'll henceforth dub it "TPG's Brazilian version" (or just "TPG" simpliciter). Its distinctive objective—largely glossed over in extant philosophical analyses—is to implement two desiderata (to be fleshed out below):

(G): TPG should be a "gauge theory of translations".

(L): TPG should be "locally Lorentz transformation invariant".

[1] Recall that curvature, familiar from *Riemannian* geometry, is the property whereby parallel transporting a vector around a closed loop doesn't preserve the direction in which that vector points. Torsion arises in extensions of Riemannian geometry developed in the early 1920s by Weyl and Cartan (see, e.g., Scholz (2012)), and was independently re-invented by Einstein in the late 1920s (see Sauer (2006)). It denotes the property whereby the operations of transporting two vectors at a point along the directions picked out by the other don't commute. Consequently, parallelograms don't close (see, e.g., Jiménez et al. (2019) for further discussion).

[2] See *inter alia* Dürr and Read (2024), March et al. (2025, 2023), Mulder and Read (2024), Read (2023), Read and Teh (2018), Weatherall (2025), Weatherall and Meskhidze (2024), Wolf and Read (2023), and Wolf et al. (2024a).

The key task before us will be to carefully reconstruct TPG's Brazilian version, and to examine whether TPG successfully achieves this goal. We'll find that to claim that it does so is dubious. In particular, the heuristics that Aldrovandi and Pereira (2013) employ arguably at least in places run afoul of mathematical incoherence. For a version of TPG that respects both (L) and (G), one must move to a different theoretical context, e.g. that of Poincaré gauge theory, and concomitantly to a different geometrical framework, such as that of Cartan geometries.[3]

The plan of the paper is as follows. In **§2**, we'll zoom in on TPG's two central desiderata, (L) and (G). **§3** will reconstruct the reasoning deployed by *inter alios* Aldrovandi and Pereira (2013) in building a theory which supposedly makes good on them. The greater transparency about TPG's construction and conceptual architecture will facilitate reflections on their coherence. In **§4**, we'll address the other above-mentioned conceptual-interpretative questions regarding TPG. **§5** will conclude with an overall methodological assessment of TPG.

2. Desiderata for and historical context of TPG

Let's put a little more flesh on the bones of the two desiderata that Brazilian TPG purports to satisfy—first from a more systematic perspective (**§2.1**), and then from a more historical perspective (**§2.2**).

2.1. Brazilian TPG as a Lorentz-invariant gauge theory.

(G) demands that TPG incorporate key elements of gauge theories (of the classical Yang-Mills type). In particular, TPGists tout it as a gauge theory for the translation group $\mathbb{T}_4$. The implementation of the gauge principle is supposed to achieve a kind of conceptual unification: one would thus be able to describe gravity within the same conceptual framework, harnessing the same principles, as for the other forces of the Standard Model (see, e.g., Aldrovandi and Pereira (2013, ch. 3); cf. Krššák et al. (2019, p. 5), and Hehl (2017, §§1–2)).[4]

(L) demands that TPG respect full invariance under local (point-dependent) Lorentz transformations (LLTs). A precursor to TPG's Brazilian version original form (discovered by Einstein (1928), systematically unpacked by Møller (1961), and gauge-theoretically re-derived by Hayashi (1977), see below) was "purely tetradic" (i.e. formulated solely in terms of (co)tetrad fields with the coefficients ${e^a}_\nu$). The components of this theory's torsionful ("Weitzenböck") spacetime connection are given by

$$\mathcal{W}^{\mu}{}_{\lambda\nu} = e^a{}_\nu \partial_\lambda e^\mu{}_a. \tag{1}$$

The Weitzenböck connection $\mathcal{W}^{\mu}{}_{\nu\lambda}$ is evidently *not* invariant under LLTs of the (co)tetrad fields (with $(\Lambda^a{}_b(x)) \in \mathrm{SO}(1,3)$, and the inverse $(\Lambda_a{}^b(x))$),

$$\left\{ \begin{array}{l} e^a{}_\mu \\ e_a{}^\mu \end{array} \right. \longrightarrow \left\{ \begin{array}{l} \Lambda^a{}_b\, e^b{}_\mu \\ \Lambda_a{}^b\, e_b{}^\mu \end{array} \right. . \tag{2}$$

[3]Only within such a framework can one make sense of both local Lorentz invariance and local translation invariance—see Huguet et al. (2021a,b) and Le Delliou et al. (2020a,b), and for discussion in the philosophical literature March et al. (2025) and Weatherall (2025).

[4]For sceptical remarks on TPG's abilities to do this which are adjacent to our own concerns, see March et al. (2025), Wallace (2015), and Weatherall (2025).

It is, however, invariant under *global* (i.e., rigid or point-*in*dependent) Lorentz transformations of the tetrad fields. As such, the theory appears to violate what one might dub the "principle of equivalence of frames": the assertion that, as far as their suitability for describing physical reality is concerned, all frames (i.e. tetrads) are on a par (more on this below). This feature might strike one as problematic. To salvage local Lorentz invariance (L), Brazilian TPG introduces—in addition to the set of (co)tetrads as auxiliary quantities—a spin connection. One then defines the components of the torsionful spacetime connection in terms of both.

2.2. **From Einstein's sickbed to Brazil.** To understand the origins of these two desiderata, it's best to regard TPG's Brazilian version as the confluence of two lines of research (cf. Aldrovandi and Pereira (2013, ch. 4.1)). The first we'll label "GR's geometric modification". This approach explores an extension of Riemannian geometry, and implements it into GR (at the level of the action or at the level of the field equations). At bottom, one generalises the framework of Riemannian geometry to so-called Riemann–Cartan geometry to allow for torsion. GR's geometric modification then draws on mathematical identities that relate the Riemannian quantities and those of the torsionful geometry (while setting the curvature to zero). The latter are plugged into the original GR expressions to obtain their equivalents in terms of a flat, torsionful connection.

The roots of this approach date back to Einstein's (1928) quest for a unified classical field theory (and more mathematical work by Weitzenböck (1923)). Einstein effectively discovered a reformulation of GR in terms of a torsionful but flat connection. Intent on exploiting the additional degrees of freedom to accommodate electromagnetism, Einstein evinces no awareness of the fact that his Lagrangian density is dynamically equivalent to the Einstein–Hilbert one for GR. To the best of our knowledge, Møller (1961, p. 27) was the first to explicitly articulate this geometric reformulation of GR (by dint of tetrads). Møller also highlights a blemish of this reformulation that determines the key task of latter-day TPG: its "surplus degrees of freedom" (op.cit., p. 28). In modern terminology, insofar as one views the torsionful connection as physically meaningful, LLT covariance is broken; GR's torsionful reformulation posits *in principle* unobservable structure by requiring a preferred class of (globally Lorentz transformation-related) reference frames (tetrads). Each class singles out a different connection: generically, two LLT-related tetrads induce distinct connections.

Some of the earlier work on TPG (e.g., Andrade and Pereira (1999); but also more recently, Krššák et al. (2019) or Hohmann (2023)) portray TPG as a geometric modification of GR: in virtue of mathematical identities, GR's mathematical structures can be reformulated in terms of a flat, torsionful connection (dispensing with a formal commitment to GR's non-flat Levi-Civita connection).

The other origin of TPG's Brazilian version stems from gravitational gauge theories for the translation group $\mathbb{T}_4$. Its clearest beginnings lie in the work of Hayashi and Nakano (1967). Cho (1976) explicitly constructs the gravitational analogue of classical Yang–Mills theory for translations in terms of fibre bundles. The natural Lagrangian density at which Cho arrives, quadratic in the field strength, turns out to be dynamically equivalent to the Einstein–Hilbert action's density. Again, insofar as one regards the field strength as a physically meaningful quantity, LLT covariance is broken: Cho's translational gauge theory of gravity then posits empirically elusive degrees of freedom: an equivalence class of globally Lorentz transformation-related tetrads (see, e.g., Hayashi (1977, p. 442) for an explicit statement, also pointing out that the same isn't true of the (LLT-invariant) Levi-Civita connection). Alongside expositions of TPG as a geometric modification of GR, we also find this gauge-theoretic approach in the early literature on Brazilian TPG (e.g., Andrade and Pereira (1997, 1999) and Andrade (2000)).

