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Reassessing the Explanatory Indispensability Argument: A Bayesian Defence of Nominalism

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Advocates of the explanatory indispensability argument for platonism say two things. First, we should believe in the parts of our best scientific theories that are explanatory. Second, mathematical objects play an explanatory role within those theories. I give a two-part response. I start by using a Bayesian framework to argue that the standards many have proposed must be met to show that mathematical objects are dispensable are too demanding. In particular, nominalistic theories may be more probable than platonistic ones even if they are extremely complicated by comparison. This is true even if there are genuine cases of mathematical explanation in science. The point made here is a matter of principle, holding regardless of how one assesses nominalistic theories already on offer. I then examine my recent nominalization of second-order impure set theory in light of the correct, laxer standards. I make a tentative case that my nominalistic theory meets those standards, which would undermine the explanatory indispensability argument. While this case is provisional, I aim to bring attention to my nominalization and others in light of the revised standards for demonstrating dispensability.
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1. Introduction

The explanatory indispensability argument is one of the most influential arguments for platonism to date. Yet despite all the attention it has received, it remains hotly contested between nominalists and platonists. Vexing questions concerning mathematical explanation and empirical confirmation have formed a Gordian Knot around the argument. No consensus about how to untangle these problems appears to be in the offing.

My purpose in this paper is to intervene on the nominalist side. Unlike earlier responses to the argument, however, my approach does not require taking a stand on many issues that are sources of intense disagreement in the literature. This paper is an attempt to cut the Gordian Knot rather than untie it, at least for the purpose of settling the debate between platonists and nominalists.

I begin with a short description of the explanatory indispensability argument. I then provide an overview of my response to it, with an intuitive characterization of the central idea. Briefly put:

on Bayesian grounds, it can be shown that many have proposed excessively demanding standards for what it takes to demonstrate that mathematical objects are dispensable. Contrary to what some have said, it is, in principle, possible for nominalists to undercut the explanatory indispensability argument even if mathematics can genuinely explain some empirical phenomena, platonism is capable of empirical confirmation, and nominalistic theories are extremely complicated compared to platonistic ones. Next, I make some clarificatory remarks. I then unveil the probabilistic apparatus used in my response and make my formal argument. Subsequently, I discuss my ([2023]) recent modal nominalization of impure second- order Zermelo-Fraenkel set theory with Choice (ZF^2CU). I proceed to make a tentative case that, in light of my earlier conclusions, my nominalization in fact undermines the explanatory indispensability argument. I do this in the hope of bringing renewed attention to nominalizations of mathematics that are already on offer, as my general argument proposes new standards for how they ought to be evaluated. I conclude by reflecting on the continued relevance of determining whether there are genuine instances of mathematics explaining empirical phenomena.

2. The Explanatory Indispensability Argument

The explanatory indispensability argument (EIA), introduced by Alan Baker and Mark Colyvan, runs as follows:[1]

1. We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
2. Mathematical objects play an indispensable explanatory role in science.
3. Therefore, we ought rationally to believe in the existence of mathematical objects.

Mathematical entities are understood to be abstract, platonic objects. It therefore serves as an argument for platonism.

The idea behind the EIA is that we should be committed to at least those entities that play an indispensable explanatory role in our best science. Advocates of the EIA then point out that mathematical entities appear to play such a role in our best scientific theories. Two purported cases are the lifecycle of cicadas and honeybees' use of hexagons when constructing their hives. Certain *Magicada* species of cicadas emerge every 13 or 17 years. This prime-numbered lifecycle is believed to aid in predator avoidance, as it doesn't align well with the life cycles of potential predators.[2] Meanwhile, honeybees instinctively opt for a hexagonal tessellation when constructing their hives. Geometrical principles entail that this pattern allows the bees to maximize storage space while minimizing the amount of wax used.[3]

[1]This is Baker's formulation ([2009], p. 613). See also Baker ([2005]) and Colyvan ([2001]).
[2]See Baker ([2005], pp. 229-36).
[3]See Aidan Lyon and Colyvan ([2008], pp. 228-29).

Some clarification is in order regarding 'indispensability'. Supporters of indispensability arguments often distinguish dispensability from mere eliminability. Colyvan ([1999], pp. 5-8) has suggested that any attempt to dispense with mathematical entities needs to produce theories that are preferable to platonistic ones. Colyvan cashes this out in terms of theoretical virtues. A nominalist rendering of a scientific theory must have a more attractive mixture of simplicity, explanatory power, and boldness when compared to its platonist formulation.[4] A nominalistic theory that scores worse on these fronts than a platonistic one has only shown that mathematical objects are eliminable, not dispensable. This may happen even if the nominalistic theory is equally as explanatory as the platonistic one: nominalist theories can still be more complicated even if there is no difference in explanatory power. Though the eliminability/dispensability distinction comes from Colyvan, in a similar vein, John P. Burgess ([1983], pp. 97-100) argues that nominalistic reconstructions of scientific theories fare worse than platonistic ones by ordinary criteria for theory choice in science. He argues that the approaches to nominalizing science offered by Charles Chihara ([1990]) in *Constructibility and Mathematical Existence* and Hartry Field ([2016]) in *Science without Numbers* are more convoluted than utilizing platonistic mathematics and confer no practical advantage.[5] Though Burgess' discussion is framed in terms of theory choice rather than indispensability, his core insight is similar to Colyvan's, as Colyvan himself remarks ([1999], p. 16, fn. 4).

Insufficient attention to the distinction between dispensability and eliminability have sometimes led advocates of the EIA to sell the argument short. For example, Baker ([2009], pp. 618-19) indicates that the EIA is only directed at those who reject that any nominalization of mathematics can be carried out. This is too quick. On its own, the fact these nominalizations can be carried out only speaks to the eliminability of abstracta, not their dispensability. To assess dispensability, we have to compare the nominalizations to platonistic mathematics. Since a cursory examination of the former makes them seem like worse theories than the latter, abstracta do not at first appear to be dispensable—not in the sense of being eliminable without yielding an overall less virtuous theory. As I will argue, it's only once one sees how these constructions emerge from principles that are already reasonable that one should forgo platonism, and seeing how they do takes a fair amount of work.

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[4]Colyvan also mentions formal elegance (p. 6), but concedes that might be explicable in terms of simplicity and explanatoriness rather than be an independent virtue (p. 17, fn. 16). For the sake of brevity, I will assume that formal elegance can be so explained. If it cannot, I can be understood as making an argument that a nominalistic theory may be more probable than a platonistic one despite being more complicated and less elegant.

[5]Burgess also criticizes the notion that mathematical language really referred to some nominalistic construction all along. He distinguishes between 'hermeneutic' nominalism, which takes the position on mathematical language Burgess criticizes, and 'revolutionary' nominalism, which concedes standard mathematics is platonistic but offers an alternative ([1983], pp. 95-96). (For context, Chihara nominalizes science by nominalizing applied mathematics as such.) For my part, I take no stance on the semantics of mathematical language. Whatever mathematical language refers to in ordinary use, one can still describe explicitly platonistic and nominalistic constructions of mathematics and take those to be different theories.

3. Overview of the Response

In this section I lay out the core components of my response to the EIA, contrasting it with existing responses, and then use an analogy to informally characterize the central idea.

Many objections to the EIA have already been lodged. Much discussion has been centered on premise (2), with several philosophers suggesting that mathematics is not really explanatory.[6] Others have doubted premise (1) on various grounds. For instance, it has been argued that there could never be empirical evidence for abstracta.[7] Consensus looks to be far away, showing just how difficult it is to untangle these matters.

I take a different tack. I hold no strong opinion on whether mathematics is genuinely explanatory in the cicada or honeycomb examples, but I do not wish to rule out the possibility that mathematics can explain empirical phenomena. I am also happy to grant that, in principle, there can be empirical evidence for abstract objects. My criticism is that, depending on how 'best scientific theories' is understood, either premise (1) is false or premise (2) is false. If theories T_1 and T_2 are exactly alike in their theoretical virtues except that T_1 is simpler and T_2 has greater universal coherence—that is to say, it fits better with what we already have reason to believe—then T_2 might be more probable than T_1.[8] If universal coherence is not a factor in deciding what counts as the best theory, premise (1) is false. A nominalistic theory might be worse than a platonistic alternative, but nonetheless be more probable. On the other hand, if universal coherence is counted as one of the virtues that make one theory better than another, then premise (2) is false. Scientists may have good practical reason to ignore the metaphysics of mathematics and work with the simplest option available—they are generally concerned with some aspect of the physical world, not reality *tout court*—but nominalistic theories can still be better overall than platonistic ones despite being more complicated. A scientific 'theory of everything' should be nominalistic rather than platonistic.

