Misrepresenting the Hole Argument?

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Abstract

Key elements of the recent dialectic surrounding the hole argument in the philosophy of general relativity are clarified by close attendance to the nature of scientific representation. I argue that a structuralist account of representation renders the purported haecceitistic differences between target systems irrelevant to the representational role of models of general relativity. Framing the hole argument in this way helps resolve the impasse in the literature between Weatherall and Pooley and Read.

1 Introduction

The hole argument poses an interpretational problem for general relativity (GR), and experts in the field cannot agree on its resolution. Nonetheless, practising physicists working with GR will likely have never heard of the supposed problem (other than, perhaps, Einstein's original formulation), let alone encounter in practice the problems of indeterminism and underdetermination that some philosophers claim it entails. This contrast in attitudes requires explanation. There have been three distinct waves of debate, as summarised in Section 3. The primary goal of this paper is to dissolve a specific impasse in the dispute within the most recent phase of debate between Weatherall (2018), who argues attendance to mathematical practice allows us to block the hole argument, and Pooley and Read (2025) who argue that Weatherall's position is not justified.

My resolution of this dispute draws on Van Fraassen (2008) account of scientific representation. I argue that the hole argument itself *is* of no importance for physics, but this *does not* mean that the recent flurry of literature has been of no importance for the philosophy of physics. Contrary to Weatherall (2018), mathematical practice alone is not sufficient to block the hole argument, and it fails to preclude other uses of isomorphic models to represent merely haecceitistic differences. Moreover, those who believe in spacetime points with haecceities should use isomorphic models to represent them, and in this limited sense Pooley and Read (2025) are correct. Nonetheless, an account of scientific representation, attentive to the ways in which models are used to represent their target systems, reveals that the hole argument assumes a representational difference that cannot make any difference to scientific practice. Surrogative reasoning about a target system is possible in virtue of the representation relation, which, I argue, should be understood as a kind of structural mapping, for which haecceitistic differences are irrelevant. Hence, scientific practice is invariant with respect to supposed haecceitistic differences between models.

The structure of the article thereafter is as follows. The following Section 2 presents a structuralist account of representation, drawing heavily on Van Fraassen (2008), that emphasises the role of 'use' in representation. Section 3 then introduces the hole argument in historical, formal, and philosophical context. Section 4 diagnoses the reasons for the impasse in the dialectic between Weatherall (2018) and Pooley and Read (2025). Thinking carefully about representation can help articulate the sense in which they are talking past each other, and shows that their dispute is not one to be resolved, but dissolved.

2 Scientific Representation

The aim of this section is to distinguish scientific representations from mere representations. The three most immediate questions one can ask of any account of scientific representation are:

1. In virtue of what do carriers (models, mathematical structures, etc.) represent their target systems?
2. How does the answer to (1) explain the ways in which carriers are used to generate and test hypotheses about their target systems?
3. What, if anything, distinguishes scientific representations from representations more broadly?

These questions align well with part of the framework for scientific representation developed in Frigg and Nguyen (2020). In particular, (2) and (3) correspond to the following:

Surrogative Reasoning Condition (SRC): "[M]odels represent in a way that allows scientists to form hypotheses about the models' target systems: they can generate claims about target systems by investigating the models that represent them."

Scientific Representational Demarcation Problem: "Do scientific representations differ from other kinds of epistemic representations [i.e., representations that meet (SRC)], and, if so, wherein does the difference lie?"
(Frigg and Nguyen 2020, 3)

I argue for a structuralist answer to (1), and use this to answer (2) and (3). The answer I present to (1) is that a necessary, but not sufficient, condition for carriers to represent their target systems is that there is some variety of structure preserving mapping (determined by the representational context) between the mathematical structure and the target system. The (SRC) is then easy to meet, as structuralist representation allows for surrogate reasoning in virtue of the structural mapping. The answer to (3) requires the most work; indeed, it is not within the scope of the current paper to defend fully this claim for non-mathematised sciences. Nonetheless, I contend that, at least for modern physics, scientific representations are distinct from representations more broadly.

Depending on the intended application, the exact variety of mapping may differ. If one wants to consider the topology of a human, then a homeomorphism to which represents them as a seven-holed doughnut will do; but, this would clearly be inadequate for representing physiological traits. Structuralist representation is not, as some are tempted to characterise it, a simple case of the carrier and target being similar (i.e., resembling each other) in some unspecified way. Similarity in an unrestricted sense is irrelevant for scientific representation. Yet, by talking about models representing systems without specifying how they are used to do so, Leibniz equivalence intuitively pushes one towards this kind of mistaken reasoning. While this view may fail to characterise representations more generally, I argue it does capture an indispensable aspect of scientific representations.

