Towards a consistent Semiclassical Theory of Gravity

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Abstract

We argue that semiclassical gravity can be rendered consistent by assuming that quantum systems only emit a gravitational field when they interact with stable determination chains (SDCs), which are specific chains of interactions modeled via decoherence and test functions obeying a set of conditions. When systems are disconnected from SDCs, they do not emit a gravitational field. This denies the universality of gravity, while upholding a version of the equivalence principle. We argue that this theory can be tested by experiments that investigate the gravitational field emitted by isolated systems like in gravcats experiments or by investigating the gravitational interactions between entangled systems like in the (Bose-Marletto-Vedral) BMV experiment. Our theory fits into a new framework which holds that in the absence of certain conditions, quantum systems cannot emit a gravitational field. There are many possible conditions for systems to emit a gravitational field, and we will adopt a subset of them. We will show how this subset of conditions provides multiple benefits beyond rendering semiclassical gravity consistent, which includes deriving the value of the cosmological constant from first principles and providing an explanation for why the vacuum does not gravitate.

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1 Introduction

It is often claimed that we need a theory of quantum gravity, and that semiclassical gravity needs to be replaced by this theory to avoid unphysical consequences. In this article, we explore new theoretical and empirical possibilities for understanding classical gravity via quantum field theory, and propose the beginnings of a theory of semiclassical gravity that avoids well-known limitations. This theory has empirical consequences, which might be tested in the future via experiments that aim to test the quantum nature of gravity. A striking one is that a system under sufficient isolation does not emit its own gravitational field. This goes against the commonly held view that gravity is universal. Furthermore, as we will see, this theory follows a different strategy from the commonly adopted ones, which either consider gravity as quantum or as a classical stochastic field that gives rise to an outcome.

This theory is as minimalistic and conservative as possible. It does not modify the fundamental equations of quantum theory (QT) and only minimally the ones of general relativity by assuming the semiclassical equations of gravity. More concretely, we will be as conservative as possible in what we see as one of the most fundamental features of general relativity, which is general covariance and the equivalence principle. Regarding the latter, we will take very seriously the idea that a generalized version of the equivalence principle applied to the quantum regime should hold in a theory of gravity. We will use a recently proposed approach to QT called Environmental Determinacy-based Quantum Theory (EnDQT) [73] and propose a version of it that gives an account of gravity. The key idea is that gravity should not be quantized, and although quantum matter field systems can be affected by gravity, they cannot act as sources of (classical) gravity unless they interact with quantum matter field systems that form certain chains of interactions. These interactions are represented via quantum field theory and are modeled via smearing/test functions widely used in algebraic treatments of QFT, where the origin of what these functions represent comes from stochastic interactions between systems, which give rise to a mean field. The theory proposed here might have concrete applications to the measurement theory in QFT [29] because it shares common tools.[1]

Besides this theory, we will propose a framework to think about gravity in this context, which involves what we will call gravitational conditions, which are the conditions for systems to emit a gravitational field. Our conditions are one among other possible ones. So, we are also presenting a set of underexplored features for future theoretical and empirical investigations. Besides arguing that the semiclassical Einstein equation can provide a consistent account of gravity and that this view can circumvent some of the common objections to the semiclassical approach, we will also defend this theory by showing how it can provide multiple benefits. For instance, it allows us to derive the value of the cosmological constant and provide an explanation for why the vacuum does not gravitate, potentially addressing the cosmological constant problem. This value

[1] We will briefly see how we can understand it via particle detector models [75, 71] in Appendix E.

comes from fluctuations in the stress-energy tensor. Interestingly, this derivation leads to the prediction that this value changes over time and that it is getting progressively smaller, in agreement with current observations that indicate that dark energy is getting progressively weaker [1]. Other conjectures concerning black holes and inflation will be presented to show the potential of this theory.

So, our goal is not to provide a complete theory of gravity but rather a theory and framework under development that we hope will lead to new and productive ways of understanding gravity, while showing that the main motivations to find a quantum theory of gravity may be undercut by rethinking core foundational assumptions.

We will start by motivating this theory by explaining two scenarios that can test it and distinguish it from quantum theories of gravity and theories where gravity leads to the collapse of quantum superpositions (Section 2). Then, we will present the basic features of EnDQT (Section 3). Afterwards, we will present three postulates that constitute the basis for the theory of classical gravity based on EnDQT (Section 4), and explain how this theory generalizes the equivalence principle. In Section 6, we will show how it may be able to deal with some of the common objections of the semiclassical approach and examine some other consequences of this theory, which include a conjecture that the core of black holes does not gravitate, and thus a singularity does not arise. In Section 7, we will show how this theory allows us to predict the value of the cosmological constant and interpret dark energy as having a time-varying value that gets increasingly smaller. Some calculations will be presented in the appendices, including how this time-varying cosmological constant value leads to some of the effects that we associate with inflation, and potentially new benefits associated with not having to postulate an inflaton field (Appendix H). For simplicity, we will focus on real scalar fields, obeying the Klein-Gordon equation. However, we conjecture that the approach developed here will be valid for other kinds of fields. Throughout this article, we will adopt the metric signature $(-+++)$. We will also assume mainly natural units ($\hbar = c = 1$). The context will make it clear when we do not.[2]

2 Semiclassical gravity and experiments to test this theory

The Einstein equations take the form

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor, defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor encoding spacetime geometry and Λ is the cosmological constant, often considered

[2] Quantum operators will be written with an hat, except in some sections.

to represent dark energy. The source of the field is the stress-energy tensor of matter fields, $T_{\mu\nu}$.

If matter and radiation fields are quantised, it is unclear what to take for the source of the gravitational field. There are multiple approaches to this problem. The simplest one replaces the right-hand side with the average of the stress-energy operator evaluated in some semiclassical state. The dynamics are governed by a modified version of Einstein's field equations [61, 78]:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle_\psi \tag{2}$$

where $\langle \hat{T}_{\mu\nu} \rangle_\psi$ is the mean value of the quantum energy-momentum tensor in a given quantum state $|\psi\rangle$. The quantum matter fields influence the curvature of spacetime via their average energy-momentum, but the gravitational field itself is not quantized and we ignore the backaction of quantum matter fluctuations onto gravity dynamics. This is a form of mean field theory and leads to well-known problems [51, 97].

One alternative is to formulate a theory of quantum gravity, which quantizes geometrical degrees of freedom or makes them emerge from some more fundamental quantized ones, and where eq. (2) is obtained in some limit.[3] Another approach is to find a consistent way to combine quantum and classical dynamics [21, 64], without reducing the latter to the former, and leading to a gravity-caused collapse process. This can be done by adding a minimum amount of noise to both the classical and quantum dynamics. Adding noise to the classical equations makes gravity stochastic, which can change in such a way that it does not lead to the collapse of the quantum states, not *revealing* where the quantum system is. However, under certain conditions determined by the system's stress-energy tensor, such noise is reduced and decoherence of the quantum degrees of freedom is increased, leading to collapse, and an outcome. Importantly, in isolated systems in a coherent superposition, the stochastic gravitational field is always present. Penrose's theory [69] , considers that superpositions of masses/energy correspond to superpositions of spacetimes, which are non-stationary and tend to collapse due to their gravitational self-energy.[4] Thus, both theories would consider that independently of their environment, an outcome eventually arises. We will call this class of theories, gravity-caused collapse theories.

In the theory that we are proposing, the stochastic gravitational field is not always present and is not directly implicated in the collapse. Systems emit such a gravitational field only under certain local decohering interactions between matter fields, even in the presence of a stochastic gravitational field. The behavior predicted by the semiclassical equations only occurs under these interactions. If they do not occur, quantum systems evolve in flat spacetime or under the gravitational field emitted by other systems, as described by flat or curved spacetime QFT, respectively. Also, if isolated from these interactions, systems can evolve

[3] E.g., [38, 39, 80].

[4] In the approach proposed here, one cannot place spacetimes in a superposition. If we put masses in a superposition, they do not generate a gravitational field.

unitarily indefinitely. Moreover, under certain assumptions, the fluctuations of the stress-energy tensor lead to dark energy effects. Other strategies will be presented that show how we can minimize the fluctuations of the stress-energy tensor by allowing gravity to arise only in contexts where such fluctuations are minimized. In this first paper, we will not focus on how to characterize more suitably the stochastic gravitational field, nor solve the semiclassical equations, although in principle they can be solved in the circumstances we know how to solve them. Rather, we will focus on the circumstances in which the gravitational field is emitted, and some distinct features of this theory.

To motivate our proposal and show how it could be tested, we will look at the gravcat experiments and the so-called BMV (Bose-Marletto-Vedral) experiments [7, 58]. Let us first consider a scenario where a system is placed in a cat state [6], which is the superposition of distinguishable coherent states,[5]

$$|\psi_{\text{cat}}\rangle = \mathcal{N}\left(|\alpha\rangle + |-\alpha\rangle\right), \quad \mathcal{N} = \frac{1}{\sqrt{2+2e^{-2|\alpha|^2}}}, \tag{3}$$

where this cat state is isolated from its environment in such a way that the components of that superposition can self-interfere under suitable conditions. Now, we place nearby a detector of the gravitational field generated by this system. According to this theory, an isolated system cannot emit its own gravitational field, and thus in no way could the detector detect the gravitational field emitted by the system. It could be subject to the gravitational field from other systems like all quantum systems are in principle subject to according to our current evidence [66, 15, 98], but not in a classical way as described by semiclassical gravity, unless it interacts with other members of the so-called stable determination chains (SDCs), which concern certain chains of interactions between systems (or more generally in the absence of processes that make systems emit a gravitational field, see below), no stochastic process happens that selects one of the states of the superposition, and systems do not emit a gravitational field. Decoherence applied to open systems is an inferential tool to infer the behavior of these chains and when the stochastic process occurs.

The above experiment can be performed in principle (see, e.g., [12, 13]), and according to this theory, the rate at which we can observe a gravitational field emitted by the system should be exclusively determined by the decoherence rate at which the target system of the experiment is decohered by the surrounding matter fields in its environment.[6][7] This contrasts with gravity-caused collapse theories, where the mass density would also be a determining factor. The absence of a gravitational field emitted by the degrees of freedom in a coherent superposition of a particle would constitute significant evidence favoring this theory.

[5] See Section 3.2.3 for a characterization of these states.

[6] See, e.g., [83] for an expression of the decoherence rate of an object in a spatial superposition due to air molecules.

[7] Some versions of this theory consider that the gravitational field is only emitted by systems in certain semiclassical states. However, the most classical states (and experimentally accessible), i.e., coherent state, will certainly be considered to gravitate. See Section 4.2.

The assumption that systems do not emit their own gravitational field may seem quite radical, but in fact, it can be considered as a generalization of the Weak Equivalence Principle (WEP). A precise statement of the WEP is due to Clifford Will [99, 55]:

"[I]f an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition."

where by an ""uncharged test body" we mean an electrically neutral body that has negligible self-gravitational energy (as estimated by Newtonian theory) and that is small enough in size so that its coupling to inhomogeneities in external fields can be ignored." [99] Note that the notion of a body with a negligible self-gravitational energy is an idealization. In principle, a probe with sufficient resolution could detect that gravitational field. The theory that we are proposing does not make that an idealization, and considers that under certain circumstances a body does not have self-gravitational energy. Thus, it takes this feature seriously and in a sense generalizes this principle to any body (not just test particles like the bodies above) and quantum phenomena by proposing what we will call the EnD Equivalence Principle:

Without being affected by other forces, any quantum system under the same gravitational field exhibits the same behavior due to this field.

The "other forces" are forces that can involve members of SDCs, which interact non-gravitationally. So, non-interacting systems (with SDCs) do not give rise to a gravitational field and behave in the same way in free fall even if they have very different masses.

Turning now to the BMV experiment, let us consider a scenario with two particles [7] that are sufficiently isolated to maintain the coherence of their orbital and spin degrees of freedom, and where these particles have an internal two-state degree of freedom (spin projection). This internal degree of freedom can be placed in an arbitrary superposition without affecting their center of mass degree of freedom. Let us suppose that the particles are free-falling. We now use a Stern-Gerlach device to subject each atom to a force that depends on the internal electronic states. As the internal states are a superposition of spin up and spin down, the momentum change of each atom is a superposition of positive and negative increments. Let us consider the states $|C\rangle, |L\rangle$, and $|R\rangle$ as concerning the center of mass degrees of freedom of the particles. If the internal state is $|\downarrow\rangle$ the particle gets a kick of $+\Delta p$, while if the particle is in state $|\uparrow\rangle$ it will get a momentum kick of $-\Delta p$. Thus, if the particle has spin-up, it will go to the left; if it has spin-down, it will go to the right,

$$|C\rangle_j \frac{1}{\sqrt{2}} (|\uparrow\rangle_j + |\downarrow\rangle_j) \rightarrow \frac{1}{\sqrt{2}} (|L,\uparrow\rangle_j + |R,\downarrow\rangle_j) . \quad (4)$$

After this, they may or may not get their position degrees of freedom entangled via a distance-dependent quantum gravitational field. Then, each particle goes over a *refocusing* Stern-Gerlach device that moves them toward the center,

$$|C\rangle_j \frac{1}{\sqrt{2}} (|\uparrow\rangle_j + |\downarrow\rangle_j) \to \frac{1}{\sqrt{2}} (|L,\uparrow\rangle_j + |R,\downarrow\rangle_j) \tag{5}$$

If there is no quantum gravitational interaction between the particles, they remain unentangled as they free fall, and we can reverse the operation in the following way:

$$|C\rangle_1 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1 + |\downarrow\rangle_1\Big) \otimes |C\rangle_2 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_2 + |\downarrow\rangle_2\Big) \tag{6}$$

$$\longrightarrow \frac{1}{\sqrt{2}}\Big(|L,\uparrow\rangle_1 + |R,\downarrow\rangle_1\Big) \otimes \frac{1}{\sqrt{2}}\Big(|L,\uparrow\rangle_2 + |R,\downarrow\rangle_2\Big) \tag{7}$$

$$\longrightarrow |C\rangle_1 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1 + |\downarrow\rangle_1\Big) \otimes |C\rangle_2 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_2 + |\downarrow\rangle_2\Big) \tag{8}$$

If gravity is quantum, gravity will mediate the entanglement between the spins of the particles, and we would obtain the following state at the end of the experiment,

$$\begin{aligned} |\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}}\Big\{&|\uparrow\rangle_1 \frac{1}{\sqrt{2}}\big(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}|\downarrow\rangle_2\big) \\ &+ |\downarrow\rangle_1 \frac{1}{\sqrt{2}}\big(e^{i\Delta\phi_{RL}}|\uparrow\rangle_2 + |\downarrow\rangle_2\big)\Big\}|C\rangle_1|C\rangle_2 \end{aligned} \tag{9}$$

with

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \quad \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)}, \quad \phi \sim \frac{Gm_1m_2\tau}{\hbar d} \tag{10}$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi, \quad \Delta\phi_{LR} = \phi_{LR} - \phi$. Measuring the spin of the particles 1 and 2 at the end of the experiment provides a way of certifying this so-called gravity-mediated entanglement, $\mathcal{W} = \left|\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle\right|$, where there will be such entanglement if $\mathcal{W} > 1$ (see also Figure 1).

According to gravity-caused collapse theories, at a certain threshold dependent on their mass/energy, we would have a collapse and not have gravity-mediated entanglement. Thus, this class of theories would consider that independently of their environment the particles would eventually collapse, and we could not unitarily reverse the final collapsed state to its initial state,[8]

$$|C\rangle_1 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1 + |\downarrow\rangle_1\Big) \otimes |C\rangle_2 \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_2 + |\downarrow\rangle_2\Big) \tag{11}$$

$$\longrightarrow \frac{1}{\sqrt{2}}\Big(|L,\uparrow\rangle_1 + |R,\downarrow\rangle_1\Big) \otimes \frac{1}{\sqrt{2}}\Big(|L,\uparrow\rangle_2 + |R,\downarrow\rangle_2\Big) \tag{12}$$

$$\longrightarrow \begin{cases} |L,\uparrow\rangle_1 \quad \text{or} \quad |R,\downarrow\rangle_1, \\ |L,\uparrow\rangle_2 \quad \text{or} \quad |C,\downarrow\rangle_2, \end{cases} \tag{13}$$

$$\longrightarrow \begin{cases} |C,\uparrow\rangle_1 \quad \text{or} \quad |C,\downarrow\rangle_1, \\ |C,\uparrow\rangle_2 \quad \text{or} \quad |C,\downarrow\rangle_2. \end{cases} \tag{14}$$

We propose an alternative route to these two classes of theories. Contrary to quantum gravity theories, no gravitation-mediated entanglement could occur. In the absence of interactions with members of SDCs, no probabilistic process happens, and systems do not source a gravitational field, which would be otherwise classical. Moreover, contrary to gravity-caused collapse theories, the *trigger* for the stochastic collapse is independent of the size of the mass density of the particles in a superposition but whether they have interacted with members of an SDC. Thus, for particles of any mass sufficiently isolated from their environment, we could in principle reverse their state to their initial state as in (8), contrary to these theories. So, according to the theory that we propose, the extent to which we cannot reverse the state of the masses is thus exclusively determined by the decoherence rates due to other particles/matter fields (see [77] for the quantification of such rates and explicit expressions).

As we can see, these experiments should be able to provide evidence for our theory and help distinguish it from quantum gravity theories and gravity-caused collapse theories. Moreover, we can also distinguish the theory proposed here from gravity-caused collapse theories by testing their domain of validity via other experiments. For instance, experiments have been proposed and done to test the Diósi-Penrose model (e.g., see [31] and references therein). If their domain of validity becomes problematic and we cannot find a satisfactory quantum theory of gravity or evidence for it, this, in principle, could be evidence supporting our theory.

[8] This matter is more subtle as discussed in [44]: "[a]s Bose et al. (2017, p. 1) put it, (...) [classical stochastic] theories imply "the breakdown of quantum mechanics itself at scales macroscopic enough to produce prominent gravitational effects." The question of course is what counts as "prominent." On the one hand, by Penrose's estimates, the proposed experiment, with gravcats of 10^{-14} kg separated by $100\,\mu$m, the gravitational collapse time should be of the order of a second, which would be fast enough for the classicality of the field to affect any observed entanglement. And so it seems it is a "prominent" effect: The quantum state will collapse, and no entanglement will be observed. However, on the other hand, should entanglement be observed, the theories do have a tunable parameter, which could be set to prevent collapse in the currently envisioned GIE experiments, although they would place a new bound on it. But so doing is to accept that the experiment witnesses a quantum superposition of the gravitational field, which is at least against the spirit of Penrose's position, and quite possibly falls afoul of the very arguments by which he motivates it."

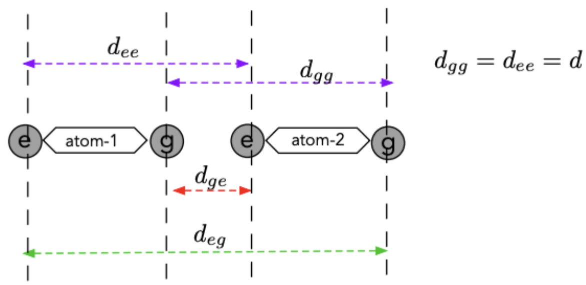


Figure 1: If there is a quantum gravitational interaction between the particles, the interaction distinguishes three paths as there are three distinct particle separations, $d_{ee} = d_{gg} = d$ and $d_{eg} > d$, $d_{ge} < d$. This will entangle the two-particle center of momentum motion in a way that depends on the mass of the particle. If we repeated the experiment with two identical particles of a different mass, the entanglement would be different. Measurements made in a free-falling frame could thus distinguish the three paths.

As we have mentioned, in this article, we will propose a series of so-called gravitational conditions, which involve SDCs, and which establish when the systems emit a gravitational field. However, note that this is only one possible set of gravitational conditions. Other theories could impose different conditions for a gravitational field to be emitted. For instance, one could appeal to SDCs with other rules (see next section), or one could have certain modifications of the dynamical equation of QT, which impose a collapse rule, such as a spontaneous collapse, and which triggers the gravitational field. One could also have a many-worlds or many-worlds-like/relational theory that says that under decoherence and branching or particular interactions, such classical field arises. Relatedly, one could have a theory that appeals to an emergent or primitive notion of agents that trigger the gravitational field. One could even appeal to hidden-variables that account for such triggering. However, many of the above classes of theories suffer from lack of experimental evidence or well-known issues, and so we are presenting a theory that also aims to circumvent them. Nevertheless, this article could be read as also opening up new so far neglected empirical and theoretical possibilities concerning how gravity arises.

3 Introduction to the framework of EnDQT

Related to gravitational conditions, there are the so-called determination conditions, which are the conditions for measurement outcomes to arise. Different interpretations or quantum theories pose different determination conditions. Although these two kinds of conditions are related, they should be kept separate because they may have different roles. We may consider that a system has a feature that we associate with a measurement outcome, but still not emit a gravitational field, as we will see. The system may act only as a test system. We will now present the main features of EnDQT in a non-relativistic setting, and its QFT version, which will provide a particular set of determination conditions.

SDCs mentioned above are like von Neumann chains [62], i.e., involving a series of intertwined unitary evolutions and stochastic processes, but done in such a way that in principle we never lose track of the systems that belong to those chains. Local interactions modeled via test functions provide a way of tracking these systems and chains. Also, we want SDCs to be compatible with relativity and with the success of decoherence in representing measurement processes, and we will see that these chains will obey the key features of both. Furthermore, SDCs aim to be applicable to cosmological contexts, not relying on anthropocentric notions. Crucially, via the rules that will be presented, which just appeal to local QFT-based decohering interactions with a certain structure, we aim to not modify significantly the quantum formalism to provide the criteria for when an outcome arises (in a single-world and non-relational way) unlike spontaneous collapse and gravity-caused collapse theories, and to not appeal to non-local, superdeterministic, or retrocausal hidden variables. Thus, we aim to be conservative and circumvent the issues of these approaches.[9]

We will now establish a set of criteria to assign definite or determinate values to observables based on SDCs. Historically, the criteria to assign determinate values to observables in QT have some underappreciated importance (see [35] for a historical overview), and come in the form of criteria such as the Eigenstate-Eigenvalue Link. This link says that a system has a determinate/definite value of an observable O if and only if the system is in a state that is an eigenstate of O. However, as it is well-known, this criterion is at odds with scientific practice because we often want to assign determinate values when systems are not in an eigenstate of some observable. Also, systems typically rapidly evolve out of those eigenstates after being measured [96]. Being in an eigenstate of a dynamical observable is better seen as an idealization. The determination conditions below aim to provide more realistic and less problematic criteria.

For pedagogical reasons, we will initially appeal to non-relativistic QT,[10] but we will see that these features become much more intuitive when expressing them using QFT. One of the main features of EnDQT comes from taking seriously the view that systems are never in eigenstates of dynamical observables, except when they are being measured, and it is the following:

Systems have, by default, indeterminate values of any non-dynamical observable,

[9] See, e.g., [26, 36, 34, 79, 33, 32, 41], and references therein.

[10] In the simplest pure-state based Hilbert space formalism, a quantum system is represented by a normalized vector within a complex, complete inner product space Hilbert space. The observables of a system are described by Hermitian operators acting on these vectors, with their eigenvalues corresponding to measurable quantities. The probability of obtaining a specific measurement outcome is determined by the squared magnitude of the inner product between the state vector and the observable's eigenstate or associated quantum state (see below what we mean by this). Additionally, the system's time evolution is governed by unitary operators, which ensure that the total probability remains conserved over time.

except under certain interactions with systems with the determination capacity.

For example, consider the dynamical observables spin in multiple directions, momentum, energy, or mass.[11] For EnDQT, systems have indeterminate values of all of these observables unless these interactions occur. More specifically:

A system X can only give rise to measurement outcomes or to another system Y having a determinate value of an observable of Y when X has the determination capacity concerning Y, which we denote as DC-Y.

Furthermore, this capacity tends to spread because, under specific conditions that will be specified below, system Y can acquire that capacity and transmit it to other systems under interactions. Moreover, only *during* interactions systems have determinate values of a certain observable. Importantly,

it is indeterministic which determinate values of the observables O_X and O_Y systems X and Y will have under these interactions among the possible ones, where the possible values are given by the eigenstates or associated quantum states of O_X and O_Y, which were in a superposition.

We mention "associated quantum states to an observable" because as we will see, for example, in the case of observables such as the ones represented via the energy-momentum operator, systems could have determinate values of energy-momentum even if they are not in an eigenstate of that observable.[12] Furthermore, as one can see, similar to, for example, the Copenhagen interpretation, EnDQT is an indeterministic theory in the circumstances where specific interactions are involved.

One way to infer whether a system X with the determination capacity concerning a system Y acts locally as a "measurement device" for the observable O_Y of another system Y is if an eigenstate or associated quantum state of an observable of X contains information of an eigenstate or associated quantum state of an observable of Y, or via the locally induced entanglement of the degrees of freedom of X with the eigenstates or associated quantum states of the observable O_Y of Y. More precisely, it is not just entanglement but an entanglement involving many degrees of freedom, which is often called environmental-induced decoherence [46, 101]. When decoherence occurs and X has the DC-Y, an indeterministic process arises that makes both X and Y have a determinate

[11] And perhaps even electric charge, if not subject to a superselection rule, and hence considered as a dynamical observable. Roughly, an observable is subject to a superselection rules when there are certain rules that forbid the preparation of their eigenstates in a coherent superposition. Electric charge is typically subject to that rule. However, we could allow for the more radical view that all observables are dynamical, and appeal to decoherence via SDCs (in a similar way to the so-called Environment-induced superselection) to account for why typically we do not see their eigenstates self-interfering.

[12] As we will see, coherent states are not eigenstates of the energy-momentum tensor operator, although we will consider that systems have a determinate energy-momentum when in those states under the interactions mentioned above.

value that can be represented by one of the eigenvalues of the observables O_X and O_Y whose eigenstates got entangled. So, we will regard the models of decoherence as inferential tools to infer when systems that have the determination capacity give rise to others having determinate values [73].[13] Together with test functions (more on this below), they will provide the main inferential tools to infer whether the conditions below are fulfilled.

More concretely, we consider that decoherence allows us to infer in open environments when SDCs act, even in the absence of knowledge about where they precisely are. This is because it is considered that these are the typical environments where they evolve. Also, it allows us to infer what the conditions are to shield systems from SDCs via the conditions to shield systems from decoherence. Furthermore, if we manage to track precisely where they are, it allows us to represent their behavior. The way we use decoherence study which states $\mathcal{S} \subset \mathcal{H}_{\mathcal{S}}$ arise stochastically from local interactions with members of SDCs is often via the locally established many records of the environment of S of those states, such that if the system starts in those states $\mathcal{S}$, at later times it is still well-approximated by another member of the set $\mathcal{S}$, and the environment contains records of them, having their states correlated with them,

$$|\alpha\rangle_S \otimes |0\rangle^{\otimes N} \xrightarrow{U} |\alpha_0\rangle_S \otimes |\varepsilon_1(\alpha)\rangle_1 \otimes \cdots \otimes |\varepsilon_N(\alpha)\rangle_N. \tag{15}$$

On the other hand, if it starts in a superposition of those states or other states, it is driven over time into a mixture of $\mathcal{S}$, and we can infer that the environment has a record of $\mathcal{S}$, where this process is quasi-irreversible,[14] and the mixture of states retains a probabilistic interpretation in terms of a diagonal mixture in that basis $\mathcal{S}$ over time. This occurs upon tracing out the states of the environment. Examining that systems are driven quasi-irreversibly locally over time into a mixture of states $\mathcal{S}$ that have a probabilistic interpretation[15] offers a related way of inferring that the system ends up in one of the states $\mathcal{S}$. Thus, the features of the dynamics of systems play a large role in this evaluation of how (as we will sometimes say) SDCs select certain states. Moreover, we can infer what the determinate values of the environment are by examining the values associated with the states that have information about the state of the target system, being correlated with that state.

[13] Note that the determination capacity can be grounded on categorical properties, but we choose to set that characterization aside here.

[14] Decoherence timescales gives us an estimate for how long does it take for such stochatic processes to occur. Such decoherence timescale typically varies inversely with the size of the bath/environment that leads to it, and thus the number of members of SDCs interacting with the system influences how much time it takes for such stochastic process to occur [94]. Decoherence is a quasi-irreversible process, in the sense that it has very high recurrence times τ_D, e.g., timescales much higher than the age of the universe or the heat death onset of the universe (i.e., Poincaré recurrence timescales). Of course, these timescales need to allow for a proper time definition, being measured along the proper time of the interacting systems that are subject to this stochastic process and where this process occurs in a bounded spacetime region inferred via test functions.

[15] Another example: having a Wigner function that is positive in some interactions involving Gaussian states.

3.1 Conditions for determinate values and the determination capacity to spread

We will now explain the conditions for the determination capacity to spread through interactions. To build some intuition, we will explain it through a non-relativistic toy model to see how this works and pretend that entanglement between two systems is enough for determinate values to arise in interactions (and not entanglement involving a collective of systems that give rise to decoherence). In parallel, we will also explain the QFT case with two systems.

Let us consider the following Hamiltonian for representing two continuous CNOT gates,

$$\hat{H}_{ABC}(t) = f_{AB}(t)\frac{\pi}{2}\Big(\frac{1-\hat{\sigma}_{zB}}{2}\Big)\hat{\sigma}_{xA} + f_{BC}(t)\frac{\pi}{2}\Big(\frac{1-\hat{\sigma}_{zC}}{2}\Big)\hat{\sigma}_{xB}, \tag{16}$$

which describes the interactions between systems A, B, and C. More about $f_{AB}(t)$ and $f_{BC}(t)$ below.

The initial state of these systems is

$$|\Psi(0)\rangle = |1\rangle_A \frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C), \tag{17}$$

where the states above are eigenstates of the observable spin-z.

In the QFT case, we could have in the Hamiltonian picture an Hamiltonian density describing the interactions between scalar fields A and B, and B and C,

$$\hat{H}_{\text{int}}(t) = \int d^3x \left[\lambda_{AB}\, f_{AB}(t,\mathbf{x})\, \hat{\phi}_A(t,\mathbf{x})\, \hat{\phi}_B(t,\mathbf{x}) + \lambda_{BC}\, f_{BC}(t,\mathbf{x})\, \hat{\phi}_B(t,\mathbf{x})\, \hat{\phi}_C(t,\mathbf{x})\right]. \tag{18}$$

λ_{AB} and λ_{BC} are coupling constants, where $x = (t, \mathbf{x})$, and $f_{AB}(t, \mathbf{x})$ and $f_{BC}(t, \mathbf{x})$ are smearing/test functions that serve to represent and infer the localization of quantum fields in a spatiotemporal region in the QFT case, and which can be used to impose energy and momentum cut-offs. $f_{AB}(t)$ and $f_{BC}(t)$ in the non-relativistic case will just localize the system in time, and provide energy cut-offs.

So, test functions have an important role in rigorous treatments of QFT, and they are used to handle divergences. However, for EnDQT they will have the extra role of providing the conditions for when systems have determinate values. More concretely, test functions provide a way to spell out the so-called no-disturbance condition. According to this condition, the interaction between A and B should not be disturbed by the interaction between B and C, where the interaction between A and B starts first and in order for that interaction to give rise to A and B having determinate values. Furthermore, test functions should obey the relativistic constraints of being compatible with general covariance. Note that in this article, we will be concerned with the interaction between quantum fields that are spatiotemporally localized due to these interactions. By this, we mean that they have determinate values in bounded spacetime regions in a local way (more on this below). Thus, in the simple case of only two interacting fields, we will consider a test function $f_{XY}(x)$, which is a function that is compactly supported within a region, or at least strongly localized around a region that smears the fields $\hat{\phi}_X(x)$ and $\hat{\phi}_Y(x)$ in that region. More concretely, to maintain general covariance, we could adopt a test bump function $f(x)$ that localizes the interaction between X and Y around a point x_j such as:[16]

$$f(x) = \begin{cases} N \exp\left(-\frac{1}{\left(1-\left(\frac{\sigma(x,x_j)}{\sigma_0}\right)^2\right)}\right), & \text{if } |\sigma(x,x_j)| < \sigma_0 \\ 0, & \textit{otherwise} \end{cases} \tag{19}$$

where $\sigma(x, x_j)$ is Synge's world function [90] and is chosen so that the integral of f over its support equals 1. The Synge's world function $\sigma(x, x_j)$ represents one-half the squared geodesic interval between point x and a center x_j. More specifically, let us consider a smooth spacetime with a Lorentzian metric g. Let x and x' be two points in spacetime, where x lies in the convex normal neighborhood U of x' (which is associated with the Levi-Civita connection of g). In this neighborhood, there exists a unique geodesic $\gamma(\lambda)$ connecting x and x', parameterized by an affine parameter λ ranging from λ_0 to λ_1. Let us assume that $\gamma(\lambda_0) = x'$ and $\gamma(\lambda_1) = x$. Synge's world function is then defined as:

$$\sigma(x, x_j) = \frac{1}{2}(\lambda_1 - \lambda_0) \int_\gamma g_{\mu\nu}(z) t^\mu t^\nu \, d\lambda, \tag{20}$$

where γ is the geodesic connecting x' and x, $t^\mu = \frac{dz^\mu}{d\lambda}$ is the tangent vector along γ, $g_{\mu\nu}(z)$ is the metric tensor evaluated along γ. In Minkowski spacetime, it simplifies to $\sigma(x, x_j) = \frac{1}{2}\eta_{\alpha\beta}(x-x_j)^\alpha(x-x_j)^\beta$ or $\sigma(x, x_j) = \frac{1}{2}\left(-(t-t_j)^2 + |x - x_j|^2\right)$. The Synge's world function as a test function has the benefit of being generally covariant. It incorporates the exact spacetime geometry through geodesic distances and is applicable in any curved spacetime without requiring specific coordinate systems. σ_0 determines the size of the support. As one can see, this function is smooth, smoothly going to zero as $\sigma(x, x_j)$ approaches σ_0. In the Minkowski spacetime, we have that

$$f(x) = \begin{cases} N \exp\left(-\frac{1}{\left(1-\left(\frac{\left[(t-t_0)^2-|\mathbf{x}-\mathbf{x_0}|^2\right]}{2\sigma_0}\right)^2\right)}\right), & \text{if } \frac{1}{2}\left|\left[(t-t_0)^2 - |\mathbf{x}-\mathbf{x_0}|^2\right]\right| < \sigma_0 \\ 0, & \textit{otherwise} \end{cases} \tag{21}$$

t_0 and $\mathbf{x_0}$ are the center of the bump function and σ_0 is its width. The bigger the σ_0, the larger are the spacetime regions in which the interactions are confined.

In the non-relativistic case that our simple example is concerned with, we

[16]Examples of test functions that are not compactly supported (contrary to bump functions) are the ones that belong to Schwartz space $\mathcal{S}(\mathbb{R}^n)$. These are functions that are infinitely differentiable and rapidly decreasing at infinity, as well as all their derivatives.

will ignore space and relativistic considerations. Thus, we will consider that

$$f_{XY}(t) = \begin{cases} \exp\left(-\frac{1}{\left(1-\left(\frac{(t-t_{XY})^2}{2\sigma_{XY}}\right)^2\right)}\right), & \text{if } \frac{1}{2}(t-t_{XY})^2 < \sigma_0 \\ 0, & \textit{otherwise} \end{cases} \tag{22}$$

where we will have $f_{AB}(t)$ and $f_{BC}(t)$, which could be normalized. t_{XY} and σ_{XY} allow us to infer the duration of the interactions between quantum systems X and Y.