The response works as follows. Suppose that there is a wholesale nominalization of scientifically-applicable mathematics with the following two features.[9] First, the nominalization satisfies *the plausibility condition*: it is entailed by principles the conjunction of which has a significantly higher subjective prior probability than platonism—prior, that is, to considering the explanatory potential of mathematics. (This description is not the proper definition of satisfying the plausibility condition, since 'significantly' is vague, but my hope is it is intuitive enough for the reader at this stage to get the sense of how my response will work. The proper definition of 'the plausibility

[6]Jody Azzouni, Otávio Bueno, and Stephen Yablo are a few examples. Mary Leng is a moderate case. See Azzouni ([2004]), Azzouni ([2012], p. 964), Bueno ([2012]), Yablo ([2005]), Yablo ([2012], pp. 1020-26), Leng ([2010], pp. 241-52), and Leng ([2012]). For a platonist reply, see Colyvan ([2010]) and Colyvan ([2012]).

[7]See Kenneth Boyce ([2018]).

[8]For a list and discussion of theoretical virtues, see Michael N. Keas ([2018]).

[9]It is worth noting that Field ([2016]) does not offer a wholesale nominalization of scientifically-applicable mathematics. Field instead nominalizes a specific scientific theory. Chihara ([1990], pp. 3-121) offers an example of what I have in mind by a wholesale nominalization, though it is not one I will focus on.

condition' will be provided in a later section.[10]) Second, the nominalization satisfies *the adequacy condition*: it has all the theoretical virtues of platonistic mathematics—including explanatory power—except that it may be more complicated.[11] Then it can be established with Bayesian reasoning that platonism remains improbable after taking into account what platonist mathematics explains. In other words, platonism is not confirmed over nominalism so long as there is a nominalization of scientifically-applicable mathematics that satisfies the plausibility and adequacy conditions. Simplicity is irrelevant: what matters is that the explanatory potential of mathematics does not confirm platonism over plausible, adequate nominalizations, and any such nominalization has a significantly higher prior probability than platonism.

Dialectically, there are two upshots to my response. The first is abstract, and the second is in relation to existing nominalist views. The abstract upshot is that Burgess and Colyvan have set the bar for dispensability too high. They do not mention universal coherence as a factor in theory choice, but it can override the support that simplicity lends a theory. Simplicity figures into assessing the likelihood of a theory by raising its prior probability, but universal coherence can raise the probability of the theory on one's background knowledge far greater.[12] This point holds regardless of whether any currently proposed nominalization of mathematics is plausible and adequate. The second upshot is that the combination of the abstract lesson with existing nominalizations of mathematics may undermine the EIA. A case can be made that some existing nominalizations of mathematics satisfy the plausibility and adequacy conditions. I will discuss my ([2023]) nominalization by way of example, and try to motivate the thought that it possesses these features. I do not take myself to be settling the issue. Rather, I see my discussion of this theory as an invitation for those in the debate over indispensability arguments to pay renewed attention to this and other nominalizations. *Pace* Colyvan and Burgess, it not enough to remark that nominalistic theories are more complicated than platonistic ones. Instead, the right grounds for criticism are that the nominalizations either are not plausible or are not adequate. Either their underlying principles are unsound, or they are lacking in some theoretical virtue besides simplicity.

Detailed discussion of my response to the EIA will be deferred to later sections, but an analogy will help to informally explain it. Suppose that Tom and Sarah wake up one morning and find that

[10]Specifically, the next to last paragraph of §5, 'Platonism, Nominalism, and Probabilities'.

[11]Note that, trivially, platonistic mathematics satisfies the adequacy condition. An adequate theory may be more complicated than platonistic mathematics, but need not be, and platonistic mathematics has all the virtues of platonistic mathematics. This observation will be useful later. Additionally, note that an adequate nominalization may have novel virtues not possessed in platonistic mathematics. All the virtues of platonistic mathematics must be there, besides simplicity; but not necessarily only those virtues.

[12]Note that in some cases, a simpler hypothesis more strongly predicts the phenomenon it can explain: the conditional probability of that phenomenon given the hypothesis is greater than the conditional probability of that phenomenon given a more complicated explanation. This is irrelevant here, however, since we are stipulating that the nominalization has the same explanatory power as platonistic mathematics. Also note that if formal elegance is viewed as an independent theoretical virtue, it seems likely it is only probabilistically relevant by raising the prior probability. Universal coherence can then override formal elegance as well.

the whole environment outside their home is wet. Now consider two hypotheses: (A) it rained, and (B) a truck with a massive sprinkler mounted on it drove through the neighborhood firing massive quantities of water everywhere. Naturally, Tom and Sarah decide that it rained. If they had even bothered to consider (B), it would be readily apparent that (B) is a much worse hypothesis. (A) is vastly simpler. Rain is a common occurrence, while a truck driving through behaving this way would be aberrant in the extreme. For (B) to occur, there would need to be people with strange psychologies going around in the neighborhood who somehow had access to the necessary technological means.[13]

But suppose that Tom and Sarah had a surveillance camera. There were many break-ins in the neighborhood lately, so they decided to take precautions. On a whim, they review the security camera footage and see that (B) took place after all. There it is, a truck driving through shooting massive quantities of water all over the neighborhood. They must have slept through it because they wore earplugs and ran a white noise machine all night. Then they turn on the news and find a reporter covering the incident.

At this point, it is apparent that Tom and Sarah have no reason to believe (A) on the basis of the wetness of their surroundings. It's not that they have evidence that (A) is false: it could have rained as well. It's just that they already know (B) took place, and (B) fully explains the condition of their environment. Nothing is added to explanation by (A). This is despite the fact that *prima facie*, (A) is a simpler, more attractive hypothesis than (B).

My position is that platonism is much like (A), while any nominalization of mathematics that satisfies the plausibility and adequacy conditions is much like (B). Moreover, I will suggest, but not definitively show, that at least one actually proposed, complicated nominalization of mathematics is plausible and adequate: my own ([2023]). As a general matter, platonistic mathematics is much simpler than mathematics' extant nominalizations. The latter do interpret mathematical axioms, but their constructions can be incredibly complicated. Nonetheless, there is independent reason to believe that those nominalistic constructions succeed. They don't rule out platonism by themselves (for reasons that will be evident in time), but they render platonism explanatorily redundant. So, the explanatory capacity of mathematics provides no reason to believe in platonism. The purpose of bringing in the Bayesian apparatus is to add some rigor to this core idea, as well as to handle cases where there is uncertainty: as if, say, the footage that Tom and Suzy had was suggestive of (B) but somewhat ambiguous. The evidence for existing nominalizations of mathematics is like this. The arguments for the underlying principles are good, but not certain.

[13]This scenario is similar to a common example involving rain and lawn sprinklers found in the discussion of Bayesian networks, but with some important differences. Crucially, unlike those examples, (A) $\wedge$ (B) has no more explanatory potential than (A) or (B) alone.

4. Clarifications on the Response

Before proceeding, several clarifications should be made. It might be wondered why I choose to work with subjective probability, what a 'construction' of mathematics is, and what I mean by the phrase 'prior to considering the explanatory potential of mathematics' in the previous section. Additionally, I sometimes speak of comparing platonistic and nominalistic accounts of mathematics, and other times speak of comparing platonistic and nominalistic scientific theories: I need to explain which I have in mind. I should also note that I will assume platonism has a low probability given our other background knowledge, at least in the sense that I will not examine what happens when it is greater than or equal to 0.5. I comment on these in turn.

The use of subjective probability is crucial. Little sense can be made of the use of objective probability when assessing platonism and nominalism. However objective probability is understood, the objective probability of platonism is either 0 or 1. Likewise for nominalism. But subjective probability is still a useful tool here. It is reasonable to have degrees of confidence toward platonism and nominalism that fall between 0 and 1. As for the use of Bayesian reasoning, standard arguments can be brought forth to the effect that having credences which violate Bayesian standards is irrational. Such people are apt to be Dutch Booked, and so on.[14] It is not my aim here to defend Bayesianism, but to apply it.

A'construction' of mathematics, which I sometimes also call an 'interpretation', is a systematic translation of mathematical assertions into an explicit ontological framework. To be satisfactory, a translation τ needs to preserve the structure of the mathematical theory. If individuals $e_1 ... e_n$ in the mathematical theory are distinct, then $\tau(e_1)...\tau(e_n)$ need to be distinct. If individuals e_1, e_2 are R-related but not R'-related in the theory, then $\tau(e_1), \tau(e_2)$ need to be $\tau(R)$-related and not $\tau(R')$-related. Etc. Admittedly, this characterization leaves some things to be desired. It gives the impression that every translation proceeds by interpreting atomic expressions and operating compositionally thereafter. Some translations do involve making uniform, wide-scope alterations of mathematical assertions too (e.g. finalizing the translation process by embedding everything within a modal operator: my own work ([2023]) is itself an example). Still, even if a translation process does involve those maneuvers, how it describes the manner in which properties and relations are distributed across individuals must be isomorphic to the reality described by the original mathematical theory.

Note that while the platonist interpretation of mathematics is trivial to introduce, it's an interpretation all the same. Real analysis does not explicitly declare that π is abstract. I will mainly use 'construction' when the translation into an ontological framework takes more work to develop.

