CAUSALLY SYMMETRIC BOHM MODEL

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§1 Introduction

The aim of this paper is to construct a version of Bohm's model that also includes the existence of backwards-in-time influences in addition to the usual forwards causation. The motivation for this extension is to remove the need in the existing model for a preferred reference frame. As is well known, Bohm's explanation for the nonlocality of Bell's theorem necessarily involves instantaneous changes being produced at space-like separations, in conflict with the "spirit" of special relativity even though these changes are not directly observable. While this mechanism is quite adequate from a purely empirical perspective, the overwhelming experimental success of special relativity (together with the theory's natural attractiveness), makes one reluctant to abandon it even at a "hidden" level. There are, of course, trade-offs to be made in formulating an alternative model and it is ultimately a matter of taste as to which is preferred. However, constructing an explicit example of a causally symmetric formalism allows the pros and cons of each version to be compared and highlights the consequences of imposing such symmetry[1]. In particular, in addition to providing a natural explanation for Bell nonlocality, the new model allows us to define and work with a mathematical description in 3-dimensional space, rather than configuration space, even in the correlated many-particle case.

The structure of the paper is as follows. In Sec. 2, the basic causally symmetric scheme is introduced in terms of initial and final boundary conditions. Sec. 3 then highlights the ways in which the corresponding initial and final wavefunctions will propagate. The basic equations of the alternative model are deduced in Sec. 4 in close analogy to the formalism of the standard Bohm model. Sec. 5 then points out how the notion of retrocausality has been given an explicit mathematical form and Sec. 6 checks some elementary matters of consistency. The discussion in Sec. 7 indicates how backwards-in-time effects provide a meaning for the notion of negative probability. Sec. 8 then explains the way in which an objection to an earlier and related model of de Broglie is now overcome. After dealing with some technical details in Sec. 9, the analysis in Sec. 10 shows how the model explains Bell's nonlocality in a way that is Lorentz invariant, as well as being local from a 4-dimensional point of view. The generalization of the formalism to n particles is given in Sec. 11, followed by an outline in Sec. 12 of ways in which the model has inherently weaker predictive power. A relativistic version is formulated in Sec. 13 for the single-particle Dirac case. Finally, conclusions are presented in Sec. 14.

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[1] This notion of causal symmetry needs to be distinguished from the more usual concept of time symmetry. Most mathematical formalisms in physics, including the Bohm model, already possess symmetry under time reversal, but this is separate from the issue of causal structure.

§2 General Structure of the Model

We will limit ourselves initially to the single-particle case for simplicity. The many-particle case will be considered later after gaining some preliminary insight from a discussion of the EPR/Bell arrangement.

Bohm's model (Ref. 1) makes the assumption that a particle always has a definite, but hidden, trajectory. It then specifies the particle's velocity in terms of the wavefunction ψ. Our aim here is to provide a consistent generalization of this formalism that incorporates backwards-in-time effects, or retrocausality, into the model. The state of the particle at any time will then be partly determined by the particle's future experiences as well as by its past. The way in which this helps with Bell's nonlocality will then be outlined in Sec. 10.

As a first step towards developing such a formalism, we must deal with the question: what aspects of a particle's future are relevant?[2] Possible factors could be the type of measurement to be performed next, the nature of the particle's interaction with the next particle it encounters and perhaps the nature of all future measurements and interactions. This seems a daunting prospect at first. However, an indication of the best way to proceed is obtained by looking at the usual way we take account of a particle's **past** experiences: we work with an initial wavefunction ψ_i which summarizes the particle's relevant past. More formally speaking, ψ_i specifies the initial boundary conditions. Therefore, by symmetry, it seems natural to supplement ψ_i with a "final" wavefunction ψ_f specifying the **final** boundary conditions. To keep the arrangement time-symmetric, the final wavefunction ψ_f will be restricted, like ψ_i, to being a solution of the time-dependent Schrödinger equation. The procedure to be followed here then is to construct a version of Bohm's model containing both ψ_i and ψ_f.

Note that the new wavefunction ψ_f being introduced here is independent of the usual wavefunction ψ_i and should not be confused with the result of evolving ψ_i deterministically to a later time. Thus, at any single time t, there are two distinct wavefunctions: (i) the initial wavefunction $\psi_i(\mathbf{x},t)$, which summarizes the initial boundary conditions existing at some earlier time t_1 and which has been evolved forwards from t_1 to t and (ii) the final wavefunction $\psi_f(\mathbf{x},t)$, which summarizes the final boundary conditions at some later time t_2 and which has been evolved back from t_2 to t. The model to be developed here will be deterministic once **both** wavefunctions are specified, together with the particle's position at one instant of time. In particular, specifying ψ_i at time t_1 and ψ_f at time t_2 will then determine the particle's velocity at any intermediate time.

Like the standard Bohm model, the causally symmetric version will be a "no collapse" model, with empty branches of wavefunctions after measurements being ignored as irrelevant. The model does not give any special status to measurement interactions, observers or the macroscopic world[3]. Indeed, it is intended to be as similar as possible to the standard Bohm formulation, apart from the obvious fact that such a retrocausal model cannot be deterministic when only the initial conditions are given.

[2] Some of the presentation in this paper has been employed previously in Ref. 2.

[3] A theory of measurement for this model will be presented in a forthcoming paper.

§3 Backwards-in-Time effects

At first sight, it may seem that the model being proposed is simply one containing a second wavefunction (acting as a hidden variable) without necessarily being retrocausal[4]. It is important, therefore, to note various ways in which ψ_i and ψ_f differ in behaviour.

(i) Consider a particle propagating from a source to a photographic plate. Its ψ_i typically spreads out forwards in time in propagating from the source to the plate. By contrast, the particle's ψ_f typically spreads out backwards in time in going from the spot on the photographic plate back to the source.

(ii) Consider a particle which is initially isolated but which then interacts with other particles before eventually being detected. Starting as a single-particle wavefunction, the particle's ψ_i will evolve forward in time to form a correlated, many-particle wavefunction in 3n-dimensional configuration space. By contrast, the particle's ψ_f will be a single-particle wavefunction at the final detection point, with the interactions making it more and more correlated in going backwards in time towards the source.

(iii) (Counterfactuality) Choosing to perform one type of measurement instead of another on a particle will be assumed to affect the form of the particle's ψ_i in the future, but not in the past. By contrast, the measurement choice will be assumed to affect the form of the particle's ψ_f in the past, but not in the future[5].

§4 Basic Mathematical Formalism

The standard version of Bohm's model will now be summarized briefly for comparison with the equations of the subsequent causally symmetric model. Strictly speaking the wavefunctions in this summary should all be written with subscripts i for "initial", to conform with the notation introduced above. For simplicity, however, the i's will not be included here.

For the single-particle case we are initially considering, Bohm's model postulates the following:

(i) For a particle with wavefunction $\psi(\mathbf{x},t)$, the probability distribution $\rho(\mathbf{x},t)$ for the position $\mathbf{x}$ of the particle at any time t is given by

$$\rho(\mathbf{x},t) = \psi^*\psi \tag{1}$$

(ii) The velocity $\mathbf{v}(\mathbf{x},t)$ of the particle is related at all times to the particle's position by

$$\mathbf{v} \equiv \frac{d\mathbf{x}}{dt} = \frac{\hbar}{2im}\frac{\psi^*\overleftrightarrow{\nabla}\psi}{\psi^*\psi} \tag{2}$$

where m is the particle's mass, $\hbar$ is Planck's constant, $\overleftrightarrow{\nabla}$ stands for $\vec{\nabla} - \overleftarrow{\nabla}$ and the grad operators $\vec{\nabla}$ and $\overleftarrow{\nabla}$ act to the right and left, respectively. As shown by Bohm, the model

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[4] It is, of course, possible to construct a two-wavefunction model that is not causally symmetric and it may be possible to construct a single-wavefunction model containing retrocausality. Neither, however, is relevant to the present aim of formulating a Lorentz invariant Bohm model.

[5] An intuitive notion of free choice is being assumed here, although it is recognised that this is an area requiring further examination.

characterized by these assumptions is consistent with all the predictions of non-relativistic quantum mechanics. Given the initial position of a particle, Equation (2) uniquely determines the particle's future trajectory and so the above scheme is deterministic. The need to resort to the usual statistical description of quantum mechanics is then attributed in this model to our inherent lack of knowledge of the particle's initial position within the wavefunction.

