Overlapping Scientific Consensus theory:
extending quasi-truth for scientific knowledge in historical sciences

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ABSTRACT:
The theory of quasi-truth was developed by Newton da Costa and collaborators as a more realistic account of truth, encompassing the incompleteness and inconsistency of scientific knowledge. Intuitively, the idea is that truth is reached when consensus is established at the end of inquiry; until that is reached, we have something less than the whole truth, we have partial or quasi-truth. Formally, the view faces some challenges that have been recently addressed in the literature; they concern a mismatch between the offered formalism and the expected claims to be formalized. In this paper we use inspiration from quasi-truth theory to develop an account of consensus in science encompassing the notion of quasi-truth. We not only present the formal system capturing the idea of a scientific consensus, but also show how quasi-truth may be represented within it too. We compare the original quasi-truth approach to ours, and argue that the latter is able to face some of the difficulties that plagued the former.

KEYWORDS: quasi-truth, historical sciences, consensus, historical announcement logic.

1. Introduction
The standard account of scientific theories, as incorporated in the semantic view of theories (see KRAUSE & ARENHART, 2017), has it that a scientific theory is a collection of models. Models, on their turn, are set theoretic structures, of the same kind that Tarski used to define truth for standard first-order formal languages. Such models are, in a sense, *complete*, they leave no question open concerning the truth or falsity of a sentence, and they are also *consistent,* they allow for no inconsistency. There is simply no question of whether an object has a certain property that is not already defined in the model. This is directly related to the fact that Tarskian theory of truth attempts to capture the Aristotelean views on truth.

A rival formal view of truth — quasi-truth — has been advanced in the works of Newton da Costa and collaborators (see MIKENBERG, DA COSTA & CHUAQUI 1986; DA COSTA &

FRENCH, 2003; BUENO & DA COSTA, 2007). Basically, the goal is to start from a pragmatic view of truth as a motivation, and to develop a formal account for such a view in the same lines as Tarski did for the Aristotelean view. Roughly speaking, the pragmatic account considers that truth is the goal of investigation, reached when consensus is achieved. Any time before that, we have incomplete knowledge, which, the story goes, is progressively filled up until the final stage of complete truth.

> If the final conception is taken to be complete or total, then our conception at any given time prior to the realization of this limit may be said to be partial. And because it is, at any given time, partial, it is, at that time, open in the sense that it may be completable in a variety of ways. (DA COSTA & FRENCH 2003, pp.13-14)

The theory of quasi-truth was advanced to account for such incompleteness of information on stages of research obtaining before the final complete consensus. The idea, in a nutshell, is that we may represent incomplete information by partial structures, structures leaving some of the information undefined, and which may be extended in many different incompatible ways; however, all of such extensions result in a structure encompassing complete information, in the style of Tarski.

Although offering a more realistic account of truth in science, allowing for the representation of the widely open character of scientific knowledge, the proposal is not without difficulties (see ARENHART & COSTA, 2021; ARENHART & KRAUSE, 2023). Part of the trouble with quasi-truth as defined by da Costa and collaborators is that it somehow collapses lack of information with excess of information (it identifies not knowing whether P is the case with the claim that P and the negation of P are both quasi-true), and also, that it cannot account for a more sophisticated notion of change of truth values in science: that is, our theories may only be completed, but they cannot change, in the sense that truth values, once attributed, cannot be revised. Quasi-truth is a matter of gaining more and more traditional truths, with no place for revision (again, see ARENHART & COSTA, 2021; ARENHART & KRAUSE, 2023).

Of particular interest for our purposes in this paper is the case of historical sciences, which seem to constitute a paradigmatic case of science dealing with incomplete information. The past only partially reveals itself through the historical sources, and there are incompatible ways to fill the gaps of missing information. One could hope to get some enlightenment of these sciences and their relation to truth by using the machinery of quasi-truth. Our aim in this paper is to do just that. With some of the criticisms to quasi-truth advanced in ARENHART & COSTA, 2021 in mind, we advance an alternative to the quasi-truth formalism that may be useful to the case of historical sciences.

A science is a historical science when (I) its propositions correspond to probable facts in a *historical time* and (II) it acquires knowledge about these facts considering at least the principle of *historical actualism*; this principle can hermetically be complemented by the principle of gradualism (CURRIE, 2019; ARENHART & COSTA, 2021). A science assumes the epistemic principle of historical actualism when: (1) it describes present or past facts within historical time; (2) the facts referred to are, for the scientific community, epistemically accessible by present objects; and (3) every probable hypothesis formulated to cognitively reconstruct what occurred is analogous to some known current fact. (COSTA, 2024b)

In short, historical actualism involves (1) historical descritibility, (2) historical accessibility, and (3) methodological actualism. In a metaphorical way, it can be said that historical actualism consists of declaring that "the present is the key to knowing the past". The instantiation of facts in historical time – according to (1) – and the accessibility of the past through present objects – following (2) – guarantee the possibility of basing hypotheses on historical sources, or, in the case of the absence of sufficient sources, of formulating hypotheses based on analogous phenomena in the present – as stated in (3).

In this paper we shall start from that understanding of historical sciences to formulate a system of temporal logic encompassing alternative timelines and alternative histories. That will be done in section 3. A historical theory may be understood either as a temporal line, or else as a bundle of such temporal lines. So, in a sense, as we mentioned, there may be alternative accounts of the past. Each such account, provided it is not forbidden by the evidence, counts as delivering a quasi-true version of the past. Our system shall make this core idea precise. The pragmatist idea of truth as a consensus may be clearly formulated in such an account. We also regiment the idea of the progress in theory formulation by closing-off some of the possible accounts when new evidence is discovered that precludes such an account. This will be done by the use of public announcement tools in the context of temporal logic. We start, however, in section 2, recovering the basics of quasi-truth theory.