In the general case, we consider that $f_{XY}(x)$ allows us to make inferences about a) when systems X and Y have determinate values of their observables when interacting, and b) how this interaction-based process of having determinate values influences other processes of having determinate values if different test functions for different interactions have some of their support in common. Information a) and b) is relevant for the no-disturbance because a) encodes the timing of the interaction between A and B, and B and C. b) encodes whether the interaction between A and B is disturbed by the interaction between B and C.

Taking into account that the interactions between system X and Y in the Schrodinger picture (neglecting the self-Hamiltonian) or in the interaction picture can be given by

$$\hat{U}(x) = \mathcal{T} \exp\left(-i \int dV \mathcal{H}_{\text{int}}(x)\right), \tag{23}$$

where $\mathcal{T}$ is the time-ordering, $dV = \sqrt{-g} d^4x$ with g being the determinant of $g_{\mu\nu}$, there are four conditions, which constitute the core of our determination conditions (and we will call them simply determination conditions), for a system B to obtain the determination capacity concerning a target system C, which we will denote as DC-C. The determination conditions are the following:

i) if A has the determination capacity concerning B (DC-B);

ii) if C interacts with B, while B is interacting with A where the interaction between A and B starts first. This is translated in the centers of the test function being time-like or light-like separated,

$$\frac{1}{2}\left[-(t_{BC} - t_{AB})^2 + |\mathbf{x}_{BC} - \mathbf{x}_{AB}|^2\right] \geq 0, \tag{24}$$

but where $t_{BC} > t_{AB}$;

iii) if B has a determinate value due to A. In the toy example below, this will be inferred by B being entangled with A, or in realistic cases, A locally decoheres B. For instance, if A is a composite system, as for example a set of modes of a field, this could also be inferred by A decohering B.[17][18]

iv) if the interactions between B and C are such that C does not disturb the interaction between A and B in such a way that A probes B and both have determinate values. Taking into account ii), another way of expressing this condition is that A should decohere B before the interaction between B and C ends, giving rise to A and B having determinate values; and C does not disturb this process so that the unitary U that describes the interaction between A and B, and B and C is $U\rho_X \otimes \rho_Y \otimes \rho_Z U^\dagger \approx U'\rho_X \otimes \rho_Y U'^\dagger \otimes \rho''_Z = \rho' \otimes \rho_Z$. U' is the unitary that describes the local decohering interaction, which typically entangles the states of X and Y, resulting in the state ρ'. ρ''_Z is the state of Z that does not get entangled with X and Y. This is the no-disturbance condition mentioned above. Considering $\hat{U}^{AB}$ as describing the interaction between A and B, and $\hat{U}^{BC}$ as describing the interaction between B and C, we sometimes may[19] express this condition as establishing that for successive systems in an SDC, it is sufficient that the following holds in the common support of the test functions $\Omega = \text{supp}(f_{AB}) \cap \text{supp}(f_{BC})$ of $f_{AB}(x)$ and $f_{BC}(x)$,

$$\left[\hat{U}^{AB}(x), \hat{U}^{BC}(x)\right] \ll 1. \tag{25}$$

Another possible way to express the above is by noticing that the commutator of the two terms in Eq.(16) is proportional to $f_{AB}(x), f_{BC}(x)$ or $f_{AB}(t), f_{BC}(t)$. Then, one could show that is sufficient that the following should hold,

$$\int dV\, f_{AB}(x) f_{BC}(x) \ll 1 \tag{26}$$

in Ω for the non-disturbance condition to be fulfilled. Given the form that test functions should have, if the two test functions $f_{AB}(t), f_{BC}(t)$ have almost disjoint regions of support, the above is guaranteed to occur.[20]

Let us then turn to the analysis of the interactions between A, B, and C in the non-relativistic toy model. Let us assume then that A has the DC-B (the condition i) is fulfilled), that C interacts with B, while B is interacting with A (the condition ii) is fulfilled). Moreover, we will consider that the interaction between A, B, and C is such that C doesn't disturb the interaction between A and B where this non-disturbing interaction is represented by the Hamiltonian

[17] See Section 4.2 and Appendix E for other ways to infer this through a mode or modes of fields that probe a target system, and the correlation functions of the latter

[18] Here we are simplifying and disregarding the possibility that determinacy comes in degrees, inferred by the degree of the distinguishability by the states of the environment of the state of the target system in local decohering interaction. In [74] this was considered for target systems. However, it was considered that the latter do not obtain the DC concerning other system.

[19] We will see further below other ways that dispense with smearing functions to some degree, to fulfill this condition.

[20] Note that the above could hold just for spatial test functions if we opted to only use them, or for both temporal and spatial test functions if they were treated separately.

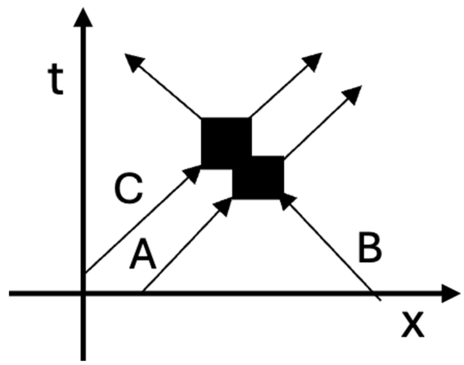


Figure 2: An SDC with systems A, B, and C in QFT interacting in overlapping regions of spacetime in agreement with $i^*) - iii^*)$.

in eq. (16). Thus, A and B get entangled at $t = 1$ and we can represent this interaction by

$$|\Psi(1)_{\text{approx}}\rangle \approx \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B - i|0\rangle_A|1\rangle_B)\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C), \tag{27}$$

and thus condition iii) is fulfilled.[21]

Note that according to ii), for B to have the DC-C, C needs to start interacting with B while A and B are interacting (between 0 and 1). Then, when the entanglement between A and B is achieved since A has the DC-B, an indeterministic process occurs that gives rise to A and B having determinate values of their spin-z observables. Let us (for example) consider that this indeterministic process gives rise to A and B having determinate values 1 and 0, respectively. We then update the state of the system to the new state that will serve as the initial state to the next interaction,

$$|\Psi(1)\rangle \approx |1\rangle_A|0\rangle_B\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C). \tag{28}$$

Then, since conditions i)-iv) are fulfilled, when B gets its states entangled with C at $t = 2$, i.e.,

$$|\Psi(2)\rangle = |1\rangle_A\frac{1}{\sqrt{2}}(|0\rangle_B|0\rangle_C - i|1\rangle_B|1\rangle_C), \tag{29}$$

it is able to give rise to C having a determinate value (1 or 0) and also has a determinate value (0 or 1), where one of the possible outcomes will again arise indeterministically.

As we have mentioned, in a realistic decoherence setting, we would not only have A but N systems A_i, which could be modes of a field, and which

[21] In Appendix A, we do a numerical study to show why the above approximation is fulfilled, given the no-disturbance condition.

interact with B with randomly distributed coupling strengths λ_{A_iB} (for instance, assuming uniformly distributed values from 0 to 1) that would also be multiplied by the above Hamiltonian of interaction. For large N and over time, systems A_i would decohere system B. This could, for example, be observed by off-diagonal terms of the reduced density operator of B going quasi-irreversibly to zero over time, or by the quasi-irreversible loss of purity of this operator. Furthermore, note that we would not just B but many N' systems B_j that interact with A_i, which will then interact with many N'' systems C_h if this chain would continue, and so on. Although it might seem like an ad-hoc condition, the no-disturbance condition can be seen as a necessary condition for this decoherence to occur because we do not want other systems to disturb this process due to A_i, and so on. Note that test functions have a widespread use in rigorous approaches to QFT, and EnDQT is an approach to QT that relies on them in a more diverse way than usual.

Note that for EnDQT, the quantum formalism (including the Hamiltonian) and quantum states have primarily a predictive and inferential role concerning the local behavior of quantum systems. Therefore, for example, there is no sense in which there is action at a distance when an agent learns about the determinate value of its entangled target system in a Bell scenario. There is just a state update concerning the outcomes that arose indeterministically at each wing in the Bell scenario.[22]

So, the above are the conditions for B to act as a "measurement device" for C; now, if C did not interact with B while B was interacting with A (i.e. if condition ii) was not fulfilled), B could not act as a measurement device for C. So, A would merely act as a preparation device for B, and in this way, we would have a measurement-based preparation. Then, when B interacts with C, they only get entangled and evolve unitarily with no indeterministic process occurring.

Furthermore, as we can see, the determination capacity spreads through interactions, and the chain that concerns the spread of this capacity is called the stable determination chain (SDC). SDCs have a structure. We can write the structure of this simple chain as $A \to B \to C$, where the arrows represent the transmission of the determination capacity, or a system giving rise to another having determinate values.

A question that one might have is when SDCs started. One option is to invoke some systems that gave rise to the SDCs, called initiators. In that case, we would add a new postulate to the ones above concerning the conditions for a system B to obtain the determination capacity concerning a target system C, which we will denote as DC-C:

v) If B is an initiator, interacting with C without the need of some other system

[22]We are adopting the view that EnDQT is a single-world theory, and that does not require some (emergent) agents. Alternative versions may deny this, and consider that SDCs involve some branching process, or that SDCs only tell us about interactions that ultimately need agents making measurements to give rise to measurement outcomes. These versions are problematic.

that allows it to have the $DC - C$.

As was argued before [73], to explain why initiators are in principle not observable currently (i.e., a measurement device or a probe seems always to need other systems that apply it or prepare it, respectively, at least according to our more direct evidence) it is because such initiator is, for example, the inflaton field and after its activity, it sits at the bottom of its potential $V(\mathbf{x}, t)$, but there are other possibilities beyond inflation, as we will discuss.[23] As it will become clearer (Section 5), the scales in which initiators exist may be used to help explain the scales in which members of SDCs operate. Of course, we might assume that SDCs go on indefinitely, and in that case we do not need to invoke initiators, and could have some kind of cyclic universe, for example. What the correct view is might end up being an empirical question. We will come back to initiators in Sections 5, 7, and Appendix H.

Notice that according to EnDQT for a system to maintain its quantum coherence, it needs to be isolated from the SDCs. The system that is isolated from SDCs could be arbitrarily large, and if that isolation was achieved, the system could in principle be maintained for an arbitrary amount of time in a superposition. This contrasts with spontaneous collapse theories, which consider that an isolated system would still collapse at some point no matter what, or gravity-caused collapse theories. where a system would collapse depending on its mass/energy. We also do not need to modify the fundamental equations of QT to represent SDCs, contrary to these theories.

3.2 The QFT case

Let us turn to the QFT case. We want to focus on spacetimes where the classical dynamics governed by the Klein-Gordon equation (see below) have a well-posed initial value formulation roughly in the sense that it admits a spacelike hypersurface where the initial data can be specified such that the entire evolution in spacetime is determined by this data. This hypersurface is a Cauchy surface and a Lorentzian manifold is globally hyperbolic if and only if it admits a smooth Cauchy hypersurface.

So, let ϕ be a real scalar field defined in a $D = n + 1$-dimensional globally hyperbolic Lorentzian spacetime $(\mathcal{M}, g_{\mu\nu})$, where n is the number of spatial dimensions. The field satisfies the Klein-Gordon equation:

$$\hat{P}\phi = 0, \quad \hat{P} = \nabla_a \nabla^a + m^2 + \xi R, \tag{30}$$

where ξ is a curvature coupling constant, R is the Ricci scalar, and ∇_a is the Levi-Civita connection corresponding to the metric $g_{\mu\nu}$. The condition of global hyperbolicity guarantees the existence of a smooth foliation by Cauchy surfaces $\{\Sigma_t\}_{t\in\mathbb{R}}$ and a diffeomorphism $\mathcal{M} \cong \mathbb{R} \times \Sigma$. In these spacetimes, the Klein-Gordon equation admits a well-posed initial value formulation,[24] and we can

[23] An initiator may be the source of its own smearing function. See Section 5.

[24] An initial value formulation involves a differential equation together with an initial condition that specifies the value of the unknown function at a given point in their domain.

meaningfully describe constant-time slices. For instance, in Minkowski spacetime, we may identify the Cauchy surfaces $\Sigma_t \cong \mathbb{R}^n$ as any spacelike hypersurface of codimension 1. Using the global inertial coordinates $(t, \mathbf{x})$, these hypersurfaces correspond to surfaces of constant t.

Although we will mostly adopt the "physicist" formalism for ease of exposition, we will always have in the background the more rigorous algebraic quantum field theoretic (AQFT) account [29] with its algebra of observables independent of a Hilbert space representation, and its smeared fields.[25] We have seen in the previous section how the test function that smears fields over a spacetime region plays an important role for EnDQT in representing how SDCs propagate.

We work on a globally-hyperbolic spacetime $(\mathcal{M}, g)$ that either contains a region with a timelike Killing vector K^a (e.g. a static patch), or possesses a preferred time function t whose asymptotic or adiabatic behaviour selects a "positive-frequency" notion (e.g. conformal time in the Poincaré patch of de Sitter). With n spatial dimensions the real scalar field admits the Fourier expansion,

$$\phi(x) = \int d^n\mathbf{k} \Big[a_{\mathbf{k}}\, u_{\mathbf{k}}(x) + a^\dagger_{\mathbf{k}}\, u^*_{\mathbf{k}}(x) \Big], \tag{31}$$

where the normalization factors were built into the mode functions.[26]
Promoting $a_{\mathbf{k}}, a^\dagger_{\mathbf{k}}$ to operators gives

$$\hat{\phi}(x) = \int d^n\mathbf{k}\, \left(\hat{a}_{\mathbf{k}}\, u_{\mathbf{k}}(x) + \hat{a}^\dagger_{\mathbf{k}}\, u^*_{\mathbf{k}}(x) \right), \quad \left[\hat{a}_{\mathbf{k}},\, \hat{a}^\dagger_{\mathbf{k}'} \right] = \delta^n_{\mathbf{k}\mathbf{k}'}\, \mathbb{I}. \tag{32}$$

The vacuum state $|0\rangle$ is defined as the state annihilated by $\hat{a}_{\mathbf{k}}|0\rangle = 0$ for all $\mathbf{k}$. Performing the quantization on a constant-time foliation $\mathbb{R} \times \Sigma_t$, where Σ_t is a spacelike Cauchy surface, we obtain the equal-time commutation relations:

$$[\hat{\phi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')] = i\delta^n_\Sigma(\mathbf{x}, \mathbf{x}')\mathbb{I}, \tag{33}$$

$$[\hat{\phi}(t, \mathbf{x}), \hat{\phi}(t, \mathbf{x}')] = [\hat{\pi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')] = 0. \tag{34}$$

Here,[27] the canonical momentum operator is defined in curved spacetime as $\pi(t, \mathbf{x}) = \sqrt{h} n^a \nabla_a \phi(t, \mathbf{x})$ where $h = \det(h_{ij})$ is the determinant of the induced metric h_{ij} on the Cauchy surface Σ_t, and n^a is the future-directed unit normal to Σ_t. In Minkowski spacetime, with Σ_t being a constant-t hypersurface, this reduces to the familiar definition $\pi = \partial_t \phi$.

3.2.1 Introduction to the partial trace and constraints on smearing functions

To infer when and how determinate values arise under interactions, it is useful to use the partial trace. However, it is not technically correct to assign a density

[25] See Appendix B for some formal details regarding the quantization of the scalar field from an AQFT perspective.

[26] For proper normalization in curved space the modes $u_{\mathbf{k}}, u_{\mathbf{k}'}$ should be orthonormal under the Klein–Gordon inner product, i.e. $(u_{\mathbf{k}}, u_{\mathbf{k}'}) = \delta^n(\mathbf{k} - \mathbf{k}')$, where $(u, v) = i \int_\Sigma d\Sigma^a \left[u^* \nabla_a v - (\nabla_a u^*) v \right]$.

[27] δ^n_Σ denotes the covariant delta distribution with respect to the volume element $\mathrm{d}^n x \sqrt{h}$.

matrix to the restriction of a vacuum state or any physical state of a QFT to any local subregion. Mathematically, this is because the local algebra of observables on a finite region of a relativistic QFT is a type III von Neumann algebra. This algebra does not admit any irreducible representation as an algebra of operators on a Hilbert space, and does not have any nontrivial faithful operation with the properties of a trace. Thus, operations like taking a partial trace over a subregion are unavailable, and the von Neumann entropies of the reduced density operator of a QFT on a given region are not well-defined. Therefore, we cannot use them to talk about the reduced state of a QFT on a local subregion.

We will thus focus on a subset of modes of real scalar fields that participate in interactions involved in SDCs, where that selection will be inferred via the smearing functions, and this will provide one possible way to go to a type I von Neumann algebra, which we will often use here. We often do this by quantizing a sum of discrete solutions to the Klein-Gordon equation in a bounded spacetime region,

$$\hat{\phi}(x) \;=\; \sum_{\alpha}\Big[\hat{a}_{\alpha}\, u_{\alpha}(x) \;+\; \hat{a}^{\dagger}_{\alpha}\, u^{*}_{\alpha}(x)\Big]. \tag{35}$$

To calculate the quantum amplitudes, let us suppose that we have the following temporal and spatial smearing gaussian functions,

$$\Lambda(x) = \chi(t)\, F(\mathbf{x}) = \frac{1}{\pi^{1/4}\sqrt{T}}\exp\Big[-\frac{(t-t_0)^2}{2T^2}\Big]\frac{1}{(\pi\sigma^2)^{3/4}}\exp\Big[-\frac{|\mathbf{x}-\mathbf{L}|^2}{2\sigma^2}\Big]. \tag{36}$$

T and σ represent the standard deviations in time and space, and characterize the region where $\Lambda(x)$ is effectively nonzero. The parameters t_0 and $\mathbf{L}$ determine the central position of the function's effective support.

There are different positions that one might take regarding the smearing functions. One is that they are fundamental and they offer ways to infer how the DC propagates and SDCs expand, not being attached to any particular system. However, one might object against that strategy because one might see the smearing functions as mysterious. Furthermore, there is a good case to be made that they come rather from a time-dependent potential so the system is implicitly open and work is being done upon it. What is doing work on it? Furthermore, one may wonder in which frame is t of the smearing functions defined.

3.2.2 Systems emitting a field that corresponds to a smearing function

Another option is that smearing functions allow us to infer the localization features of the systems belonging to SDCs, and they can be defined in their own frame. There are multiple ways to proceed. Here, we will consider that they arise from some systems D in a state $\hat{\rho}_D$, being (a possibly complex-valued). Ultimately, as we will see, they are related to the emission of a gravitational field of the localized systems. We restrict our attention to those systems whose mean field gives rise to a well-defined smearing function.[28]

Such mean-field is *emitted* by a system D,

$$f(\mathbf{x},t) = Tr(\hat{\rho}_D \hat{\phi}_D(\mathbf{x},t)), \tag{37}$$

where depending on a cutoff imposed on the integral of this equation over energy and/or momentum, $\hat{\phi}_D$ could be some modes of a real scalar quantum field, or some positive or negative momentum component of these fields, if we wanted f to be complex valued.[29] In the case of EnDQT, they arise from systems that belong to SDCs. Thus, when we consider the interaction between system A and system B, we consider a system D that belongs to an SDC that we choose to ignore,[30] and that sources the smearing function for other fields in a spacetime region. Note that D are some modes of a field and A could be some modes of the same field as D, and thus they can be regarded as effectively being the same system. See Appendix C for an example of how coherent states can source the test functions.

Thus, the idea is that SDCs also involve systems that source the test functions in certain states. Another role of test functions and systems that source them is to establish what are the scales of systems that source gravity, i.e., the gravitational scales. It is often argued that the semiclassical approach breaks at Planck scales. However, behind this idea is frequently the assumption that scales that we cannot probe in principle have a gravitational field associated with them, and this assumption can be denied. One of the effects of SDCs is to select which systems at certain scales emit a gravitational field. We hypothesize that the scales in which SDCs operate are ones much higher than the Planck scale, and thus semiclassical gravity will be enough to explain the behavior of the gravitational field.

An example that supports this perspective, which will lead to another determination condition, is the following: test functions are involved in all tests of special relativity, which respect Poincaré invariance. Thus, we want them to not spoil the commutation relations between the generators of Poincaré transformations, which is needed to preserve Poincaré invariance. This leads to constraints on the smearing functions for all local Hamiltonians in flat spacetime. In the case of spatial variance σ we get the following constraint (see Appendix C),

$$\sigma \gg 1/k_{max} \tag{38}$$

where $k_{\max} = |\mathbf{k}_{\max}|$ concerns the maximum momentum of the physical processes under study, and $L_{phys} = 1/k_{max}$ is the minimal length scale of the modes of

[28] While other emission mechanisms may exist, the results we derive apply most directly to this subclass. However, the results concerning the smearing functions for the Hamiltonian under analysis are general.

[29] The bounds derived below are applicable to this case as well.

[30] We could introduce in this definition a smearing function inside the expectation value value, inherited from D's previous interaction with members of an SDC. However, given the bounds derived below, in principle, we do not have to. Given information of D's previous interaction concerning information about how the modes of the field were filtered, we can introduce a cutoff in the integral in eq. (37) by hand. See Appendix C.

the field involved in the physical process under study (i.e., the interactions). Similarly, we get the following constraint on the variance T of the temporal smearing function (see Appendix C),

$$T \gg 1/\omega_{max} \tag{39}$$

where ω_{max} concerns the maximum energy of the process under study, and $\tau_{phys} = 1/\omega_{max}$ concerns the minimal temporal scale involved of the modes of the field in the physical process under study. Similar inequalities need to be obeyed by the IR filter, which filters out infrared modes in flat spacetime.[31] Furthermore, it can be shown that bandpass filters also obey these bounds.[32]

We will also analyze the case of de Sitter spacetime with only a temporal smearing function,

$$f_\ell(t, \mathbf{x}) = N_\ell \exp\Big[-\frac{(t-t_0)^2}{2\ell_t^2}\Big], \tag{40}$$

we obtain similar inequality $\omega_{\max} \gg 1/\ell_t$ for all local Hamiltonians in these spacetimes, taking into account the generators of symmetries of a de Sitter spacetime and their commutation relations. Note that many or perhaps most spacetimes do not have the above bounds because they lack symmetries.

Thus, smearing functions with the above features in these spacetimes will only act on systems subject to the above inequalities. Therefore, interactions that form SDCs with the above symmetric features often have to have couplings that obey the above constraints, which depend on the maximal momentum or energy of the modes they couple to. Furthermore, the spacetime symmetries of the theory impose that the emitters of the smearing function involve systems that live at much higher scales than the smaller systems that are subject to those smearings, and that impose smooth UV or IR cutoffs on these systems. Note that, considering all possible spacetimes, relativistic symmetries are the exception rather than the rule, and never hold exactly. Thus, small violations, even if they are the case, are unproblematic.

This gives rise to the following postulate that we add to postulate 2, concerning the conditions for a system B to obtain the determination capacity concerning a target system C, which we denote as DC-C:

vi) If B is interacting with the emitters of a smearing function for its interactions with A and C. A system is an emitter of a smearing function if it is in a state that can give rise to a valid smearing function, having the DC concerning the systems it interacts with, in agreement with relativistic symmetries, while having

[31] Assuming for definiteness the following smearing function, $f(\mathbf{x}, t) = g\Big[\frac{\mathbf{x}^2}{\sigma^4} - \frac{d}{\sigma^2} + \frac{t^2}{T^4} - \frac{1}{T^2}\Big] \exp\Big(-\frac{\mathbf{x}^2}{2\sigma^2} - \frac{t^2}{2T^2}\Big)$, whose Fourier transform is $\tilde{f}(\mathbf{k}, \omega) = g\big(\sigma^2 k^2 + T^2\omega^2\big) \exp\Big[-\frac{1}{2}\big(\sigma^2 k^2 + T^2\omega^2\big)\Big]$, and which filters out the long–wavelength/low–energy sector, we can see that scaling argument is identical to that given in Appendix C; hence $k_{\max}\sigma \gg 1$ and $\omega_{\max} T \gg 1$

[32] An example of that filter is $f_{\mathrm{BP}}(\mathbf{x}, t) = g\Big[1 - e^{-\mathbf{x}^2/(2\sigma_{\mathrm{IR}}^2)}\Big]\Big[1 - e^{-t^2/(2T_{\mathrm{IR}}^2)}\Big] e^{-\mathbf{x}^2/(2\sigma_{\mathrm{UV}}^2)} e^{-t^2/(2T_{\mathrm{UV}}^2)}$. where g is some normalization factor.

determinate values in the spacetime region where it emits such a function.

By valid smearing function, we mean a function that is smooth and strongly localized such as a Schwartz or bump function. In agreement with relativistic symmetries in this case means that the test function emitted by the system should obey conditions such as the above ones, which make relativistic symmetries or constraints be satisfied. Thus, we can see that being subject to a test function will depend on the features of the systems that emitters interact with, having a k_{max} obeying inequalities such as the ones above.

Moreover, note that emitters of the test functions are necessary but not sufficient to lead to this filtering out of the modes of a system S'. We also need that systems S interact with S', have the DC-S' and decohere the modes that are not filtered out. Again, effectively the emitter of the smearing function and the system S may be the same system, being different modes of the same field (even S' could be other modes of the same field).

Another feature worth noticing is that with the mean field definition that we have adopted, not all states $\hat{\rho}$ can emit a smearing function, they have to be states such that $\alpha_{\mathbf{k}} := \mathrm{Tr}\big(\hat{\rho}\,\hat{a}_{\mathbf{k}}\big) \neq 0$, for at least some modes $\mathbf{k}$. Thus, for instance, coherent states with $\alpha_{\mathbf{k}} \neq 0$ can do that, as well as squeezed coherent states and field amplitude eigenstates approximated by a gaussian. Number states, thermal states, parity symmetric and antisymmetric cat states cannot. Note that these states still need to be selected via decoherence.

Note that the smearing function is typically calculated as the expectation value of a continuum of modes. However, in agreement with the scale-dependence of SDCs, in practice, we are never working with all modes of a field. The system that emits the smearing function will have a series of modes up to some bound k_{max} in the UV filter case, which will have determinate values due to the decoherence and the filtering due to some other systems, and where these modes will participate in the *emission* of the smearing function to other systems. More concretely, although the smearing function is calculated by integrating over a continuum of modes dk, from 0 to ∞, in practice, often we integrate up to a $\Lambda = k_{max}$. Thus, depending on $\Lambda = k_{max}$, we can view the smearing function as being emitted by many single modes or even by a single-mode $\mathbf{k} \approx 0$ if $k_{max} = \Lambda \ll 1$ for a UV cut-off. Interestingly, the inequalities (38) and (39) that we derived above for the smearing functions to obey the spacetime symmetries, guarantees the validity of these cutoff-based bounded integrals.[33] A related feature is that the emitters of the smearing function can acquire the DC concerning other systems without a third system that localizes their interaction. We will come back to this kind of acquisition of the DC in the next section.

Another example that supports the scale-dependence of SDCs is that decoherence in curved spacetime tends to occur at certain scales, such as at super-Horizon scales in the case of de Sitter spacetime as we will see in Section 5, and thus the determinate values and the emission of the gravitational field of certain systems may only occur at certain scales.

[33] See at the end of Appendix C.

The last example that we would like to mention here, which supports the scale-dependence perspective, is based on how detector resolution determines whether a more massive system influences or not the decoherence of some other less massive system. More concretely, it can also be shown that smearing functions determine whether a more massive system S' in the same spacetime region of S decoheres S, or decouples from S, not decohering it. This will depend on how massive is S' compared with the temporal cutoff represented by the variance of the temporal smearing function. If its mass M is much bigger, it will be UV filtered out (See Appendix D for more details). We will see that this case is related to the previous example because the system that emits a gravitational field in the de Sitter case is also considered to be the system that emits the smearing function.

So, the point is that smearing functions emitted by systems that belong to SDCs account for a massive system decohering or not a target system, and hence it accounts for whether such higher energetic system emits a gravitational field or not. This supports the view advanced above that SDCs select systems at certain scales to emit a gravitational field. Therefore, by adopting this theory, we are able to consider that it is not necessarily the case that the gravitational field is emitted at all scales, including at the Planck scale. It will depend on the structure and elements of SDCs.[34] We will come back to this topic at the end of the next section, after examining a more concrete example.

3.2.3 SDCs in flat spacetime

We will now give an example of an SDC in flat spacetime involving systems in an inertial frame by appealing to the well-known models of decoherence. This will also make it clear how systems in a coherent state are selected, which we will appeal to.

Let us consider that we have A, B, C, and D. We consider that A is a large collection of N modes of a field in a Gibbs state:

$$\rho_A = \int \prod_i \frac{d^2\alpha_i}{\pi} \left[\prod_i \left(1 - e^{-\beta\omega_i}\right) e^{-(1-e^{-\beta\omega_i})|\alpha_i|^2}\right] \big|\{\alpha_i\}\big\rangle\big\langle\{\alpha_i\}\big| \,, \tag{41}$$

and we consider that B could be a number N' of modes B in some arbitrary state. We will focus on a single mode of B for now, where modes of B are in a state $|\psi\rangle_B$. Furthermore, D is a field that emits the smearing function, and which we will assume it is in a coherent state $|\alpha\rangle_D$ due to its interactions with other members of an SDC that we choose to ignore (we will come back on how D might have ended up in that state).[35] Moreover, D is a system (composed of multiple modes) that is interacting with multiple modes that constitute A and B, while they interact, and that can also interact with B and C, if they interact,

[34] And perhaps also on relativity, see Section 6 for a conjecture based on Black Holes concerning the minimal four-volume of elements of SDCs.

[35] Such coherent state has $\alpha_{\mathbf{k}} = \tilde{f}(\mathbf{k}, \omega_{\mathbf{k}}) = \exp\Big[-\frac{1}{2}\left(\sigma_r^2|\mathbf{k}|^2 + \sigma_t^2\omega_{\mathbf{k}}^2\right)\Big]$.

emitting the smearing function for these interactions. We assume that modes of C are in an arbitrary quantum state $|\psi\rangle_C$.

We will consider that the multiple modes that constitute field A interact with a single mode of B, where the Hamiltonian of interaction is given by

$$\hat{H}_{\text{int}} = \sum_{\mathbf{k}\neq\mathbf{k}_B} C_{\mathbf{k}}\, \hat{X}\, \hat{q}_{\mathbf{k}}, \tag{42}$$

where X and $q_{\mathbf{k}}$ are the field quadratures for the single mode of B and for the multiple modes A_i of A, respectively, and $C_{\mathbf{k}}$ are the coupling constants. Assuming the non-disturbance condition, we choose to ignore the interaction between B_i and the rest of the systems that constitute C.

To arrive at the above H_{int}, starting from flat spacetime QFT we consider the following smeared linear interaction Hamiltonian,

$$\hat{H}_{\text{int}}(t) = \lambda \int d^3x\, dt\; f(\mathbf{x},t)\, \hat{\phi}_B(\mathbf{x},t)\, \hat{\phi}_A(\mathbf{x},t), \tag{43}$$

select one of the modes of B for simplicity and insert the plane-wave decompositions in SI units,

$$\hat{\phi}_B(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sqrt{\frac{\hbar}{2\Omega}}\, \big[\hat{a}_B\, e^{i(\mathbf{k}_B\cdot\mathbf{x}-\Omega t)} + \hat{a}_B^{\dagger}\, e^{-i(\mathbf{k}_B\cdot\mathbf{x}-\Omega t)}\big], \tag{44}$$

$$\hat{\phi}_A(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}\neq\mathbf{k}_B}\sqrt{\frac{\hbar}{2\omega_{\mathbf{k}}}}\, \big[\hat{a}_{\mathbf{k}}\, e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}} t)} + \hat{a}_{\mathbf{k}}^{\dagger}\, e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k}} t)}\big], \tag{45}$$

together with the Gaussian smearing function

$$f(\mathbf{x},t) = f_r(\mathbf{x})\, f_t(t), \quad f_r(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/(2\sigma_r^2)}}{(2\pi)^{3/2}\,\sigma_r^3}, \quad f_t(t) = \frac{e^{-t^2/(2\sigma_t^2)}}{(2\pi)^{1/2}\,\sigma_t}. \tag{46}$$

We end up with four exponentials,

$$\begin{aligned} I_1(\mathbf{k}) &= \exp\Big[-\tfrac{1}{2}\big(\sigma_r^2|\mathbf{k}_B+\mathbf{k}|^2 + \sigma_t^2(\Omega+\omega_{\mathbf{k}})^2\big)\Big], \\ I_2(\mathbf{k}) &= \exp\Big[-\tfrac{1}{2}\big(\sigma_r^2|\mathbf{k}_B-\mathbf{k}|^2 + \sigma_t^2(\Omega-\omega_{\mathbf{k}})^2\big)\Big], \\ I_3(\mathbf{k}) &= I_2(\mathbf{k}), \qquad I_4(\mathbf{k}) = I_1(\mathbf{k}). \end{aligned}$$

Given relativistic symmetries-inducing bounds derived in the previous section, $|\mathbf{k}|\,\sigma_r \gg 1, \omega\,\sigma_t \gg 1$, and thus

$$\sigma_r^2|\mathbf{k}_B+\mathbf{k}|^2 \gg 1, \qquad \sigma_t^2(\Omega+\omega_{\mathbf{k}})^2 \gg 1, \tag{47}$$

so we have that $I_1 \simeq I_4 \ll 1$.

Furthermore, assuming that the emitter of the smearing function filters out every mode except those that are quasi-resonant with the environmental probes, or assuming that it operates in narrow band, we get $\big|\mathbf{k}-\mathbf{k}_B\big| \lesssim \sigma_r^{-1}, \big|\omega_{\mathbf{k}}-\Omega\big| \lesssim \sigma_t^{-1}$. Thus,

$$\sigma_r^2|\mathbf{k}_B-\mathbf{k}|^2 \ll 1, \qquad \sigma_t^2(\Omega-\omega_{\mathbf{k}})^2 \ll 1, \tag{48}$$

and therefore $I_2 = I_3 \approx 1$. So, $I_1(\mathbf{k}) = I_4(\mathbf{k}) \longrightarrow 0, I_2(\mathbf{k}) = I_3(\mathbf{k}) \longrightarrow 1$. We thus obtain,

$$\hat{H}_{\rm int} \;=\; \frac{\lambda\hbar}{V\sqrt{2\Omega}} \sum_{\substack{\mathbf{k}\neq\mathbf{k}_B \\ |\mathbf{k}|\leq k_{\max}}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \left(\hat{a}_B\,\hat{a}_{\mathbf{k}}^\dagger + \hat{a}_B^\dagger\,\hat{a}_{\mathbf{k}}\right), \tag{49}$$

which can be rewritten as

$$\hat{H}_{\rm int} = \sum_{\mathbf{k}\neq\mathbf{k}_B} C_{\mathbf{k}}\,\hat{X}\,\hat{q}_{\mathbf{k}} \tag{50}$$

with $C_{\mathbf{k}} = \lambda\hbar/[V\sqrt{2\Omega\,2\omega_{\mathbf{k}}}]$, $\hat{X} = \sqrt{\hbar/(2\Omega)}(\hat{a}_B + \hat{a}_B^\dagger)$, and $\hat{q}_{\mathbf{k}} = \sqrt{\hbar/(2\omega_{\mathbf{k}})}(\hat{a}_{\mathbf{k}} + \hat{a}_{\mathbf{k}}^\dagger)$.