The notation is usually simplified by writing the wavefunction in the polar form:

$$\psi = R\exp(iS/\hbar) \tag{3}$$

where $R(\mathbf{x},t)$ and $S(\mathbf{x},t)$ are real quantities. Eqs. (1) and (2) may then be expressed as:

$$\rho(\mathbf{x},t) = R^2 \tag{4}$$

and

$$\mathbf{v} = \nabla \mathbf{S}/m \tag{5}$$

This simplification, however, is not always available in relativistic versions of Bohm's model. In particular, it is not possible in Bohm's model for the Dirac equation (Ref. 3). In the present context, the polar notation of (3), (4) and (5) does not provide any obvious advantage and so will not be employed.

There are several arguments that lead to the choice of velocity expression in Equation (2). The most useful one for our present purposes will now be outlined to serve as a basis for obtaining a causally symmetric version. In the standard formalism for the flow of probability current, the evolution of probability density with time is analogous to the flow of a fluid. In order for the probability to be conserved at each point, it must satisfy the equation of continuity:

$$\nabla\cdot(\rho\mathbf{v}) + \frac{\partial\rho}{\partial t} = 0 \tag{6}$$

where $\rho(\mathbf{x},t)$ is the probability density for the particle to be within a volume element $d^3\mathbf{x}$ surrounding position $\mathbf{x}$ at time t, and $\mathbf{v}(\mathbf{x},t)$ is the velocity of the probability flow at that point. Bohm's model involves the extra assumption that there is a unique particle velocity specified at each point in space-time once the wavefunction is given. In these circumstances, the velocity of the probability flow at $(\mathbf{x},t)$ is the same as the particle velocity at that point. Therefore, in constructing Bohm's model, the expressions chosen in terms of ψ for the particle's position probability $\rho(\mathbf{x},t)$ and velocity $\mathbf{v}(\mathbf{x},t)$ must together satisfy Equation (6) in order to conserve probability at each point[6].

Now, starting from the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{7}$$

and its complex conjugate:

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[6] Local conservation of probability here essentially means compatibility with the existence of unbroken trajectories, so that particles need not be spontaneously appearing and disappearing.

$$-\frac{\hbar^2}{2m}\nabla^2\psi^* + V\psi^* = -i\hbar\frac{\partial\psi^*}{\partial t} \tag{8}$$

the corresponding equation of continuity resembling (6) is obtained by the familiar method of multiplying (7) by ψ^* and (8) by ψ, then subtracting the resulting two equations, to obtain:

$$\nabla\cdot\left(\frac{\hbar}{2im}\psi^*\overleftrightarrow{\nabla}\psi\right) + \frac{\partial}{\partial t}(\psi^*\psi) = 0 \tag{9}$$

This equation holds automatically for any wavefunction satisfying the Schrödinger equation. Comparing (6) and (9) then points to the expressions chosen in Eqs. (1) and (2), so that Bohm's model is thereby obtained.

The aim now is to follow an analogous path to a causally symmetric version of Bohm's model. Such a model must obviously feature both the initial and final wavefunctions ψ_i and ψ_f in its formalism. The key point to note is that the steps leading to equation (9) essentially treat ψ and ψ^* as two separate functions and do not depend critically on them being related as complex conjugates. Indeed, if one takes them as independent functions by simply putting a subscript i on ψ and a subscript f on ψ^*, the Schrödinger equation ensures that the following modified version of Equation (9) still holds:

$$\nabla\cdot\left(\frac{\hbar}{2im}\psi_f^*\overleftrightarrow{\nabla}\psi_i\right) + \frac{\partial}{\partial t}(\psi_f^*\psi_i) = 0 \tag{10}$$

This is a promising result for our purposes, since it has the form of an equation of continuity with i and f equally represented and it holds automatically for any two independent wavefunctions ψ_i and ψ_f that are both solutions of the Schrödinger equation.

Like (9), Equation (10) implies a conserved quantity. This is easily demonstrated by performing an integral d^3x over all space on each term in (10). Under the standard assumption that a wavefunction falls to zero as **x** goes to infinity, the integral of the first term in (10) is zero and we are left with:

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\psi_f^*\,\psi_i\,d^3x = 0 \tag{11}$$

This result indicates that the quantity

$$a \equiv \int_{-\infty}^{+\infty}\psi_f^*(\mathbf{x},t)\,\psi_i(\mathbf{x},t)\,d^3x \tag{12}$$

is conserved through time. In particular, if it is non-zero at one instant of time, it must continue to be non-zero for other times.

Before using the new equation of continuity further, there is a need to rectify two minor complications which have arisen in the transition from (9) to (10). First, a normalization factor given by the amplitude a in (12) needs to be introduced to ensure that total probability remains equal to one. Second, the equation is no longer real, which will be avoided simply by taking the real part of it. (This also makes it fully symmetric with respect to ψ_i and ψ_f.) Equation (10) thus becomes modified to the form[7]:

$$\nabla \cdot \mathrm{Re}\left(\frac{\hbar}{2ima}\psi_f^* \overleftrightarrow{\nabla}\psi_i\right) + \frac{\partial}{\partial t}\mathrm{Re}\left(\frac{1}{a}\psi_f^*\psi_i\right) = 0 \qquad (13)$$

Note that we have been able to move the quantity a inside the time derivative in (13) because, from (11), it is independent of t.

Equation (13) has now been put into an appropriate form to provide expressions for a causally symmetric model. Specifically, comparing Eqs. (6) and (13) points to the two identifications:

$$\rho(\mathbf{x},t) = \mathrm{Re}\left(\frac{1}{a}\psi_f^*\psi_i\right) \qquad (14)$$

and

$$\mathbf{v}(\mathbf{x},t) = \frac{\mathrm{Re}\left(\frac{\hbar}{2ima}\psi_f^* \overleftrightarrow{\nabla}\psi_i\right)}{\mathrm{Re}\left(\frac{1}{a}\psi_f^*\psi_i\right)} \qquad (15)$$

Equations (14) and (15) are the basis of the proposed causally symmetric version of Bohm's model[8]. They contain ψ_i and ψ_f on an equal footing and should be compared with equations (1) and (2) of the original model[9].

The obvious objection that can be made at this point is that the probability density (14) is not positive definite[10]. This will be dealt with in detail in Sec.7. Normally the prediction of a negative probability would be fatal for any proposed theory. It is a remarkable fact, however, that the introduction of backwards-in-time phenomena allows a natural interpretation of some negative probability expressions (precisely those that are the basis of the new model). Therefore, unlike in other cases where negative probabilities have been mooted, there is no serious problem here. Indeed, one can argue that they should be expected in any truly causally symmetric model. In any case, at this point it simply needs to be emphasized that the model does not predict negative probabilities for the **outcomes of measurements**, so no meaning need be found for such a notion. Indeed, using the word "probability" here may be a little misleading, but we will persist with it and leave the detailed explanation to Sec. 7.

The discussion in the Sec. 7 will be seen to be necessarily relativistic, whereas the considerations above have been limited to the non-relativistic case for simplicity and for

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[7] At this point we can also consider the alternative of defining the normalizing factor to be the **real part** of expression (12). However, this would lead to a different result later in Sec. 6(b).

[8] It is actually more viable for the model to define expression (15) as the mean velocity, rather than a unique value, at point **x**. Since, however, such a refinement is not helpful to the present purpose of comparing two similar models, it will be left for future work.

[9] The physical interpretation of the case where the denominator of (15) is zero is stated in Sec. 9 after the groundwork is laid in Sec. 7.

[10] It is, of course, possible to devise alternative expressions for $\rho(\mathbf{x},t)$ that **are** positive. However, then the proposed probability distribution would not satisfy an equation of continuity and so probability would not be conserved, which is a more intractable problem.

ease of comparison with Bohm's original formalism. This situation will be rectified in Sec. 13, where a Bohm model for the Dirac equation will be formulated.

§5 Retrocausal Influence on Particle Velocity

To demonstrate that the particle velocity defined by (15) really is retrocausally affected by future circumstances, consider two separate particles each having an identical initial wavefunction ψ_i from time t_1 onwards. If we choose to perform measurements of different non-commuting observables on the particles at a later time t_2, they will have different final wavefunctions ψ_f extending back from t_2 to t_1 (these being eigenfunctions of the respective observables measured). Since the velocity expression (15) is obviously dependent on ψ_f, it then follows that the velocity values at any intermediate time between t_1 and t_2 will be different for the two particles. Hence the type of measurement chosen at t_2 has a bearing on the physical reality existing at an earlier time, which constitutes retrocausality. This example also indicates the way in which our initial notion of retrocausality has been given a specific mathematical form.