Both approaches focussed on GR's gravitational sector: they primarily dealt with GR's vacuum part. To the extent that the non-vacuum case was considered, either (typically for the geometric approach) the general relativistic matter-gravity coupling (with covariant derivatives with respect to the Levi-Civita connection) seems to have been simply retained, or (typically for the gauge-theoretic approach) the minimal coupling prescription was modified by replacing the Levi-Civita connection by the gauge-covariant derivative. In the latter case, strict empirical equivalence with GR was lost.[5] Both approaches were brought together most perspicuously by Hayashi and Shirafuji (1979) (building on Hayashi (1977)). They clarified the mathematical equivalence of the gauge-theoretic and the geometric derivation.

At the heart of TPG's Brazilian version, we discern two motives that drive its further development (with the results of Hayashi and Shirafuji as the point of departure). The first is a desire for a geometrically *distinct*, but empirically equivalent alternative to GR (Arcos and Pereira (2004, 2005); see also Andrade (2000, p. 1)): Brazilian TPGists are bent on preserving GR's empirical content, but recast it via a flat, torsionful geometry. As indicated, this effectively translates into only altering the vacuum/purely gravitational part of GR, and leaving the matter coupling to the Levi-Cevita connection intact.

Given this premise, one inherits the problem of redundancy, alerted to by Møller (1961): *prima facie*, local Lorentz covariance is violated—and, even more rebarbatively, in an in-principle undetectable way. This elicits the second principal motive behind TPG: taking up the baton from Hayashi and Shirafuji's (1979) gauge-theoretic derivation of the gravitational action (or, equivalently, GR's geometric modification), TPGists seek to restore LLT covariance—in other words, to implement (L).

Arguably, the need to implement (L) is perceived as especially strong because in the (implicitly) adopted geometric framework for both the gauge-theoretic and the GR-modificatory approach frames/tetrads have only a subsidiary status: akin to potentials, they merely serve as expedient props for expressing the entities of physical-geometric substance. All (LLT-related) tetrads are on a par, in accordance with the principle of equivalence of frames, mentioned above. In the presupposed Riemann–Cartan geometries (see, e.g., Eguchi et al. (1980) and Hehl and Obukhov (2007)), that is, objective content attaches only to LLT-*invariant* structures; in particular, metric structure, torsion and curvature, are LLT-invariant.[6]

By LLT-covariantising the theory obtained through the gauge-theoretic and geometric-modificatory route, TPG's Brazilian version aspires to "*fully* settle" TPG's "basic foundations" (Aldrovandi and Pereira 2013, p. 40, our emphasis). The key conceptual innovation to that end was the introduction of what is later

[5] In fact, Obukhov and Pereira (2004) argue that even consistency is in jeopardy for that case.

[6] As such, it would be at least unnatural for a theory to break LLT-invariance—albeit not perhaps strictly metaphysically *verboten*.

referred to as the "inertial connection".[7] Regrettably, this crucial move has been largely overlooked in most philosophical analyses of TPG (e.g., Knox (2011, p. 271)) —and has led to fallacious criticism (e.g., op.cit., §2.2; or Garecki (2010)). In short: TPG's Brazilian version is, in essence, the attempt to "upgrade" a gauge theory of gravity for the translation group that reproduces GR's empirical content to a theory that is furthermore LLT-invariant, *thanks to this inertial connection.*

If successful, that alone would render TGP attractive for gauge theory aficionados. The two main gauge theories of gravity—one for the *Poincaré* group, and the other for the larger *affine* group, with both respecting (L)—are the Einstein-Cartan(-Kibble-Sciama) theory (see, e.g., Hehl et al. (1976) and Trautman (2006)) and the metric-affine gauge theory of gravity, propounded by Hehl (2023) and Hehl et al. (1995). The former reduces to GR in its standard form in the absence of spinning matter; in the presence of spinning matter, it deviates slightly from GR (see, e.g., Hehl (2017, sect.10), Hehl et al. (1976)). Hence, strictly speaking, the Einstein–Cartan theory isn't an empirically indistinguishable geometric alternative to GR, as TPGists hanker after; rather, it qualifies as a minor correction of GR *for whenever spin can't be neglected.* Mathematically, the metric-affine theory is *significantly* more complicated, detracting from its appeal.

3. Reconstructing TPG

Ordering and clarifying the arguments and conceptual manoeuvres found in the pertinent literature (and in particular Aldrovandi and Pereira (2013), but also Pereira and Obukhov (2019)), this section will provide a careful and (hopefully) more pellucid reconstruction of Brazilian TPG. First, we'll scrutinise the theory's kinematics: those parts dealing with gravity in the absence of matter sources (**§3.1**). We'll then flesh out Brazilian TPG's dynamics: how matter is supposed to couple to gravity as its source (**§3.2**). For the reader's convenience, by way of a summary, **§3.3** will compile the main steps.

3.1. Kinematic part.

The conceptual architecture of TPG's kinematics—its purely gravitational sector—is erected in two principal stages. The first draws on gauge-theoretic machinery, in an effort to implement (G); the second consists in implementing (L).

3.1.1. *Gauge-theoretic elements.*

Motivated by the considerations sketched in §2, Brazilian TPG starts off with gauge-theoretic elements. The idea is to adopt as the launching pad structures of what one might expect a gravitational gauge theory of the translation group $\mathbb{T}_4$ to look like.

In this spirit, suppose that the principal bundle structure of the theory be given locally by $P \cong M \times \mathcal{A}\mathbb{M}$. Here, $\mathcal{A}\mathbb{M}$ denotes the affine generalisation of Minkowski space.[8] In local coordinates $(x^\mu, \overline{\chi}^a)$, Brazilian TPGists (in particular Pereira and

[7] To the best of our knowledge, it isn't mentioned in the literature prior to Aldrovandi et al. (2009, §§5–6).

[8] In our presentation of these gauge-theoretic elements, we generally follow Pereira and Obukhov (2019, §1.2), while using some of the notation from Wallace (2015).

Obukhov (2019)) claim then that the connection 1-form on this bundle is given by

$$\omega^a = B^a{}_\mu dx^\mu + d\overline{\chi}^a. \tag{3}$$

Now, given some local section $\sigma : M \to P$, we obtain the connection form on the spacetime manifold M via the pull-back of the connection 1-form:

$$h^a := \sigma^* \omega^a = (B^a{}_\mu + \partial_\mu \chi^a) dx^\mu. \tag{4}$$

Here some issues already arise (identified by Huguet et al. (2021a,b) and Le Delliou et al. (2020b)). TPGists want to identify h^a with a co-frame (co-tetrad). However, coframes are usually understood to be the pullbacks (along some local section σ) of the canonical 1-form θ *on the frame bundle*. This, however, *isn't* what is going on in the above construction.[9] Indeed, one can already sense how these points might push one to a Cartan-geometric framework, in which the canonical 1-form θ is folded into the connection (see again March et al. (2025) and Weatherall (2025) for philosophical discussion of this move, the former including extensive mathematical discussion of Cartan geometries which we won't repeat here).

For the sake of charity (and fidelity to TPGists' reasoning), let's set aside these concerns, legitimate as they might be, and press on with an analysis of (3) and (4). *Qua* mathematical function that assigns to any point on M a quadruple of real numbers, $\chi^a = \overline{\chi}^a$. This difference in notation is intended to signal a difference in interpretation: as figuring in the connection 1-form on P, $\overline{\chi}^a$ denotes coordinates of points on the *affine* Minkowski space, $\mathcal{A}\mathbb{M}$, which constitutes the fibres. By contradistinction, χ^a—albeit of the same functional-mathematical form—denotes spacetime coordinates; they label *spacetime* points. That is, mathematically, descriptions in terms of χ^a contain representational surplus structure—as far as the conceptual resources devised hitherto are concerned. Unless one introduces additional assumptions that would underwrite such distinctions, $(B^a{}_\mu + \partial_\mu \chi^a)dx^\mu$ and $B^a{}_\mu dx^\mu$ should however encode the same physical content; the piece $\partial_\mu \chi^a dx^\mu$, as an *artefact* of the introduction of a physically unwarranted origin in the transition from $\mathcal{A}\mathbb{M}$ to (co)tangent space, lacks physical significance.

Following the analogy with electromagnetism, let's *pro tempore* declare h^a our provisional gravitational potential. (Eventually, we'll settle on a different candidate: a modification of h^a.) What field strength—the *physical* quantity representing the gravitational field (as opposed to the auxiliary, unphysical status of the potential)—should we then take this potential to generate? The analogy with electromagnetism suggests a *prima facie* natural candidate: the potential's exterior derivative,

$$T^a[h] := dh^a \equiv \partial_{[\mu} h^a{}_{\nu]} dx^\mu \wedge dx^\nu \equiv \partial_{[\mu} B^a{}_{\nu]} dx^\mu \wedge dx^\nu. \tag{5}$$

At first blush, three features commend this preliminary choice of the gravitational field strength:

1. It's obtained in exact analogy with the electromagnetic field-strength: $F_{\text{EM}} =$

[9] Pereira and Obukhov (2019) propose considering a principal translation bundle. As Le Delliou et al. (2020a, p.4) counter, "(i)n the translation-only gauge formalism summarized by Pereira and Obukhov (2019), the canonical form is identified with the translation connection ω_T, a one-form defined on the bundle of translations-only $P(M, \mathbb{T}_4, \pi)$. This identification is not mathematically allowed."