What I have in mind by the phrase 'prior to considering [. . .]' is this. Let 'B' stand for all of our evidence besides the knowledge that mathematics can explain certain observed phenomena.

[14] Alan Haájek ([2009]) provides a survey of Dutch Book arguments.

The probability, given B, that a nominalistic interpretation of scientifically-applicable mathematics that satisfies the plausibility condition is true is significantly greater than the probability that platonism is true given B.[15] No one is actually in possession of B, since no one has access to all of the evidence ever considered by human beings: the 'our' is meant unrestrictedly. Entertaining these conditional probabilities therefore involves some idealization. But the idealization is not pernicious. When it comes to the abstract upshot of my argument, the point is just to demonstrate that a nominalistic alternative to a platonistic theory can be significantly more probable than the platonistic theory even if it is deficient in a certain theoretical virtue compared to the platonistic theory. Since that is just a theoretical point, idealizing is harmless. Moreover, when it comes to applying the abstract lesson to existing nominalist theories, the parts of B that are relevant to assessing the likelihood that certain nominalistic interpretations of mathematics are true are accessible to philosophers. They are sketched out in later sections. More about those factors can be said than what I will say, but again, my purpose here is to bring attention to these theories rather than settle the matter.

That being said, in my Bayesian argument, what I will discuss is the probability that some nominalistic interpretation of relevant mathematics is true given B and nominalism, not the probability given B alone. This difference doesn't really matter, though. Regarding the abstract lesson, we can simply add as a further assumption that a nominalization satisfying the plausibility and adequacy conditions is not less likely to be true given B and nominalism than it is to be true given B alone. The Bayesian result would then follow, showing that Burgess and Colyvan have excessively demanding standards for nominalist theories. When it comes to existing nominalistic interpretations of mathematics, examination reveals that the reasons for accepting the principles that are used to construct them more-or-less have nothing to do with the (non-)existence of abstract objects. The two conditional probabilities, if not exactly the same, are at least very close. An implication of this is that platonism and the nominalizations of mathematics I have in mind are not contraries. It is epistemically possible that platonism is true and some nominalistic reconstruction of mathematics succeeds. This is no more puzzling than the epistemic possibility that a copy of V exists, allowing for multiple platonistic interpretations of mathematics. In the context of mathematical explanation, either possibility would amount to explanatory overdetermination, much as effects can be causally overdetermined. These different interpretations of mathematics do not combine to yield any greater explanatory force. For mathematical explanation, all that matters is that some interpretation of the relevant mathematics that satisfies the adequacy condition is available, not how many such interpretations are.

When giving my Bayesian argument, I will assume that the probability of platonism given B is less than 0.5. Many platonists will disagree. They might claim that it is part of our evidence that $1 + 1 = 2$ even without knowing that mathematics can explain some empirical observations. In

[15] This emphasis on knowledge also makes clear the importance of using subjective probability in the argument.

that case, the probability of platonism given B is already 1. Obviously, it would do no good for a nominalist to concede the truth of platonism on other grounds while responding to an argument for platonism. Likewise, many scientific claims might seem to belong to B despite being platonistic. It is plausible that people can know various quantitative measurement statements are true before having thought about the explanatory power of mathematics, but such statements entail that numbers exist.[16]

My response is that what belongs to B is more modest. What belongs to B is not $1 + 1 = 2$, but rather this: the knowledge that a certain disjunction is true, namely, the disjunction that contains as its disjuncts '$1 + 1 = 2$' and each translation of '$1 + 1 = 2$' into a nominalistic construction of mathematics that satisfies the adequacy condition. In general, for any sentence φ with mathematical content such that the knowledge that φ is true seems like it belongs to B, I suggest that instead, what belongs to B is the knowledge of the truth of the disjunction that contains as its disjuncts φ and each translation of φ into an adequate nominalistic construction of mathematics.[17] This is justified by what it takes for a construction of mathematics to satisfy the adequacy condition. If a construction is truly adequate, it must, in some sense, be capable of describing the same facts as platonistic mathematics—otherwise, it is clearly theoretically deficient. For example, it had better be that the translation of a quantitative measurement statement represents the same physical reality as the original. One would not accept a construction that offers a translation of '8 measures the mass of the bowling ball in kilograms' with a truth-condition involving a barricuda rather than a bowling ball, or involving length rather than mass. Of course, talk of 'describing the same facts' or 'representing the same physical reality' is vague: a nominalist had better supply a precise characterization of what these ultimately amount to alongside their preferred construction of mathematics. For now, though, the intuitive notion should convey that B can be metaphysically neutral with regard to platonism while still containing information that is stated with mathematical language.[18]

Platonists may resist, claiming B contains more information than I let on. Some believe that the existence of abstract mathematical objects is *a priori*, in no need of confirming evidence from the empirical world.[19] My argument will not convince *a priori* platonists. No matter. I do not aim to convince them, and it is the limits of my ambition that justifies my stance. My aim is to persuade those who hold that the explanatory potential of mathematics is the only evidence for platonism of

[16]The argument of this paragraph assumes that by mathematical language is platonistic by default. Not all agree, and I am neutral. See fn. 5.

[17]Note that this knowledge is *de dicto*. It doesn't require anyone to believe there is an adequate nominalistic construction of mathematics or be able to give specific translations.

[18]A similar point can be made when the phenomenon to be explained is mathematically described. What really needs explanation, in my view, is the truth of the disjunction that contains as its disjuncts the phenomenon's standard description and that description's translations in adequate interpretations of mathematics. E, introduced in the next section, may likewise be a disjunction that contains as its disjuncts explanations corresponding to the different interpretations of mathematics. For the most part, this is ignored to avoid unnecessary complication.

[19]Two examples are Bernard Linksy and Edward N. Zalta ([1995]).

two things: (a) demonstrating mathematical objects are dispensable is not, in principle, as difficult as some have supposed; and (b) that several nominalist proposals on offer should be reevaluated in light of that fact. As Colyvan ([2023]) points out, many do regard this explanatory potential to be the only evidence for platonism.[20] If the explanatory power of mathematics were to be removed from consideration, such individuals would have a low credence in platonism. So my assumption that the probability of platonism given B is low is dialectically justified: it's shared with the platonist audience I am trying to engage. Besides, if the *a priori* platonists are right, platonism has no need for the explanatory indispensability argument. B justifies platonism on its own.

It should be noted that assuming a theory satisfies the plausibility condition does not require assuming that it is likely to be true. It just requires that the theory has a significantly greater prior probability than platonism. It turns out that there are scenarios where one shouldn't be convinced of platonism even if the probability of every nominalization of mathematics given B is less than 0.5. These obtain when the probability of platonism given B is very low. The specific numbers will be provided in the next section, but the basic point is significant. For one, it goes even further toward the point that some philosophers ask too much of nominalists when they propose alternatives to platonist theories. For another, some philosophers may doubt many principles involved in existing nominalizations of mathematics, but be so skeptical of platonism before taking note of mathematical explanation that platonism still loses out to nominalism in the probability calculus.

Now for the matter of whether it is competing accounts of mathematics or competing scientific theories that are being evaluated. The idea is this. For the purpose of evaluation, scientific theories are reconstrued in terms of nominalistic constructions of mathematics.To keep matters simple, it is assumed that pure mathematics forms a part of standard scientific theories: scientists generally feel free to use as much mathematics as they want, so this simplification seems harmless. So pure mathematics, as a fragment of the initial, platonistic theory, combined with 'impure' statements that mix mathematical and physical vocabulary, end up being given nominalistic translations and placed within the new, nominalistic scientific theories. (For any given platonistic scientific theory, there's a bijection between its nominalistic counterparts and the nominalistic constructions of mathematics: each construction gets its own version of the scientific theory.) The purely physical parts of the initial theory are present without alteration in the new theories. The end result is that empirical confirmation of the nominalistic scientific theories also counts as empirical confirmation of the nominalistic constructions of mathematics; likewise for the empirical confirmation of the platonistic scientific theory and platonistic mathematics. At least, this is true provided platonistic mathematics and its nominalistic constructions play explanatory roles within their respective

[20]'If, as some believe, the indispensability argument is the only argument for platonism worthy of consideration, then [. . .]' (Colyvan [2023], §6).

theories.[21] I speak of nominalizations of scientific theories and nominalizations of mathematics interchangeably because they are simultaneously emprically confirmed (or empirically disconfirmed, for that matter).

With these clarifications made, I now develop the argument.

5. Platonism, Nominalism, and Probabilities

To begin, we need to introduce four propositions:

- P = mathematical platonism is true.
- M = some interpretation of scientifically-applicable mathematics that satisfies the adequacy condition is true.
- E = a description of how mathematics is able to explain various empirical phenomena.
- B = a statement of idealized background knowledge.

$\neg P$ is a statement of nominalism. B is idealized background knowledge in the manner described in the previous section.