Note further in this example that it is not possible to interpret the two ψ_f's as instead originating at the earlier time t_1, independent of the future measurements at t_2, and then propagating forwards in time. This is because these ψ_f's are eigenfunctions of two different observables that will subsequently be chosen freely[11] by the experimenter at t_2. It would be inexplicable why, for each particle, the ψ_f that arises randomly at t_1 always happens to be an eigenfunction of the correct observable to be nominated and measured later at t_2. The only explanation is that each ψ_f must be retrocausally determined by the choice at t_2.

§6 Consistency with Observation

It will be demonstrated briefly here that the probability expression (14) is quite consistent with what is observed when a measurement is actually performed. It should be kept in mind that the position probability distributions of Bohm-type models describe the position of a particle at all times, so that most of the times are **between** measurements. In terms of experimental agreement, it doesn't matter what is predicted there, since the distribution is hidden. We will now consider two simple cases to illustrate how (14) fits in with the usual quantum mechanical results.

(a) Consider a position measurement that gives a result $\mathbf{x_0}$ at time t_0. Starting at earlier times t, the particle's final wavefunction must approach the form:

$$\psi_f = \delta^3(\mathbf{x} - \mathbf{x_0}) \tag{16}$$

as t_0 gets closer. Self-consistency of the model requires that the density $\rho(\mathbf{x},t)$ also becomes a delta function at t_0. To check that this is the case, it is more convenient to switch to Dirac bra-ket notation:

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[11] See footnote 5.

$$\rho(\mathbf{x},t) = \mathrm{Re}(\tfrac{1}{a}\psi_f^*\psi_i) \\ = \mathrm{Re}\,\frac{\langle\psi_f|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{17}$$

Then, inserting the delta function (16), we have for time t_0:

$$\rho(\mathbf{x},t_0) = \mathrm{Re}\,\frac{\delta^3(\mathbf{x}-\mathbf{x}_0)\,\langle\mathbf{x}|\psi_i\rangle}{\langle\mathbf{x}_0|\psi_i\rangle} \\ = \mathrm{Re}\,\frac{\delta^3(\mathbf{x}-\mathbf{x}_0)\,\langle\mathbf{x}_0|\psi_i\rangle}{\langle\mathbf{x}_0|\psi_i\rangle} \\ = \delta^3(\mathbf{x}-\mathbf{x}_0)\text{ , as required.} \tag{18}$$

So the distribution becomes positive at point $\mathbf{x_0}$ and zero everywhere else. Any negative value for the probability therefore goes away as the time of observation approaches because the final wavefunction gradually dominates.

(b) This second case concerns the usual statistical expressions of quantum mechanics. Expression (17) actually represents the **conditional** probability density given both the initial and final states:

$$\rho(\mathbf{x}|\psi_i,\psi_f) = \mathrm{Re}\,\frac{\langle\psi_f|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{19}$$

Normally, however, the final state is not known and we require instead the conditional probability given the initial state alone. In both quantum mechanics and the standard Bohm model this probability for position has the form:

$$\rho(\mathbf{x}|\psi_i) = |\langle\mathbf{x}|\psi_i\rangle|^2 \tag{20}$$

and more generally, when ψ_f is one of the possible outcomes of a subsequent measurement, we have the result:

$$\rho(\psi_f|\psi_i) = |\langle\psi_f|\psi_i\rangle|^2 \tag{21}$$

Thus we need to check how the model fits in with these last two expressions. To proceed, we employ the following general rule involving the joint probability distribution $\rho(a,b)$ for two quantities a and b:

$$\rho(a,b) = \rho(a|b)\,\rho(b) \tag{22}$$

We then apply this to our particular case by inserting expressions (19) and (21) into the right hand side. This yields:

$$\rho(\mathbf{x},\psi_f|\psi_i) = \rho(\mathbf{x}|\psi_i,\psi_f)\,\rho(\psi_f|\psi_i) \\ = \mathrm{Re}\,\frac{\langle\psi_f|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}\,|\langle\psi_f|\psi_i\rangle|^2 \\ = \mathrm{Re}\,\langle\psi_f|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle\langle\psi_i|\psi_f\rangle \\ = \mathrm{Re}\,\langle\psi_i|\psi_f\rangle\langle\psi_f|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle \tag{23}$$

and then summing over all possible final states ψ_f (under the assumption that these constitute a complete orthonormal set[12]) we obtain:

$$\rho(\mathbf{x}|\psi_i) = \langle\psi_i|\mathbf{x}\rangle\langle\mathbf{x}|\psi_i\rangle \qquad (24)$$

in accordance with (20). Therefore, the causally symmetric expression for ρ is found to mesh neatly with the standard statistical predictions of quantum mechanics.

The plausibility of this model, however, depends mainly on the conclusions of the next section.

§ 7 Interpretation of Negative Probabilities

The aim of this section is to examine the usual formalism describing probability density for a particle's position and thereby understand the meaning of a negative value for this probability. The interpretation given below is not new, but previously it raised logical questions which cast doubt upon its viability. These now seem resolved in the context of retrocausality.

From here on we will adopt the convention of setting $\hbar = c = 1$. The relativistic formalism for probability current will now be briefly summarized. We return to the equation of continuity given earlier in Equation (6):

$$\nabla\cdot(\rho\mathbf{v}) + \frac{\partial\rho}{\partial t} = 0 \qquad (25)$$

which can be rewritten in relativistic notation as:

$$\partial_\nu(\rho_0 u^\nu) = 0 \qquad (26)$$

where:

$\rho_0(\mathbf{x},t)$ is the **rest** probability density, i.e., the probability density in the local rest frame of the probability flow at the space-time point $(\mathbf{x},t)$,

$u^\nu = \dfrac{dx^\nu}{d\tau}$ is the 4-velocity of the flow at $(\mathbf{x},t)$,

τ is the proper time taken along the 4-dimensional flow line at $(\mathbf{x},t)$,

x^ν ($\nu = 0,1,2,3$) represents the coordinates t,x,y,z,

∂_ν represents the partial derivative $\partial/\partial x^\nu$,

and a summation over ν is implied.

The quantity $\rho_0 u^\nu$ is known as the 4-current density and equation (26) states that its 4-divergence is zero. The rest density ρ_0 is an invariant, while u^ν is a 4-vector. Hence the current density $\rho_0\, u^\nu$ is a 4-vector. Comparing equations (25) and (26), the connection between the probability density ρ and the rest density ρ_0 is identified to be:

$$\rho = \rho_0 u^0 \qquad (27)$$

[12] This follows from the above assumption that the ψ_f's are the possible outcomes of a subsequent measurement.

where u^0 is the time component of the 4-velocity.

Now, the basic point is as follows: From (27), the probability density ρ is seen to be **the time component of a 4-vector** in space-time. Hence the meaning of a negative value for position probability density at a particular point is simply that the time component of the current density is negative and so **the current density 4-vector is pointing backwards in time** at that point. This is illustrated in the space-time diagram of Fig. 1.

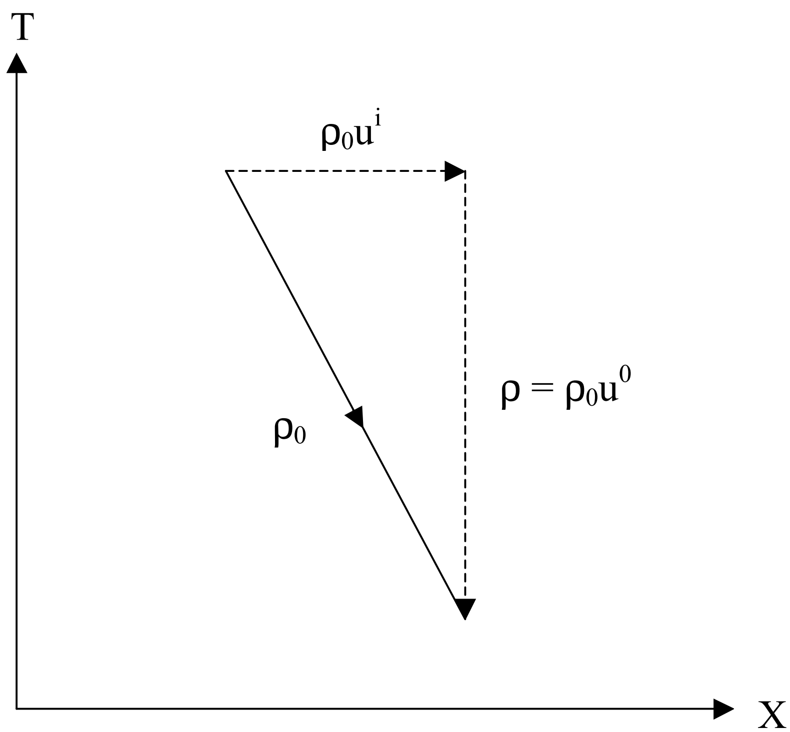

Fig. 1

Only one of the three spatial components $\rho_0 u^i$ (i = 1,2,3) of the 4-current density can be shown since the diagram is 2-dimensional. The magnitude of this 4-vector is ρ_0, which we will return to later.