$dA \equiv \partial_{[\mu}A_{\nu]}dx^\mu \wedge dx^\nu$, where $A = A_\mu dx^\mu$ denotes the electromagnetic 4-potential.

2. In line with our preceding comments on the physical redundancy of an origin in the transition from $\mathcal{A}\mathbb{M}$ to tangent space, the preliminary field-strength doesn't depend on an origin: h^a and $h^a + d\theta^a = (\partial_\mu(x^a + \theta^a) + B^a{}_\mu)dx^\mu$ have the same exterior derivative (with an θ^a an arbitrary differentiable function, corresponding to an alternative choice of origin).

3. Consequently, T^a remains invariant under local gauge transformations. The latter are changes of section,

$$\sigma(x) = (x^\mu(x), \chi^a(x)) \to (x^\mu(x), \chi^a(x) + \epsilon^a(x)), \tag{6}$$

viz. position-dependent translations on tangent space

$$\chi^a \to \chi^a + \epsilon^a(x). \tag{7}$$

It might be tempting to proceed by defining the purely gravitational Lagrangian density by further exploiting the analogy with electromagnetism (after suitably introducing a Hodge dual, see e.g. Aldrovandi and Pereira (2013, ch. 8), and more on which below). But this step would be premature given the ambitions of Brazilian TPG: its advocates demand that the LLT invariance postulate (L) be respected. Evidently, equation (5) flouts this. For a Lorentz matrix $(\Lambda^a{}_b(x)) \in$ SO(1,3), we have:

$$T^a\left[\Lambda h\right] = \Lambda^a{}_b T^b\left[h\right] + d\Lambda^a{}_b \wedge h^b, \tag{8}$$

with $d\Lambda^a{}_b \wedge h^b$, provisional field strength thus acquires a *non*-covariant term. It mars the provisional field strength's LLT-covariance.

Only once we have remedied this in a modification of our tentative gravitational field strength, (5), are we ready for the next step.

3.1.2. *Implementing LLT invariance.* We'll now unpack how TPGists such as Aldrovandi and Pereira (2013) and Pereira and Obukhov (2019) seek to implement the desideratum of LLT-invariance, (L). With the potential h^a formally constituting a co-tetrad (or so the claim goes—despite the voiced worries about the relationship, or lack thereof, between h^a and the canonical 1-form θ on the frame bundle), it accrues (or so TPGists seem to think, recall **§2.2**) objective-geometric content only up to LLTs. One may therefore plausibly require that LLTs not reflect genuine differences in the theory to be constructed.

In the present context, LLT-invariance is understood (see e.g. Krššák et al. (2019, p. 11), Aldrovandi and Pereira (2013, p. 46)) to demand covariance of h^a under LLTs acting on internal indices. We take this to imply that under

$$\begin{cases} \chi^a \mapsto \chi^a := \Lambda^a{}_b\chi^b, \\ B^a{}_\mu \mapsto \Lambda^a{}_b B^b{}_\mu \end{cases} \tag{9}$$

the field h^a (or some suitably modified version thereof) should be invariant. At first blush, this desideratum seems natural to impose. Unfortunately, however, it isn't tenable so long as one retains $B^a{}_\mu$'s transformation behaviour necessitated by ω^a as a gauge connection: *the transformations given above aren't consistent with h^a's invariance under translations (nor with dh^a's gauge-invariance).* One cannot consistently allow arbitrary concatenations (or direct products[10]) of both translations, $\chi^a \to \chi^a + \epsilon^a$, as gauge transformations (resulting in $B^a{}_\mu \to B^a{}_\mu - \partial_\mu \epsilon^a$), *and* LLTs as per (9): translations and Lorentz transformations don't commute; their concatenation would depend on their order.

Vis-à-vis this dilemma, we decide to jettison (G): the desideratum that TPG be a gauge theory of translations. Our choice chimes with the explicit admission of e.g. Kršśák et al. (2019) that the *primary* goal of the modern variant of TPG is to achieve LLT-invariance. We'll return later (**§5**) to the ramifications of abandoning translation invariance.

3.1.3. *LLT-invariance without local translation invariance.* To accomplish (L), TPGists introduce the additional structure of an "affine spin-connection 1-form" $\dot{\omega}^a{}_b \equiv \dot{\omega}^a{}_{b\mu} dx^\mu$ (see, e.g., Eguchi et al. (1980, §2)). This will allow us to declare co-tangent vectors at different points equivalent ("parallel"). TPGists define this spin connection implicitly via:

1. Relative to the co-frame (co-tetrad) h^a (from (4)), its components are supposed to vanish:
$$\dot{\omega}^a{}_{b\mu}\big|_h \equiv 0. \tag{10}$$
2. In virtue of a spin connection's transformation behaviour, relative to a frame/tetrad $h'^a = \Lambda^a{}_b h^b$ that is related to h^a via a local Lorentz transformation effected by the Lorentz matrix $(\Lambda^a{}_b(x)) \in \mathrm{SO}(1,3)$ (with the inverse $(\Lambda_a{}^b)$), its components become:
$$\dot{\omega}^a{}_{b\mu}\big|_{h'} = \Lambda^a{}_c \partial_\mu \Lambda_b{}^c. \tag{11}$$

Note that it follows directly from the first condition, together with the first Maurer–Cartan equation and its exterior derivative (i.e. the first Bianchi identity) that $\dot{\omega}^a{}_b$ has vanishing curvature,

$$\dot{\mathcal{R}}^a{}_b := d\dot{\omega}^a{}_b + \dot{\omega}^a{}_c \wedge \dot{\omega}^c{}_b \equiv 0. \tag{12}$$

(Recall that the Maurer-Cartan Equations are preserved under LLTs.) By contrast, the spin connection's torsion,

$$\dot{\Theta}^a := dh^a + \dot{\omega}^a{}_b \wedge h^b, \tag{13}$$

doesn't vanish: it coincides (LLT-invariantly) with dh^a, and thus with our tentative gravitational field strength (5).[11]

[10] Of course, they may be combined in a way that *does* yield a well-defined group action, namely via their *semi*-direct product. This yields the Poincaré group (see e.g. Weinberg (1995, ch. 2)). For now, suffice it to observe that it requires a different formal setup than the one presented here (and in the standard TPG literature).

[11] This follows from evaluating the first Maurer–Cartan Equation in the frame h^a. The result—$\dot{\Theta}^a \equiv dh^a$—is, of course, *in*dependent of the choice of frame; choosing h^a as the frame for evalu-

Associated with this spin-connection is a(n LLT-)covariant derivative (Aldrovandi and Pereira 2013, §1.2):

$$\dot{\mathcal{D}}_\mu := \partial_\mu + \frac{i}{2}\dot{\omega}^{ab}{}_\mu \mathcal{S}_{ab}, \tag{14}$$

where $\dot{\omega}^{ab}{}_\mu := \eta^{bc}\dot{\omega}^a{}_{c\mu}$ (with $(\eta_{ab}) := \mathrm{diag}(1,-1,-1,-1) = \text{const.}$), and $\mathcal{S}_{ab}$ is a representation of the Lorentz group, appropriate to the object on which the covariant derivative acts (for example, for a scalar field, we have $\mathcal{S}_{ab} = 0$).

3.1.4. *LLT-covariantising the field strength.* With the covariant derivative in place, let's return to TPGists' aim of implementing (L). The task is to find a suitable modification of the tentative gravitational field strength (5), in line with (9).

Two alternative routes suggest themselves. One might first try to redefine the field-strength as the *covariant* exterior derivative of the preliminary gravitational potential (4):

$$\begin{aligned}\tilde{T}^a := \dot{\mathcal{D}}h^a &\equiv 2(\partial_{[\mu}B^a{}_{\nu]} + \dot{\omega}^a{}_{b[\mu}h^b{}_{\nu]})dx^\mu \wedge dx^\nu \\ &\equiv 2(\dot{\mathcal{D}}_{[\mu}B^a{}_{\nu]} + \dot{\omega}^a{}_{b[\mu}\partial_{\nu]}\chi^b)dx^\mu \wedge dx^\nu.\end{aligned} \tag{15}$$

Unfortunately, $\tilde{T}^a$ fails to respect (9), as evidenced by the second term in the last line of (15). Put otherwise, $\tilde{T}^a$ differs from the field strength adopted in the TPG literature—*and* is prevented from conforming to the desideratum of LLT-invariance—by the term $\dot{\omega}^a{}_{b[\mu}\partial_{\nu]}\chi^b dx^\mu \wedge dx^\nu$.