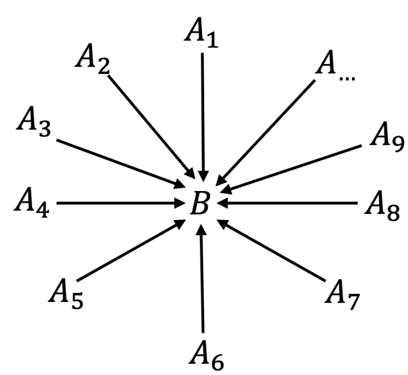


Figure 3: Multiple systems $A_1, ..., A_N$ belonging to SDCs, and interacting with system B, giving rise to B having a determinate values and emitting a gravitational field. The inference regarding how these interactions occur is made via models of decoherence. We omit system C and D in this diagram.

It was shown that an arbitrary state $|\psi\rangle$ decohering into statistical mixture of coherent states is a generic feature of free quantum systems that are linearly coupled to an environment in a Gibbs state. This environment can have a non-zero temperature, and involve an ohmic, subohmic, as well as supraohmic damping, and the interactions can have arbitrary coupling strengths [24]. Moreover, this Hamiltonian of interaction, depending on the specifics of the model [92], also allows for interactions that lead to systems in a Gibbs state. Thus, we can have multiple situations. For instance, we can have a situation where modes of a system A leaves multiple modes of a system B in a Gibbs state, where these modes then leave multiple modes of system C in a coherent state and maybe other modes of C in another Gibbs state, and so on. Moreover, these mixtures of coherent states stochastically give rise to single coherent states, which can then emit smearing functions. Thus, we have here a mechanism in which systems in a coherent state arise via SDCs.

As we can see, to give rise to a system having determinate values, it seems that a source of a smearing function is needed, as well as some system that decoheres or whose state correlates with the state of another system. Decoherence is a good way to infer these processes because it allows us to infer via the quasi-irreversibility in theory, the irreversibility in practice. Let us call *spreading of the DC by control* the spreading of the DC between systems S due to systems S', where S' emit the smearing functions that make systems S interact between each other, obey the no-disturbance condition, and transmit the DC between each other. This is the kind of spreading of the DC that we have been seeing.[36]

However, the spreading of the DC by control between systems due to some other systems is not the only way to spread the DC. First, it is not always the case that we need a different system emitting the smearing function, which is different from the ones involved in the interactions. Sometimes systems emit the smearing functions of their own interactions. As we have been seeing, emitters can emit the smearing function for their own interaction with other systems. We can always consider that modes of the system D, in the example above, are emitting a smearing function that concerns their interaction with A and B, and with B and C. There is no need for a fifth system to do that.

To see this, first notice the curious fact that, given some Hamiltonian, there may be interactions between systems where we do not know how the way they will give rise to determinate values is going to precisely occur. This is the case where we have emitters of the smearing function, as we have said. Consider A interacting with B via certain modes where $\lambda_{AB} f_{AB} \hat{\phi}_A \hat{\phi}_B$ (where we disregard which modes will interact between each other). Moreover, B might develop self-interactions through certain modes via a cubic interaction $\hat{H}_{int}(t) = \int d^3x \frac{g}{3!} \left(\hat{\phi}_{B,\mathbf{k}=0}(t) + \delta\hat{\phi}_{B,\mathbf{k}\neq 0}(\mathbf{x},t) \right)^3$ (omitting the sum over the rest of the modes $|\mathbf{k}| \neq 0$, which is inside $\delta\hat{\phi}_{B,\mathbf{k}\neq 0}$), for $g \ll 1$, which in momentum space involves terms such as one proportional to $\hat{\phi}_0(t) \sum_{\mathbf{k}\neq 0} \hat{\phi}_{B,\mathbf{k}}(\mathbf{x},t)\, \hat{\phi}_{B,-\mathbf{k}}(\mathbf{x},t)$. $\phi_{B,\mathbf{k}=0}$ is the $\mathbf{k}=0$ mode of B, which if it is decohered by A, it could act as an emitter of a temporal smearing function $f(t) = \langle \hat{\phi}_0(t) \rangle_\rho$ for interactions involving other modes $\mathbf{k} > 0$ of B. However, we do not know in which state ρ, $\hat{\phi}_0$ will end up, where this state will determine the features of $f(t)$.

So, in this case, these $\mathbf{k} \neq 0$ modes of B that are interacting with $\mathbf{k}=0$ of B (but are not interacting with modes of A), while the $\mathbf{k}=0$ of B is interacting with A, fulfill the no-disturbance condition because they do not disturb the

[36] The features that we have been seeing in fact opens up the possibility of alternative determination condition, which consider that the only systems that have the DC are the emitters of the smearing functions, not the systems S. However, this approach in so far that it is consistent, neglects the role that systems S that decohere each other have in giving rise to measurement outcomes. Also, intuitively, it seems that measurement instruments need to have some importance when measuring target systems, not only the systems that localize their interactions.

interactions between $\mathbf{k} = 0$ of B and A. So, given the determination conditions, $\mathbf{k} = 0$ of B will obtain the DC concerning $\mathbf{k} \neq 0$ of B. This non-disturbance is trivially fulfilled because there is no smearing function (yet) that makes them interact with $\mathbf{k} = 0$ of B. Let us call *transmission of the DC by osmosis* these interactions that involve systems obtaining the DC concerning some other systems without intermediaries involving emitters of the smearing functions like $\phi_{B,\mathbf{k}=0}$ that obtain the DC concerning some systems $\delta\hat{\phi}_{B,\mathbf{k}\neq 0}$ that it does not filter out.[37]

Moreover, we can consider that the emitters of the smearing function interact with all the modes of A and B up to k_{max}, which they do not filter out (UV filter) or which they filter out (IR filter). So, filtering is done via the interaction of the emitter of the smearing function with certain modes. Then, the modes that are not filtered can participate in a process that gives rise to determinate values. The modes that are filtered out cannot. It is in this sense that SDCs only exist at certain scales.

Given these determination conditions, the picture that emerges from this theory is of a tower of scale-dependent emitters of smearing functions and systems that are subject to such emission, which end up emitting the smearing function to other systems and so on.

Finally, we would like to mention that we can treat the systems that have the DC as a probe/particle detector, and use particle detector models or measurement theory in QFT (developed in algebraic QFT) to update the state of the systems. In Appendix F we do that for particle detector models.

4 The theory of gravity

We will now turn to the presentation of the theory of gravity based on EnDQT. It will involve three postulates, which are an addition to the other features of EnDQT mentioned above involving the determination conditions. Although some of these postulates may seem radical, one should notice that this is actually a very conservative theory. String theory is not being appealed to, spacetime or gravity will not be quantized, but we also do not need to view the metric and the conjugate momentum as some stochastic classical system. So, it will not be a hybrid classical-quantum theory or a gravitational causes collapse theory like the ones from Diosi and Penrose.[38] We will also show in more detail how this theory agrees and generalizes the equivalence principle, one of the basic principles of relativity.

4.1 The first postulate

We have made clearer above what our QFT setting is, now we need to make sure we make clear what we can consider to be the fundamental systems stud-

[37] Note that there might exist other mechanisms for the transmission of the DC beyond those presented here, and consistent with our desiderata.

[38] [20, 69].

ied by this theory of gravity, and what affects their evolution in the absence of interactions with members of SDCs. This will be the goal of the first postulate.

Postulate 1 Quantum systems involve sets of modes of quantum fields (henceforward quantum fields) that may occupy bounded spacetime regions, and have quantum properties, which are properties represented by observables, such as field amplitude operators and energy-momentum operators, and quantum states in agreement with QFT. In the absence of interactions with SDCs given by the determination conditions, quantum fields in R have indeterminate values of any of their dynamical observables in R. Quantum fields S in a spacetime region R that are not interacting with members of an SDC along R, evolve under the dynamical equations of QFT that quantum fields obey, such as the Klein-Gordon equation, but no determinate values arise. The above equations are in part determined by the gravitational field in R, or by a flat spacetime metric in the absence of a gravitational field. However, this field is not emitted by S because S cannot emit a gravitational field.

We will be interested in studying quantum systems that occupy bounded regions of spacetime and that establish local interactions with other systems. In the previous sections, we have seen how we can represent their interactions via test functions $f(\mathbf{x}, t)$. Furthermore, quantum fields in R are affected by the gravitational field in that region emitted by the sources of that field. However, they are not affected classically by that gravitational field in the sense of being test quantum fields that have determinate values or that give rise to a test particle obeying the geodesic equation (we will come back to this and justify it with postulate 2). The way they are affected is described by the equations that concern the evolution of the quantum fields in that region, such as the Klein-Gordon equation or the Dirac equation for flat and curved spacetimes. However, they do not emit any gravitational field of their own. We will see in the next sections how this postulate allows us to address some issues with the semiclassical approach.

To understand one of the consequences of postulate 1, let us consider two scalar fields isolated from SDCs in a spacetime region R. Let us consider that these scalar fields evolve under the same gravitational field in R determined via the metric $g_{\mu\nu}$ (e.g., the gravitational field coming from earth). Also, let us assume that under interactions with SDCs, these systems give rise to a very different determinate energy-momentum each (which could be arbitrarily different). However, their dynamics are the same, which depends on the Klein-Gordon equation for curved spacetimes that depends on the metric $g_{\mu\nu}$. Thus, this implies that systems with very different energy-momentum in the same region of spacetime R will evolve under the same gravitational field. Therefore, according to this theory, it is possible that a very massive quantum object in a coherent superposition of macroscopic states (such as a star or black hole in a superposition) evolves under the gravitational field emitted by a feather without affecting their spacetime, provided that the former object is not interacting with SDCs (because of their macroscopicity and decoherence due to the probes, this phenomenon should be physically very unlikely). Relatedly, a feather and a planet in a coherent superposition would behave similarly under the influence of the same gravitational field (assuming that no other forces intervene).

Although at first sight, these consequences seem quite radical and their features counterintuitive, we think they are not because, as we have mentioned (See Section 2 for more details), they can be regarded as a generalization of the Weak Equivalence Principle (WEP),

Without being affected by other forces, any quantum system under the same gravitational field exhibits the same behavior due to this field.

Thus, if these systems do not interact non-gravitationally with other objects, which includes not interacting with SDCs, they will evolve similarly under the same gravitational field.

Furthermore, this theory can be seen as generalizing the so-called strong equivalence principle (SEP) to QFT with no backreaction on gravity (i.e., the standard curved spacetime QFT). In its simplest formulation, the SEP states that [55]:

Locally, special relativity is at least approximately valid.

This theory states the following, which we will call the Special EnD Equivalence Principle:

When not interacting with SDCs, curved spacetime QFT, where a system does not give rise to a gravitational field that influences its evolution, is valid.

The idea is that curved spacetime QFT, where a system does not give rise to a gravitational field that influences its evolution (which is the standard idealization), is valid. Given special relativity, a special case of the above principle concerns the evolution in a local region of spacetime. This special case connects this theory with the SEP and is as follows:

When not interacting with SDCs, locally flat spacetime QFT and, hence, special relativity are at least approximately valid.

Thus, we can see that the SEP is a special case of the Special EnD Equivalence Principle.

Let us now turn to how we will understand interactions in QFT, and to the second postulate concerning how SDCs give rise to the gravitational field.

4.2 Second postulate and probing the metric through SDCs

We will now explain how a system can emit a gravitational field due to systems belonging to SDCs that probe it. We will start by showing an intuitive way of understanding how multiple systems probe a target system and give rise to that system emitting a gravitational field in a spacetime region. Then, we will present postulate 2, which establishes conditions for when systems can emit a gravitational field.

A model to understand how multiple systems belonging to SDCs that constitute probes (see Appendix E) give rise to a target system emitting a gravitational field is based on the work from [70], which was based on [81, 49, 50]. They have shown how we can infer the metric that a quantum real scalar field is subject to from local measurements by particle detectors coupled to that field, where the target system is for simplicity in a Gaussian state fulfilling the Hadamard condition. Essentially, states that fulfill this condition allow for a finite renormalized stress-energy tensor.[39] In Appendix E we can see how the reduced state of a detector contains information about a target system via two-point correlation functions. Using this feature, the inference of the metric through particle detectors involves the probes measuring two-point correlation functions, represented by the Feynman propagator and the Wightman function, and extracting geometric information from them. The central goal is to express the spacetime metric $g_{\mu\nu}$ in terms of these correlators.

More concretely, the starting point is the Feynman propagator, $G_F(x,x') = \langle 0|T\hat{\phi}(x)\hat{\phi}(x')|0\rangle$, and the Wightman function, where $\hat{\phi}(x)$ is the operator of the target quantum field at spacetime point x, and T represents the time-ordering operator. Assuming that the target system is in a vacuum state $|0\rangle$, we can express the metric $g_{\mu\nu}$ in D spatiotemporal dimensions as follows:

$$g_{\mu\nu} = -\frac{1}{2}\left(\Gamma\left(\frac{D}{2}-1\right)\frac{1}{4\pi^{D/2}}\right)^{\frac{2}{D-2}} \partial_\mu\partial_\nu\left(W_{\rho_\phi}(x,x')^{\frac{2}{2-D}}\right), \tag{51}$$

where the above equation is calculated by taking the limit $\mathbf{x}' \to \mathbf{x}$, and where Γ is a Gamma function. This equation holds for any normalized field state ρ_ϕ. Importantly, this means that the metric can be recovered regardless of the specific state of the quantum field as long as the vacuum is a Hadamard state.[40]

These detectors can be understood as modes of a field under certain conditions (Appendix E), and that is the view we will assume. Moreover, we will consider that what is going on is that these systems are giving rise to that field emitting a gravitational field, not just probing it. The detectors probe the system in separate spatiotemporal regions forming an *array* that illustrates how the metric/gravitational field with its distances arises from these interactions. But,

[39] See Appendix B for an introduction to states fulfilling the Hadamard condition.

[40] More concretely, they show that for any normalized pure state $|\psi\rangle$, we have that

$$W_\psi(x,x') = \langle\psi|\hat{\phi}(x)\hat{\phi}(x')|\psi\rangle = W(x,x') + \sum_{m=0}^{\infty} F_m(x)G_m^*(x') + \text{H.c.}$$

where $F_m(x)$ and $G_m(x')$ are state-dependent regular functions in the limit $x' \to x$. Thus, the behavior of $W_\psi(x,x')$ in this limit does not depend on the specific quantum state of the field. While the coincidence limit $x' \to x$ formally leads to a divergence, it has been demonstrated that the singular part of $W_\psi(x,x')$ coincides with that of the vacuum Wightman function $W(x,x')$. Importantly, since this result holds for pure states, it extends naturally to any normalized mixed state, as these can be expressed as convex combinations of pure states.

instead of just an array of detectors, we will consider that what we fundamentally have is an array of systems belonging to SDCs in space, interacting with a target quantum field over time. See Appendix F for more details.

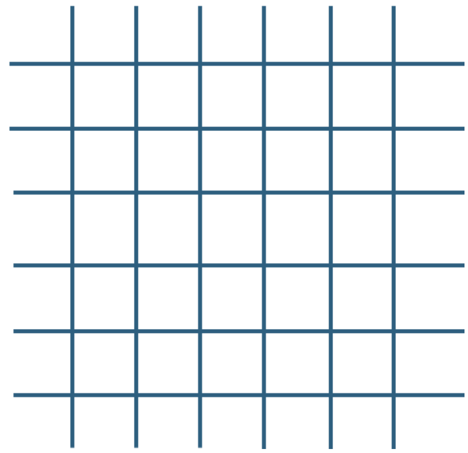


Figure 4: Two-dimensional spatial hypersurface of members of an SDC probing a scalar field at separate points of what can be illustrated as an array of detectors, giving rise to that field emitting a gravitational field in a certain region.

We will consider that settings like this one constitute environments that give rise to the target system emitting a gravitational field. The persistence of these interactions gives rise to systems emitting a gravitational field or being subject to it classically in a region R (i.e., moving along geodesics, more on this below). Note that this target quantum field can itself probe other systems, and so on, constituting an SDC.

These interactions will give rise to a set of values and correlation functions and an associated metric. In a sense, SDCs act like rods and clocks that produce a non-flat metric.

Above we have established the determinacy conditions, and now we will establish the gravitational conditions, which as a reminder, are the conditions for a system to emit a gravitational field. Postulate 2 establishes what the gravitational conditions are that we will adopt. As there are various possible determination conditions (see [74] for a discussion), there are also multiple possible gravitational conditions. We will go over some of them, and explain why we adopt the ones that we will adopt. The first point of division is whether systems in all states, as long as they yield a finite renormalized stress-energy tensor, such as Hadamard states or C^4 states, emit a gravitational field. SDCs would leave systems in those states. Another option is that only systems in more specific states can emit a gravitational field. A possible criterion to select those kinds of states could be supported by Kuo and Ford criterion [53]. Some states whose second and higher moments of the energy-momentum tensor can be neglected are coherent states. These states give trustworthy inputs to the semiclassical equations to calculate the expectation value of the energy-momentum tensor. This was argued in the paper from Chung-I Kuo and and L. H. Ford [53] for the case of flat spacetime and by [4] for the more general case of globally hyperbolic spacetimes:

$$\Delta_{\mu\nu\lambda\rho}(x,x') = \left| \frac{\langle : \hat{T}_{\mu\nu}(x)\hat{T}_{\lambda\rho}(x') : \rangle - \langle : \hat{T}_{\mu\nu}(x) : \rangle \langle : \hat{T}_{\lambda\rho}(x') : \rangle}{\langle : \hat{T}_{\mu\nu}(x)\hat{T}_{\lambda\rho}(x') : \rangle} \right|. \tag{52}$$

This estimator is understood as the ratio between the variance of the energy-momentum tensor and the expectation value of its square. If this estimator is $\Delta_{\mu\nu\lambda\rho}(x,x') \ll 1$ for all x, x', then we are inside the regime of validity of semiclassical gravity. It was found that for coherent states this condition is fulfilled.[41][42]

Note that this estimator is useful for the specific case of gaussian states (coherent states are gaussian states) because in this case, all statistical moments of quadratic observables are functions of the second and first moments, guaranteeing that the satisfaction of the criterion of Kuo-Ford ensures that the system in this state gravitates semiclassically. However, there are other states such as cat states where other moments are relevant, and so the above criterion fails. It was shown in [4] that cat states, i.e., superpositions of distinguishable coherent states, also deliver trustworthy expectation values of the stress-energy tensor in globally hyperbolic spacetimes for cat states.[43]

So, according to this gravitational criterion, only in certain contexts, such as ones where a system ends up in minimum uncertainty states, such as coherent states and/or cat states, we would have systems in those states emitting a gravitational field, or being subject classicaly to it. We will come back to this point below.

A second point of division is whether in interactions a system can have determinate values (give rise to measurement outcomes) with or without emitting a gravitational field. One option is that we might have circumstances involving the fulfillment of the determination conditions, where systems can have determinate values but without emitting a gravitational field, where, for example, these systems are in states that are not coherent states. Another option is that at least one of the systems involved in the interactions fulfilling the determination conditions must emit the gravitational field under the interactions, but the others

[41]This is because for coherent states $\langle : \hat{T}_{\mu\nu}(x)\hat{T}_{\lambda\rho}(x') : \rangle = \langle : \hat{T}_{\mu\nu}(x) : \rangle \langle : \hat{T}_{\lambda\rho}(x') : \rangle$

[42]Besides what is mentioned below, note that this criterion is informative in the case the expectation value of the stress-energy tensor is non-zero, which does not happen in the case of the Minkowski spacetime. This deficiency is not problematic for this theory because according to it and postulate 3 (see below), the Minkowski metric can be consider the default metric and not a metric that arises from the application of the Einstein Field Equations.

[43]More specifically it was shown that the cat state fulfills the above criterion when the coherent amplitude of the state becomes sufficiently large so that the overlap (inner product) between the two superposed components becomes negligible, and for any cat state where the coefficients of the superposition are chosen such that the relative phase difference between the two coherent states equals $\pi/2$. Moreover, the uncertainty and all regularized higher-order central moments of the energy-momentum density either diminish significantly in the first case or vanish entirely in the second case.

don't. For instance, the system emitting a gravitational field would be in a coherent state, where the others not necessarily so.

Another option is that all systems involved in the interactions, which fulfill the determination conditions, must emit a gravitational field, and SDCs select unproblematic states, such as Hadamard states, and C^4 states that emit such field. We will favor the adoption of this latter option (for now) via the postulate 2 (see below) because we think that it is the most conservative and open to many possible states. Furthermore, it might be the most fruitful option. As we will see, we will hypothesize that the uncertainties in the stress-energy tensor involved in states that give rise to the gravitational field can be absorbed by a negative stress-energy, giving rise to a balance, which allows us to provide an account of dark energy, and derive its value. However, we will be open-minded and consider another perspective via postulate 2' below. Despite the plurality of options seen above that this theory allows for, one should see that as unproblematic because it gives us interesting new hypotheses to study, and which may be testable with gravcats experiments (more on this below).[44]

Before stating postulate 2, let us see how to understand the conservation of the expectation value of the stress-energy tensor and determinate trajectories in spacetime, according to this theory. As it is well known, the covariant conservation of energy-momentum,

$$\nabla^{\mu} T_{\mu\nu}(x) = 0, \tag{53}$$

follows from the Bianchi identity $\nabla^{\mu} G_{\mu\nu} = 0$ together with the Einstein Field equations. Via the covariant conservation equation, we can derive the equations that represent the trajectories of bodies in spacetime such as the geodesic equation and the Mathisson–Papapetrou–Dixon equations equations [59, 68, 19]. Similarly,

$$\nabla^{\mu} \langle \hat{T}_{\mu\nu}[\phi, g_{\mu\nu}] \rangle_{ren} = 0, \tag{54}$$

where $\langle T_{\mu\nu}[\phi, g_{\mu\nu}] \rangle_{ren}$ denotes the renormalized expectation value of the energy-momentum tensor operator. However, the semiclassical covariant conservation equation and the geodesic equations in the semiclassical regime are just applicable once an outcome arises via the decohering interactions that constitute SDCs, in a spacetime region. Before that, the systems have an indeterminate value of their stress-energy tensor in agreement with the determination conditions (Section 3.1), and hence the notion of determinate trajectory given by the geodesic equations is not applicable.

[44] These gravitational conditions may involve three possibilities regarding how gravity and determinate values relate, although here we just focus in the first one because it is the most conservative: -Gravitational imperialism: all systems belonging to SDCs, when having determinate values, need also to emit a gravitational field. -Gravitational necessitism: gravity is needed at least just for one of the systems involved in interactions involved in SDCs in order for systems to have determinate values. -Gravitational dispensabilism: gravity is not needed in order for systems to have determinate values; we can have determinate values in a flat spacetime with systems fulfilling the determination conditions, with none of these systems emitting a gravitational field.

So, the postulate 2 is the following,

Postulate 2 A system S only emits a gravitational field, has determinate values, and evolves classically under a gravitational field, which can involve a determinate trajectory, when it is interacting with systems that belong to SDCs, which leads to the selection of the system as being in a certain quantum state favorable to emit a gravitational field such as Hadamard states and C^4 adiabatic states. The gravitational field sourced by S and that affects classically S is given by the semiclassical Einstein field equation with the energy-momentum tensor properly renormalized:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu}[\phi, g_{\mu\nu}]\rangle, \tag{55}$$

and thus this equation is only valid to describe how the gravitational field affects or is affected by S when S is interacting with members of an SDC.

So, SDCs select certain states that are unproblematic to emit a gravitational field, but postulate 2 leaves in which states these are more open than the alternatives, as we shall see. Also, it is more restrictive because it considers that all the systems involved in SDCs emit a gravitational field, unlike the next one.

One possible alternative to postulate 2 establishes restricted contexts $\mathcal{C}$ in which systems gravitate:

Postulate 2' A system S only emits a gravitational field and can only evolve classically under a gravitational field, which can involve a determinate trajectory, when i) it is interacting with systems that belong to SDCs, and ii) when these interactions between a target quantum matter field S and other quantum matter fields belonging to SDCs that probe the field in a region R lead S to have values that correspond to a quantum state whose second and higher moments of the energy-momentum can be neglected in the spacetime regions R where it is probed, and possibly in other contexts $\mathcal{C}$ that guarantee that the averaged stress-energy tensor provides reliable results. The gravitational field sourced by S and that affects classically S is given by the semiclassical Einstein field equation with the energy-momentum tensor properly renormalized:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu}[\phi, g_{\mu\nu}]\rangle, \tag{56}$$

and thus this equation is only valid to describe how the gravitational field affects or is affected by S when S is interacting with members of an SDC.

Behind this postulate is the hypothesis that coherent states and/or other states whose second and higher moments we can neglect are responsible for the emission of the gravitational field, or only in certain contexts where the stress-energy tensor gives trustworthy results gravitation arises. We will be neutral about which states, or more broadly contexts $\mathcal{C}$, should be the ones that give rise to a gravitational field. The advantage of this hypothesis is that it automatically considers that the only systems that emit a gravitational field have always a stress-energy tensor with low fluctuations, which guarantees that the semiclassical equation gives trustworthy results. Note that smearing functions, via the spatial and time averaging over finite intervals of spacetime, can reduce the probability of large fluctuations of the stress energy tensor [28], and thus this approach allows SDCs to reduce the high variance of these quantities, given that we consider that SDCs give rise to systems that emit smearing functions. So, postulate $2'$ could allow gravity to be emitted just in these cases, if it included them in the set of context $\mathcal{C}$. The disadvantage of Postulate 2' is that it might restrict too much the domain of relativity and in a problematic way. There might be states with high fluctuation, and where no context can reduce their variance.[45] Despite the potential advantages of postulate 2' in terms of dealing with fluctuating stress-energy tensors, we will see in Section 7 that we have other ways of dealing with those fluctuations via dark energy, which can complement postulate 2' or be adopted by someone who adopts postulate 2.

Nevertheless, it is an empirical question which one of the gravitational conditions and associated postulates is the right one, which could be decided by experiments involving preparing gravcat systems in specific states or contexts, and then measuring their potential gravitational field to see if they emit a gravitational field or not.

4.3 Postulate 3: Dark energy

We will turn to the postulate concerning the origin of dark energy, which we relate to the cosmological constant showing up in the Einstein field equations. We will also postulate what the default gravitational field is in the absence of SDCs. This is an open question, and therefore there are different versions of postulate 3. A third version of this postulate will be given in Section 7. All of these postulates are consistent with the theory, and the reason we have three different ones is that we enter in much more speculative domains.

Postulate 3 (version 1) The effects of the cosmological constant Λ are sourced by SDCs when they emit the gravitational field. Therefore, the repulsive effect of dark energy due to this constant is rather an effect of SDCs. In the absence of SDCs, spacetime is flat and there is no accelerated expansion of the universe.

According to this postulate, sourceless gravitational fields do not exist because every gravitational field is sourced by some quantum matter field. Furthermore, quantum matter fields sourced by SDCs not only give rise to a gravitational field

[45] There is also other possibilities such as systems having determinate values, but not emitting a gravitational field of their own. We will refrain from elaborating postulates regarding this because they might be too radical. The most plausible possibility is in the case of observables such as spin projection. Although radical, it is still conceivable. A contextualist postulate would claim that system have determinate values of those observable, may evolve under a gravitational field as quantum systems, but do not gravitate classically.

but also to dark energy. In the absence of these sources, spacetime is flat. For instance, when a cosmological constant $\Lambda \neq 0$ is present, the vacuum ($T_{\mu\nu} = 0$) exterior solution around a spherically symmetric mass M is the Schwarzschild–de Sitter metric:

$$ds^2 = -\left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2. \quad (57)$$

This solution includes both parameters: M (the "mass") and Λ. They appear as separate ingredients in the metric. In this perspective, the sourcing of a gravitational field by an *idealized* point-like system with mass M would always be accompanied by dark energy. When there is no matter sourcing that field, $\Lambda = 0$. The appearance of the Λ independently of a source in the Einstein Field Equations would be an idealization. Arguably, there are always some SDCs somewhere giving rise to a gravitational field and dark energy, and its persistence would be justified in this way. Furthermore, the value of Λ would be simply a brute fact (i.e., unexplainable).

Note that in this view, dark energy does not come from the vacuum fluctuations. It only comes from quantum matter fields connected with SDCs in such a way that they emit a gravitational field. An alternative view to this one that has been giving rise to many problems (leading to the so-called cosmological constant problem) is that the vacuum just happens to have an inherent energy-momentum that gravitates, and thus, the vacuum energy should explain this constant. However, given the above postulate, this theory can reject this view by considering that systems in the vacuum are not interacting with SDCs. We will explain in Section 7 this idea in more detail. There is another version of postulate 3,

Postulate 3 (version 2) Dark energy is just the default gravitational field of the universe in the absence of matter.

In this view, the gravitational field determined by the cosmological constant is the default gravitational field in the universe, and not flat spacetime, contrary to version 1 of postulate 3. Thus, in the absence of matter, we would have

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -g_{\mu\nu}\Lambda, \quad (58)$$

$$\text{or equivalently in 4D,} \quad R_{\mu\nu} = \Lambda g_{\mu\nu}, \quad R = 4\Lambda. \quad (59)$$

Note that in this view, the dark energy also does not come from the vacuum field fluctuations. The gravitational field is a self-standing entity with default gravitational field values independent of quantum fields. In common with the previous version, the value of Λ would also be a brute fact, contrary to the alternative postulate in Section 7.

Both postulates are at least, in principle (but likely not in practice), testable via the study of the gravitational field emitted by members of the SDCs. If the gravitational field emitted by SDCs involved dark energy effects, this would be evidence for postulate 3 (version 1). However, is unsatisfactory because it leaves unanswered the precise nature of dark energy. Postulate 3 (version 3) in Section 7 commits to an answer with further consequences.

5 SDCs in curved spacetime

We will consider an example of SDCs in a de Sitter spacetime, which is defined by the metric,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\mathbf{x}^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right), \tag{60}$$

where H is constant and $a(t) = e^{Ht}$, t is the cosmological time, and η is the conformal time, which satisfies $dt = a\,d\eta$. $\eta = -H^{-1}e^{-Ht} = -1/(aH)$, where $-\infty < \eta < 0$ when $-\infty < t < \infty$. So, the scale factor in conformal time is $a(\eta) = -1/(H\eta)$ and we have that late times correspond to $\eta \to 0$.

The action for scalar fields in a de Sitter spacetime is given by

$$S = -\int d^4x\,\sqrt{-g}\,\left[\frac{M_p^2}{2}\,R + \mathcal{V}_m + \tfrac{1}{2}\,g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma + \tfrac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + V(\sigma,\phi)\right]. \tag{61}$$

In this expression, $\mathcal{V}_m$ represents the system or systems whose energy–momentum dominates that spacetime region and that drives the background geometry, belonging to an SDC. This could be, for example, radiation or matter. We may take $\mathcal{V}_m$ to be a pure cosmological constant, such constant could be sourced by some system. So, it can give rise to an inflation-like phenomenon due to matter/radiation, which sources a kind of time-varying dark-energy (in section 7 and appendix H, we will provide a way of understanding this constant through this view), or this system could be a mode of the inflaton field.

For simplicity, we assume that the same system $\mathcal{V}_m$ is both sourcing the smearing function and gravity, and via a mode $\mathbf{k} = 0$ that is in a coherent state, which is a homogeneous and isotropic coherent state (see Section 3.2.2 to see how we understand this), and allows us to solve the semiclassical Einstein field equation (more on this below).

We can further interpret $\mathcal{V}_m$ as a field whose $\mathbf{k} = 0$ modes are interacting with SDCs through another field that we ignore for simplicity, or $\mathbf{k} = 0$ is one of the modes of the *initiator* mentioned in Section 3.1, which is the first system starting the SDCs with no predecessor. Multiple modes of this initiator system would exist and energy scales, but note that we will consider that there are no systems that correspond to the Planck scale, and so SDCs will not operate at that scale. So, in this understanding, the inflaton (for example) would be the first system starting the SDCs, transmitting the DC to other systems, and being active during a short amount of time, until it reaches the bottom of its potential $V(\mathbf{x}, t)$. In the typical way of understanding this scenario, S would be the homogeneous part $\phi(t)$ of the inflaton field, and $\phi(\mathbf{x}, t)$ could be a non-homogeneous part of the inflaton $\delta\hat{\phi}(\mathbf{x}, t)$, where the inflaton field would be split between the homogeneous and the non-homogeneous part, $\hat{\phi}(\mathbf{x}, t) = \phi(t) + \delta\hat{\phi}(\mathbf{x}, t)$. Then, ϕ has the DC concerning

σ (not part of the inflaton field), and then σ could continue propagating the DC to other systems. These fluctuations have ultimately important empirical consequences, explaining the temperature anisotropies in the CMB and the seeds for structure formation (galaxies, clusters, etc.).

However, let us focus on a more abstract case, and consider the background gravitational field is due to a $\mathbf{k} = 0$ mode of another real scalar field ψ, where the timescales over which ψ changes are much slower than the ones of the other fields under analysis, in such a way that we can treat the field emitted ψ as approximately constant so that it gives rise to a de Sitter spacetime. In de Sitter spacetime, the Hamiltonian corresponding to the real scalar fields ϕ and σ is given by

$$H = \int d^3x \, \sqrt{\gamma} \left[\frac{1}{2} \pi_\sigma^2 + \frac{1}{2} \gamma^{ij} \, \partial_i \sigma \, \partial_j \sigma + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \gamma^{ij} \, \partial_i \phi \, \partial_j \phi \right. \\ \left. + \frac{1}{2} \left(m_{\rm env}^2 + \xi_\phi \, R \right) \phi^2 + \frac{1}{2} \left(m_{\rm sys}^2 + \xi_\sigma \, R \right) \sigma^2 + H_{\rm int}(\sigma, \phi) \right], \tag{62}$$

where we have the spatial volume element $\sqrt{\gamma} = a^3$, m is the mass, and where ξ coupling constant to the scalar curvature. The conjugate momenta are defined by $\pi_\sigma := \dot{\sigma}$ and $\pi_\phi := \dot{\phi}$, with the dot representing differentiation with respect to cosmic time.

We consider the following potential and interaction terms:

$$V = \frac{1}{2} m_{\rm env}^2 \phi^2 + \frac{1}{2} m_{\rm sys}^2 \sigma^2 + H_{\rm int}(\sigma, \phi), \tag{63}$$

The interaction Hamiltonian that we focus on is of the form

$$H_{\rm int}(t, \mathbf{x}) = \mathcal{O}(t, \mathbf{x}) \sigma(t, \mathbf{x}), \tag{64}$$

where $\sigma(t, \mathbf{x})$ is the operator that acts on the system's Hilbert space and $\phi(t, \mathbf{x})$ acts on the environment Hilbert space. We consider both quadratic $\mathcal{O}_{\rm mix} = \mu^2 f(t) \phi(t, \mathbf{x})$ and cubic interactions $\mathcal{O}_c = g \, f(t) \phi(t, \mathbf{x})^2$, where $f(t)$ is a Gaussian temporal smearing function emitted by the background field, which is in a homogeneous and isotropic state.