To pursue this notion further, the flow line shown in Fig. 2 will be considered. Of course, such a line is generally viewed as not being physically permissible, but it will be useful as an example here. Now, since the current density 4-vector is directed backwards in time between space-time points 2 and 3, the probability density would be negative along this segment.

We can therefore draw the conclusion that negative probability would be a meaningful concept if probability flows such as that shown in Fig. 2 could occur in physics. Note that the rest density ρ_0 always remains positive. From equation (27), the density ρ simply becomes negative when the 4-velocity component u^0 becomes negative. Recall that rest density is defined to be the density in the local rest frame of the flow. Such a rest frame can always be defined. It is straightforward to extend the concept of a reference frame to motions faster than light and paths backwards in time[13].

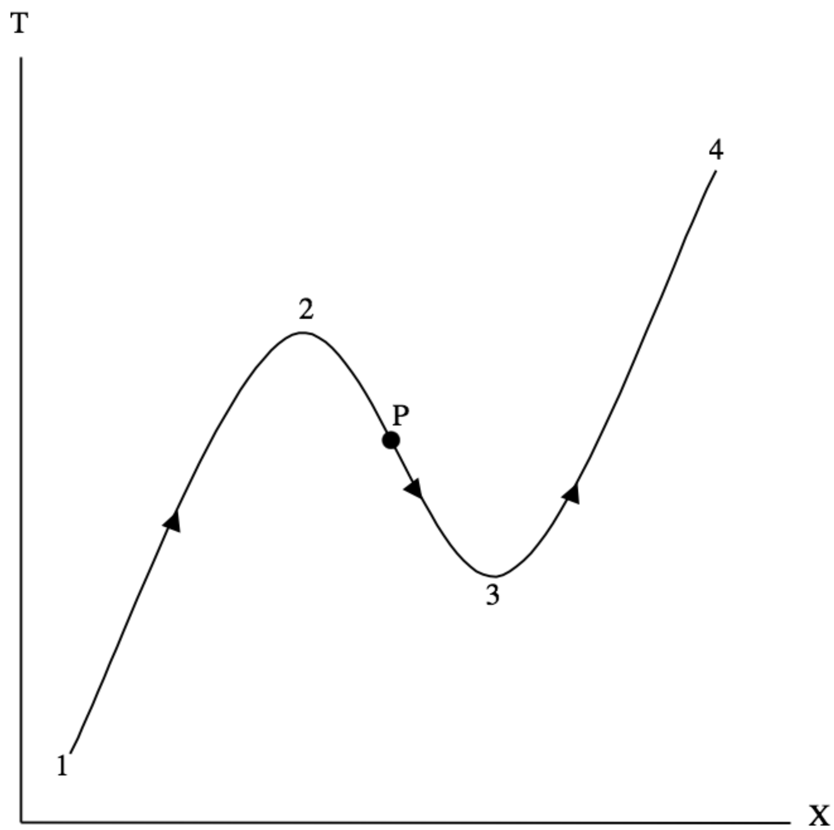

Fig. 2

To consider further the likelihood of negative probabilities being relevant in physics, it will now be more convenient to focus directly on world lines of particles, rather than on lines of probability flow. To this end, let us tentatively examine the viability of the curve in Fig. 2 as a possible world line for a particle. Such a world line is generally viewed as being ruled out for several reasons. For example, (i) the particle behaves in a way that has never been observed, (ii) the particle goes faster than light, (iii) the particle goes backwards in time, (iv) the particle could be used to create causality paradoxes, (v) the particle passes smoothly through the "light barrier". Actually, however, none of these points constitutes a fatal objection here.

In response to (i), "Doubling back" of the particle's world line only occurs at times **between measurements** and is therefore hidden. The particle returns to normal

[13] For example, the time axis of the rest frame at point P in Fig. 2 is defined to be tangential to the flow line and in the direction of the arrowhead shown, while the spatial axes of the frame are defined to lie in the 3-dimensional hyperplane orthogonal to this time axis (orthogonality being well defined in Minkowskian geometry). Further details can be found in Ref. 4.

behaviour as the time of the next measurement approaches[14]. In response to (ii), many authors have pointed out that faster-than-light particles (tachyons) are consistent with special relativity. It is simply that they have never been observed experimentally. The response to (iii) is similar to that for (ii), since faster-than-light motion becomes backwards in time when viewed from an appropriately chosen, different frame of reference. In response to (iv), the particle's motion is beyond our control between measurements and so not able to be manipulated to create causality problems. Point (v) seems at first to be the strongest objection to the world line in Fig. 2. Special relativity does not permit a particle of non-zero rest mass to travel at the speed of light, since this would require infinite energy. As will be seen in Sec. 13, however, it is surprisingly easy to construct a model which avoids this problem. One simply allows the particle's rest mass to vary appropriately with position[15] (in a way dependent on the particle's wavefunction) so that it becomes zero at the instant when the particle passes through the light barrier. Such a model was outlined in 1960 by de Broglie (Ref. 5, ch. 10) in a proposed relativistic extension of his hidden variable work[16].

Since a world line that turns backwards in time cannot definitely be ruled out on theoretical grounds, it remains now to look at whether it might be a useful notion. Referring back to expression (14) for our position probability density, we are faced with the fact that this expression can only be explained in terms of continuous and smooth world lines if we are willing to permit world lines such as in Fig. 2. These are therefore being postulated here as being an essential (and perhaps a natural?) part of a causally symmetric model.

Before proceeding on, it should be mentioned that there is another possible interpretation that could be adopted for Fig. 2, namely that it simply represents the creation of a particle-antiparticle pair at point 3, followed by particle-antiparticle annihilation at point 2. This is certainly an equivalent way of viewing the situation, although such creation and annihilation events are normally represented with sharp vertices at 2 and 3 rather than smoothly curved ones, allowing compatibility with slower-than-light propagation. This alternative description involving particle-antiparticle pairs will not be employed here for three reasons. Firstly, the points 2 and 3 at which the world line reverses its time direction are actually both frame dependent, so that different observers will not agree in specifying the precise space-time event at which creation or annihilation occurs. Secondly, the single-particle perspective involves a single proper time variable τ increasing continuously along the world line as per the arrowheads shown, whereas the creation-annihilation view would require separate proper time variables for the three particles, changing discontinuously at the two (artificially generated) vertices. Thirdly, the proposed application of such paths is intended to be in quantum mechanical scenarios where they would not be directly observable anyway, so there is no need to think in terms of "what would actually be observed".

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[14] The measurement clearly needs to have some retrocausal influence on the particle in order for this to work. However, the simple considerations of Sec. 6(a) indicate that such an evolution can occur quite naturally.

[15] Of course, by definition, rest mass does not vary with velocity. However, there is nothing to prevent us postulating that rest mass varies with position, as we are suggesting here.

[16] A suitable expression for variable rest mass is given later in Sec. 13.

§ 8 Overcoming an Objection to Negative Probabilities

The particle-antiparticle viewpoint just discussed leads into another objection that should be considered. This problem is perhaps of the sort that caused de Broglie's 1960 model to be viewed unfavourably even by fellow hidden variable advocates. To introduce the argument, the world line we have been considering is presented again in Fig. 3, but with a particular time t highlighted for attention.