We now lay out two substantive claims relating these propositions:

(1) $\Pr(M|P, B) = 1$

(2) $\Pr(E|\neg P, M, B) = \Pr(E|P, M, B) = \Pr(E|M, B)$

Justifying (1) and (2) is straightforward. Regarding (1), platonism entails that the platonistic interpretation of scientifically-applicable mathematics is true. That is part of the definition of platonism.[22] We've already noted (in fn. 11) that platonistic mathematics trivially satisfies the adequacy condition. (2) follows from (i) the definition of what it takes for an interpretation of scientifically-applicable mathematics to satisfy the adequacy condition, and (ii) the absence of any additional explanatory benefit carried by having multiple adequate interpretations (also noted in §4, 'Overview of the Response'). Regarding (i), an adequate interpretation must have the same explanatory power as platonistic mathematics, so whatever evidential force E has for platonistic mathematics will carry over to any adequate interpretation of mathematics. Meanwhile, (ii) means we needn't worry about the probability of E given P, B, and some adequate nominalization of mathematics N. If $\Pr(E|P, N, B) > \Pr(E|N, B)$, then despite (i) entailing that $\Pr(E|P, B) = \Pr(E|N, B)$, it could be that $\Pr(E|P, M, B) > \Pr(E|\neg P, M, B)$.[23] Fortunately, (ii) excludes the possibility that $\Pr(E|P, N, B) > \Pr(E|N, B)$. For like reasons, we needn't think about the

[21]Penelope Maddy ([1992], [1995]) and Elliott Sober ([1993]) influentially criticized confirmation holism. Their criticism made it seem as though tenable forms of scientific realism might reveal that empirical evidence has no bearing on platonism. The EIA was developed in response to this concern, with the thought being that explanatory parts of theories, at least, get confirmed by empirical evidence.

[22]Or at least mathematical platonism: I will say nothing about properties and the like.

[23]Note that $\Pr(E|P, B) = \Pr(E|P, M, B)$ by (1). Also note that because N is adequate, $\Pr(M|N, B) = 1$, so $\Pr(E|N, B) = \Pr(E|N, M, B)$.

probability of E given B and multiple adequate nominalizations of mathematics as a separate case from having one. M is all that matters.

For the notion that the explanatory potential of mathematics supports platonism to get off the ground, it must be that $\Pr(E|M,B) > \Pr(E|\neg M,B)$. Implicit in this is a controversial solution to a thorny problem for Bayesianism: the problem of old evidence. The problem is that the empirical observations that E concerns are already in B, which makes it appear that they cannot confirm any new theory in a Bayesian framework. Daniel Garber ([1983]) proposed that what does the confirming is E, not the observations themselves. But not all agree with Garber that claims like E can confirm theories. Personally, I am inclined to accept Stephan Hartmann and Branden Fitelson's ([2015]) Garber-style view.[24] If they are right, it could be shown that $\Pr(E|M,B) > \Pr(E|\neg M,B)$ by considering certain additional propositions and arguing they meet Hartmann and Fitelson's constraints. However, I don't need to take a stance on this. For my purposes, it's better to just take it for granted that $\Pr(E|M,B) > \Pr(E|\neg M,B)$. That there is some way to work this out is a concession to platonists. Additionally, I focus on theories that satisfy the adequacy condition rather than alternatives to platonistic mathematics that merely possess the same explanatory power because I assume that solutions in this vein can be provided for boldness and other theoretical virtues. This is another concession to platonists. If any virtue cannot be made probabilistically relevant, it can be excluded from the definition of an adequate theory, lightening the burden that nominalists must bear.[25] Simplicity is special in that it figures into the prior probability of a theory, before confirming empirical facts are taken into account.[26] Universal coherence is special in this way too: B is the only salient consideration.

Claim (1) entails that $\Pr(P,M|B) = \Pr(P|B)$, so we needn't discuss $(P,M|B)$ as a separate case from $(P|B)$. Now we turn to two applications of Bayes' theorem for the probability of a proposition given multiple conditions:

- $\Pr(P|E,B) = \dfrac{\Pr(E|P,B) \times \Pr(P|B)}{\Pr(E|B)}$
- $\Pr(\neg P,M|E,B) = \dfrac{\Pr(E|\neg P,M,B) \times \Pr(\neg P,M|B)}{\Pr(E|B)}$

Claim (1) entails that $\Pr(E|P,M,B) = \Pr(E|P,B)$. That and claim (2) together entail that $\Pr(E|\neg P,M,B) = \Pr(E|P,B)$. So given the above, where $x = \Pr(E|P,B) = \Pr(E|\neg P,M,B)$ and $y = \Pr(E|B)$:

- $\Pr(P|E,B) = \dfrac{x \times \Pr(P|B)}{y}$
- $\Pr(\neg P,M|E,B) = \dfrac{x \times \Pr(\neg P,M|B)}{y}$

[24] For criticism, see David Kinney ([2019]).

[25] I will note that I am doubtful whether boldness should be seen as an independent virtue. It strikes me that truly virtuous boldness stems from an underlying increase in explanatory power.

[26] At least this is true for the virtues listed by Colyvan ([1999], p. 6), with the caveat mentioned before (fn. 4) that formal elegance is reducible to simplicity and explanatoriness. We will see how this might be questioned later.

Thus, as long as $\Pr(\neg P, M|B) > \Pr(P|B)$, it will turn out that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$. E does not confirm P over $\neg P \wedge M$.

To gain insight about the conditions under which $\Pr(\neg P, M|B) > \Pr(P|B)$, we simplify the inequality:

Using the definition of conditional probability and chain rule on the left half:
$\Pr(\neg P|B) \times \Pr(M|\neg P, B) > \Pr(P|B)$
This becomes, by using standard principles for $\neg$:
$(1 - \Pr(P|B)) \times \Pr(M|\neg P, B) > \Pr(P|B)$
Which, by dividing both sides by $(1 - \Pr(P|B))$, comes to:
$\Pr(M|\neg P, B) > \dfrac{\Pr(P|B)}{1 - \Pr(P|B)}$

So then, $\Pr(\neg P, M|B) > \Pr(P|B)$ just in case $\Pr(M|\neg P, B) > \dfrac{\Pr(P|B)}{1 - \Pr(P|B)}$

With this in hand, we should consider some possible values for $\Pr(M|\neg P, B)$. This will tell us what $\Pr(P|B)$ needs to be for it to be the case that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$, given different degrees of credence about $\Pr(M|\neg P, B)$. Calculations are truncated to six digits.

For it to be that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$:

- If $\Pr(M|\neg P, B) = 1$, $\Pr(P|B)$ needs to be < 0.5.
- If $\Pr(M|\neg P, B) = 0.9$, $\Pr(P|B)$ needs to be < 0.473684.
- If $\Pr(M|\neg P, B) = 0.8$, $\Pr(P|B)$ needs to be < 0.444444.
- If $\Pr(M|\neg P, B) = 0.7$, $\Pr(P|B)$ needs to be < 0.411764.
- If $\Pr(M|\neg P, B) = 0.6$, $\Pr(P|B)$ needs to be < 0.375.
- If $\Pr(M|\neg P, B) = 0.5$, $\Pr(P|B)$ needs to be < 0.333333.
- If $\Pr(M|\neg P, B) = 0.4$, $\Pr(P|B)$ needs to be < 0.285714.
- If $\Pr(M|\neg P, B) = 0.3$, $\Pr(P|B)$ needs to be < 0.230769.
- If $\Pr(M|\neg P, B) = 0.2$, $\Pr(P|B)$ needs to be < 0.166666.
- If $\Pr(M|\neg P, B) = 0.1$, $\Pr(P|B)$ needs to be < 0.090909.

Here we can draw a general lesson.

Lesson: The lower the value of $\Pr(M|\neg P, B)$, the lower the upper threshold is on $\Pr(P|B)$ for it to be the case that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$.

But this is all well and good for the nominalist.

When it comes to the abstract upshot of my argument, I am now able to supply a proper definition of the plausibility condition, getting rid of the vague phrase 'significantly higher subjective prior probability'. Let N be an arbitrary nominalization of mathematics. Let $\mathcal{T}$ be the full table of values (up to some arbitrary length of decimal expansion) for $\Pr(M|\neg P, B)$ and $\Pr(P|B)$ such that each row yields the result that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$ given substantive claims (1) and (2). N is satisfies the plausibility condition just in case $\Pr(N|B)$ is one of values in the column for

$\Pr(M|\neg P, B)$ in $\mathcal{T}$ belonging to a row of $\mathcal{T}$ containing the actual value of $\Pr(P|B)$ in the column for $\Pr(P|B)$ in $\mathcal{T}$.[27] Given that N also satisfies the adequacy condition (i.e. $\Pr(M|N, B) = 1$, which entails $\Pr(M|\neg P, B) \geq \Pr(N|\neg P, B)$) and that $\Pr(N|\neg P, B) \geq \Pr(N|B)$ (another assumption noted in §4), $\Pr(M|\neg P, B) \geq \Pr(N|B)$. So, N being plausible and adequate guarantees that $\Pr(\neg P, M|E, B) > \Pr(P|E, B)$.