Following the TPG literature (e.g. Aldrovandi and Pereira (2013, §4.5) or Aldrovandi and Pereira (2015, §3.2)), let's therefore first covariantise the *potential* (4), and then redefine the field-strength in an LLT-covariant way, via the covariant exterior derivative of this LLT-covariantised alternative potential.

That is, instead of h^a, one introduces the new potential $\dot{h}^a$, with the partial derivative in (4) replaced by the covariant one associated with the spin connection:

$$\dot{h}^a := (\dot{\mathcal{D}}_\mu\chi^a + B^a{}_\mu)dx^\mu \equiv h^a + \dot{\omega}^a{}_{b\mu}\chi^b dx^\mu. \tag{16}$$

The field strength is now defined as this new potential's *covariant exterior* derivative:

$$\dot{T}^a := \dot{\mathcal{D}}\dot{h}^a \equiv 2\dot{\mathcal{D}}_{[\mu}B^a{}_{\nu]}dx^\mu \wedge dx^\nu. \tag{17}$$

The last identity follows from our earlier observation that the spin-connection $\dot{\omega}^a{}_b$ has vanishing curvature. Consequently, its associated covariant derivatives commute.

In virtue of the LLT-covariance secured by the covariant derivative, this ensures that under $\dot{h}^a \to \Lambda^a{}_b\dot{h}^b$ (or (8) directly), the new field-strength transforms covariantly, $\dot{T}^a \to \Lambda^a{}_b\dot{T}^b$. While we have thus achieved LLT-covariance, we reiterate the drawback hinted at already in **§3.1.2**: we were forced to *sacrifice gauge invariance* (invariance under translations).

ating the torsion is analogous to evaluating gauge-invariant quantities (say, the energy density of an electromagnetic or general-relativistic gravitational wave) by a convenient gauge fixing (e.g., Hobson et al. (2006, ch. 18)).

One can glean this directly from the new gravitational potential $\dot{h}^a$. In contradistinction to the original choice, h^a, under translations $\chi^a \to \chi^a + \epsilon^a(x)$, $\dot{h}^a = (\dot{\mathcal{D}}_\mu \chi^a + B^a{}_\mu)dx^\mu$ *doesn't* remain invariant, due to its construction via $\dot{\mathcal{D}}$ (rather than a partial derivative): $\dot{h}^a \to \dot{h}^a + \dot{\omega}^a{}_{b\mu}\epsilon^b dx^\mu$. The TPG literature (e.g. Aldrovandi and Pereira (2013, p. 46)) tries to address this loss of gauge-invariance by postulating that under translations, $\chi^a \to \chi^a + \epsilon^a(x)$, the B-parts of $\dot{h}^a$ transform according to $B^a{}_\mu \to B^a{}_\mu - \dot{\mathcal{D}}_\mu \epsilon^a$ (rather than $B^a{}_\mu \to B^a{}_\mu - \partial_\mu \epsilon^a$)—so as to compensate for $\dot{\mathcal{D}}_\mu \chi^a \to \dot{\mathcal{D}}_\mu \chi^a + \dot{\mathcal{D}}_\mu \epsilon^a$ in the transformation of $\dot{h}^a$. Unfortunately, this move is obscure at best, and spurious at worst. Recall that $B^a{}_\mu$'s origins lie in the gauge connection ω (as per (4)). As such, it's just *ad hoc* to change its transformation behaviour!

But this *ad hoc*ness runs deeper. It borders on incoherence: because of those origins, the additional, newly-postulated transformation behaviour requires a mathematical object that retains its identity when its components transform covariantly under LLTs as well as in the prescribed manner under translations. What object fits this profile? At best, this is unspecified: conceptually, the object remains entirely unclear. Plausibly, we need an object whose components transform suitably under *combinations* of LLTs and translations, that is, under Poincaré transformations (which are *semi-direct* products of LLTs and translations). But the formal apparatus of which the standard TPG literature avails itself doesn't allow that. So, at worst, we run into an inconsistency. It seems, again, that the mathematics pushes us here to what was naturally invited right at the start: a move to (e.g.) Cartan geometries, and Cartan connections—see Huguet et al. (2021a,b), Le Delliou et al. (2020a,b), March et al. (2025), and Weatherall (2025).

TPG's advocates bear the burden of proof to rebut this objection: it's incumbent on them to demonstrate the existence of a suitable mathematical object. For now, we'll proceed as before by *dropping* the assumption of translation invariance. With $\dot{T}^a$, we obtain a well-defined (albeit, as just explained, translation-*variant*) 2-form,

$$\dot{T} := \dot{T}^a \partial_a. \tag{18}$$

By construction, it's invariant under LLTs. In fact, by (without loss of generality) evaluating it in a frame in which $\dot{\omega}^a{}_{b\mu} = 0$ vanishes (viz. a frame that is a global Lorentz boost of h^a), we see that it coincides with the spin connection's torsion (13).

3.1.5. *Purely gravitational action.* On the basis of the gravitational potential, $\dot{h}^a$ and its induced field strength, $\dot{T}$, our next task is to construct a plausible analogue of the Lagrangian density of the source-free electromagnetic field (or classical Yang-Mills theories more generally), which is proportional to $F_{\mathrm{EM}} \wedge \star F_{\mathrm{EM}}$. For that, we need a Hodge star operator, $\star$.

The standard formal desiderata on a Hodge dual operator are reviewed by Aldrovandi and Pereira (2013, ch. 8). Crucially, a Hodge dual requires a volume element (besides the inner product on tangent space, which we naturally have with the tangent space Minkowski metric $\boldsymbol{\eta}$).

Instead of with the (alleged) tetrad h^a obtained in **§3.1.1**, we'll here have to work with our new gauge potential $\dot{h}^a$. It too induces a metric in a standard way, via

$$\mathbf{g} := \eta_{ab}\dot{h}^a \otimes \dot{h}^b \equiv \eta_{ab}\dot{h}^a{}_\mu \dot{h}^a{}_\nu dx^\mu \otimes dx^\nu. \tag{19}$$

Because of its LLT-invariance, this is precisely the same metric as would be induced by h^a (from (4)). Without loss of generality, we can evaluate the expression of $\mathbf{g}$ in terms of $\dot{h}^a$ by going to a special frame in which $\dot{\omega}^a_{b\mu} = 0$ (viz. a frame that is a global Lorentz boost of h^a, as per (4). Consequently, $\mathbf{g} \equiv \eta_{ab}h^a \otimes h^b$. Accordingly, the associated volume element is given by

$$\sqrt{|\det(\mathbf{g})|} \equiv |\dot{h}| := |\det(\dot{h}^a{}_\mu)| \equiv |\det(h^a{}_\mu)|. \tag{20}$$

With this volume element in place, the definition Hodge star $\star_{\dot{h}}$ as presented in the TPG literature—see Aldrovandi and Pereira (2013, ch. 8)—carries through. To highlight its (implicit) dependence on $\dot{h}^a$, we added the subscript.

In complete analogy with standard gauge theories, the vacuum or "purely gravitational" action (i.e. *sans* matter–gravity coupling) is then finally stipulated to be:[12]

$$\begin{aligned}
\mathcal{S}_0[\dot{h}] &:= \frac{1}{16\pi}\int \dot{T}[\dot{h}] \wedge \star_{\dot{h}}\dot{T}[\dot{h}] \\
&= \frac{1}{16\pi}\int \eta_{ab}(\dot{T}^a \wedge \star_{\dot{h}}\dot{T}^b) \\
&\equiv \frac{1}{16\pi}\int d^4x|\dot{h}|\left(\frac{1}{4}\dot{T}^\rho{}_{\mu\nu}\dot{T}_\rho{}^{\mu\nu} + \frac{1}{2}\dot{T}^\rho{}_{\mu\nu}\dot{T}_\rho{}^{\nu\mu} - \dot{T}^\rho{}_{\mu\rho}\dot{T}^{\nu\mu}{}_\nu\right).
\end{aligned} \tag{21}$$

For transparency, we made explicit the dependence of $\dot{T}$ and the Hodge star on $\dot{h}^a$ in the first line. The third line in (21) (which follows from the second through explicit computation, see e.g. Aldrovandi and Pereira (2013, §9.1)) figures spacetime-indexed terms obtained from $\dot{T}$. With the components of $\dot{h}_a$, we here define

$$\dot{T}^\lambda{}_{\mu\nu} := \dot{h}_a{}^\lambda \dot{T}^a{}_{\mu\nu}, \tag{22}$$

and raise and lower indices in the standard way, as well as contract, via $g_{\mu\nu} \equiv \eta_{ab}\dot{h}^a{}_\mu \dot{h}^b{}_\nu$ and its dual $g^{\mu\nu}$.