Comparison between our account and the original account offered by da Costa and collaborators is left for section 4. We conclude in section 5.

2. Quasi-truth theory

Let us start with a revision of the basics of standard quasi-truth. It will be important for us to recover the limitations of such an approach.

Given a domain of knowledge Δ, scientific investigation starts by selecting a set D of objects of Δ and a family R_i $(i \leq n, n \in \omega)$ of relations between such elements. By collecting these elements into a structure $\left(D, R_i\right)_{i \leq n}$ we have a structure. In the case of quasi-truth, we want the relations to incorporate the openness of knowledge and information. So each such relation is a *partial relation*, i.e. each R_i is actually a triple $R_i = \left(R_i^1, R_i^2, R_i^3\right)$ such that, intuitively, R_i^1 contains the n-tuples of D of which we know that they are related by R_i, R_i^2 contains the n-tuples of D of which it is known that they are not related by R_i, and finally, R_i^3 contains those n-tuples for which it is not defined whether they are related by R_i or not. A partial structure $\left(D, R_i\right)_{i \leq n}$ is a set D with a family of partial relations.

The idea is that we should be able to model the evolution of knowledge by shifting the elements of R_i^3 to either R_i^1 or R_i^2 as investigation progresses, so that after a reasonable number of such steps we have a total structure, i.e. a structure where for each n-ary relation, and each n-tuple of the domain, the n-tuple is either in the first or in the second component of the relation (i.e., it is defined, whether the relation holds or fail to hold for the n-tuple). In order to do that, we first must have a kind of basic knowledge guiding such extensions. That is achieved through a set P of basic propositions. The idea is that one may only extend partial relations by shifting elements from R_i^3 to the two first components when doing so is consistent with the propositions in P. A *simple pragmatic structure* is a triple $\left(D, R_i, P\right)_{i \leq n}$.

In order to formulate the notion of quasi-truth, we need one more step. Once a first-order language is given and the attribution of partial relations is made to predicate symbols, and of individuals of the domain to the individual constants of the language, we can define a A-normal structure B = $\left(D,\ R_i,\ P\right)_{i\leq n}$ of the same similarity type as A according to the following clauses:

1) The domain in both A and B is D;
2) Individual constants of the language are assigned the same objects of D in both structures;
3) Each relation R_i in B is an extension of the corresponding relation in A to a total relation (so, every relation is total)
4) B models P in the Tarskian sense.

Now, to the promised definition of quasi-truth: A sentence S of language L is quasi-true in a simple pragmatic structure A iff there is a A-normal structure B so that S is true in the Tarskian sense in B.

That definition requires truth in a full Tarskian structure. The intuitive plan is that partial information is quasi-true if there is a total extension of our actual knowledge where it counts as true. The progress of scientific knowledge could take us to one such total structure, as it were, but so far we only have partial knowledge and many possibilities concerning how to progress on truth value attribution.

The concept just defined faces some difficulties in actually achieving the proposed philosophical goals (see ARENNHART & COSTA 2021; ARENHART & KRAUSE 2023) for further discussion. We shall come back to this topic in section 4.

3. Overlapping scientific consensus theory

Although since Plato and Aristotle it has been understood that science has truth as its ultimate goal (especially universal statements), in the practice of empirical modern sciences, and especially historical sciences, what often stands out as an achievement in the scientific community is the factual consensus, which is often recognized as a basis for public policies, laws and economic choices, as is the case with consensus around climate change. Of course, a *consensus* is not understood as a simple numerical *majority* (such as 51% of scientists), although it is also not understood as *unanimity* (DELLSÉN, 2021), but something close to that: in this example, we can assume that there is about 97% consensus on anthropogenic global warming (COOK et al., 2013). Therefore, when we refer to a consensus, we are referring to an approximate consensus. In formal terms, this justifies separating disagreements between scientists into theories or "lines of interpretation", and not into the individual opinion of each scientist. Therefore, we also implicitly assume a certain possible epistemic cohesion of groups (LACKEY, 2020) and assume that knowledge-based scientific consensus exists. We will not deal with the details behind this phenomenon, but authors such as Boaz Miller (2013) have been studying the social and cognitive conditions for this type of consensus, differentiating it from cases of contingent consensus or those established by social factors such as group prejudice or the fight of a common enemy.

The *overlapping scientific consensus theory* has at its base the Peircean pragmatism that motivated Newton da Costa's theory of quasi-truth (da COSTA, 2018, p. 138–142), but it proposes to build a more comprehensive formal theory about scientific knowledge and with intermediate definitions for scientific knowledge that are situated between "truth" (in the strong sense) and "quasi-truth". Inspired by the concept of "overlapping consensus", originally proposed by John Rawls (1999) for a political philosophy context, and motivated by Ziman's studies (1968) on "public knowledge", as well as the recent area of social epistemology (O'CONNOR et al., 2024), the overlapping scientific consensus theory is based on the assumption that the scientific community can have scientific knowledge even with divergences between reasonable comprehensive theories, interpretations of equations or empirical experiments, hypotheses or specific methodologies as long as it contains some consensus, both at its base (underlapping consensus) and on its surface (overlapping consensus).

Miller (2013; 2024) identifies three conditions for knowledge being the best explanation of a consensus: (1) *social calibration*: the consensus has researchers who attribute the same meaning to the same terms and share the same fundamental background assumptions; (2) *apparent consilience of evidence*: the consensus seems to be built on an array of evidence that is drawn from a variety of techniques and methods; (3) *social diversity*: the consensus is shared by men and women from different ethnicities and places in the world, from the public and private sectors, liberals and conservatives, etc. In addition to "knowledge-indicative" accounts of consensus, as in Miller's approach (2024), there are also authors who discuss "knowledge-conductive" accounts of consensus, as in Lehrer and Wagner's approach (1981), which consists of a mathematical model of a group of rational Bayesian agents that iteratively update their degrees of belief in a given proposition and the degree of credibility they attribute to other members in relation to that proposition until all agents converge to the same degree of belief. Helen Longino (2002) offers an alternative model of knowledge-conductive accounts of consensus but emphasizing social rather than cognitive factors.