Within the interaction picture, the evolution of the full density operator $\rho(t)$ for the scalar fields is governed by the Liouville equation:

$$\partial_t \rho_I = -i \left[H_{\rm int}(t), \rho \right], \tag{65}$$

where $H_{\rm int}(t)$ denotes the interaction-picture Hamiltonian. We are interested in the reduced density matrix $\varrho(t)$ obtained by tracing out the environmental degrees of freedom:

$$\varrho(t) := \mathrm{Tr}_\phi[\rho_I(t)]. \tag{66}$$

The analysis of decoherence is given by the purity, $\gamma(t)$, defined as

$$\gamma(t) := \mathrm{Tr}_\sigma[\varrho^2(t)], \tag{67}$$

where $0 \leq \gamma \leq 1$, and we have that a state is pure if and only if $\gamma = 1$. Decoherence occurs when we end up quasi-irreversibly with a state with minimal purity under interactions (Section 3.1). To analyze the decoherence in de Sitter spacetime, we need to analyze decoherence at late times, which is when complete decoherence happens, and systems σ and ϕ are left in a state where they emit a gravitational field.

Before exiting the horizon, the system correlator oscillates rapidly (or more concretely, its modes), so when we integrate it over past times when doing perturbation theory, those oscillations largely cancel, giving a small, bounded decoherence rate. After the horizon exit, the correlator stops oscillating and instead approaches a nearly constant "frozen" real value. This leads to the decoherence of both the super-Horizon (IR) modes of the system and the environment.

Late-time calculations based on ordinary perturbation theory encounter a well-known secular problem. Every extra interaction vertex in the perturbative expansion adds an integral over past cosmic time and, once a mode has crossed the Hubble radius, its propagator stops oscillating and starts growing in such a way that invalidate the perturbative assumptions. No matter how weak the coupling is, waiting long enough makes contributions from all perturbative orders comparable, so the truncated expansion loses predictability.

Open Effective Field Theory (EFT) methods address this by starting with the so-called Nakajima–Zwanzig Equation. More concretely, one starts from the Liouville equation for the full theory and projects it onto the subsystem of interest while treating the complementary degrees of freedom as an environment. Because the Liouville equation is linear, the environmental part can be integrated formally; working to second order in the weak coupling already resums the entire tower of secular terms that plagued the naive expansion. The remaining influence of the environment on the subsystem is captured by a few time-dependent kernels that play the role of a Lamb shift and a decoherence rate. If the environmental correlator decays on a Hubble timescale those kernels can be evaluated at the current time, yielding a local Lindblad generator that drives the reduced density matrix. For gaussian states the Lindblad evolution leads to two simple first–order differential equations whose solutions remain accurate at arbitrarily late times, thereby providing reliable information on quantities such as purity long after the standard perturbation theory method has broken down (see Appendix G for some mathematical details of the calculation).[46]

We analyze a linear $\sigma\phi$ and a cubic interactions $\sigma\phi^2$, where the system σ and the environment ϕ start in the Bunch-Davies vacuum,[47] where the environment is constituted by a large (or a continuum) collection of modes that decohere a

[46]Although determining whether an evolution is Markovian is generally challenging, for Gaussian maps (which we are examining), this task becomes tractable (as discussed in X). Consider a family of linear maps $\{\mathcal{E}(t_1, t_2)\}$, valid for $t_2 \geq t_1 \geq t_0$, that are trace-preserving and describe the time evolution of a system's state $\hat{\rho}_S$ such that $\hat{\rho}_S(t_2) = \mathcal{E}(t_1, t_2)\hat{\rho}_S(t_1)$. This collection of maps is considered Markovian if it satisfies the semigroup composition rule $\mathcal{E}(t_0, t_2) = \mathcal{E}(t_1, t_2)\mathcal{E}(t_0, t_1)$ for all $t_2 \geq t_1 \geq t_0$,and if each map $\mathcal{E}(t_1, t_2)$ is completely positive, meaning it transforms positive density operators into other positive density operators, for all $t_2 \geq t_1$.

[47]Note that ϕ could have had some determinate values before this interaction.

mode of σ at the super-horizon (IR) scales, leading the target system mode $\mathbf{k}$ to be in a mixture of field amplitude states $|\sigma\rangle$,

$$\begin{aligned}\varrho_{\mathbf{k}}(t) = \frac{1}{\pi}\int_{\mathbb{C}} d^2\sigma \,\Big(A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t)\Big) \\ \times \exp\Big[- \big(A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t)\big)|\sigma|^2\Big]\,|\sigma\rangle\langle\sigma|,\end{aligned} \tag{68}$$

where $A_{\mathbf{k}} + A^*_{\mathbf{k}} - B_{\mathbf{k}} - B^*_{\mathbf{k}}$ is a fixed point of the late time evolution. On the other hand, given that the environment with its continuum of modes can be considered as a large reservoir that is not disturbed significantly by the single-mode system and the Born approximation, we assume that the environment stays in the vacuum, where at late times both systems and environment have determinate values. Only then they emit a gravitational field. It can be shown (see Appendix G) that at least in the linear and cubic Hamiltonian, this leads to Hadamard states and to a finite renormalized Stress-Energy tensor, which can then be fed into the semiclassical Einstein Field Equations to yield a solution to those equations together with the stress-energy tensor of the background. The latter system is in a coherent state. Furthermore, the states involved are all homogeneous and isotropic states.[48] It was shown by [60] that for homogeneous and isotropic quasi-free fourth-order adiabatic states (which include Hadamard states) and instantaneous vacuum states, the semiclassical Einstein equation in flat cosmological spacetimes driven by a quantum massive scalar field with arbitrary coupling to the scalar curvature has unique solutions. This can involve multiple fields sourcing the gravitational field. Importantly, given the shape of smearing functions (which tend to have small tails) and the stochastic decohering process that affects the interacting systems, we can treat the systems when the stochastic process occurs as a free/non-interacting field, as well as the emitter of the smearing function.[49] We thus see here another important role of SDCs, which is to lead to states and conditions that one can use to solve the Einstein Field Equations (Hadamard, homogeneous, and isotropic quasi-free states in the case of this scenario).

As we have been arguing, SDCs involve scale dependent phenomena, giving rise to systems emitting a gravitational field at certain scales, where some modes are filtered out with momenta $\mathbf{k}$. In our case, the system that is initially emitting the gravitational field, which gives rise to the de Sitter spacetime, filters out the UV modes.[50]

We assume each mode of σ that is decohered by a continuum of modes of ψ, where there are so many decohered modes of σ that we can treat them as a continuum of decohered modes, modeled by different decohered models. Then, we can use the states obtained via the (decohered) modes to solve the Einstein Field Equation for a flat cosmological spacetime and find the gravitational field emitted by these members of SDCs.

[48] See Appendix G for a proof concerning the state (200) being homogeneous and isotropic.
[49] See Section 3.2.2 for more on this.
[50] It could also filter IR modes, but we are idealizing.

In the semiclassical Einstein Field Equations, the stress energy tensor is calculated via unsmeared fields. However, techniques of regularization and renormalization do such smooth subtraction of UV modes, and we will regard them as implementing the filtering of the modes by other related means.[51] Importantly, regarding ϕ and σ, we proceed by considering the fields with modes bounded by infinity, and then through regularization and renormalization we focus on the stress-energy tensor for long-wavelength/lower energy modes of ϕ and σ. Regarding ψ, which emits the smearing function obeying the appropriate bounds, we can consider that its cutoff comes as a primitive fact (if it is an initiator) or from its previous interactions with members of an SDC (see Sections 3.2.2, 3.2.3, and Appendix D). Thus, we would have the following renormalized stress-energy tensors,

$$G_{ab} + \Lambda g_{ab} = 8\pi G(\langle T_{ab}\rangle_{\rho_\psi} + \langle T_{ab}\rangle_{\rho_\sigma} + \langle T_{ab}\rangle_{\rho_\phi}). \tag{69}$$

Note that on the right-hand side of the Einstein Field Equation, we include the field ψ responsible for the smearing function, which we, for simplicity, assumed is a larger system in a pure, homogeneous, and isotropic coherent state (see Appendix D), besides the two interacting systems.

As one can see, we have here an example of SDCs in curved spacetime where SDCs could further develop (ϕ and σ could be interacting with other fields while they interact with each other), and where each system involved in the interaction can emit a gravitational field. Even if systems decohere outside the horizon, they can still reenter the horizon (which we are not capturing with our simplifications), interact with other systems, and propagate the DC. Note that although the final state of σ is homogeneous and isotropic, the field amplitude states (which stochastically arise at each time) are inhomogeneous. Future work should look at whether these states can help account for the inhomogeneities that explain the origin of cosmic structure, as well as the empirical signatures that arise from them.[52]

If we assume that the fields involved belong to the inflaton field, we can assume that their influence as initiators dissolves due to their potential $V(x)$ (see above). However, we do not have to consider that they are initiators, and can have alternatively an SDC that goes on forever. Even the field that is initially emitting the gravitational field could be interacting with other systems in that spacetime region (the whole universe), which we chose to ignore, and not be simply the initiator. So, SDCs may not have an initiator. We will come back to how we can dispense with the inflaton field further below in Section 7 and Appendix H.[53]

[51] Although not typically used when regularizing the stress-energy tensor when it enters into the semiclassical equation, such smearing function technique is very much related with the point-splitting technique, and thus the equivalent adiabatic subtraction. This was first observed by deWitt [18] to our knowledge. Renormalization is related to the detector scale. These are important points deserve a more thorough exploration in future work from the point of view of this framework.

[52] This could be done along the lines of what has been done with spontaneous collapse theories [72].

[53] Furthermore, note that given the theory that we are proposing, it is possible that system

6 Answering objections to the semiclassical approach

We will start by showing how this theory answers some of the main objections concerning the semiclassical theory of gravity. Then, we will explore a few more of its consequences. In the next section, we will address another objection.

As we can see, according to this view, the gravitational field does not collapse the wavefunction of systems. Only systems belonging to SDCs can do it. However, it was argued that if a gravitational is not quantized and does not collapse the wavefunction, it can give rise to superluminal signaling, going against relativity.

This argument was posed by Eppley and Hannah [25], and explained succinctly by Callender and Huggett [11]. Suppose the gravitational field is classical and adheres to relativistic principles. In this context, it is neither quantized nor subject to uncertainty principles, and does not permit superpositions of gravitational states that would introduce indeterminacy into the classical field.

For the sake of this discussion, we temporarily adopt the standard interpretation of quantum mechanics, where measurement interactions instantaneously collapse the wavefunction into an eigenstate of the measured observable. Next, let us investigate how this classical gravitational field interacts with a quantum system. According to Eppley and Hannah, there are only two possibilities: either gravitational interactions trigger quantum state collapses, or they do not.

According to the first horn of this dilemma: if gravitational interactions do not induce wavefunction collapse, then quantum states can transmit signals faster than the speed of light, going against the principles of relativity. Eppley and Hannah, propose multiple examples to highlight this issue. One of them involves a variant of Einstein's thought experiment.

The key claim is that if gravitational interactions fail to collapse quantum states, then the interaction dynamics inherently depend on the wavefunction's shape. For instance, the way a gravitational wave scatters off a quantum particle depends on its spatial distribution, akin to its interaction with a classical mass distribution. Scattering experiments with gravitational waves thus become a tool for probing the wavefunction's properties, though they do not induce collapse. This assumption, alongside the standard collapse postulate, leads to the superluminal signaling according to the authors.

To see this more concretely, suppose that we have a rectangular box containing a single quantum particle such as an electron. The particle is in a quantum state where it is equally probable to be found in either half of the box. A barrier divides the box, leading to a superposition of states where the particle is simultaneously localized in both the left and right halves. The wavefunction in this case is given by:

$$\psi(x) = \frac{1}{\sqrt{2}}\big(\psi_L(x) + \psi_R(x)\big), \tag{70}$$

psi is emitting a gravitational field in a subregion of the whole universe, where the rest of the universe, since (let us suppose) it is not subject to SDCs, it is not subject to a gravitational field.

where $\psi_L(x)$ and $\psi_R(x)$ represent the wavefunctions confined to the left and right regions, respectively.

Now, we distribute the boxes, carrying them to spatially separated locations without observing their contents, and giving them to Alice and Bob. Assuming an instantaneous collapse interpretation, when Alice opens her box and finds it empty this can immediately influence Bob's box—even though the two boxes are spacelike separated. Assuming the collapse postulate, the wavefunction undergoes a stochastic transition upon measurement:

$$\frac{1}{\sqrt{2}}\big(\psi_L(x) + \psi_R(x)\big) \to \psi_R(x). \tag{71}$$

Now, let us consider the case where Bob employs a non-collapsing gravitational wave probe capable of interacting with the wavefunction in his box. Bob can do that by (idealizing) setting up apertures that permit gravitational waves to enter and exit the box and be detected.

Because the scattering depends on the form of the wave function in the box we have that any changes in the wave function will show up as changes in the scattering pattern that the detectors register. So, when Bob measures his system, a change in the gravitational wave will signal whether the particle is in the box or not, and this will instantaneously affect Alice's box interior, enabling superluminal communication.

There are multiple issues with this experiment. Let us set aside that according to EnDQT there would be no action at a distance in more realistic Bell scenario versions of this experiment [73].

Now, what sustains the idea that the gravitational wave reacts to the wavefunction in the box? A way of modeling gravity classically, yet coupling it to quantum matter, is via the weak-field (Newtonian) limit, where we derive the Poisson equation based on the semiclassical equation,

$$\nabla^2\Phi(\mathbf{r}, t) = 4\pi G\rho(\mathbf{r}, t). \tag{72}$$

$\Phi(\mathbf{r}, t)$ is the classical gravitational potential, and $\rho(\mathbf{r}, t)$ is the mass density, but now matter is described by a quantum wavefunction $\psi(\mathbf{r}, t)$ with

$$\rho(\mathbf{r}, t) = m\,|\psi(\mathbf{r}, t)|^2\,. \tag{73}$$

This means that the classical field Φ depends on the full spatial distribution of $|\psi|^2$. The potential and wavefunction in the Schrödinger–Newton equation obeys the above Poisson equation. So, if there is some perturbation in the gravitational field, this should come from the wavefunction part. However, given the postulate 2, the semiclassical equation is only applicable if the target system is interacting with SDCs. However, this is not the case in the scenario just described as well as in the Bell scenario version of the experiment. We want to maintain the quantum coherence of the degrees of freedom of the systems under analysis (before interacting with the measurement devices of Alice and Bob, which involve matter degrees of freedom and SDCs) and thus, we want to isolate them from SDCs.

Note that the theory that we are proposing does not need to adopt the other horn of the dilemma. In the case of this second horn, we suppose that gravitational interactions can collapse quantum states of matter like in gravitational causes collapse theories. More concretely, the idea is that if a gravitational wave of arbitrarily small momentum can be used to make a position measurement on a quantum particle (which "collapses" the wave function into an eigenstate of position) the uncertainty principle is violated. This is because the momentum imparted to the particle by the wave would violate the uncertainty principle because it could be made arbitrarily small.[54] We reject this horn too because gravitational waves are not quantum matter field degrees of freedom and are not connected with SDCs (more on gravitational waves below). More concretely, to fundamentally justify the influence of gravitational waves on the particles as a probe, we would need to use the semiclassical field equations, introducing the gravitational waves in the left-hand side of this equation. However, given postulate 2, we can only apply these equations if the particle is also interacting with members of SDCs, which it is not (before interacting with the measurement devices of Alice and Bob, which involve SDCs). Thus, we escape the difficulties concerning the violation of the Heisenberg uncertainty principle that arise from adopting the second horn.

It should by now be clear how this theory responds to the Feynman and Aharanov's thought experiment [30, 30, 3]. This thought experiment aims to show that gravity must be quantized; otherwise, the field can be measured with arbitrary precision to determine the position of a particle in a double-slit experiment. Typically, the way around this scenario is to introduce some stochasticity in the coupling between the quantum degrees of freedom and the classical ones so that we do not gain information about the quantum system (and it does not collapse). However, there is another way to proceed, which is the one adopted here. The idea is that since the quantum system that goes through the double slit does not interact with systems that belong to SDCs (because we want to maintain the system in a coherent superposition), it does not emit any gravitational field. So, the response is similar to the one given to the above dilemma.

Another objection to the semiclassical approach comes from Page and Geilker [67]. Consider a mass that is subject to gravity and exists in a superposition of two distinct position eigenstates. If the gravitational field were classical but depended on the quantum wave function, its gravitational attraction would be expected to direct toward an intermediate, "averaged" position. However, the experimental work of Page and Geilker *has shown* that this predicted behavior does not occur. But, note that this view is based on the idea that an object in such superposition should emit a gravitational field to an intermediate location. This is not the case because macroscopic systems that would form such superpositions would tend to collapse to one of the values associated with coherent states, and such states would serve as sources of the gravitational field. Furthermore, even if we had an odd environment that favors cat states, we would have to introduce

[54] There are difficulties with this argument but we do not need to go there.

the whole cat state (without decoherence) into the semiclassical equation to calculate the expectation value of such energy-momentum tensor, and this would not correspond to an intermediate average of an object being here and there.

A related objection is that semiclassical gravity and this theory are not capable of describing the Planck scale where quantum gravity effects become strong. However, note that this assumes the quantum nature of gravity, which we deny. Also, it assumes that quantum gravity occurs at the Planck scale based on a dimensional analysis and assumption regarding the fundamental constants, which is speculative and one can be skeptical about.[55] We have seen how the above theory can address that scale dependence, which establishes in which scales gravity can occur. This theory can deny that gravity occurs at the Planck scale.

Another objection is that semiclassical gravity is not able to describe the interior of black holes and the associated spacetime singularities, and this should be a task for any theory of gravity. However, it is possible that the interior of the black holes does not gravitate, and all of its mass/energy is in its shell. Many or most models of decoherence involving black holes,[56] concern decoherence occurring at the surface of the black hole where, and thus it might as well be that we have dominant gravitation on their surface. Given that whatever is going on behind the event horizon is causally disconnected from the rest of spacetime, it is quite possible that there is no gravity for matter fields beyond the horizon. This would avoid a singularity. One *may* even conjecture the lower bound in the four-volume that a member of SDC could have, which would be $\Delta V \sim \frac{R_s^4}{c}$, where R_s is the Schwarzschild radius [57]. Below this four-volume, inevitably there is gravitational collapse, and a system stops gravitating. So, given this theory, it might be the case that the interior of the black holes is smooth with no gravitation. To describe such black holes, we could perhaps use regular black holes (i.e., black holes devoid of singularities) with an asymptotically Minkowski core,[57] which is associated with the absence of gravitation.[58][59] Therefore, in principle, we do not need to appeal to a theory of quantum gravity to solve the black hole singularity problems, and there are other alternatives to further explore within the semiclassical approach.

Related with the above objection, it is often claimed that the semiclassical approach has trouble describing black hole evaporation when the Schwarzschild radius is not large compared to the Planck scale [95]. However, it is not even clear that black holes should evaporate when we examine the assumptions that go into these arguments regarding global energy-conservation of the energy-momentum tensor [14]. Thus, it is not necessarily the case that this is a problem worth

[55] See [45] for further related responses to this objection.

[56] See, e.g., [17]

[57] See, e.g., [88].

[58] This conjecture was advanced in collaboration with Gerard Milburn and future work will develop it.

[59] A possibly related idea (but not necessarily so) is that there are regular black holes whose metric involves a fundamental minimal length scale that is bigger than the Planck scale [54], and which hypothetically would be a scale where SDCs operate.

considering.[60]

We will now examine some of the consequences of these postulates. One consequence is that since gravity (when one assumes that it can only be sourced) is fully sourced only by matter fields (with their stress-energy tensor), gravitational energy-momentum (as something that emits a gravitational field per se) does not exist since pure gravitational degrees of freedom do not source gravity according to this theory. Thus, gravitational waves do not carry any energy-momentum. One can rather regard the pseudo-tensor or the radiative energy that shows up in the equations representing gravitational waves as the maximal amount of work they can do via tidal effects [84]. These tidal effects are only classically felt by systems interacting with members of SDCs.

This consequence also supports the claim that, according to this theory, gravitons do not exist. Note that the above hypothesis does not imply that gravitational waves do not exist. Rather it implies that they do not carry actual energy-momentum. This view should not be problematic because of the notorious issues involved in formulating a gravitational energy-momentum tensor, including one for gravitational waves. See [40, 22] for a more complete defense of these positions.[61]

Another consequence is that there may be no default gravitational field, and the gravitational field fully depends on matter fields. Without necessarily endorsing all the features of relationalism, this consequence can be further supported by a kind of relationalist view that would defend that matter degrees of freedom fully determines the gravitational field and thus gravitational fields can be understood via their sources. So, vacuum solutions to the Einstein Field Equations are regarded as idealizations; they do not really exist in nature. For instance, the Schwarzschild external solution should be regarded as a solution taking into account the gravitational field sourced by a system that we idealize as a point mass.[62]

[60] Another alleged limitation of the semiclassical approach is that it is not able to describe the quantum fluctuations in the inflaton field [97]. However, these models are very speculative and have their own problems. Furthermore, it is unclear whether or not this view can actually account for these fluctuations. Besides, we have seen in Section 5 and we will see in Appendix H that perhaps we can provide an alternative picture of inflation that does not appeal to such fluctuations, or that gets rid of the inflaton as traditionally conceived altogether.

[61] The interpretation of what is a gravitational wave and field gives rise to at least two distinct positions considering the ontology of this theory:
-the gravitational field emitted by SDCs affects how quantum matter fields evolve in spacetime. However, this influence of SDCs travels through spacetime via quantum matter fields since quantum matter fields are everywhere, and no determinate energy-momentum needs to be carried via gravitational waves. This view assumes that quantum fields are more fundamental than the gravitational field, assuming a kind of relationalist perspective.

-A different philosophical perspective on the ontology of this theory considers that the mathematical objects of general relativity describe the classical gravitational field. In cases such as the propagation of gravitational waves, it amounts to changes in the values of the gravitational field throughout spacetime. Thus, this view considers that the gravitational field is as fundamental as quantum matter fields, assuming a kind of substantivalist perspective.

[62] See [85] for a recent nuanced relationalist account regarding the idealizations present in vacuum solutions.

7 Derivation of the cosmological constant Λ

An issue that one might have with semiclassical gravity is that it is a mean-field theory, and so it may not account for deviations of the mean that may naturally occur. One option is to consider that these fluctuations do not gravitate. For some reason, the expectation value is enough to determine the gravitational field. Another way is to still argue for that but have a context that minimizes those fluctuations, such as to consider that only systems in certain states under some conditions that minimize the second and higher order moments of the stress-energy tensor gravitate. However, even if we impose this, one may argue that the more contextual and restrictive postulate $2'$, does not completely eliminate the fluctuations of the stress-energy tensor. It turns out that a potential solution to that problem has connections with dark energy and the cosmological constant problem. The idea is that, for example,[63] those fluctuations that contribute to gravitation are annihilated or balanced out (in a sense to be specified below), or at least at the cosmological scales such fluctuations tend to give rise to a dark energy phenomenon. Or what we consider as single events of the stress-energy tensor contain a dark energy effect.[64] The cosmological constant is used to describe how our universe is expanding at an accelerated rate. The cosmological-constant problem appears when we work within a semiclassical framework, replacing the classical stress–energy tensor $T_{\mu\nu}$ with its quantum-field-theory vacuum expectation value $\langle T_{\mu\nu} \rangle$. Each field's vacuum energy density then takes the form of a cosmological-constant term—a constant times the metric $g_{\mu\nu}$—and it is claimed that it should contribute directly to the observed value. However, the standard QFT "prediction" for the combined vacuum energies overshoots the measured cosmological constant Λ by many dozens of orders of magnitude [97].

The problem can be framed as a reductio ad absurdum that arises when we treat General Relativity as a low energy EFT [52]. Assuming the perspective adopted here, GR arises from QFT in specific circumstances, but it is not a low energy QFT. Thus, we should look elsewhere.

Moreover, treating systems that are in a vacuum and in a flat spacetime as gravitating according to the theory adopted here cannot be done because if such system gravitated, it would not be in the vacuum.[65] More generally, one should not include systems in a given state in the stress-energy tensor of the semiclassical equation indiscriminately in any curved spacetime. One should only do that if we have good reasons to consider that those systems were locally decohered in some open environment (i.e., that those systems interacted with members of SDCs), and we have a realistic decoherence model that represents that process. There is a good case to be made that no realistic decoherence model favors the vacuum. For instance, in flat spacetime, given the results from

[63] These interpretative possibilities are not exhaustive.

[64] Another solution is to adopt a stochastic gravity approach [43], but such approach might not be needed.

[65] Indeed, in Wald's fourth axiom [95], where this axiom belongs to a set of axioms that gives us a finite, well-defined, covariant, conserved, renormalizable stress-energy tensor, this tensor is set to zero in the Minkowski vacuum. Setting it to zero is equivalent to not gravitating. This is motivated by the equivalence principle.

[24], we have seen that non-zero temperature environments, irrespective of the initial state, lead systems to a mixture of coherent states, which are not vacuum states. Furthermore, in realistic environments there is not only decoherence but also diffusion, which drives the system out of the vacuum (e.g., [10, 42, 101, 82]). Even in cosmological contexts, the target system starts in the vacuum, then evolves into a mixed state, which upon interaction makes the system leave the vacuum and have a certain field amplitude value. The environment is treated as staying in the vacuum, but this is an idealization because of its size and weak interactions. The model in Section 5 illustrates this. There is so far no indication that this will change in future models of decoherence. Taking into account models of decoherence as good models to infer what we measure, we can hypothesize that what we realistically measure directly (when we infer effects of the vacuum) are not quantum fields in the vacuum but rather quantum systems that were in the vacuum or very close to it.

Thus, by adopting Postulates 2, we can deny that the vacuum gravitates based on models of decoherence, and even that we are able to measure directly the vacuum (we only measure the effects that its evolution and interactions give rise to). So, the conjecture is that if there is something that the cosmological constant problem points to, it is that we need to take into account whether systems are interacting or not with members of SDCs, being decohered by them, to consider whether they gravitate or not.

Assuming the above conjecture, we do not include the energy density of the vacuum in the semiclassical equations, and we can set the value of the cosmological constant based on other features. Furthermore, if we consider that the cosmological constant is behind dark energy, for this theory, the explanation for dark energy should only involve systems that belong to SDCs. Thus, we now pose an alternative postulate 3:

Postulate 3 (version 3) When systems are not interacting with SDCs, they do not give rise to the relativistic four-volume, and furthermore spacetime is flat. The relativistic four-volume is estimated by the number of events in a spacetime region involving quantum systems having determinate values of observables and emitting a gravitational field, which we will call *relativistic events*, and which constitutes the *relativistic spacetime*. We estimate the volume by counting the number of these events because the four-volume of relativity arises from the interactions between systems that constitute SDCs.

The intuition behind postulate 3 is that, given postulate 2 and its consequences, the full-blown notion of trajectories and four-volume from our familiar macroscopic world needs systems with determinate values of observables. So, although when we do not consider SDCs, we still have a spacetime given by a metric and a manifold, systems do not have classical trajectories, and thus, the above notion of a determinate four-volume from relativity does not really make sense. We crucially need matter fields with a determinate energy-momentum. Four-volumes will only make sense in this emergent relativistic spacetime, where the four-volume is equal to the number of relativistic events.[66]

To estimate the value of the cosmological constant (without invoking the vacuum energy), let us start by assuming that in a universe where SDCs had not yet formed, there were no systems with determinate energy-momentum and determinate values of any other dynamical observable. When SDCs began to be formed in spacetime, systems with determinate energy-momentum arose, and relativistic spacetime too, and started expanding. We may alternatively, and less speculatively, assume that at least we have SDCs at certain more macroscopic scales across the four-volume of the visible universe.

The strong energy condition roughly says that gravity must be attractive.[67] Although negative pressure violates the strong energy condition, negative energy densities of a certain magnitude over bounded spacetime regions is something that QT allows for [27], e.g., the Casimir effect. Furthermore, the cosmological constant (which we are deriving) violates the strong energy condition [16]. As we have been seeing, according to this theory, the gravitational field is determined by quantum matter fields that belong to SDCs. Let us consider that in addition to the positive energy associated with gravitational attraction, SDCs produce negative energy and pressure in the comoving reference frame associated with cosmic time, i.e., the reference frame associated with hypothetical observers who are at rest relative to the expanding universe. Furthermore, let us assume that there is a balance of energy-density, pressure, and other quantities, which arises from the interactions that produce the gravitational field. The result of this balance is what we associate with dark energy.[68]

More concretely, if the cosmological constant has a quantum origin, it could arise as an expectation value of some observable, and it can be written as a stress-energy tensor as

$$\left\langle T^{\Lambda}_{\mu\nu}\right\rangle \;=\; -\frac{c^4}{8\pi G}\left\langle \Lambda\right\rangle g_{\mu\nu}. \tag{74}$$

Now, instead of these quantities, let us consider that on the right-hand side of this equation we get a balance between a positive and negative stress-energy tensors produced by systems and their respective uncertainties,

$$G_{\mu\nu} \;=\; \frac{8\pi G}{c^4}\Big(\big(\langle T^{(\mathrm{matt}')}_{\mu\nu}\rangle \pm \Delta t^{(\mathrm{matt}')}_{\mu\nu}\big) + \big(\langle T^{\Lambda'}_{\mu\nu}\rangle \pm \Delta t^{\Lambda'}_{\mu\nu}\big)\Big). \tag{75}$$

What we mean by negative stress-energy tensor is that all the entries of $\langle T^{\Lambda}_{\mu\nu}\rangle \pm \Delta t^{\Lambda'}_{\mu\nu}$ are negative.

So, on the right-hand side of this equation we get a balance between a positive and negative energy and their respective uncertainties (with $\left\langle \rho_{\Lambda'}\right\rangle \pm \Delta\rho_{\Lambda'} < 0$)

[66] Notice that, like causal set theory, this theory assumes that the four-volume of spacetime depends on the number of events.

[67] More precisely, the strong energy condition postulates that for every timelike vector field v^μ, the trace of the tidal tensor ($T = T^a_b$) measured by the corresponding observers is always non-negative: $\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right) v^\mu v^\nu \geq 0$.

[68] See [5] for another model that makes dark energy emerge from decohering interactions, and which may be related to this one.

measured by an observer,

$$u^\mu u^\nu G_{\mu\nu} \;=\; \frac{8\pi G}{c^2}\Big((\langle\rho_{\mathrm{matt}'}\rangle \pm \Delta\rho_{matt'}) + (\langle\rho_{\Lambda'}\rangle \pm \Delta\rho_{\Lambda'})\Big). \tag{76}$$

Now, let us hypothesize at a spatiotemporal scale that SDCs probe, the balance between these quantities results in the following,

$$G_{\mu\nu} \;=\; \frac{8\pi G}{c^4}\left(\langle T_{\mu\nu}^{(\mathrm{matt})}\rangle - t_{\mu\nu}^{\Lambda}\right) \tag{77}$$

where

$$G_{\mu\nu} \;=\; \frac{8\pi G}{c^4}\left(\langle T_{\mu\nu}^{(\mathrm{matt})}\rangle - \frac{\Delta\Lambda c^4}{8\pi G} g_{\mu\nu}\right), \tag{78}$$

and where by convention we consider $\langle T_{\mu\nu}^{\Lambda}\rangle = \langle\rho_\Lambda\rangle = 0$, and $\Delta\Lambda > 0$. We can hypothesize that this relation holds at all scales that SDCs probe, or make the weaker and perhaps more realistic hypothesis that the above relation only holds at larger or cosmological scales. In either case, we can use assumptions about the macroscales/classical or cosmological scales to estimate the value Λ. Note that these assumptions might be unrealistic and we might change these hypotheses by making $\Delta\Lambda$ scale dependent[69] or dependent on other features (e.g., features of the smearing functions, quantum states selected in different physical contexts, etc.), but we will simplify for now, and assume throughout this article. The idea underlying these assumptions is that semiclassical gravity as a mean field holds its validity, not requiring us to take into account the second, third, etc. moments of the stress-energy tensor, via this balance that results in the emission of what we call a cosmological constant or dark energy. Dark energy is a manifestation of stress-energy fluctuations. We will come back to this point.[70]

So, to estimate the value of $\Delta\rho_\Lambda$ and $\Delta\Lambda$, the best way to do so is at the cosmological scales, assuming a perfect fluid. As one can see, the strong energy conditions are violated. Estimating this will involve relating the uncertainty of the four-volume of spacetime that SDCs give rise to with the uncertainty in this energy density at the macroscopic scales where systems are in a coherent state (more on this below), and we only need to estimate the second-moments to estimate the uncertainty $\Delta t_{\mu\nu}^{\Lambda}$ at a cosmic time. More precisely, let us consider that $\hat{V}$ is an observable that represents the total relativistic four-volume that systems that belong to SDCs gave rise to, where this four-volume is in the past light-cone of a spacetime point along the cosmic time. The possible determinate values of $\hat{V}$ are different possible four-volumes that could be generated by the SDCs in that past light-cone. Then, following a relation also postulated by unimodular gravity [23, 102], assuming Planck units, i.e., $m_p = c = \hbar = 1$, let us consider that

$$[\hat{\Lambda}, \hat{V}] = i, \qquad [\hat{\rho}_\Lambda, \hat{V}] = \frac{i}{8\pi}, \tag{79}$$

[69] A scale dependent view would have to investigate which states are selected via decoherence in other scales.

[70] Our derivation could have started with (77)and (78) but this would be less explanatory.

where these observables obey the following uncertainty relation,

$$\Delta\Lambda\,\Delta V \;\geq\; \frac{1}{2}, \tag{80}$$

and where this inequality can be understood in terms of energy density,

$$\Delta\rho_\Lambda\,\Delta V \;\gtrsim\; \frac{1}{16\pi}\,. \tag{81}$$

Since we want to estimate the above uncertainty at the macroscopic scale, we can saturate the above uncertainty to estimate this value, and so

$$\Delta\Lambda \sim \frac{1}{\Delta V}. \tag{82}$$

Let us then estimate the uncertainty of the four-volume of the relativistic spacetime, which SDCs give rise to, in the past light cone of our current cosmic time. It might seem a bit odd to estimate the uncertainty of something that already happened (i.e., retrodict the past four-volume of the universe), but note that it becomes more plausible if we consider that the dynamics is often fundamentally indeterministic and could be otherwise, and therefore, the four-volume could be otherwise. Furthermore, we can use this to make certain predictions, as we will see. To estimate the uncertainty of the four-volume ΔV, we will use postulate 3, which considers that the four-volume of relativistic spacetime is equal to the number of relativistic events in that volume.

It is estimated that the baryon-to-photon ratio η is roughly 6×10^{-10} [2]. Assuming this estimate, and given the overwhelmingly larger amount of photons compared to baryons, a conservative assumption is to consider the majority of the number of spacetime events throughout relativistic spacetime that arise via SDCs involves photons, or more broadly bosons (assuming that there is not any other influential matter that we haven't detected so far).