Variation of the purely gravitational action (21) with respect to $\dot{h}^a$ yields the vacuum field equations. Before turning to the full field equations—for both the vacuum and the non-vacuum case—let's consider how in TPG matter and gravity are supposed to interact.

3.2. **Adding matter: TPG's action for matter-gravity interactions.** What is TPG's action for the matter sector as the source for gravity? And how should we combine the purely gravitational action (21) and the action parts for matter to form TPG's full action (whose variation yields the complete—matter-*cum*-gravity—field equations)? As we'll elaborate below, the prescription for TPG's matter-gravity interaction is simply the familiar general-relativistic one; it's merely expressed through

[12] A subtlety concerns the fact that the trace operator, involved in defining the purely gravitational TPG Lagrangian, requires a metric. Like in electromagnetism, the most natural choice (the so-called Cartan–Killing bilinear form) can't be used. Again like in electromagnetism, it's natural to use the Minkowskian tangent space metric $\boldsymbol{\eta} := \eta_{ab}d\chi^a \otimes d\chi^b$, with the constant-component matrix $(\eta_{ab}) := \mathrm{diag}(1, -1, -1, -1)$ instead (Aldrovandi and Pereira 2013, p. 89).

suitable combinations of TPG's basic terms (*viz.*, the spin connection, the gravitational potentials, and associated field strength).

First recall the situation in GR. There, the matter/gravity coupling prescription is furnished by the so-called minimal substitution rule in the matter action.[13] It stipulates the replacement of partial derivatives in the special-relativistic by *covariant* ones action to obtain the general-relativistic (and hence generally covariant) one:

$$\partial_\mu \to \nabla_\mu := \partial_\mu - \frac{i}{2}\Omega^a{}_{b\mu} S_a{}^b. \tag{23}$$

Here $\Omega^a{}_{b\mu}$ denotes the Levi-Civita connection's spin-connection and $S_{ab} := \eta_{bc} S_a{}^c$ the representation of the object on which the derivative is supposed to act.[14]

Now to the situation in TPG. The coupling prescription is obtained in two steps. The first is to draw on the well-known fact that, within Riemann–Cartan geometry, an arbitrary compatible Lorentz connection with the coefficients $\mathcal{A}^a{}_{b\mu}$ (and the associated covariant derivative $\mathcal{D}$) decomposes into the metrically-induced Levi-Civita connection's spin-connection $\Omega^a{}_{b\mu}$ and the Lorentz connection's contortion $\mathcal{K} = \mathcal{K}(\mathcal{A})$ (see e.g. Aldrovandi and Pereira (2013, p. 10)):

$$\mathcal{A}^a{}_{b\mu} \equiv \Omega^a{}_{b\mu} + \mathcal{K}^a{}_{b\mu}, \tag{24}$$

with the contortion

$$\mathcal{K}^a{}_{b\mu} := \frac{1}{2}(\mathcal{T}_\mu{}^a{}_b + \mathcal{T}_b{}^a{}_\mu - \mathcal{T}_\mu{}^a{}_b). \tag{25}$$

It in turn is a(n LLT-invariant) function of the Lorentz connection's torsion with the coefficients (relative to a set of tetrads X^a)

$$\mathcal{T}^a_{\mu\nu} := 2(\mathcal{D}_{[\mu} X^a{}_{\nu]}) \equiv 2(\partial_{[\mu} X^a{}_{\nu]} + \mathcal{A}^a{}_{b[\mu} X^b{}_{\nu]}). \tag{26}$$

Spacetime and algebraic indices are inter-converted, as usual, in (25) via the components of X^a or its dual. Indices are raised/lowered with the components of the spacetime or the tangent space metric (their respective duals).

The Lorentz connection to which we'll apply the above decomposition is, of course, the spin connection $\dot{\omega}$, introduced in **§3.1.3**:

$$\dot{\omega}^a{}_{b\mu} \equiv \Omega^a{}_{b\mu} + \dot{K}^a{}_{b\mu}, \tag{27}$$

where the contortion $\dot{K}^a{}_{b\mu} = \frac{1}{2}(\dot{T}_\mu{}^a{}_b + \dot{T}_b{}^a{}_\mu - \dot{T}_\mu{}^a{}_b)$ turns out to be given in terms of the (suitably indexed) components of our gravitational field strength (17).

We are now ready for the second step, which yields TPG's coupling prescrip-

[13] It's known *not always* to be unambiguous. As elaborated above, by verbatim importing GR's coupling, TPG inherits these cases of occasional ambiguity. For the state-of-the-art on minimal coupling and associated ambiguities (and, indeed, how one can understand minimal coupling such that these ambiguities are overcome), see March (2025).

[14] Recall that for an arbitrary set of tetrads $X_b := X_b{}^\mu \partial_\mu$ and their dual co-tetrad $X^b := X^b{}_\mu dx^\mu$, it's defined as $\Omega^a{}_{b\mu} := X^a{}_\lambda \nabla_\mu X_b{}^\lambda \equiv X^a{}_\lambda(\partial_\mu X_b{}^\lambda + \Gamma^\lambda{}_{\mu\nu} X_b{}^\nu)$, where ∇ denotes the standard covariant derivative in GR, induced by the metric's Levi-Civita connection with the coefficients $\Gamma^\lambda{}_{\mu\nu}$. One can write $\Omega^a{}_{b\mu}$ entirely in terms of the X_b: one has $\Omega^{ab}_\mu = X^{\nu[a}(X_\nu{}^{b]}{}_{,\mu} - X_\mu{}^{b]}{}_{,\nu} + X^{\sigma|b]} X_\mu{}^c X_{\nu c,\sigma})$. It's straightforward to verify that the spin connection transforms like a Lorentz connection under LLTs (that is, under $X^a \to \Lambda^a{}_b X_b$ and $X_a \to \Lambda_a{}^b X_b$): $\Omega^a{}_{b\mu} \to \Lambda^a{}_c \Omega^c{}_{d\mu} \Lambda_b{}^d + \Lambda^a{}_c \partial_\mu \Lambda_b{}^c$.

tion. We *keep* the general-relativistic coupling fully intact but merely rewrite the Levi-Civita spin connection via the identity (27):

$$\Omega^a{}_{b\mu} \equiv \dot{\omega}^a{}_{b\mu} - \dot{K}^a{}_{b\mu}. \tag{28}$$

One thus obtains the teleparallel action for matter via the substitution in the special-relativistic matter action:

$$\partial_\mu \to \nabla_\mu = \partial_\mu - \frac{i}{2}(\dot{\omega}^a{}_{b\mu} - \dot{K}^a{}_{b\mu}))S_a{}^b. \tag{29}$$

The resulting teleparallel matter part of the action is then, just like in GR, added to the purely gravitational part. In fact, since the preceding operation amounted to merely rewriting the general-relativistic coupling procedure, TPG's coupling prescription is in a completely natural sense—that of a mathematical identity—literally *the same* as in GR.[15]

3.3. **Recapitulation.** So much for the construction of Brazilian TPG. To close this section, let's quickly recapitulate the main conceptual moves which were made *en route* to this theory:

1. Seek to build a torsionful theory of spacetime which is (i) empirically equivalent to GR and (ii) invariant under both local translations (thereby making good on relevant desiderata from the gauge paradigm) and LLTs.
2. By virtue of not using the appropriate mathematical framework (e.g. Cartan geometry), note that one cannot make good on (ii); as such, our preference would be to abandon local translation invariance.
3. Note *prima facie* problems with building a locally Lorentz invariant theory using the potentials h^a and field strengths T^a.
4. In light of these problems, build a theory—albeit in a somewhat *ad hoc* manner—with appropriately covariantised potentials $\dot{h}^a$ and field strengths $\dot{T}^a$ and an EM-like Lagrangian.
5. Reverse-engineer the matter sector of GR in order to ensure empirical equivalence with that theory.