Just as in the case of the quasi-truth theory, the theory of overlapping and underlapping consensuses also has a cognitive emphasis; social conditions are abstracted, but that does not mean they are irrelevant. Conditions (1) and (2), in the context of the philosophy of science, are directly related to what we call in this paper "underlapping consensus". *Underlapping scientific consensus* is agreement in terms of paradigm or disciplinary matrix (KUHN, 1970), underlying logic (SUPPES, 1993), scientific method (BUNGE, 1965), or epistemic virtues (TURRI, 2021) and Mertonian norms (MERTON, 1973); widely discussed topics in the philosophy of science. Regardless of how we analyze it, underlapping consensuses, even when not explicit in the scientific community, are fundamental for the establishment of theories, hypotheses and interpretations accepted among scientists, even if they are competing proposals and incompatible with each other.

On the other hand, *overlapping scientific consensus* are facts or statements claimed by all different and competing interpretations, theories, hypotheses and specific methodologies. When there is no overlapping consensus, there is *dissensus*: statements claimed by some interpretations, theories, hypotheses and specific methodologies, but rejected by others. Our formal model is neutral regarding the constructive or realistic interpretation of these "facts" (see MULLIGAN et al., 2021). However, we can easily find examples of overlapping consensus and dissensus in the historical sciences.

The predominant view until part of the 19th century was that the Earth was 6000 years old (created in 4004 before the Common Era), according to the count of Archbishop James Ussher in his book *The Annals of the World* (1658), following the biblical genealogies. With the development of modern geology, this theory was quickly defeated, and today it appears to be a consensual fact to reject the idea of an Earth just 6000 years old. This consensus is so broad that it can be supported by studies not only in geology, but in cosmology, paleontology, historical linguistics, archeology and other historical sciences that have well-known phenomena dating back more than 6000 years old. Another solid factual consensus in historical science is that Native Americans arrived in America through immigration from East Asia. This consensus is supported by different theories, methodologies and hypotheses, based on studies of indigenous languages, genetics of indigenous descendants, archaeological excavations and environmental evidence of human intervention on the continent. Although there are disagreements regarding the exact date of the first anthropic presence in America, as well as the routes that explain the dispersion of indigenous peoples on the continent, there is no line of research recognized in the scientific community that supports that indigenous people evolved as Homo sapiens on this continent instead of in Africa.

These two examples (about the age of the Earth and the origin of the Amerindians) are not isolated cases, and it should be noted that these are not trivial statements: they would probably not be consensual without modern science. Factual consensuses like these seem to deserve a higher value than quasi-truth, compared to other propositions from the scientific community; on the other hand, they also do not fit into Newton da Costa's specific rendering of the concept of quasi-truth. We will show below how a new formal model can capture these differences in scientific knowledge.

3.1. Scientific theories as chains of instants

Let $S_1, S_2, \ldots, S_n$ be the sets of all propositions consensually accepted respectively by distinct scientific theories recognized in the scientific community, the intersection between these sets results in the *overlapping consensus* of that scientific community, that is, the set of propositions accepted by all the different theoretical and methodologies of the scientific community. This account was originally proposed in temporal logic (COSTA, 2024a), as it was designed for application in historical sciences (as sciences that attempt to describe the history of the Universe on different scales), and branching temporal logic allows a modal interpretation of the concept of this "consensus", as well as allows an analysis of the progress of scientific consensus. Below we will describe this theory in general terms.

As a particular case of modal logic, a temporal logic typically interprets “possible worlds” as moments or “successive points”, at least in an ontology of instants, which will be adopted in this work, instead of an ontology of periods or temporal intervals (see BENTHEM, 1983, p. 58–79). In this way, a temporal structure $\Omega = (T, \prec)$ is defined with a non-empty set of instants $t_1, t_2, ..., t_n \in T$ (let n be a natural number) and a binary precedence relation that allows us to form chains of instants.

For our purposes, we will adopt the common principles for a branching temporal logic (see BURGESS, 1984). We assume that the precedence relation is at least *non-reflexive* and *transitive*. A total chain $t_1 \prec ... \prec t_n$ forms a *history* h (McARTHUR, 1976, p. 9), and we can note as $h(t_1, \Omega)$, a certain history starting at t_1; and $t_1 \succ ... \succ t_n$ forms a *destiny* d, *such that* $d(t_1, \Omega)$, keeping the expression $B(t_1, \Omega)$ to denote the set of all histories or destinies (a bundle) of t_1, while $B(T, \Omega)$, or just B, designates the set of all histories and destinies of all instants in a structure (COSTA, 2024, p. 175–176). When it has more than one history and two or more endings or beginnings, we say it is a *tree*, and its divergent histories are its *branches*, which is what characterizes a branching temporal structure. Each "scientific theory" (in a broad sense) as a *local* theoretical consensus in the scientific community is interpreted in this tree as one of its branches (histories and destinies). In contrast, *global* theoretical consensuses (i.e., overlapping consensuses) vertically encompass the different branches (or theories) of a tree.