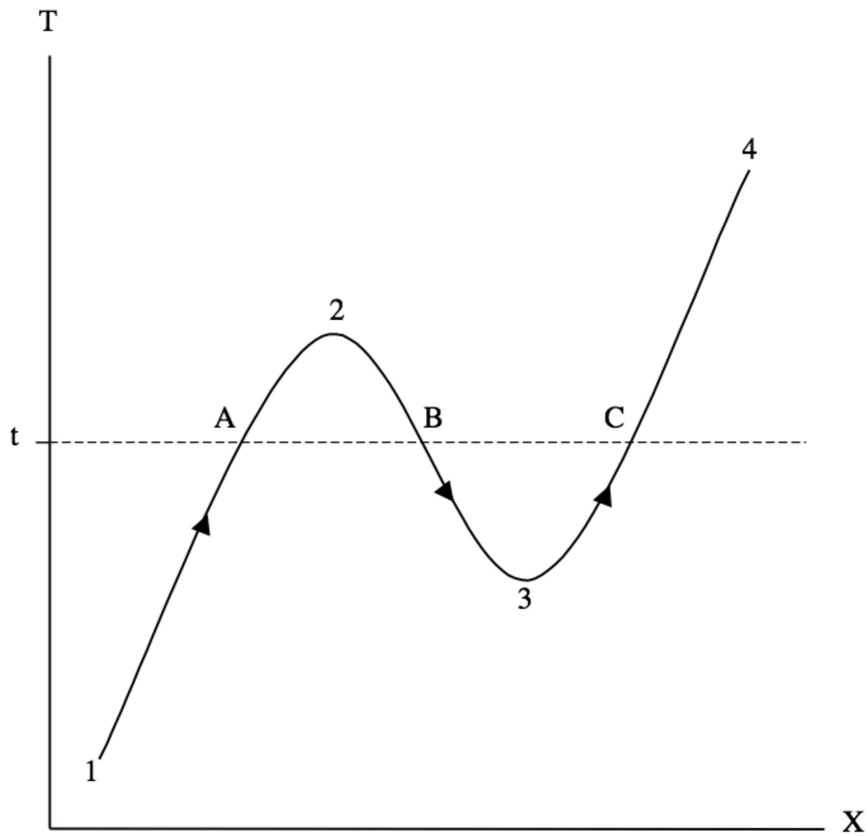

Fig. 3

Suppose a position measurement of the x coordinate is performed at this time t. For simplicity it will be assumed that the measurement is certain to detect both the particle plus any possible particle-antiparticle pair that is present. Furthermore, it will be assumed that any entity detected is absorbed by the apparatus (e.g., a photographic plate) and prevented from proceeding further. Now, what will be the result of the measurement? One possible view is that particles will be detected at each of A and C, with an antiparticle being detected at B. Another view, however, is that the only thing that will be detected is a particle at A, since the "single world line" viewpoint entails that absorption at A will prevent the particle from ever reaching B and C.

Such an argument is a legitimate objection to the de Broglie model, which involved both negative probabilities and world lines "doubling back", but did **not** involve any retrocausality. The argument is easily avoided, however, once retrocausality is included as well. In this case the backwards-in-time effect of the position measurement (i.e., the influence of ψ_f) ensures the world line must straighten out as it approaches the measurement time, so that it arrives at only one point at time t. This is demonstrated trivially in Sec.6(a), where imposing the final boundary condition that the particle is detected at one location at time t results in a probability expression that is zero everywhere else at that time, so that it does not describe the presence of any other particle or antiparticle. The probability distribution may, of course, be spread out and containing negative regions at earlier times. As the measurement time approaches, however, the final wavefunction ψ_f will dominate in (17) and the distribution will evolve gradually into a delta function. The lesson one can draw from this is that negative probabilities and "doubling back" world lines can both exist consistently in combination with retrocausality, but not without it.

Having decided to pursue a causally symmetric model, one can actually adopt a more aggressive argument in favour of the possibility of world lines such as the one in Fig. 2. Models involving retrocausality arise most naturally from assuming the block universe picture, which in turn takes time and space to be similar. In such a context one can argue as follows: One would be surprised to find a particle whose world line can only ever point in the positive x direction, without ever doubling back in the negative x direction. But if time and space are on an equal footing, should we not be surprised if a world line can only point in the positive time direction without ever doubling back? Surely such a world line should be viewed as "unnatural"? Taking this attitude, negative probabilities for a particle's position are to be expected.

§ 9 <u>Some Technical Points</u>

The sort of world lines we are considering is also reflected in the form of the 3-velocity expression (15):

$$\mathbf{v}(\mathbf{x},t) = \frac{\mathrm{Re}(\frac{\hbar}{2ima}\psi_f^* \overleftrightarrow{\nabla}\psi_i)}{\mathrm{Re}(\frac{1}{a}\psi_f^*\psi_i)}$$

This expression is infinite when its denominator $\mathrm{Re}(\frac{1}{a}\psi_f^*\psi_i)$ is zero, corresponding to points such as 2 and 3 in Fig. 2. This equation is not, however, able to indicate regions where the world line has turned backwards in time. This information is provided by the time component of the 4-velocity, which is why it is more useful in this context to work in terms of a particle's 4-velocity rather than its 3-velocity.

The 4-velocity $u^\nu = \frac{dx^\nu}{d\tau}$ is defined in terms of the proper time τ, which we are taking to be a variable that increases monotonically as we go along the world line from point 1 to point 4 in Fig. 2. It is clear that **τ needs to be always real**. This means that the usual definition:

$$\begin{aligned} d\tau &= \sqrt{dt^2 - dx^2 - dy^2 - dz^2} \\ &\equiv \sqrt{dx_\mu dx^\mu} \end{aligned} \tag{28}$$

which applies for a change $d\tau$ along a time-like segment of the world line, must be supplemented with the definition:

$$d\tau = \sqrt{-\,dx_\mu\, dx^\mu} \tag{29}$$

for the case of a space-like segment. This two-part definition for proper time is relativistically invariant because of the fact that **all observers agree as to whether any given 4-vector is time-like or space-like**. The definition can be written as:

$$d\tau = \begin{cases} \sqrt{dx_\mu\, dx^\mu} & \text{(time–like segment)} \\ \sqrt{-\,dx_\mu\, dx^\mu} & \text{(space–like segment)} \end{cases} \tag{30}$$

or, if preferred, summarized in the single expression (Ref. 4):

$$d\tau = \left| dx_\mu\, dx^\mu \right|^{1/2} \tag{31}$$

Finally, as can be seen from the 4-velocity relationship:

$$u_\nu u^\nu = \frac{dx_\nu}{d\tau}\frac{dx^\nu}{d\tau} \tag{32}$$

a consequence of (30) is that any time-like 4-velocity vector will satisfy the identity:

$$u_\nu u^\nu = 1 \tag{33}$$

whereas any space-like 4-velocity will satisfy:

$$u_\nu u^\nu = -1 \tag{34}$$

with the following identity holding for **any** 4-velocity vector:

$$\left| u_\nu u^\nu \right| = 1 \tag{35}$$

This last result will be used in Sec. 13.

§10 Explanation of Bell Nonlocality

The aim of this section is to show how causal symmetry enables us to avoid the space-like effects and preferred frame needed in the standard Bohm model's description of the EPR/Bell experiment. The usual EPR/Bell arrangement is given in Fig. 4.

An initial state decays at event D into a pair of correlated particles and measurements are subsequently performed on the particles at M_1 and M_2, respectively. To simplify the discussion, we will take the M_1 measurement to occur earlier than the M_2 measurement, as shown in the diagram. The two measurements could be taken to be time-like separated if desired.

Before the first measurement is performed, the pair of particles is described by one overall wavefunction, which we will denote by $\psi_i(\mathbf{x_1},\mathbf{x_2})$. The two single-particle wavefunctions that subsequently arise from the measurements M_1 and M_2 will be denoted by $\psi_f(\mathbf{x_1})$ and $\psi_f(\mathbf{x_2})$, respectively. (To keep the notation simple, we are using the position coordinates to distinguish between the individual states.) We will now consider the standard description of the situation as events unfold.