Van Fraassen (2008) develops an account of scientific representation that serves as a constructive starting point for the present paper:

> 'Essential to an *empiricist structuralism* is the following core construal of the slogan *all we know is structure:*
>
> *I*. Science represents the empirical phenomena as embeddable in certain *abstract structures* (theoretical models).
>
> *II*. Those abstract structures are describable only up to structural isomorphism.
>
> In the empiricist version, the structuralist slogan is clearly and substantially qualified. ... [T]he slogan must be read as meaning, at

> best, that all we know *through science* is structure.' (van Fraassen 2008, 238)

The final slogan given implies the claim that surrogative reasoning is possible in virtue of the representation relation. Beyond general structuralism, the key idea I want to borrow from van Fraassen is that for any carrier, the 'structural relationship to the phenomenon is of course not what makes it a representation, but what makes it accurate: it is its role in *use* that bestows its representational role.' (van Fraassen 2008) One need not accept, as van Fraassen claims, a variety of empiricism that epistemically privileges the human senses. One can instead adopt a form of methodological empiricism (see Ladyman forthcoming). Frigg and Nguyen arguably mischaracterise Van Fraassen when writing of him: "the crucial ingredient is now the agent's intention, and isomorphism has in fact become either a representational style or normative criterion for accurate representation" (Frigg and Nguyen 2020 ,64). Although van Fraassen takes the ways they are used to be what bestows the representational role, and it would be peculiar not to accept use as sufficient for intention, intending a carrier to represent a target system is clearly not sufficient for use, *in the context of scientific practice.*

In the context of modern physics, I prepose philosophers of science should be guided by the following five theses regarding scientific representation:

> **Thesis 1**: Scientific representation, like non-scientific representation, includes reference to physical systems via ostension, denotation, and stipulation. Reference is the most minimal form of representation.

> **Thesis 2**: Scientific representation includes structural representation, but being a structural representation is not sufficient for being a scientific representation.

Some cases of structural, but non-scientific, representations can be thought of as proto-scientific. To illustrate this point, consider the example of siteswap notation for juggling, where throws are represented by non-negative integers that denote the number of beats later the same object is thrown again. Basic arithmetic operations can be applied to discern features of a given pattern, hence surrogative reasoning is possible; but, this should not be classed as a scientific representation of the system. The same system could be scientifically represented using the language of differential equations, although this would be of no use to the performer. Moreover, not all features of the target system need to be represented: for instance, the colour of the props is irrelevant to the successful representation of a juggling pattern, and the failure to represent this does not imply a problematic indeterminacy in the models used.

> **Thesis 3**: Many scientific representations are mathematical.

Modelling in some physical contexts, including the present context of GR, is achieved via mathematical representation. Surrogative reasoning is possible in virtue of these mathematical structures. There are, of course, some edge cases as to what should be considered mathematical or scientific, such as the example considered above.[1]

> **Thesis 4**: Non-mathematical representations used in science (e.g., theoretical reference), rely on pre-existing structural models for their justification.

For example, one can refer to the perihelion precession of Mercury with confidence, but the justification for this comes from the mathematical models

[1] It would be a misguided overstatement to claim a sharp divide between mathematics and natural language and align this with a divide between scientific representations and non-scientific representations, as was arguably the case in Wallace (2024). Nonetheless, for present purposes, I am concerned with a clear case of scientific representations which are mathematical representations.

of GR, which were successfully used to predict the outcomes of observations. Scientific representations involve structural representations used for explanation and prediction. They may involve non-representational surplus structure, and often incorporate non-structural elements, such as stipulation, denotation and ostension.

> **Thesis 5**: What makes a model a *scientific* representation is its *use* in scientific practice.

Making the way in which models are used part of the defining criteria for scientific representation according to Thesis 5 requires also making the representation relation itself explicit. If we do this, we will be able to see in later sections why the metaphysical possibilities entailed by the hole argument according to Pooley and Read are not a problem for scientific representation. A general instance of including the representation relation between a model and target system is captured in the following schema:

$$\begin{tikzcd} M \arrow[r, "f"] \arrow[d, "R"'] & \tilde{M} \arrow[dl, "\tilde{R}"] \\ S & \end{tikzcd}$$

We thus have that, according to the view expressed in the schema above, if a model M (scientifically) represents a target system S via a representation relation R and f is a function corresponding to some symmetry of M such that $\tilde{M} = f(M)$, then there exists an $\tilde{R} \coloneqq R \circ f^{-1}$ such that $\tilde{M}$ (scientifically) represents S via $\tilde{R}$. Moreover, $R \coloneqq \tilde{R} \circ f$.