Relativistic spacetime and classical physics are typically concerned with particles occupying a determinate position and velocity in spacetime. However, particle number observable (i.e., the number of particles occupying a certain mode) seems to be a more appropriate observable to analyze how classicality arises from quantum field theory, since position is not an observable in QFT. Another way to argue for this is that fundamentally, our classical relativistic world seems to be constituted by particle-like systems, and thus particle number observables seem to be the most appropriate observables for the task of counting events. Thus, counting particles in spacetime regions seems to play a much more fundamental role.

We hypothesized that systems in a coherent state are responsible for emitting a gravitational field at more macroscopic scales because they are selected via SDCs.[71] In the classical limit of QT, bosons are typically in a coherent state $|\alpha\rangle$,

[71] When systems that belong to SDCs interact, it is considered an idealization that such models concern perfectly flat spacetimes when systems interact, but only spacetimes close to that.

and models of decoherence, which represent interactions between systems that belong to SDCs, consistently consider that such states are the ones selected by interactions between photons and the environment.[72] Coherent states are states that minimize the Heisenberg uncertainty relations, having that uncertainty distributed throughout position and momentum. A system in a coherent state has a particular quantum uncertainty in its particle number. Such a system in interaction with other systems that probe its particle number gives rise to particle numbers that are Poisson distributed. More precisely, the probability of the number n of particles in a single mode is

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle}\frac{\langle n\rangle^n}{n!} \tag{83}$$

which obeys the Poisson distribution, where the average number of bosons occupying a single mode and their variance is equal to each other,

$$\langle n\rangle = \langle \hat{a}^\dagger \hat{a}\rangle = \mathrm{Var(n)} = (\Delta n)^2. \tag{84}$$

Also, Poisson statistics take into account that for the large enough volume that we will consider, the events that arise are independent of each other, not affecting one another. Since we are considering the volume of the whole universe, which is large enough to guarantee this independence, it is also plausible to consider this distribution. On top of this, a Bose-Einstein condensate at high occupation numbers can be approximated as coherent states. Moreover, even for fermions, the classical limit/low density limit leads to a variance of occupation number for a mode of approximately $\sqrt{\langle n\rangle}$.

The cosmological FLRW spacetime models that estimate the value of the cosmological constant consider that matter is homogeneously distributed throughout the visible universe. Let us follow the spirit of the assumption and assume that the events that SDCs give rise to throughout the four-volume of the visible universe are homogeneously distributed.

Given these considerations, under a conservative approach we will consider that across the universe in the classical limit the standard deviation minimizing the above relation is given by $\Delta V = \sqrt{\alpha\langle n\rangle}$ where $\langle n\rangle$ is the number of events that SDCs give rise to in the four-volume of the universe and α is some positive unit constant that gives ΔV proper units of four-volume. So, we will consider that the events that arise from interactions between systems that belong to SDCs giving rise to a relativistic four-volume V will have an uncertainty given by this quantity.[73][74] It is estimated that the spacetime volume of the observable universe (determined along cosmic time) is approximately $10^{246}\ l_P^4$ (Planck volumes). Substituting this quantity in (82), we obtain $\Lambda \sim 10^{-123}\ l_P^{-2}$ (Planck units),

[72] E.g., [100] and [24].

[73] Of course, bosons can occupy the same spacetime points, which constitute the same relativistic event, but at the higher temperatures in the current universe, it is likely that we have many of them widely distributed across the four volume.

[74] We might be neglecting other possible kinds of matter, but it seems unlikely that their minimal variance would lead us to change the above conservative estimate regarding the variance.

which is remarkably the approximate value of the cosmological constant. Thus, from a first-principles approach, we have derived the value of the cosmological constant and without falling into the cosmological constant problem.

We should make clear that the four-volume of the relativistic spacetime has nothing to do with a fundamental discretization of spacetime or some fundamental minimal four-volume. That's why we distinguish between spacetime and relativistic spacetime. The fundamental spacetime is spacetime endowed with a metric, and is continuous with no minimal four-volume; the relativistic spacetime emerges from this spacetime via SDCs.

The picture that emerges from this theory is that while the universe is always expanding, there are some extra quantum effects that accelerate its expansion. Notice the different contributions of the energy-momentum tensor. When SDCs are expanding via interactions, leading relativistic spacetime to expand, there is an energy-momentum $\langle T_{\mu\nu} \rangle$ that will give rise to a gravitational field, influencing quantum systems in the spacetime. This energy-momentum leads to an "attractive force" that we commonly call gravity, and an extra pressure given by $\Delta\rho$ that contributes to the "repulsive force" that leads to an accelerated expansion of the universe. As we can see, it is not vacuum energy that drives this accelerated expansion; it is how SDCs constituted by matter fields expand, affecting the evolution of spacetime.

In summary, we have estimated the value of the cosmological constant via a heuristic and simple method and provided an explanation of its origin. This method superficially resembles in some ways the one of Sorkin in [89] because he also used the above commutativity criterion and Poisson distribution variance. However, although there is a superficial similarity, the path to arrive at this value is very different. Sorkin's approach was supported by classical dynamics. Causal set theory does not have a clear quantum dynamics. Also, it is based on a theory of quantum gravity, which is clearly not the goal of this theory. Furthermore, it is still unclear how causal sets can deal with the measurement problem, as well as provide a consistent and complete theory of quantum gravity.

One may wonder where the commutation relation between V and Λ comes from. The unimodular modification of GR [23, 102] considers that Λ and V are conjugate to each other in a similar way to energy and time in QT. We can also see that this relation is plausible by looking at the integral of the action of GR, where we find a $-\Lambda V$ term. However, it is unclear whether we have to assume unimodular gravity here, some other theory, or rather just postulate this commutation relation and interpret it as representing something fundamental regarding how SDCs give rise to spacetime volumes and the gravitational field. If we think further about the commutator between energy density and four-volume, this new commutator is plausible. It is the combination of the time and energy and position and momentum commutators. Future work should investigate this. We also might object that we are doing some fine-tuning with equation (75). Note, however, that the relative variance of the stress-energy tensor of coherent states at high-occupation numbers (which concerns the macroscales that we are interested in) is very small.

We will now explore some consequences of the derivation of Λ. First, via Λ, this theory provides a potential way to circumvent the postulation of the inflaton field. The reason for this is that the value of Λ will change with the evolution of the universe because it depends on the four-volume of the universe/relativistic spacetime, and the four-volume of the universe changes with time. According to the Big Bang model, the four-volume of the relativistic spacetime was very small at the beginning of the universe. Thus, keeping all assumptions used to derive ΔV, ΔV will also be very small, and this means that Λ will be very large. Therefore, this means that we have a very accelerated expansion at the beginning of the universe. Given the issues surrounding these inflationary models, this is another benefit that this view provides. See Appendix H for more details concerning this topic.

Second, according to this theory, the introduction of Λ in the Einstein equation is interpreted as a correction to account for these consequences of SDCs. Also, it points to the idea that the validity semiclassical equations arises from a balance between positive and negative energy-momentum, which explains why using the expectation value of the stress-energy tensor is enough for predictive purposes. Perhaps a more general Einstein equation should more explicitly take into account the dynamics of the varying value of the cosmological constant. Note that this varying value and the associated energy density give plausible results. It tells us, for example, that the smaller the four-volume of relativistic spacetime that SDCs give rise to, the higher the energy density associated with the cosmological constant. Note that $\Delta\rho$ and hence ΔV (the four-volume in a past lightone) are determined via local interactions throughout the history of SDCs. Thus, the value of the cosmological constant tends to change locally because the overall volume of the universe tends to change the energy density.

One may ask how this derivation sets this theory apart from other theories of gravity. Can they also derive the value of the cosmological constant like we did here? Certainly, the causal set approach can do that, but as we have explained, their derivation has limitations. It is unclear whether loop quantum gravity and string theory can make the same discrete spacetime assumption that we made above, as well as the assumption concerning the quantum uncertainty of the relativistic events being $\sqrt{n}$. This is because their gravitational degrees of freedom are quantum, and they should contribute to the value of dark energy irrespective of classicality. On the other hand, the assumption that we have made here is that only the "classical" physical states (i.e., the ones involving systems with determinate values) that arise via SDCs (or decoherence) contribute to determining the value of the cosmological constant, and hence gravitate. Furthermore, it is unclear how quantum gravity theories and gravity-caused collapse theories can justify in a principled way why the vacuum does not gravitate.

We think that the derivation and explanation for the value of the cosmological constant shows the potential of this theory, as well as its justification for why we should not use the value of the vacuum energy of quantum fields to calculate the value of the cosmological constant. On top of this, recent data points to a varying value of the cosmological constant across the history of the universe [1] like this theory predicts. The above style of argument regarding why the vacuum does not gravitate may help solve other problems in physics not directly related to semiclassical gravity because SDCs also determine whether a system has a determinate energy-momentum.[75]

8 Conclusion and future directions

We began proposing a theory that connects general relativity with quantum theory in a coherent way by appealing to semiclassical gravity, and we have explained how it can be empirically supported. As we can see, there are multiple ways in which this theory could be developed. We consider that we have not provided a theory that is complete, but rather one whose openness provides a series of theoretical and empirical possibilities that should be further explored and developed. We hope that this exploration leads to a more thorough connection between the fields of decoherence in curved spacetime, measurement theory in QFT, effective field theory, quantum foundations, general relativity, and cosmology. We have focused on real scalar fields for simplicity, but in principle, this theory could be extended to other fields. Of course, future work should make that extension explicit.

We will now discuss some of the challenges or shortcomings of this approach, in addition to the ones mentioned above and in the appendices. First, although we have focused on a situation and states (Hadamard states) where the semiclassical equation in principle can be solved, solving the semiclassical equations is typically a hard problem, and is outside the scope of this article. Various methods have been developed to solve it.[76] Future work will explore how this approach may help solve this equation.

Second, future work should explore how this theory works in multiple spacetimes. Essentially, one needs to explore models of decoherence in such spacetimes. Furthermore, we developed our proposal in the context of globally hyperbolic spacetimes. One may argue that these are the realistic spacetimes but future work could investigate this theory in the context of non-globally hyperbolic spacetimes.

Third, in this first paper we have not provided models representing the stochasticity of the gravitational field, and how it feeds into the dynamics of the matter fields. Here, using tools from hybrid classical-quantum theories such as in [48, 63] may be promising as effective descriptions of such stochastic gravitational field sourced by SDCs, which in turn influences the dynamics of the systems.

[75]For instance, the framework of EFT points towards new physics happening at the scale not far above the Higgs boson, but we have no evidence of new physics above the TeV. This in a nutshell is the Higgs hierarchy problem, which involves a fine-tuning problem to get renormalized Higgs mass from the predictions of EFT that predict that the Higgs mass should receive corrections due to its interactions. If SDCs cannot probe such scales and/or if we do not have a model of decoherence for such interactions, we should not infer that the existence of such large terms that need to be canceled to account for the Higgs mass. This is because there is no mechanism that renders what these terms represent determinate. This points towards the need of an integration of the theory proposed here with tools of EFE and renormalization theory to make inferences about SDCs.

[76]See [91, 47, 37] and references therein.

However, contrary to this approach, the classical stochastic gravitational field does not need to be fundamental when describing gravity. It primarily arises from interactions between quantum fields. This may surpass the difficulties of this kind of approach of deriving the Einstein Field Equations from more basic principles [65].

Fourth, future work should investigate whether the above derivation of the dark energy could fit into a broader theory, and whether such value varies with the scales being probed, as well as other features. Furthermore, one should explore the cosmological consequences of this theory, including for black holes (see Section 6 for a conjecture regarding these objects). Note that despite these conjectures, our theory at the core is very conservative and testable in the short term.

Finally, we have proposed a particular set of gravitational conditions and associated determination conditions. Future work should explore others and propose experiments to test which ones are the correct ones.

Acknowledgments

I would like to thank Gerard Milburn for his mentorship and invaluable feedback on earlier drafts.

A No-disturbance condition approximation

To see why this approximation in eq. (27) is valid and the no-disturbance condition is fulfilled, let us assume that $[t_{AB}^{\text{start}}, t_{AB}^{\text{end}}]$ is the support interval for $f_{AB}(t)$, and $[t_{BC}^{\text{start}}, t_{BC}^{\text{end}}]$ is the support interval for $f_{BC}(t)$. The overlapping interval $[t_s, t_e]$ is then $t_s = \max(t_{AB}^{\text{start}}, t_{BC}^{\text{start}}), t_e = \min(t_{AB}^{\text{end}}, t_{BC}^{\text{end}})$.

The quantity *overlap* quantifies the magnitude of the overlap between the interaction B-C and A-B within the overlapping region:

$$\text{Overlap} = \int_{t_s}^{t_e} f_{AB}(t) \cdot f_{BC}(t)\, dt.$$

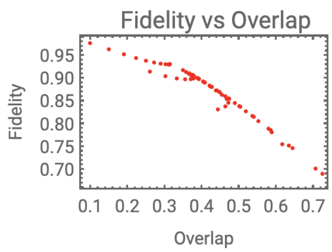
Relatedly, the quantity *strength* quantifies the relative influence of $f_{BC}(t)$ compared to $f_{AB}(t)$ within the overlapping region. It is defined as the ratio of their integrals:

$$\text{Strength} = \frac{\int_{t_s}^{t_e} f_{BC}(t)\, dt}{\int_{t_s}^{t_e} f_{AB}(t)\, dt}.$$

So, let us consider the fidelity between the above approximate state and the state $|\psi(1)_{\text{Num}}\rangle$ of the system calculated numerically, $F = |\langle\psi(1)_{\text{approx}}|\psi(1)_{\text{Num}}\rangle|^2$.[77] Looking at the plots in Figure 1, we see that this fidelity increases with strength and decreases with the amount of overlap. Thus, we will consider that there

[77]The simulations were made using the function NDSolve in Mathematica and the method ExplicitRungeKutta.

is a small overlap between the test functions concerning the interaction $A - B$ and $B - C$ because this is enough to have the no-disturbance condition fulfilled. Looking at the plot in Figure 2 we also see how the fidelity changes with the size of the common support between f_{AB} and f_{BC}, i.e., decreasing as their common support increases.



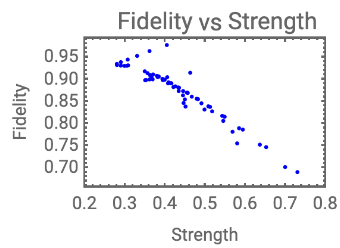


Figure 5: Strength and Overlap obtained by numerical simulations for a $t_{AB} = 0.5$ and $\sigma_{AB} = 0.13$, and for multiple values of t_{BC} and σ_{BC} within the interval $[0, 3]$ and within the common support of f_{AB} and f_{BC}. To calculate these quantities, the Schrodinger equation with the Hamiltonian in (16) was solved to yield the state $|\psi(1)_{\mathrm{Num}}\rangle$.

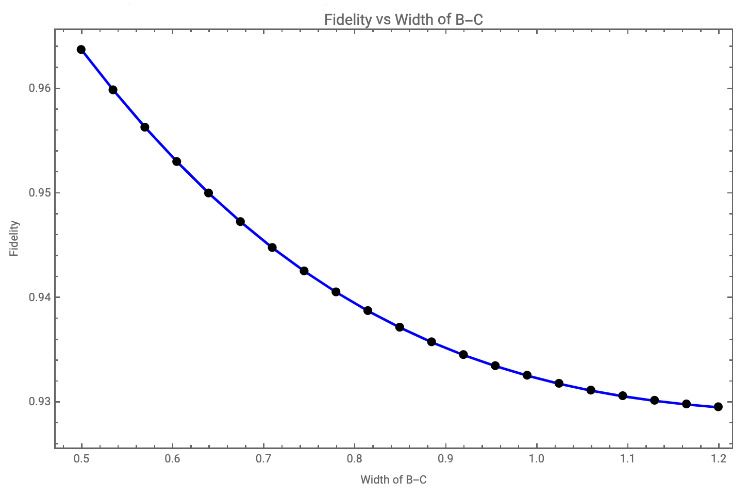


Figure 6: Fidelity as a function of σ_{BC}, assuming values between 0.5 and 1.2, and for $t_{AB} = 0.5$, $\sigma_{AB} = 0.13$, and $t_{BC} = 1.5$. We can see that the fidelity decreases as σ_{BC} increases and the size of the common support of f_{AB} and f_{BC} increases.

B Quantization of scalar fields and other definitions

For completeness, we briefly explain the quantization of the scalar field from the point of view of algebraic quantum field theory (AQFT), and explain other concepts that we will use. Let $f \in C_0^\infty(\mathcal{M})$ denote a smooth test function with compact support on $\mathcal{M}$. The advanced and retarded Green's functions, $E^\pm = E^\pm(x, y)$, correspond to the propagators associated with the Klein-Gordon operator $\hat{P}$, where $\hat{P}\phi = 0, \quad \hat{P} = \nabla_a \nabla^a + m^2 + \xi R$. Using these, we define the smeared advanced and retarded propagators as follows:

$$E^\pm f \equiv (E^\pm f)(x) := \int \mathrm{d}V'\, E^\pm(x, x') f(x'), \tag{85}$$

where the measure $\mathrm{d}V' = \mathrm{d}^D x' \sqrt{-g'}$ represents the invariant volume element, with $g' \equiv \det g_{AB}(x') < 0$. These propagators solve the inhomogeneous wave equation $\hat{P}(E^\pm f) = f$. The causal propagator is then defined as the difference between the advanced and retarded propagators: $E = E^- - E^+$. A key property is as follows: for any open neighborhood O of a Cauchy surface Σ and for any real solution $\phi \in \mathrm{Sol}_\mathbb{R}(\mathcal{M})$ to Eq. (30) with compact Cauchy data, there exists a real-valued $f \in C_0^\infty(\mathcal{M})$ with $\mathrm{supp}(f) \subset O$ such that $\phi = Ef$, where Ef is defined in an analogous way to Eq. (85).

In AQFT, the quantization of the real scalar field ϕ on $\mathcal{M}$ is understood as an $\mathbb{R}$-linear map from the space of smooth, compactly-supported test functions to a unital $*$-algebra[78] $\mathcal{A}(\mathcal{M})$ given by $\hat{\phi} : C_0^\infty(\mathcal{M}) \to \mathcal{A}(\mathcal{M}), \quad f \mapsto \hat{\phi}(f)$, that fulfills the conditions of i) Hermiticity: $\hat{\phi}(f)^\dagger = \hat{\phi}(f)$ for all $f \in C_0^\infty(\mathcal{M})$; ii) the equation for the field: $\hat{\phi}(\hat{P}f) = 0$ for all $f \in C_0^\infty(\mathcal{M})$; iii) the Canonical Commutation Relations (CCR): defining the commutator $[a, b] = ab - ba$ for $a, b \in \mathcal{A}(\mathcal{M})$, we have that $[\hat{\phi}(f), \hat{\phi}(g)] = iE(f, g)\mathbb{I}, \quad \forall f, g \in C_0^\infty(\mathcal{M})$, where $E(f, g)$ is the smeared causal propagator defined as $E(f, g) := \int \mathrm{d}V f(x)(Eg)(x)$.

The $*$-algebra $\mathcal{A}(\mathcal{M})$ is referred to as the algebra of observables for the field on $\mathcal{M}$. The smeared field operator $\hat{\phi}(f)$ can be expressed as

$$\hat{\phi}(f) = \int \mathrm{d}V \hat{\phi}(x) f(x). \tag{86}$$

The dynamics of the field are encoded in the symplectic structure. The space of solutions $\mathrm{Sol}_\mathbb{R}(\mathcal{M})$ to the Klein-Gordon equation (30) can come with a symplectic form $\sigma : \mathrm{Sol}_\mathbb{R}(\mathcal{M}) \times \mathrm{Sol}_\mathbb{R}(\mathcal{M}) \to \mathbb{R}$, defined as

$$\Omega(\phi_1, \phi_2) := \int_{\Sigma_t} \mathrm{d}\Sigma^a \left(\phi_1 \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1\right), \tag{87}$$

where $\mathrm{d}\Sigma^a = -t^a \mathrm{d}\Sigma$, $-t^a$ is the inward-directed unit normal to the Cauchy surface Σ_t, and where $\mathrm{d}\Sigma = \sqrt{h}\mathrm{d}^{D-1}x$ is the induced volume form on Σ_t. This definition is independent of the choice of Cauchy surface used in Eq. (87). The field

[78] I.e., a complex algebra equipped with involution or also known as Hermitian adjoint, and that is unital because it has the identity.

operator $\hat{\phi}(f)$ can then be expressed as a symplectically smeared field operator: $\hat{\phi}(f) = \Omega(Ef, \hat{\phi})$. The CCR algebra is reformulated as $[\Omega(Ef, \hat{\phi}), \Omega(Eg, \hat{\phi})] = i\Omega(Ef, Eg)\mathbb{I}$, where $\Omega(Ef, Eg) = E(f, g)$.

The Klein-Gordon inner product is given by[79]

$$(\phi_1, \phi_2)_{\mathrm{KG}} := i \int_{\Sigma_t} \mathrm{d}\Sigma^a \left(\phi_1^* \nabla_a \phi_2 - \phi_2 \nabla_a \phi_1^*\right). \tag{88}$$

where the element $\mathrm{d}\Sigma^a$ is given by $-t^a \mathrm{d}\Sigma$, where $-t^a$ represents the inward-pointing unit normal vector to the Cauchy surface Σ_t. Moreover, $\mathrm{d}\Sigma = \sqrt{h}\, \mathrm{d}^n x$ denotes the volume form induced on the hypersurface Σ_t. We require that the modes are normalized according to the Klein-Gordon inner product:

$$(u_{\mathbf{k}}, u_{\mathbf{k}'})_{\mathrm{KG}} = \delta^n(\mathbf{k} - \mathbf{k}'), \quad (u_{\mathbf{k}}, u^*_{\mathbf{k}'})_{\mathrm{KG}} = 0, \quad (u^*_{\mathbf{k}}, u^*_{\mathbf{k}'})_{\mathrm{KG}} = -\delta^n(\mathbf{k} - \mathbf{k}'). \tag{89}$$

Note that the equal-time CCR is not manifestly covariant because it singles out a preferred time direction from the beginning.[80] The way to do this more covariantly and arguably more satisfactorily is by using the algebraic approach as well as considering the full complexified space of solutions to the Klein-Gordon equation [95].

Turning now to Hadamard states, which we will use, for any such state, one can define a finite, locally covariant, renormalised expectation value of the stress-energy tensor, where a Hadamard state satisfies the following condition:

$$W(x, x') = \frac{\Delta^{1/2}(x, x')}{8\pi^2 \sigma(x, x')} + v(x, x') \ln|\sigma(x, x')| + h(x, x'), \tag{90}$$

where $\Delta(x, x')$ is the Van Vleck determinant, which accounts for the local geometry and geodesics between the points x and x', $\sigma(x, x')$ is Synge's world function, which as we have seen above represents half the squared geodesic distance between x and x', and $v(x, x')$ and $h(x, x')$ are regular functions, with $v(x, x')$ containing the logarithmic state dependence and $h(x, x')$ being a smooth function related to state-dependent contributions.

C Coherent states examples and bounds on smearing functions

Let us see a simple example of how a system in a coherent state can source a test function. Coherent states in the context of QFT are analogous to those of the

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[79] Note that this is defined in terms of the complex form with $(\cdot, \cdot)_{\mathrm{KG}} : \mathrm{Sol}_{\mathbb{C}}(\mathcal{M}) \times \mathrm{Sol}_{\mathbb{C}}(\mathcal{M}) \to \mathbb{C}$ where $(\phi_1, \phi_2)_{\mathrm{KG}} := i\Omega(\phi_1^*, \phi_2)$, but where the symplectic form Ω is expanded to the space of solutions $\mathrm{Sol}_{\mathbb{C}}(\mathcal{M})$ of the Klein-Gordon equation, which are complexified.

[80] A related drawback of canonical quantization is that it does not inherently show the presence of multiple unitarily inequivalent representations of the CCR algebra, which is a well-known feature of quantum field theory. Additionally, the equal-time commutation relations (CCR) are not manifestly covariant, as they inherently select a preferred time direction. As previously mentioned, a more manifestly covariant approach involves first considering the entire complexified space of solutions to the Klein-Gordon equation. However, for simplicity, we will not pursue that approach here.

harmonic oscillator and are defined as states $|\alpha(\mathbf{k})\rangle$ that satisfy the eigenstate equation:

$$\hat{a}_{\mathbf{k}}|\alpha(\mathbf{k})\rangle = \alpha(\mathbf{k})|\alpha(\mathbf{k})\rangle, \tag{91}$$

where $\alpha(\mathbf{k})$ is a complex-valued function known as the coherent amplitude distribution, which characterizes the state $|\alpha(\mathbf{k})\rangle$. Furthermore, for a coherent state, the uncertainty relations are minimized for the canonical quadrature pairs of a single mode. The vacuum state $|0\rangle$ is a coherent state with zero amplitude. Nevertheless, they typically have a non-zero mean field, which makes them ideal sources of smearing functions. As we will see, coherent states tend to be selected by SDCs, being the "most-classical" states.

We can write a multimode coherent state, which depends on the complex-valued function $\alpha(\mathbf{k})$, as a displaced vacuum:

$$|\alpha\rangle = \hat{D}[\alpha]\,|0\rangle = \exp\!\Big(\int d^n\mathbf{k}\,\big[\alpha(\mathbf{k})\,\hat{a}^\dagger_{\mathbf{k}} - \alpha^*(\mathbf{k})\,\hat{a}_{\mathbf{k}}\big]\Big)\,|0\rangle, \tag{92}$$

where $\hat{D}[\alpha]$ is the unitary displacement operator for the field.

Let us examine an example. We consider that upon decoherence in flat spacetime, a stochastic process that transitions the system to one of the terms of its reduced state (together with the environment that monitors the system), and given the shape of the smearing function with its tails, the interaction gets quickly weaker, and the system (and its environment) evolve freely approximately,[81] where its evolution is given by the free Klein-Gordon equation. From regarding ϕ_D as approximately evolving under the free Klein-Gordon equation $(\Box + m^2)\hat{\phi}_D(x) = 0$, it follows that for a test function f, the following also holds $(\Box + m^2)f = \mathrm{Tr}\big(\hat{\rho}_D\,(\Box + m^2)\,\hat{\phi}_D\big) = 0$, where $f(x,t) = \mathrm{Tr}\big(\hat{\rho}_D\,\hat{\phi}_D(x,t)\big)$. Thus, if we consider smearing functions as arising from mean fields of free scalar fields, it is plausible to consider that they should be solutions to the Klein-Gordon equation.

An ideal test function is a bump function because it is compactly supported. But the Fourier transform of this function does not have a closed analytic form. Non-compact functions such as the Gaussian presented in the previous section provide a closed-form. But this function is not a perfect solution to the free Klein-Gordon equation.

A non-compact function that is a solution to the free Klein-Gordon equation is the following,

$$\begin{aligned}\Phi(t,r) = \frac{\mathcal{A}}{4\,r}\,a^{-\frac{5}{4}}\,\Gamma\!\left(\tfrac{5}{4}\right)\Bigg[&(r+t')\;{}_1F_1\!\left(\tfrac{5}{4};\,\tfrac{3}{2};\,-\tfrac{(r+t')^2}{4a}\right)\\ &+\,(r-t')\;{}_1F_1\!\left(\tfrac{5}{4};\,\tfrac{3}{2};\,-\tfrac{(r-t')^2}{4a}\right)\Bigg],\end{aligned} \tag{93}$$

with

$$\mathcal{A} = \frac{4\sqrt{2}\,\pi\,N}{(2\pi)^{3/2}}, \qquad a = \frac{\sigma^2}{4}, \tag{94}$$

[81] This is more so for the case of bump functions.

where N is a normalization constant, and the spatial and temporal variance is proportional to σ^2. It is $\mathcal{C}^\infty$, rapidly decaying in all directions, even in $t' = t - t_0$, and spherically symmetric about $\mathbf{x}_0$.

We want to find the coherent state that gives rise to this test function. Let us obtain the result for the case for a massless scalar field that involves a continuum of modes, where the mean-field arises from

$$\langle\alpha|\hat{\phi}(t,\mathbf{x})|\alpha\rangle = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k}}[\alpha(\mathbf{k})\, e^{-ikt+i\mathbf{k}\cdot\mathbf{x}} + \alpha^*(\mathbf{k})\, e^{ikt-i\mathbf{k}\cdot\mathbf{x}}]. \quad (95)$$

If we consider

$$\alpha(\mathbf{k}) = N\, e^{-ak^2}\, e^{i\left(kt_0 - \mathbf{k}\cdot\mathbf{x}_0\right)}, \quad (96)$$

with $a = \frac{\sigma^2}{4} > 0$, the expectation value

$$\Phi(t,\mathbf{x}) = \langle\alpha|\hat{\phi}(t,\mathbf{x})|\alpha\rangle \quad (97)$$

equals the smearing function (93).[82] In the limit where $|r|,\ |t'| \ll \Delta x, \Delta x = \frac{\sigma}{2}$,[83] the above test function reduces to a Gaussian,

$$\Phi_a(t,r) \simeq N \exp\Big[-\frac{r^2}{2\lambda^2} - \frac{(t-t_0)^2}{2\lambda^2/3}\Big], \quad \lambda^2 = \frac{12a}{5} = \frac{3\sigma^2}{5}. \quad (98)$$

Turning now to the bounds on the smearing functions, consider the following single-time Poincaré algebra:

$$[H, P^i] = 0, \quad [K^i, P^j] = i\delta^{ij}H, \quad [K^i, H] = iP^i, \quad [K^i, K^j] = -i\varepsilon^{ijk}J^k. \quad (99)$$

Given, for example,

$$H_{\text{int}} = \int d^3x\, f(\mathbf{x},t)\, \hat{\mathcal{O}}_1(\mathbf{x},t)\, \hat{\mathcal{O}}_2(\mathbf{x},t), \quad f(\mathbf{x},t) = g\exp\Big[-\tfrac{\mathbf{x}^2}{2\sigma^2} - \tfrac{t^2}{2T^2}\Big]. \quad (100)$$

Then, we get

$$[H_{\text{int}}, P^i] = \int d^3x\, (\partial_i f)\, \hat{\mathcal{O}}_1\hat{\mathcal{O}}_2, \quad (101)$$

$$[K^i, H_{\text{int}}] = \int d^3x\, (t\partial_i f - x^i\partial_t f)\, \hat{\mathcal{O}}_1\hat{\mathcal{O}}_2, \quad (102)$$

$$[K^i, P^j] = \int d^3x\, \delta^{ij}\, \partial_t f\, \hat{\mathcal{O}}_1\hat{\mathcal{O}}_2, \quad (103)$$

$$[K^i, K^j] = \int d^3x\, (x^i t - x^j t)\, \partial_t f\, \hat{\mathcal{O}}_1\hat{\mathcal{O}}_2. \quad (104)$$

[82] As one can see, whether the system ends up emitting a temporal, a spatial, or a spatiotemporal smearing function depends on the state it ends up in due to decoherence by members of SDCs.

[83] As we will see, considering $|r|,\ |t'| \ll \Delta x, \Delta x = \frac{\sigma}{2}$, if we consider $|r| \approx 1/k$ and $|t| \approx 1/\omega$, will coincide with conditions for systems to emit a test function. However, in the context below k and ω concern the maximum momentum and energy, respectively, of the system subject to that test function.

The terms that spoil the Poincaré algebra commutation relations are those in which a derivative acts on the smearing function. Because

$$\partial_i f = -\frac{x_i}{\sigma^2} f, \quad \partial_t f = -\frac{t}{T^2} f, \tag{105}$$

every anomalous contribution carries either a factor t/T or x/σ. Fourier transforming gives

$$\tilde{f}(\mathbf{k}, \omega) = g \exp\Big[-\tfrac{1}{2}\big(\sigma^2 \mathbf{k}^2 + T^2 \omega^2\big)\Big] \leq \varepsilon, \quad \varepsilon \ll 1. \tag{106}$$

Considering a process of spatial width $L_{\rm phys}$ and temporal width $\tau_{\rm phys}$ (so $k_{\rm max} \sim 1/L_{\rm phys}$, $\omega_{\rm max} \sim 1/\tau_{\rm phys}$), one finds

$$k_{\rm max} \gg \Lambda_k = \frac{1}{\sigma}, \quad \omega_{\rm max} \gg \Lambda_\omega = \frac{1}{T}. \tag{107}$$

One can see that these conditions apply to any physically reasonable test function and Hamiltonian.

Now, let us show via a simple case how the inequalities (38) and (39) that we have derived for the smearing functions to obey the spacetime symmetries guarantee the validity of the cutoff-based bounded integrals. To show this, let us assume that D is a massless scalar field, where we have

$$\langle \hat{\phi} \rangle = \int_0^\infty dk\ \rho(k)\, e^{-\frac{1}{2}\Sigma^2 k^2}\, 2\cos\!\big(\mathbf{k}\cdot\mathbf{x} - kt\big), \tag{108}$$

$\rho_0 \equiv \frac{4\pi}{(2\pi)^{3/2}},\ \Sigma^2 \equiv \sigma_r^2 + \sigma_t^2,\ \rho(k) \equiv \rho_0\, k^2$. Then we obtain the difference between full and truncated integrals

$$\begin{aligned} \Delta(\mathbf{x}, t) \ &\equiv\ \langle \hat{\phi} \rangle - \langle \hat{\phi} \rangle_\Lambda \\ &= \int_0^\infty dk\, \rho(k)\, e^{-\frac{1}{2}\Sigma^2 k^2}\, 2\cos(\dots) \ - \ \int_0^\Lambda dk\, \rho(k)\, e^{-\frac{1}{2}\Sigma^2 k^2}\, 2\cos(\dots) \\ &= \int_\Lambda^\infty dk\, \rho(k)\, e^{-\frac{1}{2}\Sigma^2 k^2}\, 2\cos(\dots)\,. \end{aligned} \tag{109}$$

Note also that

$$\big|\Delta(\mathbf{x}, t)\big| \leq 2\int_\Lambda^\infty dk\, \rho(k)\, e^{-\frac{1}{2}\Sigma^2 k^2} = \frac{8\pi}{(2\pi)^{3/2}} \int_\Lambda^\infty dk\, k^2 e^{-\frac{1}{2}\Sigma^2 k^2}, \tag{110}$$

where

$$\int_\Lambda^\infty dk\, k^2 e^{-ak^2} = \frac{\sqrt{\pi}}{4a^{3/2}}\ \mathrm{erfc}\!\big(\sqrt{a}\,\Lambda\big) + \frac{\Lambda}{2a}\, e^{-a\Lambda^2}. \tag{111}$$

For large arguments ($z \gg 1$), $\mathrm{erfc}(z) \simeq \frac{e^{-z^2}}{\sqrt{\pi}\, z}$, so that, keeping only the leading term we get

$$\int_\Lambda^\infty dk\, k^2 e^{-\frac{1}{2}\Sigma^2 k^2} \lesssim \frac{\Lambda e^{-\frac{1}{2}\Sigma^2 \Lambda^2}}{\Sigma^2}. \tag{112}$$

We then obtain

$$\big|\Delta(\mathbf{x},t)\big| \lesssim \frac{8\pi}{(2\pi)^{3/2}}\frac{\Lambda e^{-a\Lambda^2}}{2a} = \frac{4\pi}{(2\pi)^{3/2}}\frac{\Lambda e^{-\frac{1}{2}\Sigma^2\Lambda^2}}{(\frac{1}{2}\Sigma^2)}. \tag{113}$$

Because the exponential dominates any power of $k_{max} = \Lambda$, we have

$$\big|\Delta(\mathbf{x},t)\big| \lesssim \exp\big[-\tfrac{1}{2}\Sigma^2\Lambda^2\big] = \exp\big[-\tfrac{1}{2}(\sigma_r^2+\sigma_t^2)\Lambda^2\big]. \tag{114}$$

Thus, whenever this cutoff satisfies (38) and (39), i.e. $k_{max} \gg 1/\sigma_r, 1/\sigma_t$, the error made by truncating the k–integral is exponentially small. This is in agreement with the bound above, and therefore, instead of integrating from 0 to ∞, we just need to integrate from 0 to k_{max}.