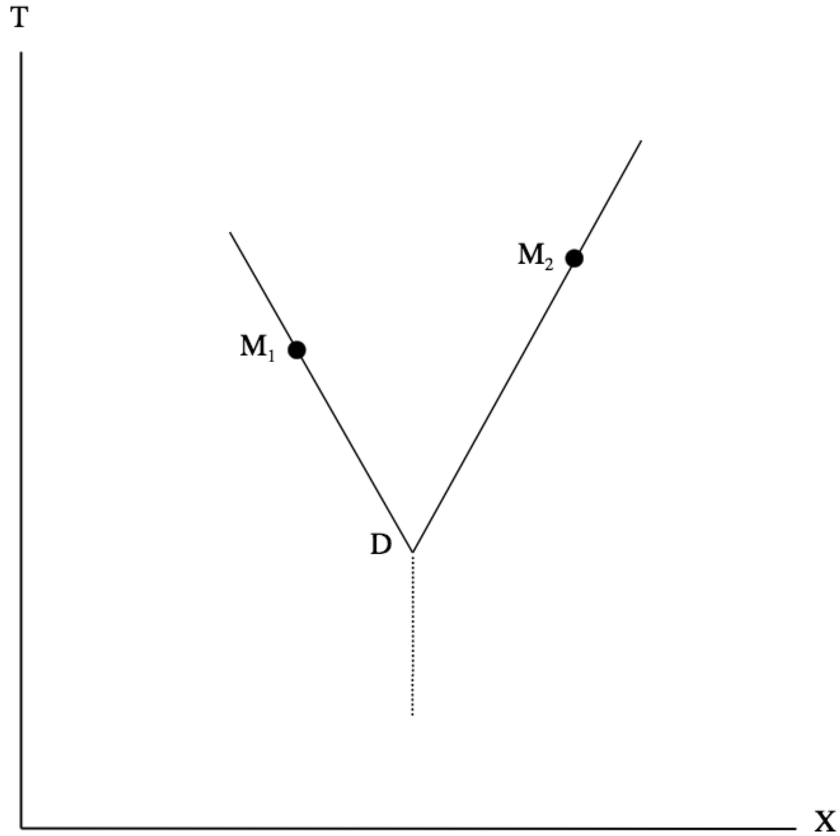

Fig. 4

Standard Quantum Mechanical Description:

Once the result of the measurement M_1 on the 1st particle is known, the state of the other particle must be updated in order to make correct statistical predictions about the result of M_2. Specifically, the 2nd particle must then be described by a single-particle wavefunction $\psi_i(\mathbf{x_2})$ given by the scalar product of the M_1 outcome $\psi_f(\mathbf{x_1})$ with the initial correlated state $\psi_i(\mathbf{x_1},\mathbf{x_2})$:

$$\psi_i(\mathbf{x_2}) = \int_{-\infty}^{+\infty} \psi_f^*(\mathbf{x_1})\, \psi_i(\mathbf{x_1},\mathbf{x_2})\, d^3x_1 \qquad (36)$$

Hence the wavefunction description of the 2nd particle changes as follows. At times between D and M_1 this particle is described by the wavefunction $\psi_i(\mathbf{x_1},\mathbf{x_2})$. Then, at times between M_1 and M_2, its appropriate wavefunction is $\psi_i(\mathbf{x_2})$, as defined in (36). Finally, after M_2, the relevant wavefunction for the 2nd particle is $\psi_f(\mathbf{x_2})$. An analogous summary can be made of the successive wavefunctions of the 1st particle.

We will now examine the further description given first by the standard Bohm model and then by the causally symmetric version in order to highlight the differences between these two models.

Standard Bohm Model:

In this case, the measurement M_1 exerts a space-like influence to cause a change in the 2^{nd} particle's trajectory compared with what it would otherwise have been. This is necessary in order to allow for an effect on the M_2 measurement result, as required by Bell's theorem.

In particular, at times between D and M_1, the 2^{nd} particle's velocity is given by inserting the wavefunction $\psi_i(\mathbf{x_1},\mathbf{x_2})$ into Equation (2) earlier and obtaining:

$$\mathbf{v_2}(\mathbf{x_1},\mathbf{x_2}) = \frac{\hbar}{2im}\frac{\psi_i^*(\mathbf{x_1},\mathbf{x_2})\overleftrightarrow{\nabla}_2\psi_i(\mathbf{x_1},\mathbf{x_2})}{\psi_i^*(\mathbf{x_1},\mathbf{x_2})\psi_i(\mathbf{x_1},\mathbf{x_2})} \tag{37}$$

whereas, at times between M_1 and M_2, the 2^{nd} particle's velocity is then given by inserting $\psi_i(\mathbf{x_2})$ instead:

$$\mathbf{v_2}(\mathbf{x_2}) = \frac{\hbar}{2im}\frac{\psi_i^*(\mathbf{x_2})\overleftrightarrow{\nabla}_2\psi_i(\mathbf{x_2})}{\psi_i^*(\mathbf{x_2})\psi_i(\mathbf{x_2})} \tag{38}$$

Note that, before the M_1 measurement, the velocities of the two particles are both calculated from the same wavefunction $\psi_i(\mathbf{x_1},\mathbf{x_2})$, which is defined in 6 dimensional configuration space. As a result, they can only be defined from a single velocity expression describing one particle moving in 6 dimensions. In particular, in order to obtain the 2^{nd} particle's velocity from (37), the 1^{st} particle's position must be specified and inserted. The situation at times after M_1 is different in that both particles have separate wavefunctions defined in 3 dimensions and so have independent velocity expressions.

Causally Symmetric Model:

In this case, we want to avoid any space-like influences between the particles. We know that the reduced wavefunction $\psi_i(\mathbf{x}_2)$ given by (36) is the correct one to use for predictions at the time of the measurement on the 2^{nd} particle. Therefore, to avoid a space-like change, we need this wavefunction to be the correct one for determining the 2^{nd} particle's velocity right back to the decay point D where the two particles separated, not just from M_1 onwards. The 2^{nd} particle will thus be guided at all times between D and M_2 via a single-particle ψ_i defined in 3 dimensions, even though this particle is initially part of a correlated pair. This possibility is available only in a causally symmetric theory, because the form of the wavefunction $\psi_i(\mathbf{x}_2)$ at times before the M_1 measurement depends on what type of measurement is subsequently chosen at M_1, which constitutes retrocausality.

Using the causally symmetric velocity expression (15) and inserting $\psi_i(\mathbf{x}_2)$ as the appropriate initial wavefunction, the specific form of the 2^{nd} particle's velocity between D and M_2 is:

$$\mathbf{v_2}(\mathbf{x_2}) = \frac{\mathrm{Re}[\frac{\hbar}{2ima}\psi_f^*(\mathbf{x_2})\overleftrightarrow{\nabla}_2\psi_i(\mathbf{x_2})]}{\mathrm{Re}[\frac{1}{a}\psi_f^*(\mathbf{x_2})\psi_i(\mathbf{x_2})]} \tag{39}$$

where $\psi_f(\mathbf{x_2})$ is the 2nd particle's final wavefunction and the amplitude a is given by:

$$a \equiv \int_{-\infty}^{+\infty} \psi_f^*(\mathbf{x_2})\, \psi_i(\mathbf{x_2})\, d^3x_2 \tag{40}$$

The above scheme is in accordance with the space-time zigzag explanation of Bell's nonlocality suggested by a number of authors (e.g., Refs. 2, 6-16). This explanation postulates the existence of a causal link along the path M_1DM_2 in Fig 4. The type of measurement performed on the 1st particle at M_1 is assumed to have a bearing on that particle's state at earlier times, i.e., between M_1 and D. This in turn affects the other particle's state forwards in time from the decay point D, thereby affecting the result of the measurement at M_2. The apparent action at a distance in 3 dimensions then becomes a local connection when viewed from a 4-dimensional viewpoint.