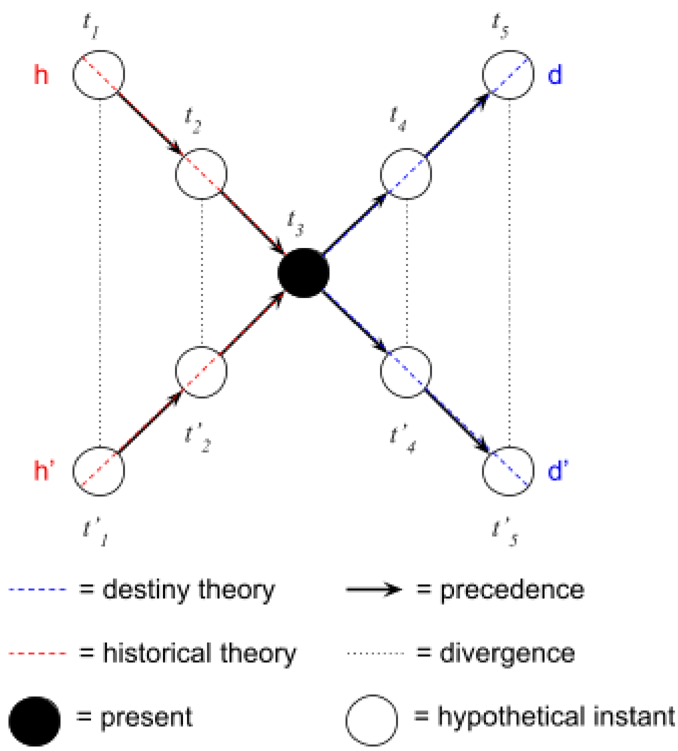

Figure 1. Diagram with divergent scientific theories about what occurred (historical theories) and what will occur (destiny theories). In the following diagrams, we use the dotted line not exhaustively, but only to emphasize relevant divergences between atomic propositions in a branching temporal structure.

It is worth noting that some historical sciences are also interested in studying the future, and not just the present and the past, therefore there are consensuses and dissensuses about what will happen, not only with respect to what occurs or has occurred. This is the case with cosmology: cosmologists study the history of the Universe from its origin to its end. There is currently a consensus among cosmologists that our Universe will expand in the short term, but there is disagreement about whether it will stop expanding in the long term.

3.2. Scientific consensus in bundles of branches

Now that we have properly defined a temporal tree Ω, we can introduce a Kripke model $M = (T, \prec, V)$, in which $\Omega = (T, \prec)$ and V is a valuation that assigns to every atomic proposition (relating to propositions stated in the scientific community) in $PROP$ the set of time instants $V(p) \subseteq T$ at which the proposition is considered true. For the purpose of fixing the language we are dealing with, we have the following definition:

$$\varphi := q \in PROP \mid \neg\varphi \mid (\varphi \wedge \psi) \mid H\varphi \mid G\varphi \mid F\varphi \mid P\varphi.$$

We call **PCL** (Peircian consensus logic) the temporal logic with a branching temporal structure for the past and the future and with the language L whose terms include the Peircian operator F (which should not be confused with the "will be the case" of usual Prior systems, this operator in Peirce's system is equivalent to $f \equiv \neg G \neg$) and its counterpart for the past, P. This logic is quite similar to **PBTL** (the Peircean branching time temporal logic) proposed by Arthur Prior (1967), inspired by Charles Peirce's notion of temporal possibility: **PBTL** basically consists of **PCL** without backward branching and without P (see THOMASON, 1984). The following are the semantic conditions for the formulas of our language:

1. $M, t \vDash q$ iff $t \in V(q)$, for $q \in PROP$;
2. $M, t \vDash \neg\varphi$ iff $M, t \nvDash \varphi$;
3. $M, t \vDash \varphi \wedge \psi$ iff $M, t \vDash \varphi$ and $M, t \vDash \psi$;

4. $M, t \vDash H\varphi$ iff $M, t_1 \vDash \varphi$ for all instants t_1 such that $t_1 \prec t$;
5. $M, t \vDash G\varphi$ iff $M, t_1 \vDash \varphi$ for all instants t_1 such that $t \prec t_1$;
6. $M, t \vDash P\varphi$ iff for all histories $h \subseteq B(t, \Omega)$, there is some instant $t_1 \in h$ such that $t \prec t_1$ e $M, t_1 \vDash \varphi$;
7. $M, t \vDash F\varphi$ iff for all destinies $d \subseteq B(t, \Omega)$, there is some instant $t_1 \in d$ such that $t_1 \prec t$ e $M, t_1 \vDash \varphi$.

Other operators:

- $(\varphi \vee \psi) \equiv \neg(\varphi \wedge \psi)$
- $(\varphi \rightarrow \psi) \equiv (\neg\varphi \vee \psi)$;
- $\bot \equiv (\varphi \wedge \neg\varphi)$;
- $F\varphi \equiv \neg G \neg\varphi$;
- $P\varphi \equiv \neg H \neg\varphi$;
- $p\varphi \equiv \neg\Box_h \neg\varphi$;
- $f\varphi \equiv \neg\Box_g \neg\varphi$.

Below we suggest how to interpret these modalities:

- $p\varphi$: "it *was* the case that φ" (in *some* history), or "there is a *past subsense* that φ";
- $h\varphi$: "it *was always* the case that φ" (in *some* history), or "there is a *historical supersense* that φ";
- $P\varphi$: "it *was* the case that φ" (in *all* histories), or "there is a *historical consensus* that φ";
- $H\varphi$: "it *was always* the case that φ" (in *all* histories);
- $f\varphi$: "it *will* be the case that φ" (in *some* destiny), or "there is a *future subsense* that φ ";
- $g\varphi$: "it *will always* the case that φ" (in *some* destiny), or "there is a *predictive supersense* that φ";
- $F\varphi$: "it *will* the case that φ" (in *all* destinies), or "there is a *predictive consensus* that φ";

- Gφ: "it *will always* be the case that φ" (in *all* destinies).

We can also define modalities equivalent to Newton da Costa's "truth" and "quasi-truth", respectively:

- $\Box\varphi \equiv (H\varphi \wedge \varphi \wedge G\varphi)$;
- $\Diamond\varphi \equiv (p\varphi \vee \varphi \vee f\varphi)$.
  - $\Box$φ: "φ is a strict pragmatic validity";
  - $\Diamond$φ: "φ is a pragmatic truth (quasi-true)".