D Smearing systems at lower scales

Consider the projection operator onto the low-energy sector P_Λ, defined for each energy eigenstate $H|E\rangle = E|E\rangle$ by:

$$\begin{cases} P_\Lambda|E\rangle = 0 & \text{if } E > \Lambda \\ P_\Lambda|E\rangle = |E\rangle & \text{if } E < \Lambda, \end{cases} \tag{115}$$

which satisfies $P_\Lambda^2 = P_\Lambda$, and where Λ is an arbitrary UV cutoff for the theory.

Burgess et al. [8] have shown via the "decoupling theorem" that integrating out heavy physics always leads to Hamiltonian evolution, and particularly, to an evolution that cannot change a pure state into a mixed state. Roughly, acting with P_Λ on the heavy states $|E\rangle$ should not lead to a non-unitary evolution afterwards.

However, recent models in flat and curved spacetime seem to imply that the purity of states depends on the mass M of heavy states of an order $\mathcal{O}\big(1/M\big)$. Consider the following Lagrangian valid to both flat and curved spacetimes,

$$\mathcal{L} = -\left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2\right] + \mathcal{L}_{\text{int}}. \tag{116}$$

Now, consider the quantity purity, which measures the degree of decoherence with $\gamma_D = 1$ corresponding to maximal purity and no-decoherence,

$$\gamma_D(t) := \text{Tr}_\sigma\left[\varrho^2(t)\right]. \tag{117}$$

In the case of

$$\mathcal{L}_{\text{int}} = -g^2\,\phi\sigma, \tag{118}$$

in the large mass limit, i.e., the limiting case where $M \gg k, m, \mu$ it can be shown that in the case that the Hamiltonian of interaction is turned instantaneously, for a mode $\mathbf{k}$ of a field σ decohered by ϕ we have

$$\gamma^s_{\mathbf{k}}(t) \simeq 1 - \frac{2\mu^4}{\omega_\sigma M^3}\sin^2\left[\frac{1}{2}M(t-t_0)\right], \tag{119}$$

or in the case that the interaction is turned on adiabatically in the remote past,

$$\bar{\gamma}^a_{\mathbf{k}}(t) \simeq 1 - \frac{\mu^4}{2\omega_\sigma M^3}. \tag{120}$$

In the case of

$$\mathcal{L}_{\rm int} = -g\,\phi^2\sigma, \tag{121}$$

it can be shown that under the above-mentioned adiabatic interaction,:

$$\gamma^a_{\mathbf{k}} \simeq 1 - \frac{g^2}{16\pi^2\omega_\sigma M}\int_1^\infty \frac{du}{u^3\sqrt{u^2-1}} = 1 - \frac{g^2}{64\pi\omega_\sigma M}. \tag{122}$$

We see above that the purity depends on the heavy mass M of the environment, and we need to make clear how we discard heavy degrees of freedom. The correct energy selection needed when discarding such degrees of freedom is automatically enforced by the so-called $i\epsilon$ prescription built into the Wightman function. We begin by specifying precisely which $i\epsilon$ prescription is intended, which is closely related prescriptions that appear in particle physics, cosmology, condensed-matter physics, quantum optics, and so on. The prescription relevant here demands that the Wightman function, $W(\mathbf{x},\, t;\, \mathbf{x}',\, t')$, be evaluated with time differences $t - t_0$ possessing a small negative imaginary component. This imaginary shift ensures convergence of the sum over intermediate states in

$$\langle 0|\phi(x)\phi(x')|0\rangle = \int d^3p\,\langle 0|\phi(0)|\mathbf{p}\rangle\,\langle \mathbf{p}|\phi(0)|0\rangle\, e^{\,i\,p(x-x')} \tag{123}$$

in purity calculations, where $p{\cdot}(x-x') = p_\mu\,(x-x')^\mu = -\omega(\mathbf{p})\,(t-t') + \mathbf{p}{\cdot}(\mathbf{x}-\mathbf{x}')$ and $\omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^2}$ is the dispersion relation of the field. The inclusion of a small negative imaginary part in $t - t'$ guarantees convergence for large $|\mathbf{p}|$. So, note that $t - t' \;\longrightarrow\; (t-t') - i\,\epsilon, \epsilon > 0$. and then

$$e^{\,ip\cdot(x-x')} = \exp\Big[-i\,\omega_p\big((t{-}t'){-}i\,\epsilon\big) + i\,\mathbf{p}{\cdot}(\mathbf{x}{-}\mathbf{x}')\Big] = e^{-\epsilon\,\omega_p}\; e^{-\,i\,\omega_p(t-t')}\; e^{+\,i\,\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}. \tag{124}$$

This role of $i\epsilon$ is analogous to its usage in quantum optics, where it regulates the finite response time of a detector, with the limit $\epsilon \to 0$ corresponding to removing any unresolved short-distance physics. In this ultraviolet (UV) interpretation, ϵ effectively acts as a temporal cutoff Λ^{-1} or is relate to an energy cutoff Λ, beyond which details of the theory are inaccessible.

So, an important function of $i\epsilon$ is to serve as a UV regulator in the Wightman function, since it suppresses contributions from energy eigenstates according to their eigenvalues—precisely what is needed when projecting out heavy modes in a decoupling basis. Furthermore, as explained fully in [8], the limits $\epsilon \to 0$ and $M \to \infty$ do not commute, and this non-commutativity is essential for making decoupling manifest. In the calculation of purity, if one first expands in powers of $1/M$ (for large M) and only afterward sends $\epsilon \to 0$, the system's state remains nearly pure up to exponentially suppressed corrections. This agrees with the expectation when using the exact (decoupled) energy eigenbasis, expressed via the "decoupling theorem" above. On the other hand, if one first takes $\epsilon \to 0$ and then performs a $1/M$ expansion, the resulting state appears mixed by an amount of order $\mathcal{O}(1/M)$. This matches the calculations of the purity that we have briefly seen through the above examples.

Physically, the above makes sense because (in the standard interpretation) $\epsilon \sim 1/\Lambda$ sets the shortest temporal resolution that the Wightman function can resolve. Heavy physics with $M > \Lambda$ produces effects too rapid for a low-energy detector to observe. Therefore, when estimating the impact of these modes, it is incorrect to take $\epsilon \to 0$ before expanding in $1/M$. In the decoupling limit—characterized by $M \gg \Lambda$—a nonzero ϵ automatically ensures that projecting out the heavy sector is equivalent to discriminating against high-energy eigenstates. On the other hand, if $M < \Lambda$, then effects of order $1/M$ can, in principle, be discerned by low-energy experiments. In that scenario, one can safely set $\epsilon \to 0$ first and only later expand in powers of $1/M$. In doing so, significant contributions to purity arise, in agreement with the decohered-basis computations, which lead to the calculation of purity, and that were discussed above.

Both approaches—working in the decoupled basis and working in the decohered basis—are valid within their own domains, and they yield different answers because they address different physical questions. Which approach applies depends on the relative size of M and Λ.

We now show that the shift $(t-t') \to (t-t') - i\varepsilon$ can be replaced by smearing each field with a temporal test function whose temporal profile encodes the same $i\varepsilon$ information. One then recovers exactly the factor $e^{-\varepsilon\omega_p}$ (or its square) that tames the ultraviolet behavior in the momentum integral. Thus, we can consider the emission of a smearing function by another system controls whether a more massive system decoheres another or not.

To begin, we introduce two smooth, rapidly–decreasing temporal smearing functions $f_\varepsilon(t)$ and $g_{\varepsilon'}(t')$, together with spatial smearing functions $F(\mathbf{x})$ and $G(\mathbf{x}')$. Then, we define the smeared field operators by

$$\Phi_f = \int dt\, d^3x\, \sqrt{-g}\, f_\varepsilon(t)\, F(\mathbf{x})\, \phi(t,\mathbf{x}), \qquad \Phi_g = \int dt'\, d^3x'\, \sqrt{-g}\, g_{\varepsilon'}(t')\, G(\mathbf{x}')\, \phi(t',\mathbf{x}'). \tag{125}$$

Their vacuum expectation value is

$$W_{f,g} = \langle 0 \mid \Phi_f\, \Phi_g \mid 0\rangle = \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'}\, f_\varepsilon(t)\, F(\mathbf{x})\, g_{\varepsilon'}(t')\, G(\mathbf{x}')\, W^+(x,x'), \tag{126}$$

where $W^+(x,x')$ is the unsmeared Wightman function of a free massive scalar field. We choose both temporal smearings to be Lorentzian of width ε and ε', respectively:

$$f_\varepsilon(t) = \frac{\varepsilon}{\pi\left(t^2+\varepsilon^2\right)}, \qquad g_{\varepsilon'}(t') = \frac{\varepsilon'}{\pi\left(t'^2+\varepsilon'^2\right)}, \tag{127}$$

with Fourier transforms

$$\widetilde{f}_{\varepsilon}(\omega) = \int_{-\infty}^{\infty} dt\; f_{\varepsilon}(t)\, e^{i\omega t} = e^{-\,\varepsilon\,|\omega|}, \qquad \widetilde{g}_{\varepsilon'}(\omega) = e^{-\,\varepsilon'\,|\omega|}. \tag{128}$$

Let us consider the Wightman function,

$$W^{+}(x, x') = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p}\; e^{-\,i\,\omega_p\,(t-t')}\; e^{\,i\,\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')}, \qquad \omega_p = \sqrt{\mathbf{p}^2 + M^2}. \tag{129}$$

Let us smear spatiotemporally this correlator, and perform the spatial integrals first, which yield

$$\int d^3x\; \sqrt{-g} F(\mathbf{x})\, e^{\,i\,\mathbf{p}\cdot\mathbf{x}} = \widetilde{F}(\mathbf{p}), \qquad \int d^3x' \sqrt{-g'}\; G(\mathbf{x}')\, e^{-\,i\,\mathbf{p}\cdot\mathbf{x}'} = \widetilde{G}^{*}(\mathbf{p}). \tag{130}$$

The remaining time integrals yield

$$\int_{-\infty}^{\infty} dt\; f_{\varepsilon}(t)\, e^{-\,i\,\omega_p\,t} = \widetilde{f}_{\varepsilon}(-\,\omega_p) = e^{-\,\varepsilon\,\omega_p}, \qquad \int_{-\infty}^{\infty} dt'\; g_{\varepsilon'}(t')\, e^{+\,i\,\omega_p\,t'} = \widetilde{g}_{\varepsilon'}(+\,\omega_p) = e^{-\,\varepsilon'\,\omega_p}. \tag{131}$$

Consequently, the smeared two–point function becomes

$$W_{f,g} = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p}\; \widetilde{F}(\mathbf{p})\, \widetilde{G}^{*}(\mathbf{p})\; e^{-(\varepsilon+\varepsilon')\,\omega_p}. \tag{132}$$

If the same spatiotemporal smearing is used on both F and G, then $W_{f,f}$ reduces to

$$W_{f,f} = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p}\; \left|\widetilde{F}(\mathbf{p})\right|^2\; e^{-\,2\,\varepsilon\,\omega_p}. \tag{133}$$

In the case of no spatial smearing $\big(F(\mathbf{x}) = G(\mathbf{x}) = \delta^{(3)}(\mathbf{x})\big)$, one recovers exactly the factor $e^{-\,2\,\varepsilon\,\omega_p}$ that arises from imposing the $i\varepsilon$ prescription directly on the time difference. Thus, the Lorentzian temporal smearing allows us to rederive the exponential damping without the above trick of shifting times into the complex plane.

Now, smearing functions are emitted by a mean-field of some other field. Thus, we need to find a state ρ of a real scalar field ϕ such that we obtain the temporal smearing

$$f_{\epsilon}(t) \;=\; \big\langle \phi(t) \big\rangle_{\rho} \;=\; \mathrm{Tr}\big[\rho\, \phi(t)\big] \;=\; \frac{\epsilon}{\pi\,\big(t^2 + \epsilon^2\big)}\,. \tag{134}$$

For simplicity we work in $1+1\,$D, set the spatial point $x = 0$, and take ϕ massless $\big(\omega_k = |k|\big)$. The coherent state $|\alpha\rangle \;=\; \exp\!\left[\int_0^\infty dk\; \big(\alpha_k\, a_k^{\dagger} - \alpha_k^{*}\, a_k\big)\right] |0\rangle$ has the classical expectation value

$$\big\langle \phi(t) \big\rangle_{\alpha} \;=\; \int_0^\infty \frac{dk}{\sqrt{4\pi\, k}}\; \big[\alpha_k\, e^{-\,i\,k\,t} + \alpha_k^{*}\, e^{\,i\,k\,t}\big]. \tag{135}$$

To match the $i\epsilon$ prescription we need

$$\alpha_k \;=\; \sqrt{\frac{k}{\pi}}\,\widetilde{f}_\epsilon(k) \;=\; \sqrt{\frac{k}{\pi}}\,e^{-\epsilon\,k}\,. \tag{136}$$

Plugging this into the expression for coherent states gives

$$\begin{aligned}\big\langle\phi(t)\big\rangle_\alpha &= \int_0^\infty dk\Big[e^{-\epsilon\,k}\,e^{-\,i\,k\,t} \;+\; e^{-\epsilon\,k}\,e^{\,i\,k\,t}\Big]\\ &= \frac{\epsilon}{\pi\left(t^2+\epsilon^2\right)},\end{aligned} \tag{137}$$

which yields our Lorentzian. Hence, the system in the state

$$\rho = |\alpha\rangle\langle\alpha| \quad \text{with} \quad \alpha_k = \sqrt{\frac{k}{\pi}}\,e^{-\epsilon\,k} \tag{138}$$

generates the test function that yields the canonical $i\epsilon$ prescription.

Following the same logic applied in $3+1$ D, it can be shown that a single coherent state of a massless real scalar can be tuned so that its time-dependent expectation value at the spatial origin reproduces the Lorentzian test function that encodes the $i\epsilon$-prescription.

We expand the free, massless field at $\mathbf{x}=0$ in the usual basis:

$$\phi(t) \;\equiv\; \phi(t,\mathbf{0}) \;=\; \int\frac{d^3k}{(2\pi)^{3/2}\,\sqrt{2\,k}}\Big[a_{\mathbf{k}}\,e^{-\,i\,k\,t} \;+\; a^\dagger_{\mathbf{k}}\,e^{\,i\,k\,t}\Big], \qquad k\equiv|\mathbf{k}|. \tag{139}$$

Now, we choose a real coherent profile $\alpha_{\mathbf{k}} = \alpha(k) = \alpha^*(k)$ and define

$$|\alpha\rangle = \exp\Big[\int_0^\infty dk k^2\;\alpha(k)\int\frac{d\Omega_{\hat{\mathbf{k}}}}{(2\pi)^{3/2}}\,\big(a^\dagger_{\mathbf{k}} - a_{\mathbf{k}}\big)\Big]|0\rangle. \tag{140}$$

Because $a_{\mathbf{k}}|\alpha\rangle = \alpha(k)|\alpha\rangle$, the one–point function at the spatial origin is

$$\big\langle\phi(t)\big\rangle_\alpha = \int\frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}}\;2\,\alpha(k)\,\cos(kt). \tag{141}$$

Using $d^3k = k^2\,dk\,d\Omega$ and $\int d\Omega = 4\pi$, this becomes

$$\big\langle\phi(t)\big\rangle_\alpha = \frac{8\pi}{(2\pi)^{3/2}\sqrt{2}}\int_0^\infty dk\;k^{3/2}\,\alpha(k)\,\cos(kt) = \frac{2}{\sqrt{\pi}}\int_0^\infty dk\;k^{3/2}\,\alpha(k)\,\cos(kt). \tag{142}$$

We wish to obtain the Lorentzian temporal profile

$$\big\langle\phi(t)\big\rangle_\alpha = \frac{\varepsilon}{\pi\left(t^2+\varepsilon^2\right)}, \qquad \varepsilon>0. \tag{143}$$

This is achieved by choosing

$$\alpha(k) = \frac{1}{2\sqrt{\pi}}\,\frac{e^{-\varepsilon k}}{k^{3/2}}. \tag{144}$$

Indeed,

$$\big\langle\phi(t)\big\rangle_\alpha = \frac{2}{\sqrt{\pi}}\,\frac{1}{2\sqrt{\pi}}\int_0^\infty dk\; e^{-\varepsilon k}\;\cos(kt) \tag{145}$$

$$= \frac{1}{\pi}\,\frac{\varepsilon}{\varepsilon^2+t^2}, \tag{146}$$

because

$$\int_0^\infty dk\; e^{-\varepsilon k}\cos(kt) = \frac{\varepsilon}{\varepsilon^2+t^2}. \tag{147}$$

Gaussian states can also source the smearing function. Let ρ be a Gaussian density operator—for example, the thermal state

$$\rho \;=\; Z^{-1}\,\exp\!\Big[-\beta\int d^3k\;\omega_k\,a^\dagger_{\mathbf{k}}a_{\mathbf{k}}\Big]. \tag{148}$$

Applying the displacement operator $D[\alpha] = \exp\!\Big[\int d^3k\big(\alpha_{\mathbf{k}}a^\dagger_{\mathbf{k}} - \alpha^*_{\mathbf{k}}a_{\mathbf{k}}\big)\Big]$ with the same coherent profile

$$\alpha_{\mathbf{k}} = \frac{1}{2\sqrt{\pi}}\,\frac{e^{-\varepsilon\,k}}{k^{3/2}}, \quad k = |\mathbf{k}|. \tag{149}$$

Then, we define the mixed state $\rho \;=\; D[\alpha]\rho_G D^\dagger[\alpha]$. Because $D^\dagger\,a_{\mathbf{k}}D = a_{\mathbf{k}} + \alpha_{\mathbf{k}}$, the mean field is shifted and we obtain:

$$\big\langle\phi(t,\mathbf{0})\big\rangle_\rho \;=\; \big\langle\phi(t,\mathbf{0})\big\rangle_{D[\alpha]|0\rangle} \;=\; \frac{\varepsilon}{\pi\,(t^2+\varepsilon^2)}. \tag{150}$$

The above Lorentzian function does not have a rapid decay and thus it is not what we often associate with a smearing function, such as Schwartz functions. Furthermore, the coherent states that give rise to it via a mean field are not homogeneous and isotropic. Thus, they do not serve to provide to the Einstein Field Equations in the FLRW spacetime, which needs such homogeneous and isotropic states, as we discuss in Section 5.

However, we can achieve this by replacing the single-pole Lorentzians by order-n super-Lorentzian functions

$$f_{n,\gamma}(t) = \frac{C_n}{\big[t^2+(\gamma/2)^2\big]^n}, \qquad g_{n',\gamma'}(t) = \frac{C_{n'}}{\big[t^2+(\gamma'/2)^2\big]^{n'}}, \tag{151}$$

where

$$C_n = \frac{\Gamma(n)}{\sqrt{\pi}\,\Gamma\big(n-\frac{1}{2}\big)}\left(\tfrac{\gamma}{2}\right)^{2n-1}, \qquad C_{n'} = \frac{\Gamma(n')}{\sqrt{\pi}\,\Gamma\big(n'-\frac{1}{2}\big)}\left(\tfrac{\gamma'}{2}\right)^{2n'-1}. \tag{152}$$

Their Fourier transforms (for $\omega \neq 0$) are given by

$$\widetilde{f}_{n,\gamma}(\omega) = \mathcal{A}_n\,|\omega|^{\,n-1}\,e^{-\frac{\gamma}{2}\,|\omega|}, \qquad \widetilde{g}_{n',\gamma'}(\omega) = \mathcal{A}_{n'}\,|\omega|^{\,n'-1}\,e^{-\frac{\gamma'}{2}\,|\omega|}, \tag{153}$$

with

$$\mathcal{A}_n = \frac{\sqrt{\pi}\, 2^{\,1-2n}\, \Gamma\big(n-\frac{1}{2}\big)}{\Gamma(n)} \Big(\tfrac{\gamma}{2}\Big)^{\,1-2n}, \qquad \mathcal{A}_{n'} = \frac{\sqrt{\pi}\, 2^{\,1-2n'}\, \Gamma\big(n'-\frac{1}{2}\big)}{\Gamma(n')} \Big(\tfrac{\gamma'}{2}\Big)^{\,1-2n'}. \tag{154}$$

Proceeding exactly as in the original calculation, one finds that the smeared two-point function becomes

$$W_{f,g} = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p}\, \widetilde{F}(\mathbf{p})\, \widetilde{G}^*(\mathbf{p})\, \big[\mathcal{A}_n\, \mathcal{A}_{n'}\, \omega_p^{\,n+n'-2}\big]\, e^{-\frac{\gamma+\gamma'}{2}\,\omega_p}, \tag{155}$$

so the exponential damping that codifies the $i\varepsilon$ can be seen here. With identical smearings ($n = n'$, $\gamma = \gamma'$, $F = G$), this reduces to

$$W_{f,f} = \int \frac{d^3p}{(2\pi)^3\, 2\omega_p}\, \big|\widetilde{F}(\mathbf{p})\big|^2\, \big(\mathcal{A}_n\big)^2\, \omega_p^{\,2n-2}\, e^{-\gamma\,\omega_p}. \tag{156}$$

For $n = 1$, one recovers the factor $e^{-2\varepsilon\,\omega_p}$. For $n > 1$, the same exponential is multiplied by ω_p^{2n-2}, which yields stronger ultraviolet suppression. In the limit $n \to \infty$ with

$$\gamma_n = \frac{2\sigma}{\sqrt{n}}, \tag{157}$$

we obtain the following temporal gaussian,

$$f_{n,\gamma_n}(t) = \frac{C_n}{\big[t^2 + (\gamma_n/2)^2\big]^n} \;\longrightarrow\; \frac{1}{\sqrt{4\pi\,\sigma^2}}\, e^{-\,t^2/4\,\sigma^2} \tag{158}$$

where σ is temporal (associated with γ and thus with the cutoff Λ mentioned above, while the corresponding momentum-space factor tends to $\exp\big(-\sigma\,|\omega|^2\big)$. Thus a super-Lorentzian not only reproduces the $i\varepsilon$ prescription but also allow us to recover in the limit of large n the Schwartz-class smearing functions. Furthermore, in the limit $\sigma \gg 1$, we get a quasi-uniform function, which we want in order to consider that this field emits a constant gravitational field that gives rise to a de Sitter spacetime. This function can be emitted as a mean-field by a scalar field in a $|\alpha\rangle_{\mathbf{k}=0}$ state, which is a homogeneous and isotropic coherent state.

E Measurement theory in QFT from SDCs

We will briefly show in this section how this theory fits with measurement theory in QFT, in particular, particle detector models.[84] We will focus on two real scalar fields A and B, where A will be decomposed into modes. We will assume that we will assume that the modes of A belong to an SDC and already has the $DC-B$ in agreement with the determination conditions explained in Section 3.

[84]We will follow closely the calculations and results obtained in [71] with some appropriate adaptations.

We will assume that the target system B and its modes is initially in a zero-mean Gaussian state, as well as A. Gaussian states are quantum states whose Wigner functions are Gaussian. They are completely characterized by their first and second moments (mean values and covariance matrices). Examples of these states are thermal, coherent states, and squeezed states. Furthermore, we will assume that the vacuum of the states under study fulfills the Hadamard condition. As is well-known, in QFT there are many unitary inequivalent Hilbert space representations. However, the consensus is to select a subclass of states known as Hadamard states that fulfill the idea that all states should look similar locally, and closer to flat space QFT as possible.

Omitting the systems that give rise to a gravitational field, in the covariant picture, the interaction between system A and system B is described by the following Lagrangian density,

$$\mathcal{L} = \frac{1}{2}(\nabla_\mu \phi_A)(\nabla^\mu \phi_A) + \frac{1}{2}(\nabla_\mu \phi_B)(\nabla^\mu \phi_B) - \lambda_{AB} f \phi_A \phi_B \tag{159}$$

where λ_{AB} is the coupling constant with dimensions of energy squared and f is a dimensionless smooth, real-valued coupling/test function with support in some compact coupling spacetime region R.[85]

We adopt the canonical picture in 3+1 globally hyperbolic spacetime, where we regard a $(3+1)$-dimensional spacetime $\mathcal{M}$ as foliated by a family of spacelike 3-dimensional hypersurfaces Σ_t, labeling the hypersurfaces by a time parameter t, and assume the following split of the metric,

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{160}$$

where h_{ij} is the spatial metric; N is the lapse function which describes how much proper time elapses between two hypersurfaces along the direction normal to the spatial slice; and N^i is the shift vector, which describes how the spatial coordinates change when moving from one hypersurface to another.

So, we have the following evolution,

$$\hat{U} = \mathcal{T} \exp\left(-i \int dt \hat{H}_{\text{int}}(t)\right), \tag{161}$$

where $\mathcal{T}\exp$ denotes a time-ordered exponential concerning any a time parameter and

$$H_{\text{int}}(t) = \lambda_{AB} \int_{\Sigma_t} d^3x \sqrt{h}\, \chi(t) F(\mathbf{x}) \phi_A(t, \mathbf{x}) \phi_B(t, \mathbf{x}). \tag{162}$$

with λ_{AB} being a coupling constant, and $\chi(t)$ and $F(\mathbf{x})$ being the temporal and spatial smearing functions, respectively, over the spacelike hypersurfaces Σ_t. Furthermore, they fulfill the no-disturbance conditions jointly with other temporal and spatial smearing functions concerning other interactions.

Assuming that λ_{AB} is sufficiently small we can have the following Dyson expansion,

$$\hat{U} = 1 + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3), \tag{163}$$

[85]We thus express the smearing function for each mode A and B in terms of this function.

where

$$\hat{U}^{(1)} = -i \int dt \hat{H}_{\rm int}(t), \tag{164}$$

and

$$\hat{U}^{(2)} = -\int dt dt' \, \hat{H}_{\rm int}(t) \hat{H}_{\rm int}(t') \theta(t - t'), \tag{165}$$

with and $\theta(t)$ being the Heaviside theta function and where t is any time coordinate.

Now, let us consider the initial state of the systems, where we focus on the interaction of one of the modes of A, which was previously decomposed into finite modes.

$$\hat{\rho}_0 = |0_A\rangle\langle 0_A| \otimes \hat{\rho}_B. \tag{166}$$

The interaction between other N modes of A with the DC-B are omitted. We could also consider A as a series of modes, which we are idealizing as a simple system, which it will decohere B. On the other hand, B could be a single mode or a whole continuum of modes that we choose to not decompose for simplicity. In Sections 5 and G, we will see a model in de Sitter spacetime where A decoheres a single mode of B in a more complex situation. Taking into account that

$$\hat{\rho}_f = \hat{U} \hat{\rho}_0 \hat{U}^\dagger, \tag{167}$$

we get that the final states of the fields are represented by

$$\hat{\rho}_f = \hat{\rho}_0 + \hat{\rho}^{(1)} + \hat{\rho}^{(2)} + \mathcal{O}(\lambda^3), \tag{168}$$

where

$$\begin{aligned} \hat{\rho}^{(1)} &= \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \\ \text{and} \quad \hat{\rho}^{(2)} &= \hat{U}^{(2)} \hat{\rho}_0 + \hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger} + \hat{\rho}_0 \hat{U}^{(2)\dagger}. \end{aligned} \tag{169}$$

More concretely,

$$\begin{aligned} \hat{\rho}^{(2)} = \lambda_{AB}^2 \int dV dV' \Big[& \hat{M}(t,\mathbf{x}) \hat{\phi}_B(t,\mathbf{x}) \hat{\rho}_0 \hat{\phi}_B(t',\mathbf{x}') \hat{M}(t',\mathbf{x}') \\ & - \hat{M}(t,\mathbf{x}) \hat{M}(t',\mathbf{x}') \hat{\phi}_B(t,\mathbf{x}) \hat{\phi}_B(t',\mathbf{x}') \hat{\rho}_0 \theta(t-t') \\ & - \hat{\rho}_0 \hat{M}(t,\mathbf{x}) \hat{\phi}_B(t',\mathbf{x}') \hat{\phi}_B(t,\mathbf{x}) \hat{M}(t',\mathbf{x}') \theta(t'-t) \Big]. \end{aligned} \tag{170}$$

with $\hat{M}(t,\mathbf{x}) = \chi(t) F(\mathbf{x}) \phi_A(t,\mathbf{x})$.

We will now partial trace the final state over the degrees of freedom of B, focusing only on mode A, i.e., $\hat{\rho}_A = Tr_B \hat{\rho}_f$, to see how system A probes the field B.

Since B starts as a zero-mean Gaussian, we have that $\mathrm{tr}_B \left(\hat{\phi}_B(t,\mathbf{x}) \hat{\rho}_B \right) = \langle \hat{\phi}(t,\mathbf{x}) \rangle_{\rho_B} = 0$. Moreover, given that $W(x,x') = \langle \hat{\phi}(x) \hat{\phi}(x') \rangle_{\rho_B}$ we have that

$$\mathrm{tr}_B\left(\hat{\rho}^{(2)}\right) = \lambda_{AB}^2 \int dt\, dt'\, W(x,x') \Big[\hat{M}(t',x')|0_A\rangle\langle 0_A|\hat{M}(t,x)
- \hat{M}(t,x)\hat{M}(t',x')|0_A\rangle\langle 0_A|\theta(t-t')
- |0_A\rangle\langle 0_A|\hat{M}(t,x)\hat{M}(t',x')\theta(t'-t) \Big].$$

Thus, we have

$$\hat{\rho}_A = |0_A\rangle\langle 0_A| + \lambda_{AB}^2 \int \mathrm{d}V\, \mathrm{d}V'\, W(x,x') \Big[\hat{M}(t',\mathbf{x}')|0_A\rangle\langle 0_A|\hat{M}(t,\mathbf{x})
- \hat{M}(t,\mathbf{x})\hat{M}(t',\mathbf{x}')|0_A\rangle\langle 0_A|\theta(t-t')
- |0_A\rangle\langle 0_A|\hat{M}(t,\mathbf{x})\hat{M}(t',\mathbf{x}')\theta(t'-t) \Big] + \mathcal{O}(\lambda^4). \tag{171}$$

As we can see, $A's$ final state contains information of the field's values of B through its correlation function, probing B, and (assuming decoherence) we infer that it gives rise to B having determinate field values over the spacetime regions x and x'.

A closer comparison to particle detector models becomes possible if the spacetime is static and the metric is such that we have a separation between space and time. This also allows us to make clearer how systems probe each other. So, in this case, the solutions $u_k(x)$ decompose as $u_k(x) = e^{-i\omega_k t}\Phi_k(\mathbf{x})$. Then, writing $\zeta(x) = \chi(t)F(\mathbf{x})\Phi_k(\mathbf{x})$, the interaction Hamiltonian becomes

$$\hat{H}_{\mathrm{static}}(x) = \lambda_{AB}\left(\zeta(x)e^{-i\omega_k t}\hat{a}_k + \zeta^*(x)e^{i\omega_k t}\hat{a}_k^\dagger\right)\hat{\phi}_B(x). \tag{172}$$

To obtain the expression of a particle detector evolving in a given "trajectory," let us consider x_0 as the spatial coordinate that concerns the center of $\Phi_k(\mathbf{x})$. More concretely, let us consider a particle detector whose center of mass has the trajectory given by the Fermi normal coordinates $z(\tau) = (\gamma\tau, x_0)$, where τ is the proper time and γ is the redshift factor relative to t. Then, the proper energy gap is defined as $\Omega = \gamma\omega_k$, so the effective interaction Hamiltonian becomes

$$\hat{H}_{\mathrm{eff}}(x) = \lambda_{AB}\left(\zeta(x)e^{-i\Omega\tau}\hat{a}_k + \zeta^*(x)e^{i\Omega\tau}\hat{a}_k^\dagger\right)\hat{\phi}_B(x). \tag{173}$$

This corresponds to the interaction Hamiltonian of a harmonic oscillator detector with an energy gap Ω, interacting with a scalar field $\hat{\phi}(x)$. By appropriately balancing the units of $F(\mathbf{x})$, the switching function, and the coupling strength, one can match this model to the harmonic oscillator Unruh-DeWitt detector (UdW) model. Note that UdWs are idealized quantum two-level systems that couple locally to the quantum field and evolve under their proper time. For "one-particle" excitations in mode $\mathbf{k}$, the Hamiltonian can be restricted to a two-level system spanned by $\{|0_{\mathbf{k}}\rangle, |1_{\mathbf{k}}\rangle\}$, reducing to the leading-order interaction of a two-level detector A with a scalar field.

It can be shown [75] that quantum field theories, which assume the principle of microcausality (where observables commute at spacelike separated points) will generate time-evolution operators that remain independent of the specific time parameter used for time ordering. However, in the above approximation using smeared operators this is not what happens. To see this, let us first observe that the algebra of creation and anhiliation operators restricted to act on a two-dimensional is isomorphic to the ladder operators $\hat{\sigma}_+$ and $\hat{\sigma}_-$ also acting on such a space. Given that the monopole moment operator is $\hat{\mu}(\tau) = e^{i\Omega\tau}\hat{\sigma}_+ + e^{-i\Omega\tau}\hat{\sigma}_-$, it can be shown that $[\hat{H}_{eff}(x), \hat{H}_{eff}(x')] = \lambda^2\Lambda(x)\Lambda(x')[\hat{\mu}(\tau(x)), \hat{\mu}(\tau(x'))]\hat{\phi}(x)\hat{\phi}(x')$ for spacelike separated regions x and x' due to $[\hat{\mu}(\tau), \hat{\mu}(\tau')] = 2i\sin(\Omega(\tau - \tau'))\hat{\sigma}_z$ just vanishes for certain times. However, in the cases where covariance violation does occur at the leading order, this is due to the spatial smearing of the detector. To see this more intuitively, note that $\hat{H}_{eff}$ couples non-locally a single quantum degree of freedom of the detector to multiple spacelike separated points. Consequently, the effect is suppressed as the smearing decays over time. Furthermore, when we have pointlike detectors (which arise as a limit of very sharply localized smearing functions), we also obtain full covariance in the sense above.

Given this, [75] smeared particle detector models lead to a quantifiable breaking of covariance because, in a covariant formalism, the time evolution operator concerning the same Hamiltonian should yield the same results regardless of the reference frame used, i.e., $\hat{\mathcal{U}}_\tau = \hat{\mathcal{U}}_t$. Particle detector models provide us a measurement theory for QFT with a series of update rules for different measurements, which we can in principle use in this framework. Nevertheless, since the theory presented here starts fundamentally from quantum fields, and particle detectors arise from it, we consider that this breaking of covariance is merely an emergent feature, not present in the fundamental theory, and which can be under control. Future work should explore the application of particle detector models to this theory in more detail, as well as how the covariant algebraic QFT framework [29] applies to it, which in principle can also be done.