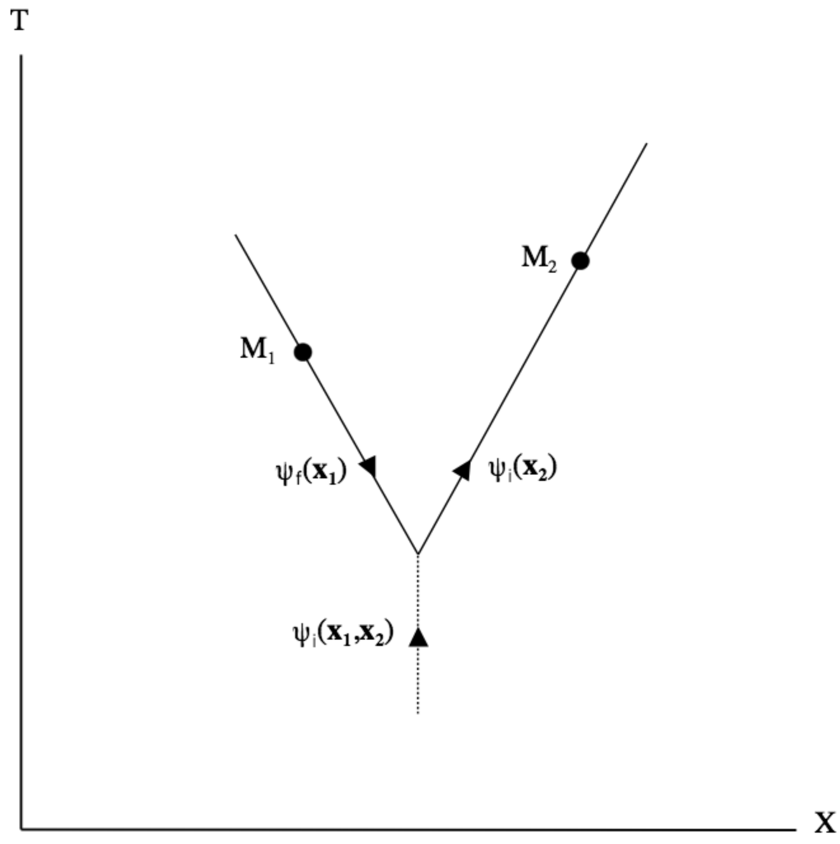

Fig. 5

In the present model, this general scheme has been given an explicit mathematical form. Recall that the causally symmetric model entails the 1st particle having (in addition to its initial wavefunction) a final wavefunction $\psi_f(\mathbf{x_1})$ evolving back from M_1 to D. Referring to Fig.5, the arrowheads indicate the way in which the wavefunctions arising from the initial and left hand branches combine to produce a wavefunction for the right hand branch. In particular, the initial wavefunction $\psi_i(\mathbf{x_1},\mathbf{x_2})$ that arises from the decay of the original system combines with the 1st particle's final wavefunction $\psi_f(\mathbf{x_1})$ via the scalar product in (36) to give the 2nd particle's initial wavefunction $\psi_i(\mathbf{x_2})$:

$$\psi_i(\mathbf{x_2}) = \int_{-\infty}^{+\infty} \psi_f^*(\mathbf{x_1})\ \psi_i(\mathbf{x_1},\mathbf{x_2})\ d^3x_1 \tag{41}$$

§11 Many-Particle Case

The considerations of the previous section can be generalized in a straightforward way to the case of n particles. Suppose we have a set of particles which have previously interacted and are therefore described by the configuration space wavefunction $\psi_i(\mathbf{x_1},\ldots,\mathbf{x_n})$. Suppose further that measurements are performed at time t on all particles except the j^{th} one. Generalizing equation (41), the standard quantum mechanical description tells us that the j^{th} particle should be described from time t onwards by the following 3-dimensional wavefunction:

$$\psi_i(\mathbf{x_j}) = \int_{-\infty}^{+\infty} \psi_f^*(\mathbf{x_1})\ldots\psi_f^*(\mathbf{x_{j-1}})\ \psi_f^*(\mathbf{x_{j+1}})\ldots\psi_f^*(\mathbf{x_n}) \times\ \psi_i(\mathbf{x_1},\ldots,\mathbf{x_j},\ldots,\mathbf{x_n})\ \ d^3x_1\ldots d^3x_{j-1}\ d^3x_{j+1}\ldots d^3x_n \tag{42}$$

where the various ψ_f's describe the measurement outcomes for the other particles.

Now, to avoid any space-like action at a distance when the particles are widely separated, we need the j^{th} particle's velocity to depend on (42) **before** the measurements as well as after (rather than depending on $\psi_i(\mathbf{x_1},\ldots,\mathbf{x_n})$ beforehand). Specifically, from expression (15), the j^{th} particle's velocity must be given by:

$$\mathbf{v_j}(\mathbf{x_j}) = \frac{\mathrm{Re}[\frac{\hbar}{2ima}\psi_f^*(\mathbf{x_j})\overleftrightarrow{\nabla}_\mathbf{j}\psi_i(\mathbf{x_j})]}{\mathrm{Re}[\frac{1}{a}\psi_f^*(\mathbf{x_j})\psi_i(\mathbf{x_j})]} \tag{43}$$

where $\psi_i(\mathbf{x_j})$ is given by (42) and $\psi_f(\mathbf{x_j})$ is this particle's final wavefunction. Hence, as in the two-particle case of the previous section, the velocity is defined in 3-dimensional space rather than configuration space.

A separate initial wavefunction similar to (42) can be introduced for each of the n correlated particles. Such 3-dimensional wavefunctions are easier to imagine as physically real than a wavefunction in 3n dimensions.

The above formulation assumes that the system's **final** wavefunction is factorizable into single-particle wavefunctions (because we are assuming n–1 measurements are performed). It is therefore not the most general case. To proceed further, we will insert (42) into (43) to write the j^{th} velocity as:

$$\mathbf{v_j}(\mathbf{x_j}) = \frac{\mathrm{Re}[\frac{\hbar}{2ima}\int_{-\infty}^{+\infty}\psi_f^*(\mathbf{x_1})\cdots\psi_f^*(\mathbf{x_n})\overleftrightarrow{\nabla}_\mathbf{j}\psi_i(\mathbf{x_1},\cdots,\mathbf{x_n})\ d^3x_1\ldots d^3x_{j\text{-}1}d^3x_{j+1}d^3x_n]}{\mathrm{Re}[\frac{1}{a}\int_{-\infty}^{+\infty}\psi_f^*(\mathbf{x_1})\cdots\psi_f^*(\mathbf{x_n})\ \psi_i(\mathbf{x_1},\cdots,\mathbf{x_n})\ d^3x_1\ldots d^3x_{j\text{-}1}d^3x_{j+1}d^3x_n]} \tag{44}$$

Now, the more general expression we are seeking must reduce to (44) when the final wavefunction of the system is factorizable. Hence the obvious generalization is[17]:

$$\mathbf{v_j}(\mathbf{x_j}) = \frac{\mathrm{Re}[\frac{\hbar}{2ima}\int_{-\infty}^{+\infty}\psi_f^*(\mathbf{x_1},...,\mathbf{x_n})\vec{\nabla}_\mathbf{j}\psi_i(\mathbf{x_1},...,\mathbf{x_n})\, d^3x_1...d^3x_{j-1}d^3x_{j+1}d^3x_n]}{\mathrm{Re}[\frac{1}{a}\int_{-\infty}^{+\infty}\psi_f^*(\mathbf{x_1},...,\mathbf{x_n})\,\psi_i(\mathbf{x_1},...,\mathbf{x_n})\, d^3x_1...d^3x_{j-1}d^3x_{j+1}d^3x_n]} \tag{45}$$

Since all the coordinates apart from $\mathbf{x_j}$ are integrated out, each particle's velocity continues to be expressible separately in 3-dimensional space for this general situation.

As discussed in Sec. 3(ii), the ψ_f of each particle will tend to become more correlated in going towards the past (i.e., the opposite of what occurs for each ψ_i), so the situation of the system having a factorizable final wavefunction, as described in Eqs. (42) to (44), is expected to be a common one. Concerning the non-factorizable case of Equation (45), however, it should be noted that a separate initial wavefunction in 3 dimensions cannot necessarily be assigned to each particle. By examining (42) and (43) one sees that the condition for the j^{th} particle to be able to be assigned its own ψ_i despite there being initial correlations is that this particle must have a separate final wavefunction:

$$\psi_f(\mathbf{x_1},...,\mathbf{x_n}) = \psi_f(\mathbf{x_1},...,\mathbf{x_{j-1}},\mathbf{x_{j+1}},...,\mathbf{x_n})\,\psi_f(\mathbf{x_j}) \tag{46}$$

§12 <u>Predictive Limitations and Theory of Measurement</u>

We will now consider some weaker points of the causally symmetric model.

Firstly, in the standard Bohm model the outcome of any measurement on a particle can be predicted once the particle's wavefunction and (hidden) initial position are specified. By contrast, this cannot be the case in any retrocausal model because the future is part of the cause, rather than just being the effect. The most that a causally symmetric model can do is predict the intermediate situation deterministically once the initial and final boundary conditions are known.