One can see both TPG's flirtations with mathematical incoherence in (2), and its *ad hoc*ness in (3) and (4). With these points in mind, let's now embark on a more thoroughgoing appraisal of the theory.

4. Illuminating TPG's interpretative structure

Complementing our reconstruction (and rectification) of TPG's formalism, this section will present and substantiate its interpretation: what is TPG about? What are its basic *physical* quantities, and spacetime structure? As a corollary, we'll clarify whether to view TPG as identical with, or distinct from, GR.[16]

[15]For further discussion of this coupling prescription, see Mulder and Read (2024).

[16]This question is also discussed by March et al. (2025, 2023), Weatherall and Meskhidze (2024), and Wolf et al. (2024b).

TPG, we submit, should be interpreted as a theory about the metric g and a metrically compatible connection $\dot{\Gamma}$ (or, equivalently, its associated spin-connection $\dot{\omega}$), inducing the covariant derivative $\dot{\mathcal{D}}$. TPG, we'll argue, is a gravitational theory about spacetime, represented by the metric-affine manifold $\langle M, g, \dot{\Gamma}\rangle$.

Both the metric and the connection represent gravitational degrees of freedom. Like in GR, the former encodes chronogeometric manifestations of gravity. Conceptually independent from the metric, the affine structure, induced by $\dot{\Gamma}$, plays a double role. One is to define the sense of parallel transport (and, accordingly, a covariant derivative) of which one avails oneself in order to formulate TPG (as we'll spell out below). Its second role is to yield TPG's gravitational field strength $\dot{T}$—which corresponds to $\dot{\Gamma}$'s torsion. In this sense, TPG also geometrises gravity (i.e., models it as a manifestation of TPG's geometry). In contrast to GR's orthodox interpretation (see, e.g., Duerr (2020)), though, TPG doesn't reduce the effects of a gravitational force to inertial phenomena.[17] Contrary to the received opinion amongst TPGists, TPG *isn't* a force theory of gravity either. In fact, as we'll expand on below, it turns out to be questionable whether it possesses inertial structure at all (at least in its *ordinary* sense).

It would be facile to simply *stipulate* the foregoing interpretative labels as a priori verbal decrees. We need reasons that motivate an interpretation: it must help us shed light on the theory's organic structure. A fully-fledged account of semantics lies outside of the present paper's ambit. Instead, in line with a focus on a theory's pragmatics (see Curiel (2019, 2023) for further details), our working hypothesis will be to treat an interpretation of a physical quantity as reasonable, provided that we can discern a perspicuous role it plays in the theory's overarching structure. Rather than an idle postulate or an inscrutably opaque epithet, a formal term's physical interpretation elucidates its function within the theory. It redounds to our understanding of how the theory's elements hang together (cf. Elgin (2000, 2006)). The interpretation sketched above delivers this. To that end, we'll inspect TPG's fundamental structure from three different angles—the field equations, the action, and the equations of motion—and show that TPG's proposed interpretation satisfies our adequacy criterion.

TPG's field equations—obtained from varying the action with respect to the gravitational potential $\dot{h}^a{}_\mu$, and identical to the Einstein equations but written in terms of TPG variables—take the following form (Krššák et al. 2019, p. 17):[18]

$$\dot{\mathcal{D}}_\sigma\left(\dot{h}\dot{S}_a{}^{\rho\sigma}\right) = \kappa\dot{h}\dot{\Sigma}_a{}^\rho + \kappa\dot{h}\Theta_a{}^\rho, \tag{30}$$

[17] Interestingly, TPG's *does* unify gravity and inertia in a way that Einstein praises GR for (see Lehmkuhl (2014) and Lehmkuhl (2009)). The difference is that whereas, on Einstein's view, GR unifies both but dispenses with the gravitational force as an independent entity, TPG can be seen as achieving the unification by dispensing with inertia as an independent entity.

[18] Two (slightly different) variants of an alternate—but mathematically equivalent—form of the field equations are given by Krššák et al. (2019, p. 17). Both forms involve, however, pseudo-tensors, *non*-geometric objects (in a precise technical sense) defy a natural interpretation (see e.g., Duerr (2021, §3.3)). We therefore don't consider these forms of the field equations as suitable for identifying physical quantities with perspicuous roles.

where

$$\dot{S}_a{}^{\varrho\sigma} := \frac{1}{2}\left(\dot{T}_a{}^{\varrho\sigma} + \dot{T}_a{}^{\varrho\sigma} - \dot{T}_a{}^{\varrho\sigma}\right) - \dot{h}_a{}^{\sigma}\dot{T}_\theta{}^{\theta\varrho} + \dot{h}_a{}^{\varrho}\dot{T}_\theta{}^{\theta\sigma},$$

$$\dot{\Sigma}_a{}^{\rho} := \frac{1}{\kappa}\dot{h}_a{}^{\lambda}\dot{S}_c{}^{\nu\rho}\dot{T}^c{}_{\nu\lambda} - \frac{\dot{h}_a{}^{\rho}}{\dot{h}}\dot{\mathcal{L}},$$

and

$$\Theta_a{}^{\rho} := -\frac{1}{h}\frac{\delta\mathcal{L}_s}{\delta\dot{h}^a{}_{\rho}}$$

is the standard (general-relativistic) energy-stress tensor for matter, where $\mathcal{L}_s$ is the Lagrangian of a general source field ($\mathcal{L} = \dot{\mathcal{L}} + \mathcal{L}_s$).

The field equations' structure naturally reflects our proposed interpretation: it parallels the "fairly generic template" (Pitts 2015, p. 9) for field-theoretic dynamics, instantiated by, for instance, the inhomogeneous Maxwell equations, or the Gauss law for Newtonian Gravity. That is, the LHS figures a differential operator (*viz.*, a covariant divergence[19]), acting on a linear combination of TPG's field strength. The RHS represents the source terms for the dynamics: energy-momentum due to matter and gravity. Here, the energy-stress tensor of ordinary matter (defined in the standard way, i.e. as the variation of the matter Lagrangian with respect to the metric) is unsurprising. Upon a little reflection, including gravitational energy-stress is likewise plausible. Roughly, assuming that energy-stress in *whatever* form creates a gravitational field, we may expect also gravitational energy itself to contribute to it (cf. Hobson et al. (2006, p. 471) for GR).[20]

A compelling motivation for interpreting $\dot{T}$ as a field strength can be gleaned directly from TPG's Lagrangian for its action (21). Recall that it was explicitly constructed in analogy with Yang–Mills theories, where the Lagrangian is quadratic in the field strength. We may thus conclude that $\dot{T}$ indeed plays the role of a field strength in a clearly discernible (if only analogical) sense.

Talk of TPG's gravitational field strength, however, ought to be divorced from the notion of a gravitational *force*, exerted by a gravitational *field*. By the former we mean a physical entity that deflects test particles from inertial trajectories; a field signifies a spatiotemporally extended physical entity—something on top of spacetime, which depends on the latter, but not *vice versa*. TPG posits neither (pace, e.g., Aldrovandi and Pereira (2013, p. 65) and Pereira (2014, §4.2)).

[19]Note that this covariant divergence operator encodes geometric structure, given by the connection $\dot{\Gamma}$. This is no different from other field theories, with their laws implicitly or explicitly being committed to geometric structure (typically: Euclidean or Minkowksian, cf. Pooley (2013)).

[20]For the sake of the argument, we'll grant that $\dot{\Sigma}_a{}^{\varrho}$ may be interpreted as gravitational energy-momentum. *Prima facie* this seems plausible. First, it's composed of squares of the field strength, as one would expect. Secondly, and more precisely, it's closely related to the natural candidate for gravitational energy, obtained from Noether's theorem: the canonical energy-stress associated with the gravitational field. $\dot{\Sigma}_a{}^{\varrho}$ differs from the latter merely by having the problematic part *removed* that would turn it into a pseudotensor. Thirdly, from a particle physics perspective gravity—as a long-range force—is associated with a massless field (in the technical, field-theoretic sense). One would therefore expect the gravitational energy-momentum tensor's trace to vanish—as is indeed the case (Krššák et al. 2019, p. 17). Notwithstanding such initial plausibility, a more detailed analysis is necessary that takes into consideration desiderata such as conservation laws, reasonable results for gravitational waves in vacuum, static or stationary spacetimes, or energy conditions.