F How SDCs allow us to infer a gravitational field and give rise to it

In this section, we give an intuition about how a classical metric may arise via how probes interact and decohere a quantum field (51). First, note that particle detectors interact with the quantum field at specific spacetime points, giving rise to detection events. By analyzing the probabilities associated with the detection events, we can extract both the real and imaginary parts of the Wightman function.

Assuming a fast switching interaction, represented by a delta coupling,[86]

[86] We should regard the delta coupling as a mathematical tool that represents very rapid interactions. This coupling leads to divergences in the models as prior work has investigated [76, 86, 87]. However, these divergences are restricted to the local terms related to each

between the detector and the field, occurring at two distinct times $\tau_i = t_1$ and $\tau_i = t_2$,[87] and for timelike separated events, the two-point correlation function between detectors at points x_1 and x_2 is expressed through measurable quantities as:

$$\cos(\Omega\Delta t)\Re\langle\hat{\phi}(x_1)\hat{\phi}(x_2)\rangle+\sin(\Omega\Delta t)\Im\langle\hat{\phi}(x_1)\hat{\phi}(x_2)\rangle = L_{ii}-P_i(x_1)-P_i(x_2), \quad (174)$$

where Ω is the energy gap of the detector, Δt is the time difference between events, and $P_i(x_1)$ and $P_i(x_2)$ are the individual detection probabilities at points x_1 and x_2, respectively, L_{ii} is the probability that the detectors at two different spacetime points x_1 and x_2 fire together due to their interaction with the quantum field. Assuming point-like detectors again, the correlation function between detectors i and k that are spacelike separated can be given by the expression

$$C(i,k) = 4\lambda^2 \sin(\Omega(t_i+\tau_0))\sin(\Omega(t_k+\tau_0))\langle\hat{\phi}(x_i)\hat{\phi}(x_k)\rangle, \quad (175)$$

where $\tau_0 = \frac{\tau_i^0+\tau_k^0}{2}$, and where the proper time at which each detector interacts is labeled $\tau_i = \tau(x_i)$, leading to $z(\tau_i) = x_i$. So, we have a way of inferring the correlators from the probabilities of the detectors having determinate values. Note that pointlike interactions are unphysical and this only an approximation to the more realistic smeared interaction in spacetime.

Now, to get the above first and second order derivative of the correlator, and hence the metric, the key is to *place* an array of probes throughout space. This grid allows one to measure the field correlations across multiple points over time, providing a detailed map of the quantum field's behavior.

More concretely, in this setup, each detector interacts with the quantum field at specific spacetime points, which are labeled by multi-indices that correspond to coordinates in the spacetime. Let's break it down further with emphasis on the labelling of detectors and their corresponding spacetime positions. The set of detectors is parametrized by $j := (j_0, j_1, ..., j_n)$, where each index corresponds to the detector's position in spacetime. The index j_0 represents the time coordinate, while the remaining $j_1, j_2, \ldots, j_n$ represent the spatial coordinates. In total, there are N^n detectors, where N is the number of detectors along each spatial direction and n is the spatial dimension. The spacetime position of a detector is

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individual interaction and have no impact on the correlations between detectors, which are the primary focus for reconstructing the metric. For instance, if one replaces Dirac deltas with sharply peaked Gaussians, the results for detector correlations would remain largely unchanged. This approach avoids divergences in the system but increases the complexity of the calculations, which extends beyond the scope of this work. For practical purposes, one may adopt a spacetime smearing function $\Lambda(x)$, centered at x_i and normalized to have a unit integral and consider the family of smeared spacetime functions that become sharply peaked at a certain limit, approximating the idealized delta-coupling.

[87]While a perfect delta-coupling interaction is unrealistic (see previous footnote, it serves as an approximation for small systems that can interact with the quantum field over times comparable to the light-crossing time. To model this fast interaction, the delta-coupling assumption leads to the following spacetime smearing function: $\Lambda_i(x) = \frac{\delta(x-z_i(t_1))}{\sqrt{-g}} + \frac{\delta(x-z_i(t_2))}{\sqrt{-g}}$. Note that the authors also consider temporal smearing.

denoted by its coordinates $x_j^\mu = (x_{j_0}^0, x_{j_1}^1, \dots, x_{j_n}^n)$. Each detector interacts with the quantum field at specific times $x_{j_0}^0$ and spatial positions $(x_{j_1}^1, \dots, x_{j_n}^n)$.

Once the interactions between the quantum field and the detectors occur, the Wightman function $W(x, x')$, which encodes the two-point correlation between spacetime points, can be computed from the detector readings. The derivative of this function at the positions corresponding to two detectors labeled j and l is discretized as:

$$\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'^\nu} W^{\frac{2}{2-D}}(x, x')\bigg|_{x=x_j, x'=x_l} \approx \frac{W^{\frac{2}{2-D}}(x_{j+1\nu}, x_{l+1\mu}) - W^{\frac{2}{2-D}}(x_j, x_{l+1\mu})}{(x_j^{\mu+1_\mu} - x_j^\mu)(x_l^{\nu+1_\nu} - x_l^\nu)} - \frac{W^{\frac{2}{2-D}}(x_{j+1\nu}, x_l) - W^{\frac{2}{2-D}}(x_j, x_l)}{(x_j^{\mu+1_\mu} - x_j^\mu)(x_l^{\nu+1_\nu} - x_l^\nu)}. \tag{176}$$

Here, the spacetime positions $x_{i+1_\mu}^\mu$ and $x_{l+1_\nu}^\nu$ refer to the locations of detectors separated by a small coordinate distance L in the μ- and ν-directions. The parameter L represents the coordinate separation between detectors in each direction, including time. This means that the detectors are spaced at regular intervals in both the spatial and temporal directions, allowing for a systematic sampling of the quantum field at different spacetime points. The coordinates of the nearby detectors can then be written as $x_{i+1_\mu}^\mu = x_i^\mu + L1_\mu$.

This arrangement of detectors allows one approximate derivative of the Wightman function, which is needed to recover the spacetime metric. Thus, by measuring the Wightman function at the positions of the detectors, we can obtain the metric tensor in eq. (51). More concretely, by refining the detector grid and taking the limit where $L \to 0$, we can infer the metric via eq. (51). The *precision* of the metric recovery depends on the detector spacing and the resolution of the measurements.

For instance, in the case of the hyperbolic static Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime example, the metric is expressed in terms of comoving coordinates. Detectors are placed at spacetime intervals L in the η-direction (conformal time) and spatial directions like χ. The Wightman function for this spacetime was calculated explicitly:

$$W(x, x') = \frac{i\mu(\chi - \chi')H_1^{(2)}(\mu[(\eta - \eta')^2 - (\chi - \chi')^2])}{8\pi a^2 \sinh(\chi - \chi')[(\eta - \eta')^2 - (\chi - \chi')^2]}, \tag{177}$$

and its derivatives were used to recover the metric components by employing the discrete approximation of the Wightman function through detector readings. The *precision* of the recovery of this metric depends on the detector spacing and the resolution of the measurements, where the lower the spacing, the more accurate the recovery.

Finally, it was proposed [70] the following setup to recover the spacetime metric using local measurements of a quantum field at different spacetime points: couple local detectors to the target quantum field, measure the correlations between detectors that are located at different spacetime points, and use these correlations to calculate the quantum field's two-point function concerning the different events and determine the spacetime metric by applying the coincidence limit described in eq. (51).

Note that it was assumed that the probes are fixed in space evolving over time, but this is an idealization. What we consider that we have is that interactions with SDCs give rise to values in an extended region of spacetime. However, systems belonging to SDCs can be approximated as evolving particle detectors in a fixed spatial region (see Section E).

G Decoherence in the FLRW spacetime, symmetric, and Hadamard states

As a reminder, we will adopt the following Gaussian test function emitted by $\mathcal{V}_m$ (see Appendix D),

$$f(t) = N \exp\left[-\frac{(t-t_c)^2}{2\sigma_t^2}\right], \tag{178}$$

obeying the bound in Section 3.2.2, i.e., $\omega_{max}\sigma_t \gg 1$, where ω_{max} is the maximum energy of the mode involved in these interactions between fields. This test function arises from a field in a $\mathbf{k}=0$ coherent state, which is a homogeneous and isotropic state. Being homogeneous and isotropic state is relevant because the system can emit a gravitational field in agreement with the symmetries of the de Sitter spacetime. Furthermore, the temporal Gaussian smearing function will be approximated by a super-Lorentzian function, as we have seen in Section 3.2.2.

The interaction Hamiltonian that we will analyze will be of the form

$$H_{\text{int}}(t,\mathbf{x}) = \mathcal{O}(t,\mathbf{x})\sigma(t,\mathbf{x}), \tag{179}$$

where $\sigma(t,\mathbf{x})$ is the operator that acts on the system's Hilbert space and $\phi(t,\mathbf{x})$ acts on the environment Hilbert space. We consider both quadratic $\mathcal{O}_{\text{mix}} = \mu^2 f(t)\phi(t,\mathbf{x})$ and cubic interactions $\mathcal{O}_c = g f(t)\phi(t,\mathbf{x})^2$. Using Open EFT techniques summarized in the main text, under the Born approximation, to the second-order in perturbation theory, this yields the following non-Markovian equation that we want to use to infer the behavior of the system at late times,

$$\begin{aligned}\partial_t \varrho(t) = &-i\int d^3x\, a^3(t)\, \big[\sigma(t,\mathbf{x}),\rho(t)\big]\, \langle\!\langle \mathcal{O}(t,\mathbf{x})\rangle\!\rangle \\ &-(i)^2\int d^3x\, a^3(t)\int d^3y\, a^3(s)\int_{t_0}^{t} ds\, \Big\{\big[\sigma(t,\mathbf{x}),\sigma(s,\mathbf{y})\,\rho(s)\big]\, W(t,\mathbf{x};s,\mathbf{y}) \\ &-\big[\sigma(t,\mathbf{x})\,\rho(s),\sigma(s,\mathbf{y})\big]\, W^*(t,\mathbf{x};s,\mathbf{y})\Big\} + \mathcal{O}\big(V_{\text{int}}^3\big),\end{aligned} \tag{180}$$

where

$$W(t,\mathbf{x};s,\mathbf{y}) = \langle\!\langle \delta\mathcal{O}(t,\mathbf{x})\,\delta\mathcal{O}(s,\mathbf{y})\rangle\!\rangle, \quad \delta\mathcal{O} = \mathcal{O} - \langle\!\langle \mathcal{O}\rangle\!\rangle, \tag{181}$$

and $\langle\langle X\rangle\rangle = \mathrm{Tr}_{\mathrm{env}}[X\,\rho_{\mathrm{env}}]$ is the vacuum expectation value. The first-order term merely generates unitary evolution under the Hamiltonian of interaction Hamiltonian $V_{\mathrm{eff}} = \langle\langle V_{\mathrm{int}}\rangle\rangle$ and therefore cannot produce decoherence. Thus, our attention is directed to the second-order contribution, which yields the dominant decoherence effect.

This expression can be simplified into a Lindblad equation depending on how sharply peaked in time the environmental correlator, which depending on the Hamiltonian of interaction can be expressed as

$$\langle\langle \delta\mathcal{O}(t,x)\,\delta\mathcal{O}(s,y)\rangle\rangle = \mathcal{W}(t,x;\,s,y) = \begin{cases}\mu^4\,W(t,x;\,s,y)\\ 2\,g^2\,\big[\,W(t,x;\,s,y)\big]^2.\end{cases} \tag{182}$$

with $W(t,\mathbf{x};s,\mathbf{y}) = \left\langle\!\!\left\langle \delta\mathcal{O}(t,\mathbf{x})\,\delta\mathcal{O}(s,\mathbf{y})\right\rangle\!\!\right\rangle$.

An alternative to the expression above requires more than simply expanding in V_{int} using perturbation theory. What is additionally needed is a clear separation of scales, or so-called hierarchy of scales, which allow us to consider that the bath changes much faster than the system changes (which concerns the decoherence timescale τ), and which allow us to implement the Markovian approximation. In the present context that separation is provided by the ratio of the Hubble scale (which determines the size of environmental correlations) to the decoherence timescale τ (which depends on $\mu \ll H$ or $g \ll H$). If the correlator $\langle\langle \delta\mathcal{O}(t,x)\,\delta\mathcal{O}(s,y)\rangle\rangle$ decays rapidly for $H|t-s| \gg 1$, the evolution for time intervals exceeding H^{-1} allows the remainder of the integrand of eq. (180) to be expanded as a Taylor series around $s = t$. Successive terms are suppressed given that $(H\partial_t)^n \ll 1$ when acting on what remains of the integrand. This leads to an overall contribution diminished by powers of $(H\tau)^{-1}$, corresponding to $H^{-1} \ll \tau$. Thus, if the correlators in (182) are sharply peaked, we can expand them, dropping the subdominating terms, neglect the memory effects and treat the evolution as Markovian,

$$\partial_t\varrho(t) \approx -\,i\big[V_{\mathrm{eff}}(t),\varrho(t)\big]\; - \int d^3x\,d^3y\;a^6(t)\,\kappa(t,\mathbf{x},\mathbf{y})\Big[\Big\{\sigma(t,\mathbf{x})\,\sigma(t,\mathbf{y}),\varrho(t)\Big\} - \,2\,\sigma(t,\mathbf{y})\,\varrho(t)\,\sigma(t,\mathbf{x})\Big], \tag{183}$$

with

$$V_{\mathrm{eff}}(t) = \int d^3x\,a^3(t)\,\sigma(t,\mathbf{x})\,\langle\langle\mathcal{O}(t,\mathbf{x})\rangle\rangle + \int d^3x\,d^3y\,a^6(t)\,h(t,\mathbf{x},\mathbf{y})\,\sigma(t,\mathbf{x})\,\sigma(t,\mathbf{y}), \tag{184}$$

where we have the expression for the Lamb-shift and dissipator kernels,

$$\kappa(t,\mathbf{x},\mathbf{y}) = \frac{1}{2}\left[C(t,\mathbf{x},\mathbf{y}) + C^*(t,\mathbf{y},\mathbf{x})\right] \quad \text{and} \quad h(t,\mathbf{x},\mathbf{y}) = -\frac{i}{2}\left[C(t,\mathbf{x},\mathbf{y}) - C^*(t,\mathbf{y},\mathbf{x})\right] \tag{185}$$

and

$$C(t,\mathbf{x},\mathbf{y}) := \int_{t_0}^{t} \mathrm{d}s \, \langle\!\langle \delta\mathcal{O}(t,\mathbf{x})\, \delta\mathcal{O}(s,\mathbf{y}) \rangle\!\rangle, \tag{186}$$

and where we consider $H^{-1}, \tau \ll \sigma_t$, so that $f(s) \to f(t)$, and we have a relatively constant temporal envelope.

We will not focus on that, but we can have another simplification if the correlation function as a function of position also falls off sufficiently quickly as a function of position. If the falloff is sufficiently steep the spatial integrals are well-approximated by expanding any fields evaluated at position y in powers of $|y-x|$ and the leading order evolution equation becomes local in space. In a sense, beside Markovianity (or a notion of "temporal locality"), "spatial locality" can also arise upon decoherence in this picture.

We will now express the equation (183) in the k-space,[88] First note that translation invariance leads to the following expression for the field operator

$$\phi(t,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[v_k(t)\, c_{\mathbf{k}} + v_k^*(t)\, c_{-\mathbf{k}}^* \right] e^{i\mathbf{k}\cdot\mathbf{x}} \,, \tag{187}$$

and similarly for $\sigma(t,\mathbf{x})$ in terms of $a_{\mathbf{k}}$ and $a_{-\mathbf{k}}^*$ and mode functions $u_k(t)$. As usual, the ladder operators satisfy

$$[a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger] = \delta^3(\mathbf{p}-\mathbf{q}) \quad \text{and} \quad [c_{\mathbf{p}}, c_{\mathbf{q}}^\dagger] = \delta^3(\mathbf{p}-\mathbf{q}). \tag{188}$$

Now, we have

$$\partial_t \varrho(t) = -i\Big[V_{\text{eff}}(t), \varrho(t)\Big] - a^6(t) \int \mathrm{d}^3\mathbf{k}\, \kappa_{\mathbf{k}}(t) \Bigg[\{\sigma_{\mathbf{k}}(t)\sigma_{-\mathbf{k}}(t), \varrho(t)\} - \sigma_{\mathbf{k}}(t)\varrho(t)\sigma_{-\mathbf{k}}(t) - \sigma_{-\mathbf{k}}(t)\varrho(t)\sigma_{\mathbf{k}}(t) \Bigg] \tag{189}$$

with

$$\sigma_{\mathbf{k}}(t) = \sigma_{-\mathbf{k}}^*(t) = u_{\mathbf{k}}(t)\, a_{\mathbf{k}} + u_{\mathbf{k}}^*(t)\, a_{-\mathbf{k}}^\dagger, \tag{190}$$

$$\kappa_{\mathbf{k}}^f(t) = \int_{t_0}^{t} ds \; \Re\big[\, W_{\mathbf{k}}(t,s)\big] \,, \tag{191}$$

[88] We should distinguish between the co-moving wavelength and momentum, where the co-moving momentum is $k = |\mathbf{k}|$ and the corresponding co-moving wavelength is $\lambda_{\text{com}} = \frac{2\pi}{k}$, from the physical wavelength and momentum, which are time-dependent: $\lambda_{\text{phys}}(t) = \frac{2\pi}{p(t)} = \frac{2\pi\, a(t)}{k}$, with $p(t) = \frac{k}{a(t)}$. The co-moving momentum k is most convenient for solving the field equations on an expanding background, since each Fourier mode decouples and k remains constant in time. The physical momentum $p(t)$, on the other hand, redshifts with the expansion and is what we compare to physical scales like the Hubble radius. Hubble crossing occurs when the physical wavelength equals the Hubble radius, i.e., $\lambda_{\text{phys}}(t_*) = \frac{1}{H} \iff \frac{2\pi\, a(t_*)}{k} = \frac{1}{H} \iff k = 2\pi\, a(t_*)\, H$. Equivalently, in terms of physical momentum, this crossing condition is: $p(t_*) = \frac{k}{a(t_*)} = 2\pi\, H$.

with,

$$V_{\rm eff}(t) = (2\pi)^{3/2} a^3(t)\, \langle\!\langle \mathcal{O}(t,\mathbf{x}) \rangle\!\rangle\, \sigma_{k=0}(t) + \int d^3k\, a^6(t)\, h^f_{\mathbf{k}}(t)\, \sigma_{\mathbf{k}}(t)\, \sigma_{-\mathbf{k}}(t)\,, \tag{192}$$

and

$$h^f_{\mathbf{k}}(t) = \int_{t_0}^{t} ds\, \Im\big[\, W_{\mathbf{k}}(t,s)\big]\,. \tag{193}$$

Having the right-hand sides of (192) and (184) be quadratic in $\sigma_{\mathbf{k}}$ ensures that there is no mode–mixing, so that the state for each momentum mode $\mathbf{k}$ remains uncorrelated as time evolves, provided this was true of the initial conditions. In particular, if one starts with $\varrho(t_0) = \bigotimes_{\mathbf{k}} \varrho_{\mathbf{k}}(t_0)$, then the factorized form is preserved, $\varrho(t) = \bigotimes_{\mathbf{k}} \varrho_{\mathbf{k}}(t)$, and thus (183) can be written as a separate evolution equation for each mode's density matrix:

$$\partial_t \varrho_{\mathbf{k}}(t) = -i\Big[V_{\rm eff}(t), \varrho_{\mathbf{k}}(t)\Big] - a^6(t)\, \kappa_{\mathbf{k}}(t) \Bigg[\{\sigma_{\mathbf{k}}(t)\sigma_{-\mathbf{k}}(t), \varrho_{\mathbf{k}}(t)\} \\ - \sigma_{\mathbf{k}}(t)\varrho_{\mathbf{k}}(t)\sigma_{-\mathbf{k}}(t) - \sigma_{-\mathbf{k}}(t)\varrho_{\mathbf{k}}(t)\sigma_{\mathbf{k}}(t)\Bigg]. \tag{194}$$

A consequence of (189) and (190), which is quadratic in $\sigma_{\mathbf{k}}$ is that if systems start as a Gaussian, stay as a Gaussian. Thus, we can solve the evolution of (194) through the following Gaussian ansatz written in the field amplitude basis $\{|\sigma\rangle, |\tilde{\sigma}\rangle\}$,

$$\langle \sigma | \varrho_{\mathbf{k}}(t) | \tilde{\sigma}\rangle = \mathcal{Z}_{\mathbf{k}}(t) \exp\big[-A_{\mathbf{k}}(t)\, \sigma^*\sigma - A^*_{\mathbf{k}}(t)\, \tilde{\sigma}^*\tilde{\sigma} + B_{\mathbf{k}}(t)\, \sigma\tilde{\sigma} + B^*_{\mathbf{k}}(t)\, \sigma^*\tilde{\sigma}^* \big], \tag{195}$$

with the following evolution that is equivalent to the evolution in eq. (194),

$$\begin{aligned} \partial_t A_{\mathbf{k}} &= -\frac{i}{a^3}\big(A^2_{\mathbf{k}} - |B_{\mathbf{k}}|^2\big) + a^3 \Big[i\big(m^2 + \tfrac{k^2}{a^2}\big) + a^3 h_{\mathbf{k}} \Big] + a^6\, \kappa_{\mathbf{k}}(t), \\ \partial_t B_{\mathbf{k}} &= -\frac{i}{a^3}\big(A_{\mathbf{k}} - A^*_{\mathbf{k}}\big) B^*_{\mathbf{k}} + a^6\, \kappa_{\mathbf{k}}(t), \end{aligned} \tag{196}$$

where $A_k(t_0) = A_{k0}$ and $B_k(t_0) = B_{k0}$ with $A_{\mathbf{k}0} + A^*_{\mathbf{k}0} = \frac{1}{|u_{\mathbf{k}}|^2}$ and $B_{\mathbf{k}0} = 0$, whose exact solution yields the expression for purity at late times,

$$\gamma_{\mathbf{k}}(t) \;:=\; \mathrm{Tr}\big[\rho^2_{\mathbf{k}}(t)\big] \;=\; \int d\sigma\, d\sigma^*\, \langle \sigma \,|\, \rho^2_{\mathbf{k}}(t) \,|\, \sigma\rangle \;=\; \frac{1-\mathcal{R}_{\mathbf{k}}}{1+\mathcal{R}_{\mathbf{k}}}, \quad \mathcal{R}_{\mathbf{k}} := \frac{\mathcal{B}_{\mathbf{k}} + \mathcal{B}^*_{\mathbf{k}}}{\mathcal{A}_{\mathbf{k}} + \mathcal{A}^*_{\mathbf{k}}}\,. \tag{197}$$

The evolution of purity at late times is given by

$$\begin{aligned} \partial_t \gamma_{\mathbf{k}} &= 2\, \frac{(B_{\mathbf{k}} + B^*_{\mathbf{k}})\, \partial_t(A_{\mathbf{k}} + A^*_{\mathbf{k}}) - (A_{\mathbf{k}} + A^*_{\mathbf{k}})\, \partial_t(B_{\mathbf{k}} + B^*_{\mathbf{k}})}{(A_{\mathbf{k}} + A^*_{\mathbf{k}} + B_{\mathbf{k}} + B^*_{\mathbf{k}})^2} \\ &= -\, \frac{4\, a^6\, \kappa_{\mathbf{k}}\, \gamma_{\mathbf{k}}}{A_{\mathbf{k}} + A^*_{\mathbf{k}} + B_{\mathbf{k}} + B^*_{\mathbf{k}}}\,. \end{aligned} \tag{198}$$

Through the above equations, it can be shown [9] that at late times ($-k\eta \ll 1$ or in cosmic time, $t \gg t_* + \frac{1}{H}$ with t_* being the Hubble crossing time with $t_* = \frac{1}{H}\ln(\frac{k}{H})$), given the system initially in a Bunch-Davies vacuum, the purity becomes minimal, and the system decoheres. Furthermore, upon decoherence, we get that $\varrho_{\mathbf{k}}$ becomes a mixture of field amplitude states whose diagonal terms are given by,

$$\langle\sigma|\varrho_{\mathbf{k}}|\sigma\rangle = \mathcal{Z}_{\mathbf{k}} \exp[-(A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t))|\sigma|^2], \tag{199}$$

with $|\sigma\rangle$ being the field amplitude basis, and where properly normalized we have

$$\begin{aligned}\varrho_{\mathbf{k}} = \frac{1}{\pi}\int_{\mathbb{C}} d^2\sigma \left(A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t)\right) \\ \times \exp\left[-\left(A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t)\right)|\sigma|^2\right]|\sigma\rangle\langle\sigma|,\end{aligned} \tag{200}$$

Note that $A_{\mathbf{k}} + A^*_{\mathbf{k}} - B_{\mathbf{k}} - B^*_{\mathbf{k}}$ is a fixed point of the late time evolution, which is found by solving equations (196), and yields a finite value. This leads to a stochastic process that probabilistically selects one of the terms of this mixture.[89]

We can use the state above to calculate the purity at late time for the diverse fields. For example, we end up with the following expression for the purity of the target field for the case of a massless environment,

$$\gamma_k(\eta) \;\simeq\; \left[1 \;+\; \frac{g^2}{32\pi^2\, H^2 \nu_{\rm sys}}\left|\, 2^{\nu_{\rm sys}}\Gamma(\nu_{\rm sys})\right|^2 (-k\eta)^{-2\nu_{\rm sys}}\right]^{-1}. \tag{201}$$

We will now show that the following single–mode Gaussian density operator is homogeneous and isotropic,

$$\varrho_{\mathbf{k}}(t) = \frac{\alpha_k(t)}{\pi}\int_{\mathbb{C}} d^2\sigma\, \exp\left[-\alpha_k(t)\,|\sigma|^2\right]|\sigma\rangle_{\mathbf{k}}\langle\sigma|, \tag{202}$$

where

$$\alpha_k(t) = A_{\mathbf{k}}(t) + A^*_{\mathbf{k}}(t) - B_{\mathbf{k}}(t) - B^*_{\mathbf{k}}(t) \tag{203}$$

is a positive real function that depends only on the magnitude $k = |\mathbf{k}|$. The full multimode state is $\varrho = \bigotimes_{\mathbf{k}} \varrho_{\mathbf{k}}(t)$.

The field–amplitude operator for a real scalar field is

$$\hat{\sigma}_{\mathbf{k}}(t) = u_k(t)\,\hat{a}_{\mathbf{k}} + u^*_k(t)\,\hat{a}^\dagger_{-\mathbf{k}}, \tag{204}$$

with commutator $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{q}}] = \delta^3(\mathbf{k} - \mathbf{q})$. The field amplitude state $|\sigma\rangle_{\mathbf{k}}$ is its eigenvector,

$$\hat{\sigma}_{\mathbf{k}}(t)\,|\sigma\rangle_{\mathbf{k}} = \sigma\,|\sigma\rangle_{\mathbf{k}}, \tag{205}$$

and obeys the resolution of the identity

$$\int_{\mathbb{C}} \frac{d^2\sigma}{\pi}\,|\sigma\rangle_{\mathbf{k}}\langle\sigma| = \mathbb{I}_{\mathbf{k}}. \tag{206}$$

[89] Notice that when we have decoherence, we have approximatelly Markovian dynamics. This was discussed in X.

To check translation invariance, let $\hat{T}(\mathbf{a}) = e^{-i\mathbf{a}\cdot\hat{\mathbf{P}}}$. It acts on the ladder operators as

$$\hat{T}(\mathbf{a})\,\hat{a}_{\mathbf{k}}\,\hat{T}^{\dagger}(\mathbf{a}) = e^{-i\mathbf{k}\cdot\mathbf{a}}\,\hat{a}_{\mathbf{k}}, \qquad \hat{T}(\mathbf{a})\,\hat{a}^{\dagger}_{\mathbf{k}}\,\hat{T}^{\dagger}(\mathbf{a}) = e^{+i\mathbf{k}\cdot\mathbf{a}}\,\hat{a}^{\dagger}_{\mathbf{k}}, \tag{207}$$

and hence on the amplitude operator

$$\hat{T}(\mathbf{a})\,\hat{\sigma}_{\mathbf{k}}(t)\,\hat{T}^{\dagger}(\mathbf{a}) = e^{-i\mathbf{k}\cdot\mathbf{a}}\,u_k\,\hat{a}_{\mathbf{k}} + e^{+i\mathbf{k}\cdot\mathbf{a}}\,u_k^*\,\hat{a}^{\dagger}_{-\mathbf{k}}. \tag{208}$$

Acting with $\hat{T}(\mathbf{a})$ on $\varrho_{\mathbf{k}}(t)$ amounts to replacing each projector $|\sigma\rangle\langle\sigma|$ by $\left|e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma\right\rangle\left\langle e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma\right|$. Changing variables $\sigma' = e^{-i\mathbf{k}\cdot\mathbf{a}}\sigma$, with $|\sigma'| = |\sigma|$ and unit Jacobian, shows that

$$\hat{T}(\mathbf{a})\,\varrho_{\mathbf{k}}(t)\,\hat{T}^{\dagger}(\mathbf{a}) = \varrho_{\mathbf{k}}(t), \tag{209}$$

for every $\mathbf{a}$. Thus the full state satisfies

$$\hat{T}(\mathbf{a})\,\varrho\,\hat{T}^{\dagger}(\mathbf{a}) = \varrho, \tag{210}$$

which is the operator statement of statistical homogeneity.

To check rotation invariance, let $\hat{R}(\Lambda)$ implement $\mathbf{k} \mapsto \Lambda\mathbf{k}$. Then

$$\hat{R}(\Lambda)\,\hat{\sigma}_{\mathbf{k}}(t)\,\hat{R}^{\dagger}(\Lambda) = \hat{\sigma}_{\Lambda\mathbf{k}}(t), \tag{211}$$

and acting on each mode's density operator gives

$$\hat{R}(\Lambda)\,\varrho_{\mathbf{k}}(t)\,\hat{R}^{\dagger}(\Lambda) = \frac{\alpha_k}{\pi}\int d^2\sigma\, e^{-\alpha_k|\sigma|^2}\,|\sigma\rangle_{\Lambda\mathbf{k}}\langle\sigma|_{\Lambda\mathbf{k}}. \tag{212}$$

Relabelling $\mathbf{k}' = \Lambda\mathbf{k}$ in the tensor product $\bigotimes_{\mathbf{k}}\varrho_{\mathbf{k}}(t)$ and using $\alpha_{|\Lambda^{-1}\mathbf{k}'|} = \alpha_{|\mathbf{k}'|}$ shows

$$\hat{R}(\Lambda)\,\varrho\,\hat{R}^{\dagger}(\Lambda) = \varrho \quad \forall\,\Lambda \in SO(3), \tag{213}$$

which shows that the state is isotropic. Hence, the mixed state (202) is both homogeneous and isotropic.

Now, we want that interaction between systems, where some of them belong to SDCs, to give rise to them emitting a gravitational field. This interaction is modelled via decoherence. We will focus on the case of the de Sitter spacetime at late times, where decoherence occurs at super-horizon scales. However, for simplicity, we will begin with the case of flat spacetime and later show that this proof also holds for a de Sitter spacetime and for other scalar fields. We will assume weak interactions, as is assumed in the framework that we use to simplify the mode equations.

A key step is to see if the state,

$$\langle\sigma_{\mathbf{k}}(\eta)|\hat{\rho}_{\mathbf{k}}(\eta)|\tilde{\sigma}_{\mathbf{k}}\rangle = Z_k(\eta)\exp\left[-A_{\mathbf{k}}(\eta)|\sigma_{\mathbf{k}}|^2 - A^*_{\mathbf{k}}(\eta)|\tilde{\sigma}_{\mathbf{k}}|^2 + B_{\mathbf{k}}(\eta)\sigma_{\mathbf{k}}\tilde{\sigma}_{\mathbf{k}} + B^*_{\mathbf{k}}(\eta)\sigma^*_{\mathbf{k}}\tilde{\sigma}^*_{\mathbf{k}}\right] \tag{214}$$

upon decoherence at late times is Hadamard where $Z_k(\eta) = \frac{C_k(\eta)}{\pi}$, $\quad C_k(\eta) = A_k + A_k^* - B_k - B_k^*$. This is because in order to have a renormalizable expectation value of the stress energy tensor of a system in the state ρ (which can emit a gravitational field), the state should be Hadamard or at least differ from a Hadamard state by a C^4 function at increasingly lower distances. The above state ρ was derived under the assumption of approximate Markovian evolution at late times, but we will not make that assumption in our Hadamard test. We will use the exact mode equations to analyze the the decohered states (not the master equations), and not make any assumptions about non-Markovianity.

A test to see if ρ (or more precisely what ρ approximates) is Hadamard using the Hamiltonian formalism does not involve having to calculate the unequal time correlation function and compare the two-point correlation function of ρ with the two-point correlation function of some other Hadamard state (such as the Bunch-Davies vacuum), and see if their difference (i.e., $W_\psi(t,x;t,x') - W_{BD}(t,x;t,x')$ yields a continuously differentiable function when $x \to x'$ and $t \to t'$. A different related Hadamard test easier to implement to our case compares $W_\rho(t,x;t,x') - W_{BD}(t,x;t,x')$ at a single t. However, to implement this test we should also compare $\partial_t(W_\psi(t,x;t',x') - W_{BD}(t,x;t',x'))_{t=t'}$ and $\partial_t\partial_{t'}(W_\psi(t,x;t',x') - W_{BD}(t,x;t',x'))_{t=t'}$ at some given time slice, and find that they are smooth. If this is the case a quasi-free state $|\psi\rangle$ is Hadamard since $W_\psi(t,x;t,x') - W_{BD}(t,x;t,x')$ satisfies the equation of motion in both (t,x) and (t',x'). This feature is because smooth initial data for the equation of motion implies a smooth solution.[90]

Let us begin with the linear interaction $\sigma\phi$.[91] We start from the exact reduced Gaussian density matrix for a single momentum pair $(\mathbf{k}, -\mathbf{k})$ obtained after tracing out ϕ [8]:

$$\langle\sigma_\mathbf{k}|\rho_{\mathrm{red},\mathbf{k}}(t)|\tilde\sigma_\mathbf{k}\rangle = \mathcal{N}_\sigma \exp\Big[-\alpha_\mathbf{k}\,\sigma_\mathbf{k}\sigma_{-\mathbf{k}} - \alpha^*_\mathbf{k}\,\tilde\sigma_\mathbf{k}\tilde\sigma_{-\mathbf{k}} + \beta_\mathbf{k}\big(\sigma_\mathbf{k}\tilde\sigma_{-\mathbf{k}} + \tilde\sigma_\mathbf{k}\sigma_{-\mathbf{k}}\big)\Big]. \tag{215}$$

We consider the following three terms, which arises from the so-called Ricatti trick [8] to solve the dynamical equations for the above density operator,

$$\alpha_\mathbf{k}(t) = \Gamma^{2,\sigma}_\mathbf{k}(t) - \frac{\big|\Gamma^{2,\mathrm{mix}}_\mathbf{k}(t)\big|^2}{\Gamma^{2,\phi}_\mathbf{k}(t)}, \qquad \beta_\mathbf{k}(t) = \frac{\big|\Gamma^{2,\mathrm{mix}}_\mathbf{k}(t)\big|^2}{2\,\Gamma^{2,\phi}_\mathbf{k}(t)}, \tag{216}$$

$$\Gamma^{2,\mathrm{mix}}_\mathbf{k}(t) = i\,\frac{\mu^2}{\omega_{k,\sigma} + \omega_{k,\phi}}\Big[1 - e^{-i(\omega_{k,\sigma}+\omega_{k,\phi})(t-t_0)}\Big]. \tag{217}$$

Because the fields are real, $\sigma_{-\mathbf{k}} = \sigma^*_\mathbf{k}$. Setting $\tilde\sigma_\mathbf{k} = \sigma_\mathbf{k} \equiv z$ and noting

$$\alpha_\mathbf{k} + \alpha^*_\mathbf{k} = 2\alpha_{\mathbf{k},R}, \tag{218}$$

(where the subscript R stands for real) the diagonal of (215) reduces to the following complex Gaussian

$$\rho_{\mathrm{diag}}(z) = \langle z|\rho_{\mathrm{red},\mathbf{k}}(t)|z\rangle = \mathcal{N}_\sigma \exp\big[-2(\alpha_{\mathbf{k},R} - \beta_\mathbf{k})|z|^2\big]. \tag{219}$$

[90] A slight subtlety here is that on a general spacetime, the singular behavior of equal time correlation functions of Hadamard states does depend on the geometry in a neighborhood of such an equal time surface (for instance via time derivatives of the metric), and for this reason, checking that a state is Hadamard from its equal time correlation functions is not always convenient. However, if we already know the correlation functions of a reference Hadamard state (as we do), this subtlety is already taken care of.