Concerning the relation of our approach and da Costa's original notion, in particular what is called 'the logic of quasi-truth' (see DA COSTA, BUENO & FRENCH 1998, DA COSTA 2018, p.144), we have the following results:

Proposition: Propositional quasi-truth theory is equivalent to a **PCL** fragment without the operators H, G, P e F, but only with $\Box$ and $\Diamond$ as primary modal operators.

Proof: By definition (DA COSTA, 2018, p. 144), *strict pragmatic validity* in quasi-truth theory is defined by alethic necessity or, in terms of temporal semantics, $M, t \vDash \varphi$ at *any* instant of time t' such that $t' \prec t \vee t' = t \vee t \prec t$. This requirement is strictly satisfied in the abbreviated definition $\Box\varphi$. On the other hand, also by definition, pragmatic truth in quasi-truth theory is defined by alethic possibility or, in terms of temporal semantics, $M, t \vDash \varphi$ at some instant of time t' such that $t' \prec t \vee t' = t \vee t \prec t'$. This requirement is strictly satisfied in the abbreviated definition $\Diamond\varphi$. ■

Although the above modalities (analogous to those of the quasi-truth theory) can translate part of scientific knowledge in temporal terms, one of them is extremely strong ($\Box$) and the other is extremely weak ($\Diamond$), so that they hardly capture real scientific knowledge. In contrast to these "extreme modalities", the scientific consensus theory offers us intermediate modalities for scientific knowledge in historical terms:

- $\bigcirc\varphi \equiv (P\varphi \vee \varphi \vee F\varphi)$;
- $\triangle\varphi \equiv (h\varphi \wedge \varphi \wedge g\varphi)$.

- ◯φ: "there is a *factual consensus* that φ" (in all lines of research in the scientific community, it is accepted that φ is true or was the case or will be);
- △φ: "there is a *natural law* that φ" (in at least one of the scientific community's lines of research, φ is true, always has been and always will be).

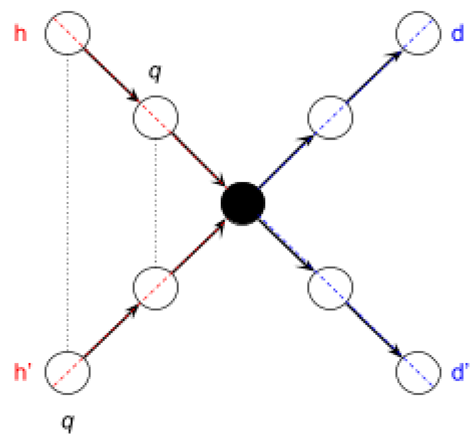

Figure 2. A model with a factual consensus ◯q (specifically a historical consensus).

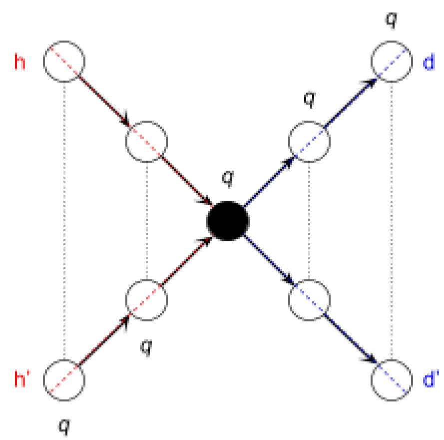

Figure 3. A model with a natural law △q.

Directly from these definitions, it can easily be demonstrated that □φ → ◯φ, □φ → △φ, □φ → ◇φ; ◯φ → ◇φ; and △φ → ◇φ. These implications are summarized in the following diagram.

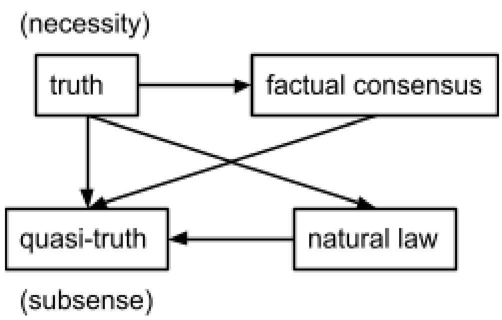

Figure 4. Implications between truth, quasi-truth, factual consensus and natural law.

3.3. The debranching process: "vertical progress" of scientific knowledge

From a static point of view, it can be argued that in the above system a consensus implies "factual truth": if all theories according to reasonable interpretations at present are represented in a tree, without exception, and if a given proposition is true in all of them, then it can be considered as fact. However, as these theories are those that are reasonable at a specific moment (in a relative present time), we must take into account that they can also change, and therefore new consensuses can also emerge, or eventually even some consensuses can be dissolved. To add this dynamic point of view, we will extend the language of our system with the "historical announcement" modality [·], which formally behaves exactly the same as the "public announcement" in public announcement logic (see BALTAG & RENNE, 2016). The new language we shall be dealing with is defined as follows:

$$\varphi := q \in PROP \mid \neg\varphi \mid (\varphi \wedge \psi) \mid H\varphi \mid G\varphi \mid P\varphi \mid F\varphi \mid [\varphi]\psi.$$

We call **HAL^{PC}** (historical announcement logic + **PCL**) the temporal logic with a branching temporal structure for the past and the future and with the new language L^* whose terms include the dynamic operator [·]. **HAL** basically consists of **HAL^{PC}** without $P\varphi$ and F (COSTA, 2024a). The following are the semantic conditions (in simplified form) for $[\varphi]\psi$:

For any formula φ, truth set $[\![\varphi]\!]$ and tree Ω, let V be a binary valuation for atomic propositions, $V: T \times PROP \mapsto \{0, 1\}$, given a model $M = (B, V)$, a updated model M with respect to φ is a model $M|_{\varphi} = (T^!, \prec^!, V^!)$, in which $T^! = [\![\varphi]\!]^M = \{t \in T: M, t \models \varphi\}$; $\prec^! = \prec \cap \left([\![\varphi]\!]^M \times [\![\varphi]\!]^M\right)$; and for each $t \in T^!$, $V^!(q, t) = V(q, t)$ and $V^!(\bot, t) = V(\bot, t)$.