As for existing nominalizations, I will make a tentative case that mine ([2023]) satisfies the adequacy and plausibility conditions. I will not explicitly use the proper definition of being plausible, for doing so would not be illuminating. Instead, I will argue my nominalization is plausible by offering reasons for thinking that its probability given B is quite high, compared to the low probability of platonism given B. 'High' and 'low' here serve as schematic notions. My audience should assess the force of the arguments I make on behalf of my nominalistic construal of mathematics, figure out their credence in $(M|\neg P, B)$ in light of that assessment, and compare that credence to their credence in $(P|B)$. But to put my cards on the table, in my view, $\Pr(M|\neg P, B) > 0.5$ and $\Pr(P|B) < 0.05$. Platonism is a fantastical thesis about the world, and my nominalistic interpretation of mathematics, among others, has solid foundations.

6. Nominalizing Mathematics

Having given my general argument, I am now in a position to give a provisional evaluation of existing nominalizations of mathematics. I will use my recent modal nominalization of $\mathrm{ZF}^2\mathrm{CU}$ ([2023]), which I will call 'counterfactual megethology' owing to its utilization of counterfactuals and reliance on plural and mereological resources, as my example.[28] I begin by motivating this choice. I then provide a brief overview of the philosophical principles counterfactual megethology relies on and a sketch of how counterfactual megethology works. In the next section, I will offer a provisional argument that counterfactual megethology satisfies the plausibility and adequacy conditions.

Several wholesale nominalizations of mathematics have already been developed. Pioneering work was done by Chihara ([1990]) in *Constructibility and Mathematical Existence*, mentioned earlier, and Geoffrey Hellman ([1989]) in *Mathematics without Numbers*. Chihara's constructibility theory nominalizes a version of type theory in terms of possible open sentence-tokens, while Hellman's modal-structuralism nominalizes mathematical theories in terms of possible structures satisfying their axioms. The argument I will give could have been framed in terms of the Chihara

[27] The plausibility condition was originally formulated in terms of principles that entail a nominalization, but no matter. That was always a choice of convenience, designed to help indicate why anyone would think a complicated nominalization might have a high prior probability.

[28] As Lewis ([1993], p. 3) states, '[m]egethology is the result of adding plural quantification [. . .] to the language of mereology'.

([1990]) and Hellman ([1989]) theories, but I focus on counterfactual megethology because it sidesteps an important difficulty encountering those earlier proposals.[29] In particular, unlike Chihara's 1990 and Hellman's 1989 theories, counterfactual megethology has no problem holding facts about the physical world 'fixed' in relevant possibilities. As we noted before, for a nominalization of an application of mathematics to get the right results, it is crucial that the nominalization 'describes the same facts' as standard mathematical assertions about the physical world.[30] As will be evident, counterfactual megethology is designed to tackle this problem in a straightforward way. This is not to say that Chihara's and Hellman's modal theories could not solve this problem. It is just that engaging in that issue is a distraction from the core argument I wish to make.

Now for the summary of counterfactual megethology. For the full details, I refer the readers to my article ([2023]).

Counterfactual megethology is, like Hellman's theory, a form of modal-structuralism: it nominalizes ZF^2CU in terms of possible structures satisfying ZF^2CU's axioms. The key difference between Hellman's modal-structuralism and counterfactual megethology is that the former makes use of restricted necessity operators rather than counterfactuals. In addition to using counterfactuals, counterfactual megethology makes use of the following principles: classical mereology, including mereological universalism; primitive plural quantification, including full plural comprehension; a plural-mereological analogue of Global Choice; and a hypothetical inaccessible infinity of simples. To develop counterfactual megethology, I start by observing a result proven by David Lewis ([1993]), namely, that given the first three principles, the existence of an inaccessible infinity of simples yields a construction of ZF^2CU. My manner of construction can be summarized as follows.[31] First, from an infinity of simples (defined as simples that form some composites C such that each composite among C is a proper part of some other composite among C; capital letters will be used as plural variables from now on), arbitrary ordered pairs can be constructed using mereological and plural resources. Several methods for doing so were described in Burgess, A. P. Hazen, and Lewis' Appendix to Lewis' *Parts of Classes*.[32] Plurally quantifying over ordered pairs plays the role of relations. With the analog of Global Choice, it can shown that some relation (that

[29]Chihara ([1990]) and Hellman ([1989]) have been discussed in the literature on the EIA before, but in different terms. For example, Christopher Pincock ([2004]) argues that the Hellman ([1989]) theory would undermine the EIA. Bueno and Colyvan ([2011], pp. 348-52) reply that that Pincock's argument relies on an impoverished view of how mathematics connects to the world. In particular, they argue that mathematical inferences need to be preserved, going beyond what Pincock requires in his 'mapping' view (Bueno and Colyvan [2011], pp. 352-70). Whether Bueno and Colyvan are right about Pincock's mapping conception or not, however, Hellman's 1989 account is capable of carrying out any mathematical inference that could be done platonistically. Likewise, the theory I will discuss can carry out scientifically-applicable mathematical inferences. Nevertheless, both theories face the objection that they are more complicated than platonist treatments of mathematics.

[30]See Mary Leng ([2017], pp. 140-41) for more details.

[31]See pp. 158-63 of my article ([2023]). There, and consequently here, I follow a suggestion from Hellman in correspondence.

[32]See Burgess, Hazen, and Lewis ([1991]). To be clear, with the Burgess-Hazen-Lewis methods, we get this result: for all x, y, there exists the pair (x, y) and the pair (y, x).

is, some plurality of ordered pairs), $>$, well-orders all the simples. With that in mind, the inaccessibility of some simples X is defined this way: there are some Y among X such that Y are infinite and there is no surjection from Y to X, and for any simples Z such that there is no surjection from Z to X, (a) there are more simples in X than there are composites made up of simples in Z, and (b) for any function on Z, there is some x among X such that x is greater under $>$ than anything mapped to by the function. ('Surjection' and 'function' here should be understood as referring to pluralities of ordered pairs that collectively meet the criteria for surjections and functions, respectively. Any talk of functions should be so understood from here on out.) This allows for the formation of a cumulative hierarchy like that of set theory. At the bottom, the objects that are to be regarded as urelements are mapped to simples. For successor stages, we consider the composites of the simples at the previous levels of the hierarchy and map them to new simples.[33] For limit stages, we consider the composites of all the simples at the stages below it and map the composites which did not appear in earlier stages to new simples, keeping the mapping the same as before for what already appeared in earlier stages. Thus, simples act like singleton sets under the mapping, while composites of them act like unions.

Now, 'ZF^{2}CU' is ambiguous. To be explicit, what is meant is the conjunction of these axioms, with the arbitrary ordered pairs generated by the Burgess-Hazen-Lewis methods assumed in the background:[34]

Extensionality: No two sets have the same members.

Pairing: If each of x and y is an urelement or a set, then there exists a set of x and y.

Replacement: If there are some ordered pairs whereby each member of a set x is paired with exactly one thing among some urelements and/or sets Y, and if for each thing among Y there is a member of x that is paired with it, then there is a set of all and only the things among Y.

Foundation: No set intersects each of its own members.

Choice: If x is a set and there are some ordered pairs whereby each member of x is paired with at least one thing and no two members of x are paired with the same thing, then there is a set y such that each member of x is paired with exactly one member of y.

Power Set: If x is a set, there is a set of all subsets of x.

Union: If x is a set, there is a set of all members of members of x.

Infinity: There are some sets S such that for any x and y among S, either x is a proper subset of y or y is a proper subset of x.

[33]Keeping in mind that a simple is a composite of itself with itself.

[34]Besides Urelement, more-or-less these axioms are formally stated by Lewis ([1991], pp. 100-07). One caveat is that Lewis' axioms apply to proper classes as well as sets. Since counterfactual megethology, unlike Lewis' megethology, doesn't mention proper classes, the axioms shouldn't mention proper classes. Plurals are used instead where required. That said, it would be easy to introduce proper classes to counterfactual megethology if desired, in which case Lewis' exact axioms would do.

Urelement: For all x, x is an urelement if and only if x is actually part of our spacetime (i.e. letting 'α' be a name for our spacetime and '@' be the standard actuality operator from modal logic, $@[x$ is part of $\alpha])$; and for any Y of urelements, there exists a set of all and only the things among Y.

While ZF^{2}CU leaves open some questions regarding large cardinals, it contains all mathematics that is needed for known scientific applications. Certainly the mathematics involved in the cicada and honeycomb examples frequently discussed in connection with the EIA lives within ZF^{2}CU. Indeed, given the structure of our spacetime, it's difficult to imagine a possible scientific application needing more than what is in ZF^{2}CU.