To see why, let's re-examine the principal reason that TPGists tend to adduce in favour of TPG's force-theoretic interpretation. It's based on a consideration of the equation of motion for test particles. With the 4-velocity u, the equations of motion for test-particles are given by (see Aldrovandi and Pereira (2013, p. 64), also for slightly alternate forms):

$$u^{\mu}\dot{\mathcal{D}}_{\mu}u_{\nu} = \dot{T}^{\lambda}{}_{\nu\mu}u^{\mu}u_{\lambda}. \tag{31}$$

From this, Aldrovandi et al. (2009, p. 7) educe "a separation between inertia and gravitation [...]. As a consequence, the right-hand side of the equation of motion [...] represents the purely gravitational force [...]. The inertial effects coming from the frame non-inertiality are represented by the connection term of the left-hand side [...]." The equation of motion in TPG is explicitly interpreted as "a force equation describing the interaction of a spinless particle with the gravitational field. According to this description, the only effect of the gravitational field is to induce a torsion in spacetime, which will then be the responsible for determining the trajectory of the particle" (Andrade and Pereira 1999, p. 14). Torsion here is supposed to "(act) as a force [...], quite analogous to the Lorentz force equations of electrodynamics" (Aldrovandi and Pereira 2013, p. 66).

On two grounds, we reject this interpretation. First, the "separation between inertia and gravity" is specious: *both* sides contain gravitational degrees of freedom. Even though the LHS depends only on the symmetric part of $\dot{\Gamma}$ (and not directly on its anti-symmetric part, and hence torsion), one cannot pronounce the LHS "*purely* inertial", i.e. gravity-free.

To see why, let's tease out the allegedly "purely inertial" contributions on the LHS of the equation of motion in a given gravitational scenario (say, a Kerr spacetime) by ignoring gravity (i.e. by setting the alleged gravitational force—the torsion—to zero). For vanishing torsion, it follows straightforwardly that consistency with the field equations requires that the LHS reduce to the geodesic equation for *Minkowksi* spacetime. This differs *ex hypothesi* from the LHS of the equation of motion in the scenario we considered. Consequently, gravitational effects must be present on the original LHS, too.

TPGists' second reason for asserting a separation of gravity and inertia is that "[i]n the presence of gravity, there is also a preferred class of frames: the class whose anholonomy is related to gravitation only, not [to] inertial effects" (Aldrovandi et al. 2009, p. 3). The argument seems to be that the spin-connection $\dot{\omega}$ (or, equivalently, the spacetime connection $\dot{\Gamma}$) vanishes in a privileged class of (global Lorentz transformation-related) frame, a class that TPGists identify as inertial. Recall now that relative to another Lorentz-boosted frame, by construction the spin-connection takes a non-vanishing form (see (11)); it explicitly depends only on the Lorentz transformation in question. From this, TPGists infer that $\dot{\omega}$ encodes the accelerative effects of this Lorentz boost, and hence inertial effects (cf. also Aldrovandi and Pereira (2013, ch. 2.4)). This argument is irredeemably flawed. For *every* spin-connection ϖ in a Riemann–Cartan geometry, so-called normal frames exist. In them the connection coefficients vanish: $\varpi'^{a}{}_{b\mu} = 0$ (e.g., Hehl (2017, p. 157)). Relative to a frame obtained from a Lorentz transformation of the normal frame via Λ, the connection coefficients take the form $\varpi''^{a}{}_{b\mu} = \Lambda^{a}{}_{c}\partial_{\mu}\Lambda^{c}{}_{b}$—exactly like in TPG. But there is no reason to assume that ϖ is "purely inertial" in origin;

our argument was of complete mathematical generality.

The upshot of the preceding critique is to dismiss the alleged separation of inertia and gravity that TPGists impute to the equation of motion for test-particles. On the received (Aristotelian–Galilean–Newtonian–Einsteinian) view, inertial structure and the notion of a force are defined correlatively (diSalle 2009; Petkov 2012): the former denotes a physically privileged path structure, corresponding to natural states of motion, in which bodies persist whenever unaffected by interactions; forces cause—and are appealed to in order to explain—deviations from inertial states of motion. It seems doubtful that this scheme applies to TPG: as we saw above, gravity enters both sides of the equation of motion for test-particles under the influence of gravity. While $\dot{\Gamma}$ defines a physically distinguished—even if not easily operationalisable (Mulder and Read 2024, p. 126)—affine/path structure, we can't unambiguously identify it with inertial motion, from which gravity may cause deflections. In TPG, the gravitational field strength $\dot{T}$ and TPG's physically privileged affine structure are too intertwined to underwrite the customary distinction between forces and inertial structure. It's much more natural to say that in TPG, $\dot{\Gamma}$ (in tandem with g) is endowed with gravitational significance *en entier*—not merely its anti-symmetric part (i.e. its torsion $\dot{T}$).[21]

In this sense, TPG represents gravity by *geometrising* it (pace TPGists). Following Lehmkuhl (2009, ch. 9), we must however distinguish the sense of geometrisation, characteristic of TPG, from that traditionally attributed to GR. The latter is essentially eliminative: such geometrisation reduces physical degrees of freedom to manifestations of inertial structure. As just argued, TPG doesn't implement this kind of geometrisation. The geometrisation salient to TPG accounts for some gravitational degrees of freedom in terms of geometric properties of an augmented spacetime structure (augmented, that is, *vis-à-vis* a Riemannian geometry, whose non-topological structure is exhaustively characterised by its metric). In TPG, gravitational effects are conceived of as manifestations of the spacetime structure, represented by the metric-affine manifold $\langle M, g, \dot{\Gamma} \rangle$.

One might perhaps object: what about the Levi Civita connection Γ? Why not include it (such that TPG is associated with the metric-*bi*affine spacetime $\langle M, g, \dot{\Gamma}, \Gamma \rangle$)? After all, as we remarked in **§3.2**, TPG's matter-gravity coupling *effectively*—even if only obliquely, through suitable combinations of $\dot{\Gamma}$ and g—employs the Levi-Civita connection. TPGists routinely parade the proof that test particles satisfying TPG's equations of motion satisfy the geodesic equation for the Levi-Civita connection (e.g., Aldrovandi and Pereira (2013, §6.2.3)). Of course, at the level of mathematics one can do this, since Γ is uniquely definable from g (cf. Wolf et al. (2024b)); at the level of interpretation, however, we'll dig our heels in, for we descry no incoherence in insisting that *fundamentally* TPG is committed to the spacetime structure represented by $\langle M, g, \dot{\Gamma} \rangle$, whilst at the same time conceding that matter-gravity coupling can be effectively described by additional geometric structure.[22]

Having established TPG's interpretation as a theory about the metric and a torsionful connection, we can now also settle the issue of TPG's theoretical equiv-

[21] Evidently, our arguments here go beyond those of Mulder and Read (2024), who have it that TPG does have a standard of inertial motion in the sense that it is built around a connection $\dot{\Gamma}$ with geodesics. This is true, but our current discussion shows that there is more to the story.

[22] Cf. Cheng and Read (2021), Passon (2006), and Wallace (2020).

alence with GR: are GR and TPG different formulations of the *same* theory, or should we regard them as empirically indistinguishable, but distinct? For our purposes (cf. Dürr and Read (2024, §2.1)), we'll adopt the view that theories count as distinct, if they admit of distinct, coherent interpretations. In contrast to merely notational ("synonymous") variants of each other, distinct theories limn the world in ontologically perspicuous, individually intelligible and coherent ways (see Butterfield (2018), Coffey (2014), and Møller-Nielsen (2017) for details and independent arguments).

From this stance towards theory individuation, GR and TPG's distinctness ensues (*contra*, e.g., Garecki (2010) and Knox (2011)—but in agreement with Mulder and Read (2024) and Wolf et al. (2024b)). The former is about a spacetime represented by $\langle M, g, \Gamma\rangle$ with the curved, but torsion-free Levi-Civita connection Γ. By contradistinction, TPG is about a spacetime represented by $\langle M, g, \dot{\Gamma}\rangle$, with the flat but torsionful connection $\dot{\Gamma}$. TPG and GR are thus empirically equivalent, genuine geometric *rivals*. As Järv and Kuusk (2023) rightly observe, this makes TPG an interesting case for geometric conventionalists (Duerr and Ben-Menahem 2022; Dürr and Read 2024): TPG is a geometrically alternative theory, empirically no less adequate than GR.

If it's an alternative, ought we maybe to plump for it—perhaps even prefer TPG over GR? So far, we refrained from judgement whether it's a *methodologically good* choice. We'll next turn to that question.

5. Concluding assessments

Having clarified TPG's conceptual foundations and interpretation, we'd finally like to know what to make of it: how does TPG fare *vis-à-vis* customary criteria for theory choice? This section will assess TPG's methodological status. We'll commence with TPG's liabilities before turning to its prospects.