8. $M, t \models [\varphi]\psi$ iff if $M, t \models \varphi$, then $M|_{\varphi}, t \models \psi$;

- $[\varphi]\psi$: "after any *historical announcement* that φ is the case, ψ is the case".

We call a "historical announcement" simply a "public announcement", now applied to instants of time rather in the place of epistemic states (van DITMARSCH et al., 2007). Intuitively, a historical announcement consists of a formula that updates a model based on a condition. In the sequence of diagrams below we use the scheme "[φ]ψ" to update a model. For this example, we built a structure with only historical theories.

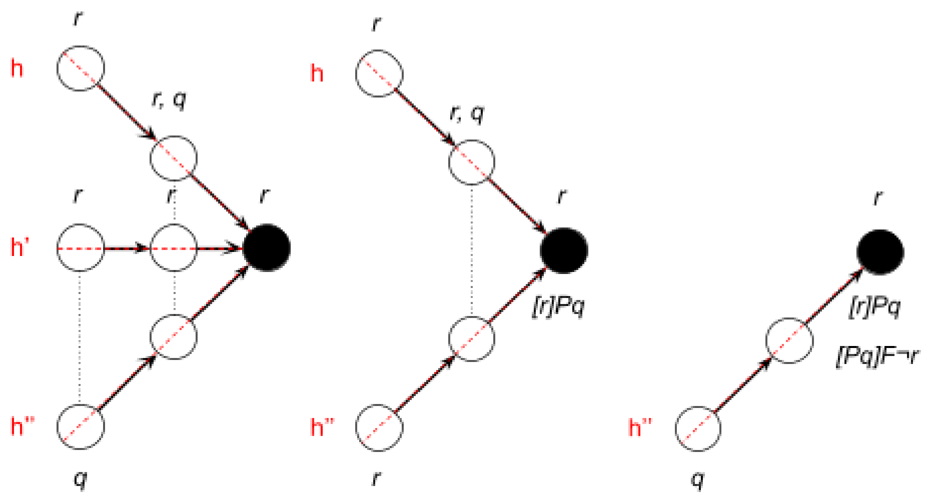

Figure 5. From left to right: A model with three competing historical theories about what happened in the past, which were elaborated from the point of view of current sources and methods (present instant); and two updated models after two respectively historical announcements.

Depending on how it is most convenient to interpret them in the real practice of scientists, we can understand these announcements in different ways, for example, they can be articles published in scientific journals, conferences, doctoral thesis, etc. Jonas Arenhart and Vítor Costa (2021) offer a good example of what we can interpret with a historical announcement:

> Greco-Roman sculptures have long been believed to be intentionally monochromatic. Culturally, this not only affected our knowledge of Greece, but also our actions in relation to this knowledge: it fatally influenced the sculptures of various artistic movements in the West, especially in the Renaissance and the neoclassicism. In addition, historiographical interpretations were built with this premise. In 1764 Johann Winckelmann—often considered the "father of art history"— wrote that "white is the color that shows the most rays of light, and thus is most easily perceived." Because of this, he believed that "a beautiful body will be all the more beautiful the whiter it is." [...] We now know, however, that the Greek sculptures were highly polychrome (with colors that were lost over time). The beauty in the saturation of the sculptures is attested in Greece in some ancient texts, as in a passage by *Helena* from Euripides—"If only I could shed my beauty and assume an uglier aspect [...] the way you would wipe color off a statue."—and among the Romans the coloring process (whose pigments were extracted from minerals, beeswax and egg yolks) is detailed by Vitruvius and Pliny the Elder. Moreover, perhaps the most fatal information for Winckelmann's interpretation is that it is now possible to know the colors of yore in Greek sculptures by exposing them to ultraviolet light. (ARENHART & COSTA, 2021, p. 131)

4. Comparing the two formal theories for the historical sciences

The overlapping scientific consensus theory is more expressive and accurate with the pragmatic point of view. Although it can be argued that scientists seek truth, in practice they do not find mere quasi-truth, but rather factual consensus and natural laws. However, even in these cases, consensuses are not infallible and natural laws can face exceptions and reformulations (in general, a natural law is conceived within a theory, and does not necessarily apply in the same way to all, as one could argue in relation to the law of gravitation in relativistic physics and quantum physics). Therefore, we will discuss in this section how the greater expression and pragmatic precision of the overlapping scientific consensus theory can overcome several problems of applying the theory of quasi-truth to the historical sciences suggested by Arenhart and Costa (2021).

4.1. Fixing inadequacies of the quasi-truth theory

As we mentioned, the definition of quasi-truth by da Costa and collaborators faces some difficulties already mapped in the literature. We start now by recalling three basic inadequacies of the account, and compare how the overlapping scientific consensus theory deals with them. The inadequacies are the following ones:

I. Divergence inadequacy;
II. Progress inadequacy;
III. Possibility inadequacy.

> (I) "Every proposition about which we have no knowledge is quasi-true, and its negation is also quasi-true. This makes room for a quite unrestricted assumption of plausibility of every proposition about which we have no knowledge, irrespective of whether it has any kind of plausibility against the background knowledge already assumed." (ARENHART & COSTA, 2021, p. 124–125)
>
> (II) "Changes from ignorance states to knowledge states are always carried over in a single step: from a sps where knowledge is incomplete to a total structure where it is totally determined (this issue relates to what has been discussed in (i))." (ARENHART & COSTA, 2021, p. 126)
>
> (III) "The formalism identifies lack of information about a sentence S with the possibility of S. We cannot have lack of knowledge or lack of information of necessary sentences; everything we ignore is possible." (ARENHART & COSTA, 2021, p. 127)