Taking the guarantee that an inaccessible infinity of simples generates a hierarchy of composites satisfying the axioms of ZF^{2}CU as my launching point, I interpret ZF^{2}CU in terms of claims about what would be the case if there were an inaccessible infinity of simples and certain other conditions were met that ensured facts about our spacetime are held 'fixed'—in particular, that those simples were spatiotemporally disconnected from our spacetime, keeping them from causally interacting with it. Since facts about our spacetime are held 'fixed', for any φ about our spacetime that lacks mathematical content, it turns out that $\ulcorner\varphi \leftrightarrow$ (there is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime) $\Box\!\!\rightarrow \varphi\urcorner$ is true. This means that counterfactual megethology allows for any mathematical argument to be emulated in a new, counterfactual-megethological argument through the following process.[35]

First, for every premise of the original argument φ about our spacetime lacking mathematical content, add φ to the new argument and infer $\ulcorner$(there is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime) $\Box\!\!\rightarrow \varphi\urcorner$ from the corresponding biconditional. Second, for every premise of the original argument with mathematical content ψ, take $\ulcorner$(there is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime) $\Box\!\!\rightarrow T(\psi)\urcorner$ as a premise in the new argument. $T(\psi)$ is the reinterpretation of ψ in terms of the inaccessible infinity of simples, composites of which function like sets as described. Third, carry out deductions in the new argument using all these counterfactuals. It is a general truth that if χ is a conclusion of some mathematical argument, $\ulcorner$(there is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime) $\Box\!\!\rightarrow T'(\chi)\urcorner$ can be proven in the corresponding argument produced by following the first two steps—a principle I call 'preservation'. '$T'(...)$' is defined as follows: for any sentence ϕ, if ϕ has mathematical content, $T'(\phi)$ is $T(\phi)$; otherwise, $T'(\phi)$ is ϕ. (Note that a mathematical argument may include premises that are not mathematical in nature, as can happen in applications of mathematics.) Fourth and finally, if the conclusion σ of the original argument is about our spacetime and lacks mathematical content (i.e. $T'(\sigma)$ is σ), infer σ from from the new counterfactual conclusion (i.e. $\ulcorner$(there is an inaccessible infinity of simples spatiotemporally

[35]More details are found in my article ([2023], pp. 163-64, 168-72).

disconnected from our spacetime) $\Box\!\!\rightarrow \sigma\urcorner$) and the corresponding biconditional. Whatever conclusion about our spacetime we reached with mathematics in the original argument, we can reach with the new, counterfactual-megethological argument as well.

Of course, merely being able to carry out mathematical reasoning is not enough. It needs to be that the translation of an assertion with mathematical content, pure (e.g. '$1{+}1 = 2$') or applied (e.g. '8 measures the mass of the bowling ball in kg'), 'describes the same facts' as the original. Even if the translations allow for all the formal derivations we might like, they will not be satisfactory if their truth-conditions are off the mark. I call my own notion along the lines of 'representing the same physical reality' that applies in cases of successful representation 'correctness' ([2023], p. 165). A mathematical sentence φ is correct just in case (i) φ is a theorem of $\mathsf{ZF}^2\mathsf{CU}$, or (ii) there is some nominalistic ψ such that ψ is true and it is *a priori* that if platonism is true, then $\ulcorner\varphi \leftrightarrow \psi\urcorner$ is true. ψ need not be *a priori*, but the conditional is *a priori*.

The central idea is this.[36] In the case where φ is a mathematical theorem, I show that the translation of φ is a theorem of counterfactual megethology as well. Thus, they trivially 'describe the same facts' with regard to our spacetime: neither specifies anything about it. As for when φ is an applied mathematical statement, I argue that $\ulcorner$(there is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime) $\Box\!\!\rightarrow T(\varphi)\urcorner$, if true, meets the conditions needed for ψ in (ii). (Recall that here, $T(\varphi)$ reinterprets the mathematical content of φ in terms of the inaccessible infinity of simples.) The thought is that platonists and nominalists alike can agree that how mathematical entities do relate to our spacetime if they exist is just how their replacements built out of extra simples spatiotemporally disconnected from us would relate to our spacetime were they to exist. The physical aspects of our spacetime wouldn't change. Moreover, the same mathematical structures would be present in the hypothetical scenario, just with the simples and the composites they form serving as their ontological basis instead of abstracta. So of course if φ is true given mathematical objects exist, the parallel assertion using the simples-and-composites-thereof ontological basis would be true if there were the extra simples. In this sense, they 'describe the same facts'. The importance of specifying the conditional is *a priori* is to highlight that this knowledge in no way depends on prior knowledge regarding the existence of abstracta (which I assume is not *a priori*).

So counterfactual megethology allows for mathematical inferences to be emulated, and my notion of correctness helps illustrate that counterfactual-megethological translations of mathematical claims have the truth-conditions we need them to have for scientific applications.[37] Or so say I: one can look at the details in my article and assess the matter for themselves. For now, I will take this much for granted. Moreover, counterfactual megethology is nominalistic in that the hypothetical inaccessible infinity of simples could all be physical objects rather than abstracta and

[36] See my article ([2023], pp. 172-3).
[37] Together, these lead to what I call a 'safety result' ([2023], pp. 165-66, 172-3).

inaccessibility is defined without a background set theory. Since the extra simples are hypothetical, they do not constitute a new ontological commitment. That said, it should be noted that the nominalistic acceptability of the underlying principles may be brought into question. For instance, plural quantification might seem like quantification over classes in disguise, not a genuine primitive. One might also suspect that platonistic resources will be found in the correct analysis of modality. Many nominalists have regarded plural quantification and modality to be nominalistically acceptable, however.[38] For present purposes, I will assume those nominalists are right, but I note this is controversial.[39] With these caveats made, the task is now to see whether, given these claims are right, counterfactual megethology satisfies the plausibility and adequacy conditions.

7. Assessing the Nominalization

I now suggest that counterfactual megethology satisfies the plausibility and adequacy conditions. I will start by arguing it is adequate. Recall, however, that my case is intended to be provisional: much more ink would need to be spilled to settle the issue. I make it to draw attention to this theory and other nominalizations of mathematics in light of my Bayesian argument's implications for simplicity.

To begin, I'll focus on explanatory power. There are many accounts of mathematical explanation. To keep matters manageable, I will focus one that is easily tied to my work on counterfactual megethology. Baker ([2005]) offers an account of mathematical explanation wherein mathematics explains empirical phenomena by playing an indispensable role in deductions of empirical phenomena. The Cicada example demonstrates how this works. Baker provides the following deduction ([2005], p. 233):

(1) Having a life-cycle period which minimizes intersection with other (nearby/lower) periods is evolutionarily advantageous.
(2) Prime periods minimize intersection (compared to non-prime periods).
(3) *Hence* organisms with periodic life-cycles are likely to evolve periods that are prime. [1, 2]
(4) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.
(5) *Hence* cicadas in ecosystem-type, E, are likely to evolve 17-year periods. [3, 4]

[38] Hellman endorses the pairing methods in the Appendix to *Parts of Classes*, for instance, which uses plural quantification ([1996]). His *Mathematics without Numbers* is also thoroughly modal ([1989]).

[39] For those skeptical of modality's nominalistic bona fides, Hellman ([1996], pp. 112-13) shows that a construction of fourth-order Peano Arithmetic (PA^4) can made with a continuum of simples and the four central principles I use. It seems likely that spacetime points constitute a concrete continuum if spacetime substantivalism is true. We know from reverse mathematics research that even PA^4 is strong enough for current scientific applications of mathematics (see Solomon Feferman ([1998]) and Stephen G. Simpson ([1999])). PA^4 is much weaker than ZF^2CU, however. Since higher-level mathematics is not conservative with respect to lower-level mathematics, it is conceivable that some scientific application of higher-level mathematics may be found. For that reason, I focus on my theory.

E is schematic for the conditions of a specified ecosystem-type. Now, for the deduction to be strictly logically valid, there need to be some premises connecting evolutionary advantages and biological constraints to likelihoods. These seem to be left implicit. No matter: they are unproblematic claims and could be explicitly added in. Doing so seems excessive for present purposes.[40] For Baker, the key point is that premise (2) is needed to justify interim conclusion (3). The essential role of premise (2) makes mathematics explanatory.