For the sake of charity, we'll assume that the mathematical queries raised in our reconstruction can be addressed. That lingering doubts exist detracts, of course, from TPG's present appeal.

As our above analysis of TPG's conceptual setup disclosed, consistency demands that TPGists renounce either Local Lorentz Covariance, (L), or the claims to a gauge theory, (G). This dilemma undermines TPG's main innovation and selling point since the work of Hayashi (1977) (recall **§2.2**).

We argued for relinquishing (G) as the lesser evil.[23] First, as reported, TPGists themselves tend to prioritise (L). Secondly, the gauge-theoretic elements utilised in TPG's construction were already highly revisionary—if not verging on *ad hoc*—*vis-à-vis* more standard gauge theories. Hence, if one decided to keep (G), the gains

[23] That said, sacrificing (L)—that is, breaking LLT-covariance—shouldn't be regarded as anathema; it wouldn't be an *egregious* choice. First, it effectively amounts to singling out a class of globally Lorentz transformation related frames. To accept their status as physical doesn't seem significantly worse than accepting absolute velocities in Newtonian physics. Albeit perhaps philosophically not optimal (see, e.g., Pooley (2013)), we have learnt to live with them. Secondly, treating TPG, like GR, as a low-energy effective theory that receives higher-order correction terms, we may conclude that TPG is best seen as a limit of some $f(T)$ Gravity (or even something more general). Within these theories, the loss of LLT-covariance is harmless: the dynamical symmetries of such theories will generically be only global Lorentz transformations, not local ones (cf. Pooley (2013, §4)).

in coherence with the gauge paradigm would be small. Thirdly and finally, as we'll elaborate presently, in terms of 'number' of *redundant* (or underdetermined) degrees of freedom, abandoning (L) amounts to a degeneracy of ∞^6 options (Garecki 2010, p. 12), corresponding to the six free functions characterising a local Lorentz matrix $(\Lambda^a{}_b) \in \mathrm{SO}(1,3)$. By contrast, abandoning (G) amounts to "merely" a degeneracy of ∞^4, corresponding to the four free functions characterising a translation. The balance favours trading (G) for (L): we "save" ∞^2 redundant degrees of freedom.

Given that choice, a main demerit of TPG comes to the fore: TPG postulates surplus structure.[24] That is, TPG presupposes more structure (viz. the choice of an origin on tangent space) than its dynamical laws "see" and "make use of". Hence, TPG presupposes more structure than is in principle observable: different choices of an origin are equally possible. Correlatively, we may say that TPG is in principle *underdetermined* up to a choice of a tangent space origin, a choice which is immaterial for the theory's further assertions about the world (analogously to absolute velocities in Newtonian mechanics).

Another line of critique targets TPG's *ad hoc*ness. We already noted the tinkering—departures from standard gauge-theoretic constructions—requisite for TPG's construction (at the purely gravitational level). That, in the end, we were forced to jettison a central tenet of the gauge paradigm—gauge invariance of the field strength—only exacerbates the sense of artificiality. Regrettably, it's not confined to the gravitational level. TPG's (*non*-minimal) coupling prescription not only contravenes what one would expect in a gauge theory of gravity. It wouldn't be unfair to characterise it as transplanted from GR (after suitably re-written in TPG's geometric terms); TPG's coupling prescription is *engineered* to reproduce GR's empirical content. Arcos and Pereira (2005, p. 9), for instance, admit that their "basic guideline will be to find a coupling prescription that results [in] equivalen[ce] to the coupling prescription of general relativity". TPGists "graft" (as Lakatos (1989) would aptly put it) GR's coupling prescription on TPG. In this sense, TPG's matter-gravity coupling counts as *ad hoc*: it doesn't emanate from the heuristic resources inherent to TPG but has to imported from GR. Note that ad-hocness of this Lakatosian ilk—the charge of heuristic patchiness (for Lakatos portending an unhealthy idea, e.g., op.cit., p.5; p. 112, fn. 2)—isn't to be conflated with inconsistency; rather it articulates a methodologically suspect form of contrivance.[25]

[24] We stress that such symmetry-related redundancy doesn't necessarily constitute a fatal flaw of a theory. It may perhaps be viewed as a strong *motivation* for looking for an alternative, purged of the redundancy. With GR, such an unimpeachable alternative to TPG is on the table. Nonetheless, we plead for permissive pluralism: a symmetry-free alternative may be desirable, but *not mandatory* (see, e.g., Møller-Nielsen (2017) and Read and Møller-Nielsen (2020) for details).

[25] The comparison with Einstein-Cartan theory is instructive. Here the coupling prescription, *in line* with what one would expect in a standard gauge theory, would be

$$\partial_\mu \to \mathcal{D}_\mu := \partial_\mu - \frac{i}{2}\mathcal{A}^{ab}_\mu S_{ab} \equiv \nabla_\mu + \frac{i}{2}\mathcal{K}^{ab}_\mu S_{ab}, \tag{32}$$

where $\mathcal{A}^{ab}_\mu$ denotes the Einstein-Cartan spin-connection (the theory's gauge connection), S_{ab} a suitable representation of the Lorentz group (appropriate to the object the derivative is supposed to act upon), ∇_μ the covariant derivative associated with the metric's Levi-Civita connection, and $\mathcal{K}^{ab}_\mu$ the contorsion of the Einstein-Cartan spin-connection. For vanishing spin, however, the torsion and contorsion vanish; the coupling then reduces to the general-relativistic coupling.

Note that, albeit minimal with respect to the theory's *natural* connection, the coupling in

After these setbacks for TPG's advocates, let's turn to its plausible promises. Four stand out as (even if at this stage somewhat tentative). They, to our minds, *justify* further research:

1. *Gravitational energy.* In GR, the issue of how to define gravitational energy (or whether the notion itself makes sense in GR) is notoriously controversial (see, e.g., de Haro (2022), Duerr (2020), and Szabados (2009)). At first blush, in TPG, the problem seems more tractable (Aldrovandi and Pereira 2013, ch. 10): as we saw, a *bona fide* tensor emerges naturally within the field equations that is related to canonical energy-stress, associated with gravity. Whether an interpretation as representing gravitational energy-stress is ultimately satisfactory, though, remains to be seen, as we indicated. In particular, further research is necessary to investigate whether TPG's gravitational energy turns out to be useful for calculations, explanations or interpretations (e.g., for understanding energy extraction processes in astrophysics, or shedding new light on asymptotic flatness).

2. *Potentially fruitful extensions.* TPG suggests two generalisations that *prima facie* deserve further pursuit. One concerns so-called $f(T)$ gravity. They are the TPG analogues of GR's generalisations to a general (rather than merely a linear) function of the Ricci scalar, so-called $f(R)$ gravity: the gravitational action in $f(T)$ gravity is taken to be a general (rather than merely a linear) function of the torsion (see, e.g., Bahamonde et al. (2023), Cai (2016), and Paliathanasis (2016)). Such models have been mooted as alternatives to Dark Energy and inflation. Like $f(R)$ Gravity (over which $f(T)$ gravity has some advantages), they are worthy of exploration as effective or toy models in their own right. A second line of further inquiry concerns Kaluza–Klein theories, attempts to unify electromagnetism and gravity by allowing extra dimensions. For such extensions, TPG has been argued to trump GR (see, e.g., Aldrovandi and Pereira (2013, ch. 16)).

3. *Newtonian Teleparallel Gravity.* Recent literature has studied the Newtonian limit of TPG (e.g., Read and Teh (2018), Schwartz (2023), Schwartz and Blanckenburg (2024), Vigneron et al. (2025), and Wolf et al. (2024a)): it yields a geometrised version of Newtonian Gravity, of significant interest for mathematical and conceptual reasons, as well as applications in condensed matter physics or string theory.

4. *Improved action principle.* Certain natural technical advantages of the TPG action have been claimed over the Einstein–Hilbert action (Hammad 2019; Krššák and Pereira 2015). These may carry over to applications for black hole horizons and gravitational thermodynamics (see Wolf and Read (2023) for some discussion).

With a modicum of charity, we may conclude that TPG receives a *mixed* score in terms of its merits and shortcomings. It would strike us as gung-ho to assert TPG's superiority over GR. Entirely reasonable, however, seems the more modest

Einstein–Cartan theory is non-minimal with respect to the *Levi-Civita connection*, thanks to the mathematical identity in the last equation. This stands in contrast to TPG, where the natural connection doesn't couple minimally.

claim about its in-principle viability and its status as an interesting, geometric alternative to GR that—warts and all—deserves further research.

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