Item (I) is a difficulty for quasi-truth due to the fact that quasi-truth seems to be attributed to any proposition whose alethic status is not known to us now. That is, quasi-truth ends up being attributed too indiscriminately. The situation is improved, at least partially, in **PCL**, with the corresponding definition of quasi-truth. In this case, it cannot happen that a proposition is quasi-true and still has no evidence to it: it must be true sometime. Notice that this definition of quasi-truth allows that some proposition and its negation may both be quasi-true (it is enough that they are verified in distant instants of time). A better solution, however, is available in $\mathbf{HAL}^{\mathbf{PC}}$, which allows us to single out with an announcement which one from a pair of contradictories is the actual case.

Inadequacy II is no longer a problem for the current account. Both **PCL** and $\mathbf{HAL}^{\mathbf{PC}}$ have resources to account for progressive change of truth values as time changes. Furthermore, given the revisability of the truth attribution (see the discussion of item iii) of section 4.2), there is no actual sense to be made of a final truth attribution for sentences.

Concerning III, we still have that logical truths are quasi-true, by default. However, it is now possible, in **PCL** already, to represent sentences that are not quasi-true, being it enough that they are not true at some instant of time in a branch. That accounts for the fact that some implausible sentences, although not being logical falsities, are just so off the mark that they would not count as quasi-true, even though we do not have evidence for their falsity.

4.2. Fixing limitations of the quasi-truth theory

In addition to the inadequacies of quasi-truth theory analyzed in the previous subsection, this theory also had three limitations in the way it captured scientific knowledge, limitations that are largely overcome by the formalism of the overlapping scientific consensus theory:

i. Dynamic limitation;
ii. Counterfactual limitation;
iii. Historical limitation.

> (i) "The formalism is not suitable to allow that new information (or new knowledge) leads us to revise what is already taken as known." (ARENHART & COSTA, 2021, p. 130)
>
> (ii) "Counterfactual histories, or alternative histories, cannot be appropriately taken into account by the formalism of quasi-truth. One cannot change the collection P of the sentences that are accepted beforehand, given that these sentences inform and somehow put a limit to the allowable extensions of the sps in which we evaluate our sentences. This puts a barrier on the kinds of historical investigations that can be accommodated by the apparatus of quasi-truth." (ARENHART & COSTA, 2021, p. 133)
>
> (iii) "As it is currently conceived, the language of quasi-truth theory is restricted to the interpretation of sentences attributing simple quasi-truth to a sentence, without any iteration of quasi-truth and mixing with temporal and epistemic operators. A

> simple example indicates that such mixing of quasi-truth with such operators is of utmost importance, as in "it's quasi-true 'that it was quasi-true that p'", where p can be a proposition; for example, "the earth revolves around the sun". Propositions like that are very common in History of Science and other areas of the Intellectual History. A context of application of the above sentence appears when we say that Aristarchus anticipated Copernicus's theory when, in his time, the dominant idea was that the earth revolved around the sun. In general, these sentences are interpreted in epistemic and temporal logics as KPKp ("we know we knew that p"; PK = 'we knew' or 'it was the case that we know'). However, if the theory of quasi-truth formally represents the meaning of pragmatic truth, even if partially, then it should be able to interpret sentences as these." (ARENHART & COSTA, 2021, p. 134–135)

The last limitation (iii) can be easily overcome with the **PCL** system alone, while limitation (i) can be overcome by the **HAL**$^{\mathbf{PC}}$ extension. Only limitation (ii) remains, since it is a property that goes beyond the motivations of both the quasi-truth theory and the overlapping scientific consensus theory, since these formal theories fundamentally aim to capture only the *factual knowledge* of science.

For the comparison of case (i), we should note that the quasi-truth theory, as well as the **PCL** system (in which the quasi-truth theory is just a fragment), consists of a *static* logic, unlike **HAL** and **HAL**$^{\mathbf{PC}}$, which allow *updated* models $M|_{\varphi}$ relative to any formula φ.

For the comparison of case (iii), it should be noted that the possible reiterations of the Newton da Costa operators are equivalent precisely to the following 4 combinations (or combinations from these): $\Box\Diamond\varphi$; $\Diamond\Box\varphi$; $\Diamond\Diamond\varphi$; and $\Box\Box\varphi$. None of these combinations allows us to infer that a quasi-truth or a truth was the case and then ceased to be so or vice versa; that is, none of them gives a *historical fallibility* to scientific knowledge. On the other hand, the **PCL** system allows 16 possible combinations of two modalities based on the historical knowledge operators $\bigcirc, \Delta, \Diamond, \Box$. And we get 64 possible combinations based on the eight operators H, G, P, F, f, p, h, g. Many combinations of these 8 operators or any of these with the 4 complex knowledge operators express historically fallible knowledge (which was or will be the case in different senses). Examples: $p\bigcirc\varphi \wedge \neg\bigcirc\varphi$ ("there was a consensus that φ, but there isn't anymore"); and $p\bigcirc\Delta\varphi \wedge \neg\bigcirc\Delta\varphi$ ("there is a consensus that there was a natural law that φ, but there is also a consensus that $\neg\Delta\varphi$").