Now consider what happens to the argument if we go through the procedure for carrying out mathematical reasoning in counterfactual megethology that was mentioned earlier. To start, we have to note that the pairing methods from Burgess, Hazen, and Lewis don't uniquely identify certain objects as ordered pairs. Instead, for any a, b, a statement of the form ...(a, b)... acquires the following form: $\forall O \forall A \forall B \forall C \forall D \forall E \forall F$ meeting such and such plural-mereological conditions supplied by Burgess, Hazen, and Lewis, ...(a, b) with respect to $OABCDEF$... .[41] In other words, statements about specific ordered pairs become universal generalizations over all the different ways of constructing that ordered pair. This means that instead of talking about a unique hierarchy of composites that satisfy the axioms of $\mathrm{ZF}^2\mathrm{CU}$, counterfactual-megethological translations describe what is true of all ways of forming a hierarchy of composites that satisfy the axioms of $\mathrm{ZF}^2\mathrm{CU}$. With that in mind, it will be useful to introduce some shorthand corresponding to this part of the translation. Here is the abbreviation:

$\mathrm{ZF}^2\mathrm{CU}$-Generalization: For all $OA...FXf$ such that $OA...F$ meet the Burgess-Hazen-Lewis conditions for defining ordered pairs with respect to $OA...F$ and X and f satisfy the axioms of $\mathrm{ZF}^2\mathrm{CU}$ with respect to $OA...F$ (where X take the place of the objects in the domain and f take the place of $\in$), [. . .]

Here 'f' is also a plural variable, but since it is a variable for some ordered pairs with respect to $OA...F$ that collectively play the role of a function, using 'f' feels appropriate for clarity. For convenience, it will also be useful to have an additional abbreviation:

Inaccessible Simples: There is an inaccessible infinity of simples spatiotemporally disconnected from our spacetime.

With these in hand, going through the steps of the process to introduce counterfactuals described in the last section leads to the following argument:

[40] Baker offers a more elaborate deduction ([2016], pp. 8-9), but owing to its complexity, I stick with his original formulation. What I say about counterfactual megethology in relation to the original argument applies equally to its relationship with the elaborate argument.

[41] Here I have in mind Lewis' final pairing method ([1993], pp. 18-19). This is a mix of the methods originally proposed by Burgess, Hazen, and Lewis ([1991]).

(1′) [Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^{2}CU-Generalization]: having a life-cycle period which minimizes intersection with respect to $OA...FXf$ with other (nearby/lower) periods is evolutionarily advantageous.[42]

(2′) [Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^{2}CU-Generalization]: prime periods with respect to $OA...FXf$ minimize intersection with respect to $OA...FXf$ (compared to non-prime periods with respect to $OA...FXf$).

(3′) *Hence*: [Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^{2}CU-Generalization]: organisms with periodic life-cycles are likely to evolve periods that are prime with respect to $OA...FXf$. [1′, 2′]

(4′) [Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^{2}CU-Generalization]: cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14-with-respect-to-$OA...FXf$ years to 18-with-respect-to-$OA...FXf$ years.

(5′) *Hence*: [Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^{2}CU-Generalization]: cicadas in ecosystem-type, E, are likely to evolve 17-with-respect-to-$OA...FXf$-year periods. [3′, 4′]

That (1′) and (2′), combined with the counterfactual-megethological translations of the implicit premises of Baker's argument, entail (3′) follows from the preservation principle (mentioned in the last section). Likewise for it being the case that (3′), (4′), and the counterfactual-megethological translations of Baker's implicit premises entail (5′). A few details can be provided to give a sense of how the proof of (3′) from (1′), (2′), and the translations of the implicit premises goes. It is easiest to grasp this by looking at a list of the steps.

- By deduction within counterfactual conditionals, if we can prove some φ from the consequents of (1′), (2′), and the translations of the implicit premises in Baker's argument, we can infer ⌜[Inaccessible Simples] $\Box\!\!\rightarrow$ φ⌝. With that in mind, we start by reasoning from these consequents.
- With plural Universal Instantiation, for the now-assumed consequents of the premises, replace 'O' with 'O', 'A' with 'A', ..., 'F' with 'F', 'X' with 'X', and 'f' with 'f'. These unitalicized, sans-sarif letters are plural constants. Drop the quantifiers that bound 'O', 'A', ..., 'F', 'X', and 'f'.
- After doing all the preceeding, one is left with statements of the form '*If* $\mathsf{OA...F}$ meet the Burgess-Hazen-Lewis conditions for defining ordered pairs with respect to $\mathsf{OA...F}$ and X and f satisfy the axioms of ZF^{2}CU with respect to $\mathsf{OA...F}$, *then*...'. So, now assume the antecedent of these statements for conditional proof for material conditionals. Infer the consequents of these statements using *modus ponens*.
- With the statements one has just inferred, one has effectively introduced statements with the same logical form as (1), (2), and the original implicit premises. By 'effectively', I mean that for the purposes of carrying out this proof, one has restored the original arity

[42]Even if Baker doesn't think premise (1) is explanatory, it still seems that it has mathematical content—minimization is mentioned, after all, and that is a quantitative notion.

of each open formula, the original number of quantifiers, the original number of logical connectives (counting by type and token) and their placement, etc. One can now derive 'organisms with periodic life-cycles are likely to evolve periods that are prime with respect to OA...FXf' with a proof that looks more or less exactly like the proof of (3) from (1), (2), and the original implicit premises.[43]

- Discharging the assumption made for conditional proof for material conditionals, infer '*If* OA...F meet the Burgess-Hazen-Lewis conditions for defining ordered pairs with respect to OA...F and X and f satisfy the axioms of ZF^2CU with respect to OA...F, *then* organisms with periodic life-cycles are likely to evolve periods that are prime with respect to OA...FXf'.
- Using plural Universal Generalization, infer '[ZF^2CU-Generalization]: organisms with periodic life-cycles are likely to evolve periods that are prime with respect to $OA...FXf$'.
- By deduction within counterfactual conditionals, infer '[Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^2CU-Generalization]: organisms with periodic life-cycles are likely to evolve periods that are prime with respect to $OA...FXf$'.

The same basic approach works for the proof of (5′) from (4′) and (3′).

But matters do not quite end here. Counterfactual megethology claims the following biconditional is true: [cicadas in ecosystem-type, E, are likely to evolve 17-year periods $\leftrightarrow$ ([Inaccessible Simples] $\Box\!\!\rightarrow$ [ZF^2CU-Generalization]: cicadas in ecosystem-type, E, are likely to evolve 17-with-respect-to-$OA...F$-year periods)]. So given (5′), one can conclude that cicadas in ecosystem-type, E, are likely to evolve 17-year periods. In other words, one can reach (5), the conclusion of the original argument. (Note that while it would be tedious, the notion of a '17- year period' could be defined with 'year' and logical vocabulary alone.)

Notice that (2′) plays an essential role in the deduction. If (2) is explanatory in Baker's original argument owing to its ineliminable role in the original argument, (2′) is likewise explanatory in the revised argument owing to its ineliminable role in the revised argument—and ultimately explanatory toward (5), given that (5) can be inferred from (5′) and the aforementioned biconditional. Additionally, I would argue that for any premise or conclusion of the original argument (N) that has mathematical content, it is *a priori* that if platonism is true, then (N) is true if and only if (N') is true ([2023], pp. 171-72). This means that the original premise or conclusion is 'correct' in my sense just in case (N') is true. Thus, (N) and (N') 'describe the same facts', meeting that desiderata for a successful nominalist construction of mathematics. Here one can see the crucial importance of the fact that [Inaccessible Simples] logically entails there are some $OA...FXf$ such that $OA...F$ meet the Burgess-Hazen-Lewis conditions for defining ordered pairs with respect to $OA...F$ and

[43]To get the feel for why this is the case, consider how nothing logically changes between the proof of $\forall x(Gx \vee Hx)$ from $\forall x Gx$ and the proof of $\forall x(Gx\mathrm{abc} \vee Hx\mathrm{abc})$ from $\forall x Gx\mathrm{abc}$. Since no logical interaction is happening with the constants and their introduction is uniform, the steps of each proof are the same. The statements we've arrived at by following the process so far stand to the original premises much as '$\forall x Gx\mathrm{abc}$' stands to '$\forall x Gx$'.

X and f satisfy the axioms of ZF^{2}CU with respect to $OA...F$. Without that logical entailment, the consequent of (N') might be vacuously true at the nearest possibility where [Inaccessible Simples] holds.

Of course, the Cicada example is just one case. However, my 'safety result' shows that for any explanatory mathematical proof in science, there's a parallel proof in counterfactual megethology ([2023], pp. 164-67). For the same reason as the Cicada case, using Baker's account of explanation, these parallels will offer explanations just as much as the platonistic proofs do.

Two concerns are worth anticipating here. First, a parody argument might be given against the notion that counterfactual megethology can explain anything. Consider the deductions we can make with respect to empirical observations using the standard model of particle physics. Whatever deductions one can make with respect to observations using the standard model, one could just as well make from the assumption that the physical world behaves as if the standard model is true. Obviously that claim is explanatorily deficient, and the counterfactuals of counterfactual megethology seem suspiciously like this 'as if' claim.[44] In response, I would say that the 'as if' claim should itself be explained by the truth of the standard model. There must be a reason for why the world behaves as if the standard model is true, and the only plausible explanation we know of is that the standard model is true. By contrast, we don't need to postulate a real inaccessible infinity of simples to explain the truth of claims made within counterfactual megethology. Instead, the explanation consists in the fact that the existence of an inaccessible infinity of simples logically entails the existence of a plural-mereological hierarchy embodying the axioms of ZF^{2}CU, plus the stipulation that the hypothetical extra simples are spatiotemporally disconnected from us in the relevant possibility. In the end, the counterfactuals of counterfactual megethology and the 'as if' claim aren't very similar.