A simple example can help illustrate why this claim is almost trivial. Consider a Euclidean plane E^2 equipped with the standard Euclidean metric. This can then be used to represent via some R a basic system of particle dynamics

M, with the state of a particle represented by a point $\mathbf{p}(t) \in E^2$, and its evolution is represented by a curve $\gamma(t) \subset E^2$. Next, define $\tilde{E}^2$ as the result of an isomorphism acting on E^2, e.g., rigid rotation of E^2, such that $\tilde{E}^2 = f(E^2)$. Evidently, there exists an $\tilde{R} := R \circ f^{-1}$ such that $\tilde{E}^2$ also represents M via $\tilde{R}$. The fact that $\tilde{E}^2$ could also be used to represent a distinct system via a different representation relation is of no consequence. Indeed, the isomorphism f is of no substance here, as one could freely use E^2 to represent a variety of physical systems by varying the representation relation alone. One is able to use mathematically equivalent models to represent distinct physical systems, and one is able to use mathematically inequivalent models to represent physical systems that are qualitatively the same. However, in both these instances, for it to matter, you need to be able to distinguish by another means, i.e., there exists a procedure for knowing you are using or representing one and not the other. The hole argument is not like this. That is, the purported metaphysical differences between worlds cannot be tracked by a representation relation used in practice.

3 A Brief History and Overview of the Hole Argument

I begin this section with a contemporary presentation of the hole argument, before briefly covering some relevant history of how the analysis of hole argument has developed.

Providing a neutral formulation of the hole argument is a challenge, as it's easy to slip into interpretational talk. The argument as presented here is representative of the standard formulations in the literature and is sufficient for the present analysis. Consider a Lorentzian manifold, (M, g_{ab}) Figure 1. Next

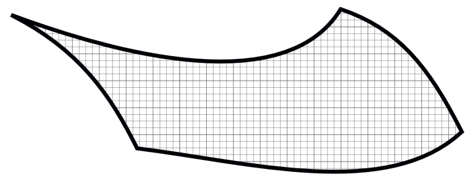

Figure 1: The manifold M.

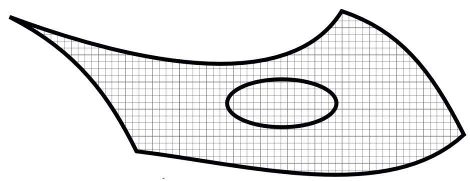

Figure 2: The 'hole' pre-diffeomorphism.

specify a diffeomorphism $d : M \to M$ which acts as the identity outside some open set $O \subset M$ with compact closure (the 'hole')Figure 2, but on O, d is not the identity. Thus, d is not the identity map on M. From this, one can generate an isometric Lorentzian manifold $(M, \tilde{g}_{ab})$ where the underlying manifold M is the same, but the metric is defined by the pushforward map determined by the diffeomorphism acting on the original metric: $\tilde{g}_{ab} = d_*(g_{ab})$. The problem is as follows: (M, g_{ab}) and $(M, \tilde{g}_{ab})$ are isometric, meaning they are equivalent as Lorentzian manifolds, and agree on all invariant structure. But, each appears to assigns different metrical properties to points within the open set O Figure 3. The laws of GR do not, from any proper initial data hypersurface outside of O, uniquely determine whether (M, g_{ab}) or $(M, \tilde{g}_{ab})$ develops. Therefore, the argument goes, the hole argument demonstrates a failure of determinism for GR.

The hole argument as detailed above is purely formal, so it is difficult to see why one should be concerned with the conclusion, as the notion of determinism isn't prima facie the familiar kind of determinism from contemporary metaphysics. The argument is typically made less abstract in the following way,

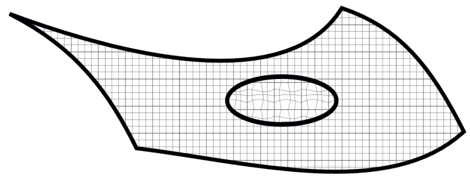

Figure 3: The 'hole' post-diffeomorphism.

but this kind of thinking is what, I argue, gives rise to the problems present in the literature. If one supposes (M, g_{ab}) and $(M, \tilde{g}_{ab})$ can be used to represent distinct physical possibilities, as a manifold substantivalist does, then the hole argument is said to show that for any initial state of the universe at a time t, there fails to be a single unique state for all future times. Again, this reasoning isn't without issue.