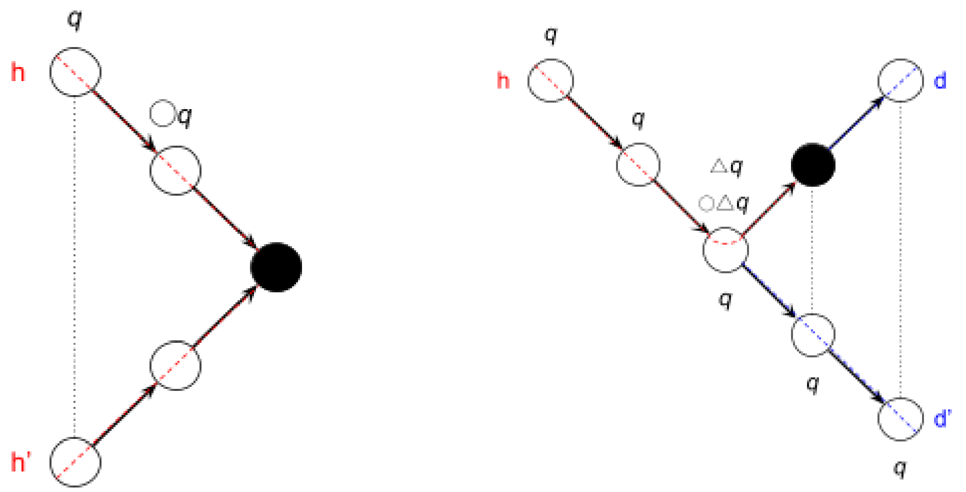

Figure 6. From left to right: An example of $p\bigcirc\varphi \wedge \neg\bigcirc\varphi$ representing a past consensus that is no longer current; an example of $p\bigcirc\Delta\varphi \wedge \neg\bigcirc\Delta\varphi$ representing that there was a consensus about q being a natural law, but that consensus is no longer current.

4.3. Other strength of the overlapping scientific consensus theory

The theory of overlapping scientific consensus captures not only consensus among scientists, but also dissensus:

- Countersense: $h\varphi \wedge h\neg\varphi$ or $g\varphi \wedge g\neg\varphi$;
- Divergence: $p\varphi \wedge h\neg\varphi$ or $f\varphi \wedge g\neg\varphi$.

In a *strict sense*, a *factual dissensus* designates a divergence (a countersense is a special case of divergence):

- $p\varphi \wedge h\neg\varphi$ or $f\varphi \wedge g\neg\varphi$: "it is the case that φ at some instant in one history, but is not the case at any instant in another history", or "there is a *factual dissensus* that φ".

On the other hand, the quasi-truth theory only captures epistemic contingencies in the scientific community, in temporal terms:

- Contingency: $p\varphi \wedge p\neg\varphi$ or $f\varphi \wedge f\neg\varphi$.

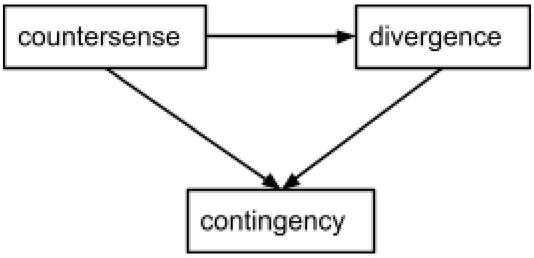

Figura 7. Diagram with implications between the three types of logical tensions: countersense, divergence and contingency (from strongest to weakest).

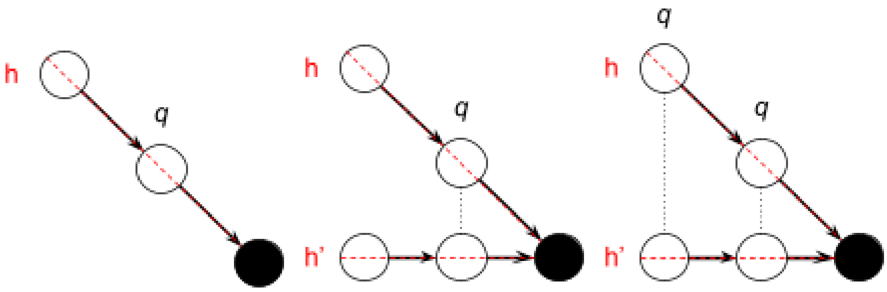

Figure 8. From left to right: A historical contingency; a "weak" historical divergence; and a historical countersense (i.e., a "strong" historical divergence). From the first example, note that a contingency does not imply divergence, as a contingency can occur in the same history or destiny, therefore history h is enough for the contingency.

However, it is worth reaffirming that the "consensus" in this system is a consensus of theories, which does not translate into unanimity. In fact, against unanimity, Dellsén (2021) argues that the presence of some scientific disagreement against a consensus is normally a stronger indicator of the veracity of the consensual belief than if there had been no such disagreement. This perspective is compatible with the overlapping scientific consensus theory, since marginal disagreements, at the individual level, are often not enough to establish a rival theory recognized in the scientific community.

5. Conclusion

Based on a previous analysis of the limitations and inadequacies of quasi-truth theory for historical sciences and on a recent proposal for dynamic historical logic with branching temporal logic, we present a formalism that overcomes virtually all the problems identified in quasi-truth theory in its attempt to capture fallible scientific knowledge about particular facts and general laws. Furthermore, the formalism we present encompasses the results of quasi-truth theory, which can be obtained as a fragment of our system from the operators □ e ◇, which can be defined in shorthand with the operators H and G (and their duals p and f).

In addition to overcoming the inadequacies and limitations of quasi-truth theory, our theory is able to capture different types of consensus and divergence in the scientific community, as studied in social epistemology and philosophy of science, and for this reason we call our interpretation of the Peircean consensus logic **PCL** the "theory of overlapping scientific consensus". Noting that **PCL** can be extended by the dynamic logic **HAL**, resulting in **HAL**^{PC}, which captures the way in which sciences revise over time their consensus and divergence on both particular facts and natural laws.

With all these results, the **PCL** and **HAL**^{PC} systems prove to be highly effective for the philosophy of historical sciences and for the social epistemology of these sciences, but also promising for science in general, deserving a further study dedicated to this generalized application, as well as to first-order extension results.

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