[91] We will ignore test functions for simplicity.

It is convenient to introduce

$$C_{\mathbf{k}}(t) \equiv 2\big[\alpha_{\mathbf{k},R}(t) - \beta_{\mathbf{k}}(t)\big], \qquad A_{\mathbf{k}} > 0. \tag{220}$$

Complex Gaussian integrals then give

$$I_0 = \int d^2z\, e^{-A|z|^2} = \frac{\pi}{A}, \qquad I_1 = \int d^2z\, |z|^2 e^{-A|z|^2} = \frac{\pi}{A^2}, \tag{221}$$

and the normalisation is fixed by $\mathrm{Tr}\,\rho = 1$ to be

$$\mathcal{N}_\sigma = \frac{C_{\mathbf{k}}(t)}{\pi}. \tag{222}$$

Inserting (221) and (222) into (219) one obtains the equal–time two–point function of the reduced state,

$$\langle \sigma_{\mathbf{k}} \sigma_{-\mathbf{k}} \rangle_\rho = \frac{1}{C_{\mathbf{k}}(t)} = \frac{1}{2\big[\alpha_{\mathbf{k},R}(t) - \beta_{\mathbf{k}}(t)\big]}. \tag{223}$$

For the free Bunch–Davies (BD) vacuum the equal–time Wightman function is

$$G_{\mathrm{BD}}(t;\mathbf{x},\mathbf{x}') = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}{2\,\omega_\sigma(\mathbf{k})}, \qquad \omega_\sigma(\mathbf{k}) = \sqrt{k^2+m^2}. \tag{224}$$

Subtracting (224) from the reduced–state propagator built out of (223) gives

$$\Delta G(t;\Delta\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\Delta\mathbf{x}} \Big[\frac{1}{2\big[\alpha_{\mathbf{k},R}(t) - \beta_{\mathbf{k}}(t)\big]} - \frac{1}{2\,\omega_\sigma(\mathbf{k})}\Big]. \tag{225}$$

In order to estimate the large–k tail we need the explicit free–mode kernels. Given

$$\Gamma^2_{\mathbf{k}}(t) = -\,i\,\frac{\dot u_{\mathbf{k}}(t)}{u_{\mathbf{k}}(t)}, \qquad u_{\mathbf{k}}(t) = \frac{e^{-i\omega_{k,\phi}t}}{\sqrt{2\omega_{k,\phi}}}, \qquad \omega_{k,\phi} = \sqrt{k^2+M^2}, \tag{226}$$

by differentiating

$$\dot u_{\mathbf{k}}(t) = \frac{1}{\sqrt{2\,\omega_{k,\phi}}}\big(-i\,\omega_{k,\phi}\big)\, e^{-i\,\omega_{k,\phi}t}, \tag{227}$$

$$\frac{\dot u_{\mathbf{k}}(t)}{u_{\mathbf{k}}(t)} = -\,i\,\omega_{k,\phi}. \tag{228}$$

we obtain that the diagonal kernels are real and equal to the frequencies,

$$\Gamma^{2,\phi}_{\mathbf{k}}(t) = \omega_{k,\phi}, \qquad \Gamma^{2,\sigma}_{\mathbf{k}}(t) = \omega_{k,\sigma}. \tag{229}$$

With (229), and given

$$\alpha_{\mathbf{k},R}(t) = \mathrm{Re}\,\Gamma^{2,\sigma}_{\mathbf{k}}(t) - \frac{\Big|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\Big|^2\ \mathrm{Re}\,\Gamma^{2,\phi}_{\mathbf{k}}(t)}{\Big|\Gamma^{2,\phi}_{\mathbf{k}}(t)\Big|^2} \tag{230}$$

the real part of (216) collapses to

$$\alpha_{\mathbf{k},R}(t) = \omega_{k,\sigma} - \frac{\left|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\right|^2}{\omega_{k,\phi}}, \qquad \beta_{\mathbf{k}}(t) = \frac{\left|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\right|^2}{2\,\omega_{k,\phi}}. \tag{231}$$

The exact mixing kernel (217) and the elementary inequality

$$|1-e^{i\theta}| = 2\left|\sin\tfrac{\theta}{2}\right| \le 2 \tag{232}$$

imply the bound

$$\left|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\right| \le \frac{2\mu^2}{\omega_{k,\sigma}+\omega_{k,\phi}} \implies \left|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\right| \lesssim \frac{\mu^2}{\omega_{k,\sigma}+\omega_{k,\phi}}. \tag{233}$$

For $|\mathbf{k}| \gg M, m$ the frequencies admit the large–momentum expansions

$$\omega_{k,\sigma} = k\Big[1+\frac{m^2}{2k^2}+\dots\Big], \qquad \omega_{k,\phi} = k\Big[1+\frac{M^2}{2k^2}+\dots\Big], \tag{234}$$

so that

$$\omega_{k,\sigma}+\omega_{k,\phi} = 2k\Big[1+\mathcal{O}(k^{-2})\Big], \qquad \left|\Gamma^{2,\mathrm{mix}}_{\mathbf{k}}(t)\right| \lesssim \frac{\mu^2}{2k}\Big[1+\mathcal{O}(k^{-2})\Big]. \tag{235}$$

Combining (231) with (235) one finds

$$\alpha_{\mathbf{k},R}(t)-\beta_{\mathbf{k}}(t) = k\Big[1+\mathcal{O}\big(\tfrac{\mu^4}{k^4}\big)\Big], \qquad \frac{1}{2[\alpha_{\mathbf{k},R}-\beta_{\mathbf{k}}]} - \frac{1}{2\omega_\sigma} = \mathcal{O}(k^{-5}). \tag{236}$$

Because the integrand of (225) falls as k^{-5}, every spatial derivative adds a factor k^n yet the momentum integral $\int d^3k\, k^{n-5}$ remains convergent for all integers n. Hence

$$\Delta G(t;\Delta\mathbf{x}) \in C^\infty \quad \text{as } \Delta\mathbf{x}\to 0. \tag{237}$$

Now, we need to look at the following difference in a time-slice:

$$\Delta W(t,t';\Delta\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\mathbf{k}\cdot\Delta\mathbf{x}}\, F_{\mathbf{k}}(t,t'), \qquad F_{\mathbf{k}}(t,t) = \mathcal{O}(k^{-5}).$$

A single time derivative on a mode function brings down $\pm i\omega_k \sim k$, so at the coincident slice $t'=t$

$$\partial_t F_{\mathbf{k}}(t,t')\big|_{t=t'} = \mathcal{O}(k^{-4}), \qquad \partial_t\partial_{t'} F_{\mathbf{k}}(t,t')\big|_{t=t'} = \mathcal{O}(k^{-3}), \tag{238}$$

both of which are still absolutely integrable in three dimensions. Thus, it is easy to see that applying the first and second time derivatives yields

$$\partial_t \Delta W(t,t';\Delta\mathbf{x})\big|_{t=t'} \in C^\infty, \qquad \partial_t\partial_{t'} \Delta W(t,t';\Delta\mathbf{x})\big|_{t=t'} \in C^\infty. \tag{239}$$

Therefore, smooth initial data for ΔW, $\partial_t\Delta W$ and $\partial_t\partial_{t'}\Delta W$ on the Cauchy slice $t=t'$ propagates as smooth solutions of the Klein–Gordon equation.

Moreover, establishes that the reduced Gaussian state constructed in (215) is Hadamard.

Let us turn to the cubic interaction Hamiltonian $\sigma\phi^2$. This case cannot be solved exactly and we need to use perturbation theory. To begin, split the Hamiltonian into a free quadratic part and a weak interaction,

$$H = H_0 + g\, H_{\text{int}}. \tag{240}$$

Let $\Psi_0[\phi, \sigma; t]$ satisfy the free Schrödinger equation

$$i\,\partial_t \Psi_0[\phi, \sigma; t] = H_0\, \Psi_0[\phi, \sigma; t]. \tag{241}$$

To first order in the coupling constant g the full wave-functional may be written as

$$\Psi[\phi, \sigma; t] = \big(1 - g\, \Delta[\phi, \sigma; t]\big)\, \Psi_0[\phi, \sigma; t], \tag{242}$$

which, when inserted into the interacting Schrödinger equation and terms of order g are collected, yields

$$\big(i\,\partial_t \Delta\big)\, \Psi_0 = [H_0, \Delta]\, \Psi_0 - H_{\text{int}}\, \Psi_0. \tag{243}$$

The pure density matrix in the field basis reads

$$\langle \phi, \sigma|\rho(t)|\tilde{\phi}, \tilde{\sigma}\rangle = \langle \phi, \sigma|\Psi(t)\rangle\langle\Psi(t)|\tilde{\phi}, \tilde{\sigma}\rangle, \tag{244}$$

and tracing out the environment ϕ gives

$$\langle\sigma|\rho_{\text{red}}(t)|\tilde{\sigma}\rangle = \int \mathcal{D}\phi\, (1 - g\, \Delta[\phi, \sigma; t])\, \Psi_0[\phi, \sigma; t]\, \Psi_0^*[\phi, \tilde{\sigma}; t]\, (1 - g\, \Delta^*[\phi, \tilde{\sigma}; t]). \tag{245}$$

We express the free wave-functional in terms of Gaussians for each field,

$$\Psi_0[\phi, \sigma; t] = \Psi_G[\phi]\, \Psi_G[\sigma], \qquad \Psi_G[\phi] = N_G(t)\, \exp\Big[-\tfrac{1}{2}\sum_q \Gamma^{\phi}_{2,q}(t)\, \phi_q \phi_{-q}\Big], \tag{246}$$

so that (245) becomes

$$\langle\sigma|\rho_{\text{red}}(t)|\tilde{\sigma}\rangle = \int \prod_k \mathcal{D}^2\phi_k\, \big(1 - \Delta[\phi, \sigma]\big)\big(1 - \Delta^*[\phi, \tilde{\sigma}]\big)\, \big|\Psi_G[\phi]\big|^2\, \Psi_G[\sigma]\, \Psi_G^*[\tilde{\sigma}], \tag{247}$$

where $\Delta[\phi, \sigma] \equiv \Delta[\phi, \sigma; t]$. Finally, a cubic-order ansatz for the non-Gaussian correction is

$$\Delta[\phi, \sigma; t] = \sum_q \Gamma^1_q(t)\, \sigma_{-q} + \tfrac{1}{2} \sum_{q_1, q_2, q_3} \Gamma^3_{q_1, q_2, q_3}(t)\, \phi_{q_1}\, \phi_{q_2}\, \sigma_{q_3} \tag{248}$$

with

$$\Gamma^3_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3}(t) = \left(-\frac{2i\, g}{\sqrt{V}}\right) \frac{1}{u_{\mathbf{q}_1}(t)\, u_{\mathbf{q}_2}(t)\, v_{\mathbf{q}_3}(t)} \int_{t_0}^{t} dt'\; u_{\mathbf{q}_1}(t')\, u_{\mathbf{q}_2}(t')\, v_{\mathbf{q}_3}(t')\,. \tag{249}$$

The reduced density matrix obtained in the Schrödinger picture is

$$\langle\sigma_{\mathbf{k}}|\rho_{\mathrm{red},\mathbf{k}}(t)|\tilde{\sigma}_{\mathbf{k}}\rangle = \mathcal{N}_k(t)\,\exp\Big[-\tfrac{1}{2}(\sigma_{\mathbf{k}},\tilde{\sigma}_{\mathbf{k}})\mathbf{R}_k(t)\begin{pmatrix}\sigma_{-\mathbf{k}}\\ \tilde{\sigma}_{-\mathbf{k}}\end{pmatrix}\Big], \tag{250}$$

$$\mathbf{R}_k(t) = \begin{pmatrix}\Gamma^{2,\sigma}_{\mathbf{k}}(t)-\mathcal{M}_{\mathbf{k}}(t) & -\mathcal{M}_{\mathbf{k}}(t)\\ -\mathcal{M}_{\mathbf{k}}(t) & \Gamma^{2,\sigma*}_{\mathbf{k}}(t)-\mathcal{M}_{\mathbf{k}}(t)\end{pmatrix}, \qquad \mathcal{M}_{\mathbf{k}}(t) = \sum_{\mathbf{q}_1,\mathbf{q}_2}\frac{\big|\Gamma^3_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{k}}(t)\big|^2}{2\,\Gamma^{2,\phi}_{R,\mathbf{q}_1}(t)\,\Gamma^{2,\phi}_{R,\mathbf{q}_2}(t)}. \tag{251}$$

With $\alpha_{\mathbf{k}} = \Gamma^{2,\sigma}_{\mathbf{k}} - \mathcal{M}_{\mathbf{k}}$, $\beta_{\mathbf{k}} = \mathcal{M}_{\mathbf{k}}$, the diagonal element that fixes the two–point function of the field σ is

$$A_{\mathbf{k}}(t) = 2\big[\alpha_{\mathbf{k},R}(t) - \beta_{\mathbf{k}}(t)\big], \qquad \langle\sigma_{\mathbf{k}}\sigma_{-\mathbf{k}}\rangle = \frac{1}{A_{\mathbf{k}}(t)} = \frac{1}{2\big[\alpha_{\mathbf{k},R}(t)-\beta_{\mathbf{k}}(t)\big]}. \tag{252}$$

We are interested in the ultraviolet regime $|\mathbf{k}| \gg m, M$. Then

$$\Gamma^{2,\sigma}_{\mathbf{k}} \simeq \omega_\sigma(\mathbf{k}), \qquad \omega_\sigma(\mathbf{k}) = k\Big[1 + \frac{m^2}{2k^2} + \dots\Big], \tag{253}$$

while $|\Gamma^3|^2 \propto g^2/k^2$. Using this and (249) for $|\mathbf{k}| \gg m, M$ again,[92] we obtain

$$\mathcal{M}_{\mathbf{k}}(t) = \frac{g^2 C(t)}{k}\Big[1 + \mathcal{O}(k^{-2})\Big], \tag{254}$$

where $C(t)$ is a k-independent variable, and therefore

$$\alpha_{\mathbf{k},R}(t) - \beta_{\mathbf{k}}(t) = \omega_\sigma(\mathbf{k}) - \frac{g^2 C(t)}{k} = k\Big[1 + \mathcal{O}(k^{-2})\Big]. \tag{255}$$

Its reciprocal expands as

$$\frac{1}{2[\alpha_{\mathbf{k},R} - \beta_{\mathbf{k}}]} = \frac{1}{2k}\Big[1 - \frac{m^2}{2k^2} + \frac{g^2 C(t)}{k^2} + \mathcal{O}(k^{-4})\Big], \tag{256}$$

whereas

$$\frac{1}{2\omega_\sigma(\mathbf{k})} = \frac{1}{2k}\Big[1 - \frac{m^2}{2k^2} + \frac{3m^4}{8k^4} + \mathcal{O}(k^{-6})\Big]. \tag{257}$$

The difference entering the two–point function of the reduced state is

$$F_k(t) = \frac{1}{2[\alpha_{\mathbf{k},R} - \beta_{\mathbf{k}}]} - \frac{1}{2\omega_\sigma(\mathbf{k})} = \frac{g^2 C(t)}{2\,k^3} + \mathcal{O}(k^{-5}), \tag{258}$$

[92] Given the following dynamical equation $i\partial_t\big(u_{\mathbf{q}_1}u_{\mathbf{q}_2}v_{\mathbf{k}}\,\Gamma^3_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{k}}\big) = (-2g/\sqrt{V})\,u_{\mathbf{q}_1}u_{\mathbf{q}_2}v_{\mathbf{k}}\,\delta_{\mathbf{q}_1+\mathbf{q}_2+\mathbf{k},\mathbf{0}}$ (see eq. (B.49d) in [8]) and given With $\kappa_s = \kappa_t = 0$, we obtain eq. (249). Because $|1-e^{i\theta}| \le 2$ we have $|\Gamma^3_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{k}}(t)| \le 2g/(\sqrt{V}\,\Omega_{12k})$ with $\Omega_{12k} = \omega_{\mathbf{q}_1,\phi} + \omega_{\mathbf{q}_2,\phi} + \omega_{\mathbf{k},\sigma}$; in the ultraviolet $\Omega_{12k} \simeq k$, so $|\Gamma^3|^2 \le 4g^2/(Vk^2)$. Inserting this bound in $\mathcal{M}_{\mathbf{k}}(t) = \sum_{\mathbf{q}_1,\mathbf{q}_2}|\Gamma^3_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{k}}(t)|^2/\big[2\,\Gamma^{2,\phi}_{R,\mathbf{q}_1}(t)\,\Gamma^{2,\phi}_{R,\mathbf{q}_2}(t)\big]$ and using $\Gamma^{2,\phi}_{R,q} \simeq \omega_{\mathbf{q},\phi}$ gives $\mathcal{M}_{\mathbf{k}}(t) = g^2 C(t)/k\,[1 + \mathcal{O}(k^{-2})]$, where $C(t) = \frac{1}{2}\sum_{\mathbf{q}_1,\mathbf{q}_2} 1/[\omega^2_{\mathbf{q}_1,\phi}\,\omega^2_{\mathbf{q}_2,\phi}]$ is k-independent and finite. Thus the leading behaviour is $\mathcal{M}_{\mathbf{k}}(t) \propto g^2/k$.

so the ultraviolet tail falls as k^{-3}.

The equal–time difference of Wightman functions is

$$\Delta G(t,r) = \frac{1}{2\pi^2 r}\int_0^\infty dk\; k\,\sin(kr)\,F_k(t), \qquad r = |\Delta\mathbf{x}|. \tag{259}$$

Although the integral above diverges at $k = 0$, at $k = 0$ the *exact* kernel is finite.[93]

For the ultraviolet portion, we insert the leading term in (258): $\Delta G_{\rm UV}(t,r) = \frac{g^2 C(t)}{4\pi^2 r}\int_1^\infty \frac{\sin(kr)}{k^2}\,dk = \frac{g^2 C(t)}{16\pi} + \mathcal{O}(r^2)$, since $\displaystyle\int_1^\infty \sin(kr)/k^2\,dk = \pi r/2 + \mathcal{O}(r^3)$. Terms beyond the $1/k^3$ tail in (258) give only higher powers of r^2. Putting both regions together, $\Delta G(t,r) = \mathrm{const}(t) + \mathcal{O}(r^2), \quad r\to 0$. The spatial or time derivatives act by multiplying the integrand by extra powers of k. Thus, $\Delta G, \quad \partial_t\Delta G, \quad \partial_t\partial_{t'}\Delta G \in C^\infty \quad (r\to 0)$, and the reduced Gaussian state (250) differs from the Bunch–Davies vacuum by a C^∞ function at coincidence, and is therefore Hadamard.

To adapt these results to de Sitter spacetime case, we should notice that in flat spacetime, given the ultraviolet limit we have

$$\Gamma_k^{2,\sigma} = k + \frac{M^2}{2k} + \mathcal{O}(k^{-3}), \qquad \Gamma_k^{2,\phi} = k + \frac{m^2}{2k} + \mathcal{O}(k^{-3}), \tag{260}$$

Now, in de Sitter spacetime case we write the metric $ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$ with $a(\eta) = -1/(H\eta)$ and rescale the fields $\tilde\sigma = a\sigma$, $\tilde\phi = a\phi$. Focusing for now in minimally coupled massive scalar fields, their mode equations read as

$$\tilde v_k'' + \Big[k^2 + a^2 M^2 - \frac{a''}{a}\Big]\tilde v_k = 0, \qquad \tilde u_k'' + \Big[k^2 + a^2 m^2 - \frac{a''}{a}\Big]\tilde u_k = 0, \tag{261}$$

with $a''/a = 2(aH)^2$. In the UV at sub-Horizon scales, we have $k \gg aH$, which gives

$$\tilde\Gamma_k^{2,\sigma} = k + \frac{a^2M^2}{2k} + \mathcal{O}(k^{-3}), \qquad \tilde\Gamma_k^{2,\phi} = k + \frac{a^2m^2}{2k} + \mathcal{O}(k^{-3}), \tag{262}$$

identical in structure to (260) apart from the smooth replacements $M^2 \to a^2M^2$, $m^2 \to a^2m^2$. The modulus square of the cubic kernel still carries the single factor

[93] Both potentially problematic factors in $F_k(t) = \frac{1}{2\left[\Gamma^{2,\sigma}_{\mathbf{k},R}(t) - \mathcal{M}_{\mathbf{k}}(t)\right]} - \frac{1}{2\,\omega_\sigma(\mathbf{k})}$ remain finite when $k\to 0$. The field σ frequency satisfies $\Gamma^{2,\sigma}_{\mathbf{k}}(t) \to \omega_\sigma(0) = m \neq 0$, and the cubic self-energy $\mathcal{M}_{\mathbf{k}}(t) = \sum_{\mathbf{q}_1,\mathbf{q}_2} \frac{\left|\Gamma^3_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{k}}(t)\right|^2}{2\,\Gamma^{2,\phi}_{R,\mathbf{q}_1}(t)\,\Gamma^{2,\phi}_{R,\mathbf{q}_2}(t)}$ is a bounded phase-space sum because $|\Gamma^3|^2$ is itself bounded and the denominator $\omega_\sigma(\mathbf{k}) + \omega_\phi(\mathbf{q}_1) + \omega_\phi(\mathbf{q}_2) \geq m + 2M > 0$. Hence $\mathcal{M}_{\mathbf{k}}(t) \to \mathcal{M}_0(t) < \infty$. Because both terms remain finite, the combination $\Gamma^{2,\sigma}_{\mathbf{k},R}(t) - \mathcal{M}_{\mathbf{k}}(t)$ tends to a nonzero constant and $F_k(t) \to F_0(t)$. In the equal-time integrand $k\,\sin(k\,r)\,F_k(t) = F_0(t)\,r\,k^2 + \mathcal{O}(k^4)$, the factor k^2 makes $\int_0^\varepsilon dk\,k^2$ finite, and every extra derivative adds further powers of k, preserving convergence at the lower limit. Neverthless, to check if the state is Hadamard, the infrared sector is irrelevant: Hadamard singularities concern the large-k (high-frequency) part of the spectrum, which determines the universal short-distance structure.

k^{-2}, so the self-energy retains

$$\tilde{\mathcal{M}}_k(\eta) = \frac{g^2 C(\eta)}{k} + \mathcal{O}(k^{-3}), \qquad C(\eta) = a^2(\eta) \times (\text{finite angular integral}). \tag{263}$$

With these ingredients one finds, as in flat space,

$$\tilde{\alpha}_{k,R} - \tilde{\beta}_k = k + \frac{a^2 M^2}{2k} - \frac{g^2 C(\eta)}{k} + \mathcal{O}(k^{-3}), \tag{264}$$

and therefore

$$\frac{1}{2[\tilde{\alpha}_{k,R} - \tilde{\beta}_k]} - \frac{1}{2\tilde{\omega}_\sigma(k)} = \frac{g^2 C(\eta)}{2k^3} + \mathcal{O}(k^{-5}). \tag{265}$$

The equal-time integrand similarly decays as k^{-5}; spatial derivatives raise the power of k but the momentum integrals remain convergent. The equal-time difference obeys the same estimates for its first and second conformal-time derivatives. Thus, the reduced Gaussian state of the field σ is Hadamard in de Sitter just as it is in Minkowski space. A similar argument could be given for massive or massless, conformally coupled or minimally coupled scalar fields, and the linear interaction case. Note that although this result was derived assuming perturbation theory and in this scenario, we get secular effects that invalidate the perturbative expansion, it was shown in [9] that in these situations the secular effects concern the IR modes and super-Horizon scales $|k\eta| \ll 1$, not the UV modes.

H Accounting for the universe's expansion and inflation via the time-varying dark energy

We will now briefly provide an effective toy model to explain how this theory may be able to account for the expansion of the universe, and inflation via the time-varying dark energy. The goal is to provide further arguments for this view, our derivation of the cosmological constant, and its time-varying features which depend on the four-volume of the universe.[94]

Consider the following FLRW metric,

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right], \tag{266}$$

[94] Note that at a certain scales in the early universe, we have coherent states, and we may argue that due to the smearing functions that they give rise to, we at least often need them for local decohering interactions and gravitation to occur. See Section 5 for a toy model. Therefore one could use those states to estimate the value of the cosmological constant even in the early universe due to their association with gravitation in a similar way to what we have seen in Section 7. Thus, the inference concerning the inflation-like effect of Λ in the early universe can in principle still hold. More on this at the end of this section.

where t is the cosmic time, $a(t)$ is the scale factor, k is the spatial curvature constant where $k = 0$ (flat), $k = +1$ (closed), $k = -1$ (open). (r, θ, ϕ) are comoving spatial coordinates.

According to this theory, we can assume that at $t = 0$, no quantum systems that belong to SDCs interacted, and because the gravitational field arises from these interactions, the above FLRW metric is not applicable. Since there are no interactions and we are modeling the whole universe, no metric except the flat metric is applicable,

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{267}$$

For simplicity, we can assume that at $t = 0$, we have only a target real scalar field ϕ_1, and a set of probes that have the DC concerning ϕ_1 (DC-ϕ_1). At this point, we have two choices. One of them assumes the cosmological constant as a brute fact that arises from SDCs. Once the system starts interacting, the cosmological constant kicks in, and its description is done via the Einstein Field Equations. One of the issues of this option is that we would also need to assume the inflaton or some other field that explains the accelerated expansion of the universe.

The second option does not require us to postulate this additional field that explains the expansion of the universe. It rather postulates a time-varying cosmological constant, as we have explained briefly in Section 7. This second view goes like this: once we have the first interaction between the ϕ_1 and the probes, a small four-volume will arise. Given that, in Planck units,

$$\Lambda \approx \frac{1}{\Delta V}, \tag{268}$$

we will get a high value of the cosmological constant and, therefore, a rapid expansion of the universe. This higher volume of the universe allows us to treat the energy-momentum tensor at higher scales as classical. We can suppose that once ϕ_1 or its mode in a homogeneous and isotropic coherent state $|\alpha\rangle_{\mathbf{k}=0}$ and that once they have a determinate energy-momentum tensor, it gives rise to a perfect fluid that leads to the FLRW metric (see Sections 5 and G). Then, we can can run the story briefly presented in Sections 5 and 7.

Note also that posing such special initial conditions at the beginning of the universe is common, and currently, any theory them. However, we think we may end up (under a more realistic and detailed model) having an advantage compared with these other theories because we do not have to postulate dark energy as a primitive or the inflaton field, and the prospects of this proposal are positive in terms of one day providing similar benefits to inflation without its issues. To see why, let us consider the two main problems inflation claims to solve: the flatness and the horizon problem.

The flatness problem arises from the observation that the current universe appears very close to being spatially flat (i.e., having zero curvature). Consider the Friedmann equation that governs the expansion of the universe, and which can be derived from the FLRW metric and the Einstein Field Equations with a perfect fluid as a source (restoring SI units):

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_R) - \frac{\kappa c^2}{a(t)^2} \tag{269}$$

where H is the Hubble parameter, which measures the expansion rate of the universe. The terms ρ_M and ρ_R represent the energy densities of matter and radiation, respectively. The parameter κ is the curvature of the universe, with $\kappa = 0$ for a flat universe, $\kappa > 0$ for a closed universe, and $\kappa < 0$ for an open universe. The scale factor, a, roughly describes the size of the universe at a given time.

The curvature term $-\kappa c^2/a(t)^2$ falls off as a^{-2}, while the energy densities of matter and radiation decay more rapidly with the scale factor. Specifically, $\rho_M \propto a^{-3}$ for matter, and $\rho_R \propto a^{-4}$ for radiation. This seems to imply that, as the universe expands and the scale factor a increases, the relative contribution of the curvature term becomes increasingly dominant over the energy densities of matter and radiation. Thus, the fact that we observe the universe to be so close to flat today suggests that the universe must have been very finely tuned to be near-flat in the early universe. This is because any small deviation from flatness would have grown over time, making the universe today either highly curved or very open.

The horizon problem is roughly the following: if we observe two widely separated parts of the Cosmic Microwave Background (CMB), we will see that we have distinct patches of the CMB that were causally disconnected at recombination (i.e., the period where protons and electrons combined to become atoms of hydrogen). However, we observe with high precision that they have a similar temperature. The problem is to explain how they have the same temperature if they were never in causally contact.

Now, let us turn to the Friedman equation with the cosmological constant,

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho_{matter/radiation} - \frac{\kappa c^2}{a(t)^2} + \frac{\Lambda c^2}{3}, \tag{270}$$

Let us consider that in the beginning of the universe $\Lambda \gg 1$, and we can treat it approximately as a large constant in this short period, and so this model is effective. In the early universe, due to its small volume and the (determinate) energy density of matter/radiation being low (because not many systems with determinate values are arising), it is thus plausible that

$$\frac{\Lambda c^2}{3} \gg \frac{8\pi G}{3}\rho, \kappa c^2. \tag{271}$$

Then, we obtain that

$$a(t) \approx A e^{\sqrt{\frac{\Lambda c^2}{3}}t}.\text{[95]} \tag{272}$$

[95] Note that the scale factor can be very small in the early universe, but the cosmological constant can be arbitrarily very large in such a way that it compensates that.

where A is a constant of integration.

This exponential expansion is similar to the exponential expansion predicted by inflation. This expansion, in principle, allows this theory to potentially address the horizon problem. The explanation goes like this: before the onset of inflation, the universe was much smaller and denser. In this phase, the entire region that would later become the observable universe was contained within a single causally connected patch. This means that any two points within this region could influence each other and reach thermal equilibrium. Exponential expansion stretched these regions beyond the current particle horizon. The particle horizon is the maximum distance from which particles could have traveled to an observer in the age of the universe. This means regions that were once close enough to interact and equilibrate went far apart, beyond each other's ability to *communicate*. In the case of this theory, this exponential expansion is due to the SDCs.

Let us turn to the sketch of the potential resolution of the flatness problem. To see how this theory might be able to deal with this problem, let us rewrite the Friedmann equation in the way below,

$$\Omega_{total} - 1 = \frac{kc^2}{a(t)^2 H^2} \tag{273}$$

where $\Omega(t) = \frac{\rho(t)}{\rho_{crit}(t)}$ with $\rho_{crit}(t)$ being the critical density defined as $3\tilde{m}_P^2 H^2(t)$, and we consider ρ to include the dark energy density. When the actual density and the critical density are equal, the geometry of the universe is flat. Thus, we consider that $\Omega = \Omega_{radiation} + \Omega_{matter} + \Omega_\Lambda$ (note that following the standard approach, we are including dark energy as part of the energy density of the universe). As we can see, in order for the universe to be flat ($k = 0$), $\Omega_{total} = 1$. Since $a(t) \approx e^{\sqrt{\frac{\Lambda c^2}{3}}t}$, we can see that with enough e-folds the early Friedmann universe, in principle, can become flat regardless of the initial densities of matter/energy.

Another problem that we will not go into deeply here, which inflation addresses, is the following: inflation is typically considered to have been driven by a scalar field ϕ which is the inflaton. It is hypothesized that the zero-point fluctuations of the quantized inflaton scalar field in some regions (i.e., fluctuations of the field in the vacuum state) and the associated energy-momentum fluctuations and gravitational field, amplified by the rapid expansion of inflation, attracted more matter than in other regions. Then, it is hypothesized that this phenomenon gave rise to the unevenly distributed cosmic structure in our universe (e.g., galaxy, galaxy clusters, etc.) [56]. Such explanation can, in principle, also be given via the above picture if we take into account that SDCs involve quantum fields that are subject to quantum fluctuations, which, upon stochastic processes, give rise to inhomogeneous states, as we have seen in Section 5.

Furthermore, note that the inflaton field is often treated classically, and the effects of these fluctuations are observed via slight temperature anisotropies in the Cosmic Microwave Background. There is also the problem of explaining how these quantum fluctuations became classical during the early stages of the evolution of the universe. Adopting this theory helps address this problem given that SDCs involve indeterministic processes that give rise to classicality. Furthermore, although this theory proposes a time-varying cosmological constant, current evidence is hinting towards a possible time-varying dark energy as we have mentioned before.

Whether this approach is better than the competing ones will need to be settled via a more physically realistic and detailed model, but we think it is promising. Several models impose a varying cosmological constant, such as quintessence models [93], and try to unify inflation and dark energy, such as inflationary quintessence models. However, to our knowledge, none predicts the precise value of the cosmological constant based on QT and with an economic ontology. For example, quintessence models add a new quantum field and, hence, a new particle (so far unobserved). This theory just starts from the basic principles of QT while solving the measurement problem, including the measurement problem that occurs right at the beginning of the universe. More concretely, note that in the models that are based on the inflaton or some other field, one needs to explain why (loosely speaking) there was a collapse of the quantum state at the beginning of the universe to account for inhomogeneities of matter distribution that gave rise to cosmic structures. Otherwise, all inflation gives us is a superposition of quantum states. Decoherence per se, which many appeal to, to solve this problem, does not solve the problem because it is a vaguely defined physical process. This theory, in principle, does not fall into this problem since it establishes clear criteria for when determinate values arise. Furthermore, if we adopt this approach, we do not need to fall into the issues of eternal inflation and the multiverse problem.

Future work should develop a more accurate cosmological model, which in principle would be able to address the cosmological singularity problem,[96] and further develop its empirical signatures in the Cosmic Microwave Background. Also, note that our dark energy cancellation hypothesis (Section 5) allows that in the early universe, we have significant fluctuations in the stress-energy tensor because the early universe has a very small past four-volume, balancing them. Future work should make this calculation in more detail, and examine how much fluctuations of the stress-energy tensor could be canceled.

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