Secondly, the standard Bohm model also has an appealing theory of measurement which allows the probabilities for all observables other than position to be deduced once the position probability distribution $|\psi(\mathbf{x})|^2$ is assumed. Such a derivation, however, is not possible from the position distribution (17) of the causally symmetric model. An extra assumption needs to be postulated for the model to incorporate the correct general probability expression. The simplest approach is simply to add (21) as an extra postulate of the model:

$$\rho(\psi_f|\psi_i) = |\langle\psi_f|\psi_i\rangle|^2 \tag{47}$$

However, this leaves one to ponder why all final states ψ_f are not equally likely, rather than being distributed according to (47). Some sort of derivation of (47) from more basic ideas would be desirable. It should be kept in mind, however, that this is a shortcoming of most interpretations of quantum mechanics, not just of the present model.

[17] It is straightforward to show that this velocity expression is consistent with an equation of continuity, in analogy to the discussion of Sec. 4.

On a brighter note, the particular model presented here avoids the well-known measurement problem of quantum mechanics, whereby the usual formalism appears to predict that measurements will not have definite outcomes. On this point, the standard and causally symmetric versions of Bohm's model are on an equal footing since they have the same underlying ontology of localized particles. This fact, together with the requirement that any measurement process must spatially separate its possible outcomes in order to distinguish between them, ensures that a definite result is found (i.e., a detection in one channel only).

§13 <u>Causally Symmetric Model for the Dirac Equation</u>

A relativistic version of the causally symmetric model developed above will now be formulated for the single-particle Dirac case. Taking ħ = c = 1, the Dirac equation has the form:

$$\gamma^\mu \partial_\mu \psi + im\psi = 0 \tag{48}$$

and its hermitean conjugate is:

$$\partial_\mu \bar{\psi} \gamma^\mu - im\bar{\psi} = 0 \tag{49}$$

where:

$$\bar{\psi} = \psi^\dagger \gamma^0 \tag{50}$$

We proceed in the usual way to an equation of continuity. Multiplying (48) from the left by $\bar{\psi}$ and (49) from the right by ψ and then subtracting yields the familiar result:

$$\partial_\nu(\bar{\psi}\gamma^\nu\psi) = 0 \tag{51}$$

Now, it is easily checked that the derivation of equation (51) from (48) and (49) would remain valid if ψ and $\bar{\psi}$ were two independent functions rather than being related as hermitean conjugates. We can therefore proceed as in the non-relativistic case of Sec. 4 to modify (51) by (i) replacing ψ with ψ_i and $\bar{\psi}$ with $\bar{\psi}_f$, (ii) introducing a normalizing constant a, and (iii) taking the real part, to obtain the following equation of continuity:

$$\partial_\nu \, \mathrm{Re}(\tfrac{1}{a}\bar{\psi}_f \gamma^\nu \psi_i) = 0 \tag{52}$$

with:

$$a \equiv \int_{-\infty}^{+\infty} \bar{\psi}_f(\mathbf{x},t)\gamma^0\psi_i(\mathbf{x},t)\, d^3x \tag{53}$$

Equation (52) will hold automatically provided ψ_i and ψ_f are solutions of the Dirac equation and so is suitable to serve as a probability-conserving starting point for a causally symmetric model. As mentioned in Sec. 9, it will be more useful here to work in terms of the particle's 4-velocity $u^\nu = \frac{dx^\nu}{d\tau}$, rather than its 3-velocity $\mathbf{v} = \frac{d\mathbf{x}}{dt}$. Thus, comparing (52) with (26) points to the identification:

$$\rho_0 u^\nu = \mathrm{Re}(\tfrac{1}{a}\bar{\psi}_f \gamma^\nu \psi_i) \tag{54}$$

which provides a suitable current density expression for the model. In particular, the causally symmetric probability density for the particle's position is:

$$\rho \equiv \rho_0 u^0 = \mathrm{Re}(\tfrac{1}{a}\overline{\psi}_f \gamma^0 \psi_i) \tag{55}$$

Now, referring back to (35), we have the identity:

$$\rho_0 = \left| (\rho_0 u_\alpha)(\rho_0 u^\alpha) \right|^{½} \tag{56}$$

Hence, inserting (54) into (56), the rest density is found to be:

$$\rho_0 = \left| \mathrm{Re}(\tfrac{1}{a}\overline{\psi}_f \gamma_\alpha \psi_i)\ \mathrm{Re}(\tfrac{1}{a}\overline{\psi}_f \gamma^\alpha \psi_i) \right|^{½} \tag{57}$$

Combining this result with (54) then yields the following for the particle's 4-velocity:

$$u^\nu = \frac{\mathrm{Re}(\tfrac{1}{a}\overline{\psi}_f \gamma^\nu \psi_i)}{\rho_0} \tag{58}$$

where ρ_0 in the denominator is understood to be the expression in (57). Equations (55) and (58) are the basis of our causally symmetric Bohm model for the Dirac case.

As mentioned in Sec. 7, it is also necessary to introduce a "variable rest mass" for the particle in order to ensure the viability of this model. The need for this can be appreciated by examining equation (58). The essential point to note is that the components of the 4-velocity u will all become infinite at locations where the denominator ρ_0 becomes zero. Now, 4-velocity components are infinite only when the corresponding 3-velocity is equal to the speed of light (since the infinite value arises from the increment $d\tau$ in the definition $u = \frac{dx}{d\tau}$ being zero along a null line in space-time). Therefore, the particle must be travelling at the speed of light at any point where expression (57) for ρ_0 becomes zero. To be compatible with special relativity, the particle's rest mass must therefore be zero at such times. Now, there is some freedom in choosing an appropriate expression for the particle's rest mass. It can be any function with dimensions of mass that is zero when ρ_0 is zero. Denoting this variable rest mass by M to distinguish it from the usual rest mass m in equation (48), a simple choice of definition is:

$$M = m\,\frac{\rho_0}{\mathrm{Re}(\overline{\psi}_f \psi_i)} \tag{59}$$

This is analogous to the expression chosen by de Broglie in his model (Ref. 5, ch. 16). Equation (59) needs to be included with (55) and (58) in defining a causally symmetric model for the Dirac case.

Finally, having defined the rest mass, it is perhaps of interest to write down the expression for the particle's momentum p by combining (58) and (59):

$$p^\nu \equiv M u^\nu = m\,\frac{\mathrm{Re}(\tfrac{1}{a}\overline{\psi}_f \gamma^\nu \psi_i)}{\mathrm{Re}(\overline{\psi}_f \psi_i)} \tag{60}$$

§14 Conclusions

In this paper, a causally symmetric version of Bohm's model has been formulated. The aim has been for the advantages and disadvantages of such symmetry to be illustrated via a comparison of two otherwise similar models.

The advantages provided by causal symmetry are as follows. The theory can be Lorentz invariant, with no need for a preferred reference frame at the hidden level. Also, the apparent non-locality highlighted by Bell's theorem can be given a local explanation from a 4-dimensional viewpoint. For the many-particle case, where the usual description is in terms of a single, correlated wavefunction defined in 3n-dimensional configuration space, causal symmetry allows each particle's velocity to be described by a separate expression in 3-dimensions. In particular, each particle can be described as being guided by its own 3-dimensional initial wavefunction $\psi_i(\mathbf{x})$, as long as the particle has a separate final wavefunction $\psi_f(\mathbf{x})$.

Causal symmetry also provides a viable physical meaning for the notion of negative probabilities, thereby allowing an interpretation for the Klein-Gordon probability expression between measurements. It even implies a possible reason for why tachyons are not observed directly and allows them to exist without causal loop problems.

On the other hand, the disadvantages are as follows. The causally symmetric model is not deterministic from the initial boundary conditions (although it becomes deterministic if the final boundary conditions are specified as well). Furthermore, unlike the theory of measurement of the standard Bohm model, it is apparently not able to predict the probability distribution for a general observable from just a knowledge of the position distribution.

Although a logical explanation can be given for negative probabilities, this notion may nevertheless not appeal to everyone's taste. The same applies to the idea of rest mass varying with position, despite being logically permissible. Finally, the equations of the causally symmetric version are not quite as simple as those of the original model.

Perhaps the main value to be gained from this formulation is that options which previously were suspected to be impossible (such as Lorentz invariance and 3-dimensional descriptions) are seen to be still available within such an underlying picture. Consequently, one should not lose heart in looking for an ontological model that retains desirable features.

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