Second, it might be objected that I've only addressed one account of mathematical explanation. What about, for instance, accounts that model mathematical explanation using counterpossibles or structural equations? Colyvan himself favors such an account.[45] I think this is a fair point. That is why I take my case here to be preliminary. Truly settling the issue of whether counterfactual megethology can accomodate all cases of mathematical explanation would require an exhaustive analysis of all the accounts of mathematical explanation and how counterfactual megethology fares with them. This cannot be undertaken here. But in addressing Baker's account, I believe I've made a good first volley.

So goes the situation for explanatory power. I take it that my discussion, however, elucidates why counterfactual megethology is as bold as platonistic mathematics. Given the preservation principle, whatever empirical phenomena that a platonistic scientific theory will predict will also

[44] Cian Dorr ([2010]) considers this concern for modal nominalist approaches as well. He offers a different, but related response.

[45] See Colyvan's collaborative work with Sam Baron and David Ripley: Baron and Colyvan ([2016]), Baron, Colyvan, and Ripley ([2017], [2020]).

be predicted by a version of that scientific theory that makes use of counterfactual megethology. So if the platonistic theory predicts an exciting, as-yet-undiscovered phenomenon, so will the counterfactual-megethological version of the theory.

With counterfactual megethology's explanatory power and boldness secure, based on Colyvan's list of theoretical virtues, counterfactual megethology would seem to satisfy the adequacy condition.[46] To see if it satisfies the plausibility condition, I turn to simplicity and universal coherence. When it comes to simplicity, there is no question that platonistic mathematics fares better than counterfactual megethology. The Burgess-Hazen-Lewis pairing method effectively causes every mathematical predicate to gain an additional nine places of adicity, and every mathematical assertion turns into the universal closure of the open formula that results from adding nine free variables to the end of every predicate in the assertion's original formulation. The new predicates also have a complicated underlying plural-mereological definition. One might think that this complexity undermines the claim that counterfactual megethology is plausible. I say that it does not. Here is where universal coherence enters into the picture.

Let us set aside B for a moment, and consider only the actual background knowledge, B_α, that a newcomer to counterfactual megethology is likely to have. In light of the complexity of this theory, I think it's reasonable for the newcomer to have credences such that $\Pr(\text{counterfactual megethology}|\neg P, B_\alpha)$ is quite low. But B_α is not B. The reason that $\Pr(\text{counterfactual megethology}|\neg P, B)$ is high is that the assumptions I mentioned on which it depends have robust philosophical justifications. The complexity doesn't really cut against counterfactual megethology's probability when the soundness of the foundations on which it is built is considered. In the end, universal coherence trumps simplicity.

Justifying the belief that it's (logically or metaphysically) possible for there to be an inaccessible infinity of simples turns on Humean modal recombination. The possibility is an instance of the Humean principle for any things, there can be as many duplicates of those things as can possibly be described without lapsing into logical incoherence.[47] Unless there is a reason to think there

[46]That said, one might object that the focus on explanatory power and boldness is overly narrow. A more complete list of virtues might point toward defects in counterfactual megethology that are unrelated to these virtues or to simplicity. For instance, counterfactual megethology seems parasitic on platonistic set theory. Regardless of how I define 'inaccessible', the concept of an inaccessible does not come out of a vacuum, but rather arises from considering models of ZF. A person might suspect that this parasitism makes platonistic set theory conceptually preferable. Additionally, it may just appear inherently implausible that the truth of, say, the intermediate value theorem would depend on what would happen if there many extra simples. These sorts of concerns could put into question whether counterfactual megethology is adequate after all. (Thanks to an anonymous reviewer for raising both of these issues.)

I do not think this matter can be sorted out here. Instead, I take myself to have shown that leaning on simplicity is not enough for the platonist to make their case. More sophisticated arguments along these lines are necessary. Platonists need to articulate the viciousness of parasitism in such a way that it doesn't collapse back into concerns about simplicity (nominalistic 'parasites' tend to be more complicated than their platonist 'hosts', but this would need to be an extra defect). Concerns about utilizing modality need to be carefully articulated. This extra burden in defending the EIA is not always recognized.

[47]See Daniel Nolan ([1996]) for a Humean perspective.

can't be so many simples, there isn't any reason to rule out that possibility. And these simples could just be, e.g., positrons spread throughout inaccessibly-many spatiotemporally-disconnected duplicates of our spacetime. This example scenario undermines several potential worries. The simples needn't occupy a common space, so there is no reason to think that limits on the possible size of spacetimes prohibit their existence. Nor do there need to be some sort of special, non-physical simples for there to be so many. Their existence needn't even contradict any physical laws, for the duplicate spacetimes would have all the same physical laws as our own. There don't appear to be any grounds for doubt.[48]

While there are holdouts (as mentioned earlier), the legitimacy of primitive plural quantification has generally been conceded since George Boolos ([1984]) influentially argued on its behalf. Of all the principles counterfactual megethology relies on, this is the least controversial.

Classical mereology can be motivated by the typical considerations. The most controversial aspect of it is mereological universalism. A standard argument for universalism goes as follows. Of the views regarding when composition occurs, only universalism and mereological nihilism avoid positing metaphysical vagueness in the world. Nihilism, however, is inconsistent with our own existence if we are material objects. It is the view that no objects have proper parts, and we are not particles. Moreover, 'gunk'—objects composed of things that are composed of yet further things, *ad infinitum*—certainly seems possible, but nihilism is inconsistent with it. This makes trouble if the principles of composition are necessary, which many have supposed them to be.[49]

As for the plural-mereological analogue of Global Choice, surely the plausibility of Global Choice is not dependent on mathematical ontology. Anyone who finds Global Choice acceptable in the platonist setting should find it acceptable in the plural-mereological one, for all the same reasons.

Many philosophers have not been convinced by the arguments for these principles. Despite this, they lend a certain credibility to them. Moreoever, although the canonical presentation of counterfactual megethology makes use of classical mereology, embracing superplural resources eliminates the need for it. Superplural quantifiers plurally quantify over pluralities, and superplural terms plurally refer to pluralities.[50] With superplurals, we could plurally quantify over and refer to pluralities of simples instead of composites of simples. It is true that superplurals have often been viewed with greater skepticism than ordinary plurals. However, this skepticism has largely been based on the presumption that superplurals do not appear in natural language. Recently, Salvatore Florio and Øystein Linnebo ([2021, pp. 174-201) have identified many instances of

[48]See my article ([2023], p. 157) for more on this line of argument.

[49]Both sorts of arguments are advanced by James Van Cleve ([2008], pp. 325-31).

[50]So to speak. This needs some care, since 'plurality' is a singular noun. The advocates of superplurals do not mean to be speaking of some special objects known as pluralities. But for present purposes, we just need some handy shorthand.

apparent superplural reference and quantification across multiple natural languages. The skeptic's case looks to be built on a mistaken belief.

So counterfactual megethology doesn't strictly require classical mereology. It could make do with superplural resources. That at least one of these two tools is legitimate seems likely (especially when compared to platonism).

To sum up, counterfactual megethology is built on a combination of principles that seem likely to be true. This is why $\mathrm{Pr}($counterfactual megethology$|\neg P, B)$ is high. *Prima facie*, then, counterfactual megethology satisfies the plausibility and adequacy conditions. There is more to discuss, but this serves as an opening strike. And this is to say nothing of the other nominalizations of mathematics that have been offered, which only serve to raise $\mathrm{Pr}(M|\neg P, B)$. To mention two examples, I did not explore Chihara's work ([1990]) or the recent nominalist proposal of Sharon Berry ([2022]). Chihara and Berry start from different bases.

8. Conclusion

I have argued for two main conclusions. The first is a theoretical one: Burgess and Colyvan have set the bar for dispensability too high. Bayesian reasoning establishes that the ability of mathematics to explain empirical phenomena does not favor platonism over nominalism provided that there is at least one nominalization of mathematics that satisfies the adequacy and plausibility conditions. This is true even if the nominalization is more complicated than platonistic mathematics. The second conclusion is that, in light of the first conclusion, a tentative case can be made that an existing nominalization of mathematics already suffices to show that mathematical objects are dispensable from science.

I will finish by discussing the continued importance of mathematical explanation. Even if it can be decisively shown that some nominalizations of mathematics are plausible and adequate, it is still philosophically important to determine whether there are genuine cases of mathematical explanation in science. It is interesting in its own right, for one thing. When it comes to metaphysics, though, it's important because genuine cases confirm the nominalizations. Despite being plausible, these theories do rely on principles that can be brought into question. If it turns out that some empirical phenomena are genuinely explained by mathematics, those principles have scientific confirmation. If, on the other hand, no empirical phenomena are genuinely explained by mathematics, they retain a measure of doubt.

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