The hole argument was originally devised by Einstein in 1913, during his struggle to formulate a generally covariant theory of gravitation. He wondered if the reason he couldn't find such a theory was because it wasn't possible. The hole argument was posed to support this, by aiming to show that any generally covariant theory of gravitation would violate determinism; hence, no such theory was admissible. During 1915, Einstein realised this was mistaken, and successfully discovered the generally covariant theory in GR. What went wrong with the hole argument, he claimed, was that it failed to adhere to the principle known as the 'point coincidence argument,' which states that only the spacetime coincidences, the intersection of field events, are physically meaningful, and these are preserved under diffeomorphism. Thus, there is no failure of determinism.

Over the following century, the hole argument has been become detached from its practical origins, and elevated to the status of a stable topic for graduate seminars in philosophy. In Earman and Norton's revival of the hole argument, the target was (manifold) substantivalism, a view which few, if any, defend outright. Manifold substantivalism is the view that the bare manifold M represents spacetime, and that points of the manifold p represent spacetime points independently of the metric. (Earman and Norton, 1987) They contend that, unless one adopt the interpretative principle of Leibniz equivalence, the substantivalist is committed to a kind of radical indeterminism. The idea is that if (M, g_{ab}) and $(M, \tilde{g}_{ab})$ are taken to be capable of representing distinct physical possibilities, say W and $\tilde{W}$, then as the area outside the hole $M - O$ is identical for both, the theory seems to fail to determine whether W or $\tilde{W}$ will be realised from $M - O$. Earman and Norton argued that this is resolved if one accepts Leibniz equivalence: "If two distributions of fields are related by a smooth transformation, then they represent the same physical systems" (Norton 2023).

It is unsurprising that this wasn't accepted as the final word on the hole argument; the principle they advocate is too imprecise to be of use, and the implications of the argument were seen as significant, even by some who had never seriously considered the viability of substantivalism. Indeed, in Weatherall's paper, he claims to be able to remain "essentially neutral on the metaphysics of space and time" (Weatherall 2018, 330).Weatherall may well be overstating the extent to which his analysis remains metaphysically neutral, but it is true that the most recent wave of literature has largely diverted from the metaphysical concerns of Earman and Norton (Fletcher 2020, Ladyman and Presnel 2020, Roberts 2015, and Thebault and Gryb 2016).

One of the primary aims of the current paper is to stress that, as a consequence of Thesis 5, one cannot justifiably talk about models either representing or failing to represent the same physical system, in the way Earman and Norton did, without talking about the ways in which they are *used* to do so. Applying this reasoning to the hole argument will, I argue, entail methodological principles for doing scientific metaphysics, but without defending any particular metaphysics of spacetime. Nonetheless, the conclusions of this paper do imply that certain metaphysical positions are outside the scope of the *philosophy of science*, at least where the discipline pays sufficient attention to representational practice in science.

4 Weatherall vs Pooley and Read

This section examines the impasse between Weatherall, who attempts to block the hole argument while remaining neutral on the metaphysics, and Pooley and Read, who argue that Weatherall's argument illegitimately precludes metaphysical views. Weatherall's approach is novel, in that attempts to block the argument at the level of the underlying mathematics, prior to the mathematics being problematically interpreted. He writes "I do not intend to argue that there is a mathematical solution to an interpretational problem. Rather, I will argue that the mathematical argument that allegedly generates the interpretational problem is misleading" (Weatherall 2018, 330-331). Once one identifies this mistake, he claims, one is no longer forced to pick between the metaphysical theses of Leibniz equivalence and substantivalism. This is the sense in which he can claim to remain neutral regarding the metaphysical theses, as the hole argument is supposedly blocked before they're even invoked:

> '(1) our interpretations of physical theories should be guided by the formalism of those theories; and (2) insofar as they are so guided, we need to be sure we are using the formalism correctly, consistently, and according to our best understanding of the mathematics. ... Insofar as one wants to claim Lorentzian manifolds are physically equivalent ... one has to use d to establish a standard of comparison between points. And relative to this standard, the two Lorentzian

manifolds agree on the metric at every point—there is no ambiguity, and no indeterminism. ... the Hole Argument seems to be blocked.' (Weatherall 2018, 339)

Weatherall is claiming that the supposed threat of indeterminism only arises from an incorrect understanding of the mathematics; the standard of comparison between points is given by the isometry, and according to this standard, no interpretive problems result. I am broadly sympathetic to this strategy, but without further justification a metaphysician who regularly treats mathematically equivalent models as metaphysically distinct is given no compelling reason to stop doing so, regardless of their technical proficiency. Pooley and Read identify several lines of argument in Weatherall's paper, and present counterarguments to each. My presentation of Weatherall above amounts to what they term 'the argument from mathematical structuralism,' and it is their response to this that creates the dialectic which, I claim, an account of scientific representation can be used to achieve a resolution. Pooley and Read counter as follows:

> 'Weatherall's claim that the hole argument is blocked when models are compared using $[d_*]$ is also questionable, for the interpretation of models that such a comparison might mandate does nothing, by itself, to eliminate the relevant space of metaphysical possibilities.' (Pooley and Read 2025, 24)

One can then ask what these metaphysical possibilities are, and whether there exist alternative motivations for eliminating them. In particular, can the putative difference between the two models in the hole argument scientifically represent a metaphysical difference between worlds? Given that M and $\tilde{M}$ are said to be empirically equivalent, the only possible kinds of differences between the worlds they represent are non-qualitative. Haecceitism is the thesis that worlds can differ in non-qualitative ways. Worlds can be numerically distinct while being qualitatively the same. Haecceities are non-qualitative properties P that supposedly make each individual the one it is. Pa is a haecceity of a only if ∀x $(Px \leftrightarrow (x = a))$, but the condition on the right hand side is not sufficient. Haecceities are usually understood as sufficient for haecceitism. Pooley and Read claim that Weatherall excluding these properties outright is without justification:

By precluding diffeomorphic models representing different states of affairs, Weatherall is illegitimately ruling out possible metaphysical views. Pooley and Read substantiate this point:

> 'A substantivalist who believes that possible spacetimes can differ merely haecceitistically is after a way to represent the structure of such a spacetime, which (let us suppose) really is no richer than that of a Lorentzian manifold, and at the same time talk about two possibilities that exemplify the very same structure but differ over which individuals possess which particular properties. How better to do this than to use isomorphic models of the appropriate structural type that differ merely over which of the base elements of their sets are assigned the structural properties common to both models?' (Pooley and Read 2025, 38)

Mathematical practice alone doesn't prevent one using isomorphic mathematical models to represent different metaphysical states of affairs. Weatherall is right to argue that the problem needs to be evaluated with respect to practice, but it is *physical* practice that should drive one's metaphysics.

5 Conclusion: Dissolving the Disputes

Returning to the hole argument, it follows that it is not intrinsic to p and $\tilde{p}$ what they represent. For either (M, g_{ab}) or $(\tilde{M}, g_{ab})$ to function as a model of a spacetime, one needs to include a representation relation connecting the carrier with the target system. Absent this, p and $\tilde{p}$ are just mathematical points without representational content. Moreover, when considering an isomorphism acting on a model, one must specify whether a corresponding transformation acts on the representation relation or the latter is left unchanged. As described above, there then exists a representation relation such that the original target system is scientifically represented.

The dialectic between Weatherall and Pooley and Read reached an impasse, as each presents a position which is defensible when understood in the relevant context. As the contexts are distinct, however, their arguments fail to properly connect with each other. Progress can be made by thinking about how we understand scientific representation. Pooley and Read are correct to claim that one is free to stipulate representational differences, but these will fail to align with Thesis 5, and consequently transcend what is relevant to scientific practice. Models only represent their target systems insofar as they are used to do so, which (often implicitly) involves specifying a representation relation. To reiterate, the hole argument assumes a difference in what represents that can't make *any difference to scientific practice*, even though it might be of interest to some metaphysicians.

Mathematical practice alone fails to preclude the possibility of the hole diffeomorphic models representing haecceitistic differences. Nonetheless, an account of scientific representation attentive to practice, that satisfies Theses 1-5 above, reveals that the argument turns on a distinction without a relevant representational difference. The purported metaphysical differences between the worlds represented by isomorphic models would be haecceitistic differences. Surrogative reasoning is possible in virtue of structural reasoning, for which haecceitistic differences are irrelevant. Making the representation relation explicit can help to show why scientific representations are invariant with respect to supposed haecceitistic differences between models.

When engaging in the metaphysics of physics, or naturalised metaphysics more generally, careful attention to the way representations are used can help to dissolve metaphysical worries that are beyond the scope of scientifically informed metaphysical inquiry.

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